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ON-GROUND ATTITUDE AND TORQUE RECONSTRUCTION FOR THE GAIA MISSION

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The work presented in this paper concerns the accurate On-Ground Attitude (OGA) reconstruction for the astrometry spacecraft Gaia in the presence of disturbance and control torques acting on the spacecraft. The reconstruction of the expected environmental torques which influence the spacecraft dynamics will be also investigated.

The telemetry data from the spacecraft will include the on-board real time attitude which is of order of several arcsec. This *raw* attitude is the starting point for the further attitude reconstruction. The OGA will use the inputs from the field coordinates of known stars (attitude stars) and also the field coordinate differences of objects on the Sky Mapper (SM) and Astrometric Field (AF) payload instruments to improve this raw attitude. The on-board attitude determination uses Kalman Filter (KF) to minimize the attitude errors and produce more accurate attitude estimation than the pure star tracker measurement. Therefore the first approach for the OGA will be an adapted version of KF. Furthermore, we will design a batch least squares algorithm to investigate having more accurate OGA estimation. Finally, a comparison between these different attitude determination techniques in terms of accuracy, robustness, speed and memory required will be evaluated in order to choose the best attitude algorithm for the OGA. The expected resulting accuracy for the OGA determination will be on the order of milli-arcsec.

INTRODUCTION

The principal feature of the Gaia astrometry mission is to accurately measure the positions, distances, space motions, and many physical characteristics of some one billion stars in our Galaxy and beyond on orbit at the vicinity of the second Lagrange point L2. In order to achieve the targeted measurement accuracies of the Gaia imaged stars, the real time on-board attitude should be improved using the on-ground attitude reconstruction.[2]

The main objective of the work presented here is to reconstruct the *non-real-time* On-Ground Attitude (OGA) with very high accuracy for further processing. The accuracy requirements for the attitude reconstruction for the Gaia on-ground data analysis would enforce studying different attitude estimation techniques and chosing the best one in terms of accuracy, robustness and speed. The current goal for the OGA accuracy is about 50 milli-arcsec for the along and across scan attitude accuracy. These techniques shall be part of the Initial Data Treatment (IDT) chain.

The Gaia scientific payload is used as a high accuracy stellar gyro for achieving the attitude determination and enabling Time Delay Integration (TDI) mode. The star speed is calculated along and across scan from Sky Mapper (SM) and Astrometric Field (AF) measurements. When considering

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the two telescopes, one obtains four angular speed measurements, from which the spacecraft rigid body angular speed vector can be calculated. It should be mentioned that SM-1 is seen by telescope No. 1 and SM-2 by telescope No. 2. The Sky Mapper measurements consist of the determination of the centroid crossing times of the stars over each of the SM CCD arrays.[3]

The Gaia Initial Data Treatment (IDT) will process the newly arrived telemetry data and also the various pieces of auxiliary data from the Gaia spacecraft. The IDT will have many processes and subtasks, one of them is the attitude reconstruction or the OGA determination.

In order to match the new observations reliably with the existing source list, it is essential to have an accurate attitude. It is therefore suggested to carry out an early attitude updating during the IDT. This will require a cross matching with a dedicated Attitude Star Catalog, followed by an attitude updating.

At this stage we are only aiming for attitude accuracy of a tens of milliarcsec level, so some simplifications may be allowed.

The IDT will have to run in several concurrent processes, and the attitude improvement is done separately for each process. The same time interval will therefore be processed several times, and we need only to update a given attitude interval if it has not already been updated in a previous process.

Through understanding of the dynamical behavior of the spacecraft, the accuracy of the reconstructed attitude and torques can be improved, and is therefore of great relevance for obtaining the highest accuracies and best overall statistical properties for the main scientific products of the mission; the Astrometric parameters of the target stars.

Given all these mission data and parameters, the attitude data simulation, in the form of the spacecraft quaternion and angular velocities, could be simulated for any time span. In order to simulate the algorithms for OGA determination, we have to simulate attitude observations with the following requirements:

- The simulated data shall be provided for the length of at least one great circle (6 hours).
- The data shall contain the true (ideal) attitude quaternion as well as the true rate in the body fixed frame. Also, the raw (measured) attitude quaternion.
- The simulated observations of attitude stars shall contain:
  - The time of observation.
  - The indicator for the FOV.
  - The position on the sky of the observed star (the right ascension α, and declination δ).
  - The field coordinates of the star (η and ζ) for SM and AF1-9 and G-magnitude.
  - The transit ID and the Attitude Star Catalog (ASC).
  - Cross-match table.
  - The attitude quaternion at each observation time will be calculated using the B-Spline as described in the next section.

GAIA ATTITUDE PRESENTATION

The Gaia spacecraft attitude is considered to be represented by the quaternion vector defined as

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \tag{1}$$

with

$$\mathbf{q}_{13} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \hat{\mathbf{n}} \sin(\tfrac{\theta}{2}) \text{ and } q_4 = \cos(\tfrac{\theta}{2}) \tag{2}$$

where $\hat{\mathbf{n}}$ is a unit vector corresponding to the axis of rotation, and θ is the rotation angle.

The elements of the quaternion satisfy a unit norm constraint

$$\mathbf{q}^T\mathbf{q} = \mathbf{q}_{13}^T\mathbf{q}_{13} + q_4^2 = 1. \tag{3}$$

The frequency of the on-board attitude (1 Hz) is different from the frequency of the star centroiding and crossing observation times in the SM and AF which measured in an uneven sequences and in order of nanosec. In order to start the OGA determination, the attitude quaternion at every time of observation should be evaluated by interpolating the on-board attitude.

Both the on-board real-time attitude and the on-ground reconstructed attitude, and perhaps also the nominal scanning law attitude,[4] can be represented by means of cubic splines followed by normalization (to validate the unit length condition). So, according to[5] at any time interval $[t_{beg}, t_{end}]$ there is a vector $\mathbf{s}(t)$ consists of four cubic spline functions $s_m(t), m = 1, .., 4$ from this vector the attitude quaternion at time t is calculated from

$$\mathbf{q}(t) = \mathbf{s}(t)/norm\,(\mathbf{s}(t))\,. \tag{4}$$

Also we can derive the instantaneous scan rate (ω) in the fields of view by computing the time derivative of the quaternion at time t from

$$\dot{\mathbf{q}}(t) = \left[\dot{\mathbf{s}} - \mathbf{q}\mathbf{q}^T\dot{\mathbf{s}}\right]/norm\,(\mathbf{s}(t))\,. \tag{5}$$

In this case the scan rate at any given time is derived from

$$\omega(t) = \frac{2}{norm(\mathbf{q})} \begin{bmatrix} q_4 & q_3 & -q_2 & -q_1 \\ -q_3 & q_4 & q_1 & -q_2 \\ q_2 & -q_1 & q_4 & -q_3 \end{bmatrix} \dot{\mathbf{q}}(t). \tag{6}$$

Each of the functions $s_m(t)$ is a cubic spline defined on $[t_{beg}, t_{end}]$, it is therefore continuous in this interval and its first two derivatives $\dot{s}_m(t)$ and $\ddot{s}_m(t)$ are also continuous. The third derivatives is discontinuous at discrete points, called the knots, and constant between the knots. The frequency of knots determines the flexibility of the spline, or in other words, more knots gives more flexible spline. The used knot sequence is used for all the four components of $\mathbf{s}(t)$, with a knot interval of 15 sec.

A cubic spline can be represented in a number of different ways, e.g. by means of its value and second derivative at each knot. However, the method proposed here is to write the spline as a linear combination of non-negative local basis functions, called B-splines (See[6]). Let $B_n(t), n = 1, ..., N$, be the N B-splines defined on the attitude interval $[t_{beg}, t_{end}]$. Because the B-splines are linearly independent basis functions, N is also the number of degrees of freedom of the spline. Then

$$s_m(t) = \sum_{n=1}^{N} S_{mn} B_n(t), \qquad m = 1, .., 4 \tag{7}$$

and

$$\dot{s}_m(t) = \sum_{n=1}^{N} S_{mn} \dot{B}_n(t), \qquad m = 1, .., 4. \tag{8}$$

The B-splines B_n (and their derivatives $\dot{B}_n(t)$) are uniquely defined by the knot sequence

$$\tau = \{\tau_{-1}, \tau_0, \tau_1, \tau_2 \equiv t_{beg}, \tau_3, ..., \tau_{N-2}, \tau_{N-1} \equiv t_{end}, \tau_N, \tau_{N+1}, \tau_{N+2}\} \tag{9}$$

The knot sequence can be evaluated for any time t according to a simple and stable recurrence relation. Moreover, it should be non-decreasing ($\tau_i <= \tau_{i+1}$, $i = -1, ..., N+2$). At any given time $t \in [\tau_n, \tau_{n+1}]$ the only non-zero B-splines are B_{n-1}, B_n, B_{n+1} and B_{n+2}. Therefore, for the whole attitude interval $[t_{beg}, t_{end}]$ the non-zero B-splines are $B_1, B_2, ..., B_N$. The additional knots on either side of the attitude interval are needed to define B_1, B_2, B_3 and B_{N-2}, B_{N-1}, B_N, and can be chosen arbitrary as long as τ is non-decreasing.[6]

External Torque Estimation

The first task in the OGA determination is to reconstruct the external torque as a continuous function of time from the telemetry attitude dynamics. In order to achieve this task, we consider the spacecraft as a rigid body then we can express the angular velocity using the following equation;

$$\dot{\omega}(t) = I_{sc}^{-1}(T_e - \omega \times I_{sc}\omega) \tag{10}$$

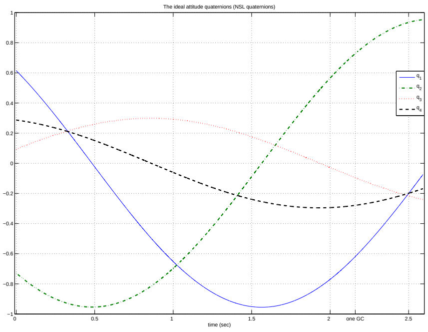
where I_{sc} is the moment of inertia of the Gaia S/C and T_e is the total disturbance and control torques acting on the spacecraft.

At each observation time the attitude quaternion is calculated from Eq. 4 and the time derivative of the quaternion from Eq. 5. By using Eq. 6 the angular rates at each observation time are determined.

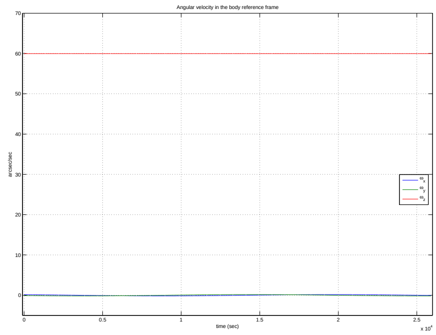
Now to calculate the estimated external torque, we could rewrite Eq. 10 in the form

$$T_e(t_{obs}) = I_{sc}\dot{\omega} + \omega \times I_{sc}\omega. \tag{11}$$

The Nominal Scanning Law (NSL)[4] is used to simulate the attitude quaternion for more than one great circle (6 hours) as illustrated in figure (1(a)). Consequently, when Eq. 6 is used to calculate the angular rates at each time step the resulting rates are illustrated in figure (1(b)). On the other hand, figure (2) demonstrates the resulting external torques which are supposed to be the sum of all the environmental disturbance torques, internal torques and control torques acting on the Gaia spacecraft while following the NSL.



(a) The simulated quaternion using NSL.



(b) The simulated quaternion using NSL.

Figure 1 The simulated attitude and rates using the B-Splines.

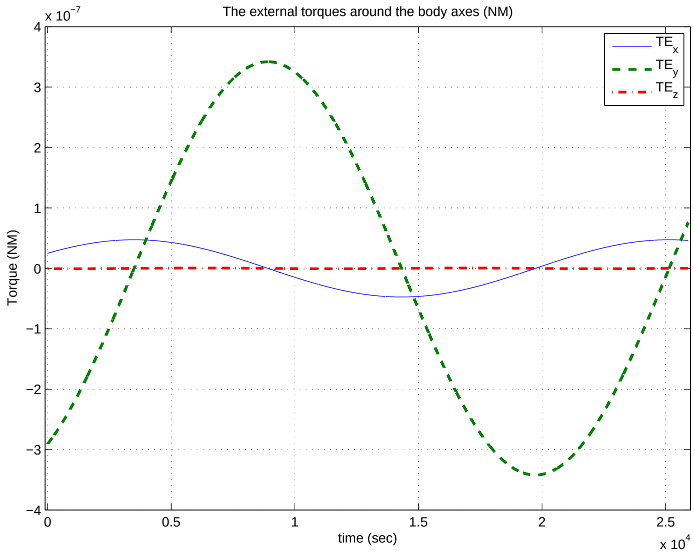


Figure 2 The external (disturbance and control) torques acting on the S/C.

THE KALMAN FILTER MODEL

The Kalman filter state vector for the Gaia OGA is consisting of the 4 components of the attitude quaternion and the three components for the angular rates.

$$x = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & \omega_x & \omega_y & \omega_z \end{bmatrix}^T. \tag{12}$$

The system model equation has the following form

$$\dot{x}(t) = f(x(t), t) + G(x(t), t)w(t). \tag{13}$$

Also, the measurement model at time t_k is given by

$$Y_k = h(x(t_k), t) + \nu(t) \tag{14}$$

where $w(t)$ is the process noise and $\nu(t)$ is the measurement noise, both are discrete Gaussian white-noise processes

$$\begin{array}{l} w(t) \simeq N(0, Q(t)), \\ \nu(t) \simeq N(0, R(t)). \end{array} \tag{15}$$

The Kalman filter dynamic equations of state are given by

$$\begin{array}{l} \dot{\mathbf{q}}(t) = \frac{1}{2}\Omega(\omega)\mathbf{q}(t) \\ \dot{\omega}(t) = I_{sc}^{-1}(T_e - \omega \times I_{sc}\omega) \end{array} \tag{16}$$

where

$$\mathbf{\Omega}(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}. \tag{17}$$

Also the external torque T_e is calculated at each observation time from Eq. 11.

The Kalman filter propagation equations are defined by

$$\begin{array}{l} \dot{x}(t) = F(t)x(t) + G(t)w(t) \\ \dot{P} = FP + PF^T + GQG^T \end{array} \tag{18}$$

where P is the covariance matrix.

The F matrix is given by

$$F = \begin{bmatrix} 0.5\Omega(\omega) & -0.5\Xi(\mathbf{q}) \\ 0_{3x4} & F_{\dot{\omega}\omega} \end{bmatrix} \tag{19}$$

where

$$\Xi(\mathbf{q}) = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{20}$$

$$F_{\dot{\omega}\omega} = -I_{sc}^{-1}\left([\omega\times]\, I_{sc} - [I_{sc}\omega\times]\right). \tag{21}$$

The matrix $[\underline{a}\times]$ is the skew symmetric matrix of the vector $\underline{a}$.

Also, the Kalman filter update equations are defined by

$$\begin{array}{l} \hat{x}_k(+) = \hat{x}_k(-) + K_k\left[Y_k - h_k(\hat{x}_k(-))\right] \\ P_k(+) = \left[I - K_k H_k(\hat{x}_k(-))\right] P_k(-) \\ K_k = P_k(-)H_k^T\left[H_k P_k(-) H_k^T + R_k\right]^{-1}. \end{array} \tag{22}$$

The measurement (Y_k) is essentially an association of a certain time instant t with certain field coordinate angles of the measured star. Each measurement vector consists of the along scan angle (η_m) and the accross scan angle (ζ_m) for a certain field of view in the instrument frame.

The calculated field angles at each observation time are given by

$$h(x_t) = \left[\begin{array}{cc} \eta_c^i & \zeta_c^i \end{array}\right] \tag{23}$$

where $i = 1$ or 2 is field of view number. These field angles depend on the basic angle between the two fields of view, which is equal to 106.5^o.[1]

The measurement sensitivity matrix is given by

$$H_k = \frac{\partial h(x_t)}{\partial x(t)} = \left[\begin{array}{cc} \dfrac{\partial \eta_k}{\partial \mathbf{q}} & 0_{1x3} \\ \dfrac{\partial \zeta_k}{\partial \mathbf{q}} & 0_{1x3} \end{array}\right]. \tag{24}$$

The Kalman Filter Results

The Kalman filter algorithm is designed and implemented using the above system model and equations. The measurement noise matrix R is chosen such that;

$$R = \left[\begin{array}{cc} \sigma_\eta^2 & 0 \\ 0 & \sigma_\zeta^2 \end{array}\right] \tag{25}$$

The standard deviation for the field angle errors along scan (σ_η) is considered to be 0.1 marcsec and for across scan (σ_ζ) is considered to be 0.5 marcsec. A total of 10^5 of the field angles measurements were simulated for about 7 hours, corresponding to about 4 measurements per sec.

Figure 3 illustrates the attitude estimation errors for the along scan and across scan directions. The along scan direction is around the rotation axis or z-axis while the across scan direction is considered to be the average of the x and y axes or the axes perpendicular to the spin axis. The estimated external torques (which illustrated in figure (2)) are used to obtain the propagated states ($\mathbf{q}$ and ω) at each observation time. In the states update step the Gaia scientific instrument measurements (the field angles) are combined with the Kalman filter simulation.

Using different initial values, the resulting attitude determination errors (3 σ) were always less than 2 marcsec along scan and 20 marcsec across scan. The estimated angular rate errors are also

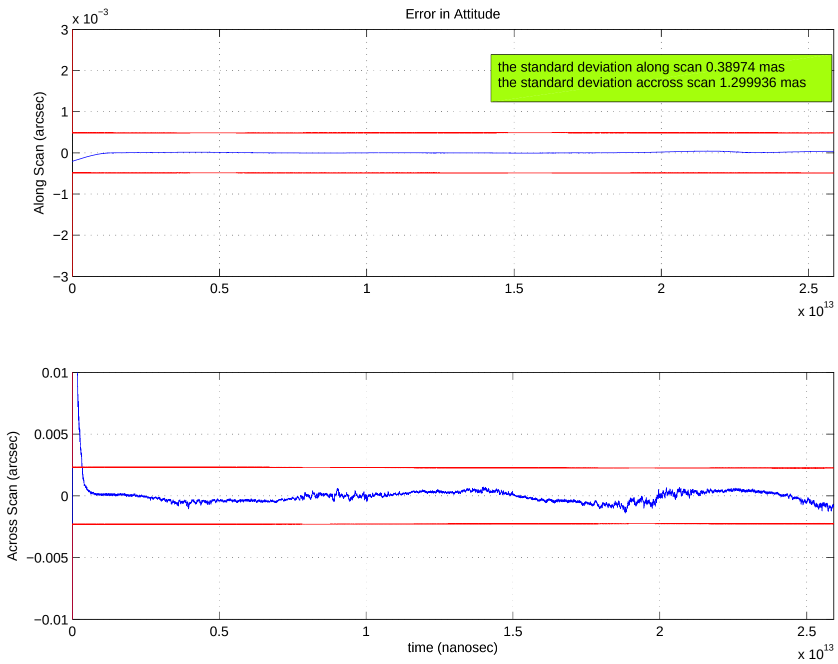


Figure 3 The attitude errors using Kalman Filter.

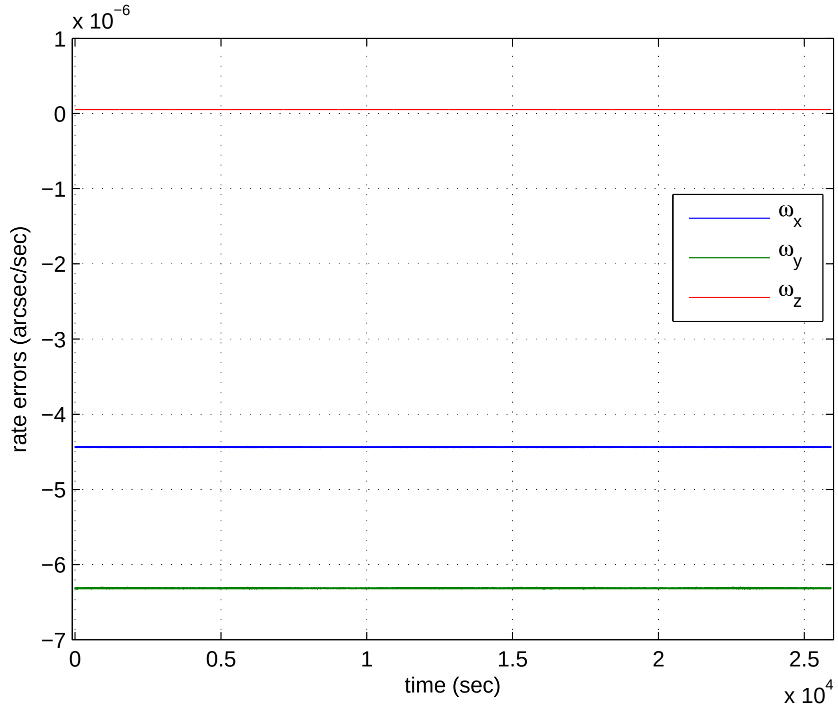


Figure 4 The angular rates errors using Kalman Filter.

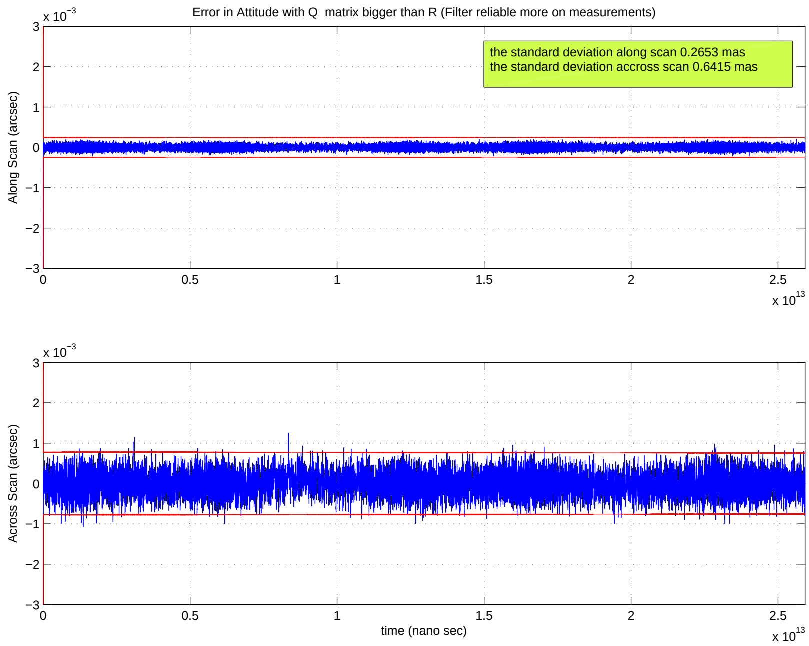


Figure 5 The attitude errors using different tuning.

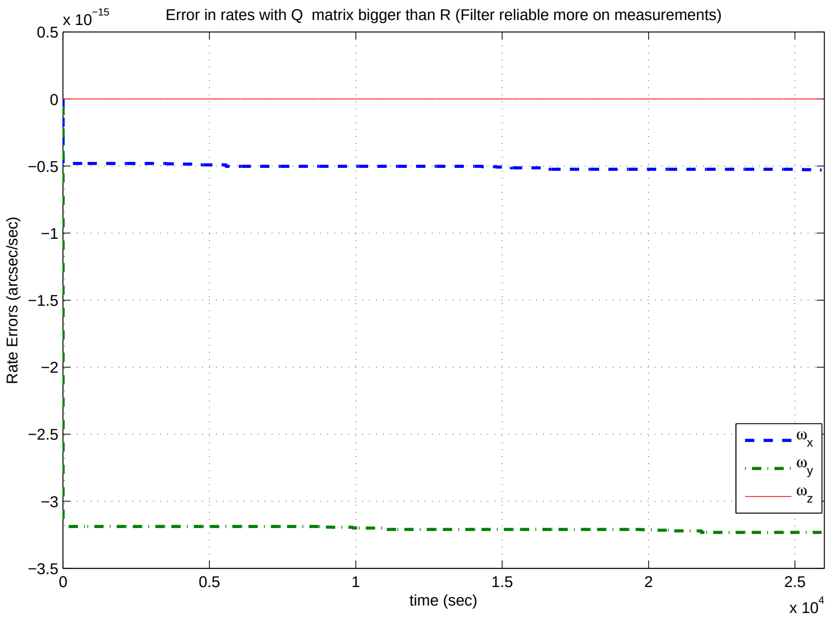


Figure 6 The angular rates errors using different tuning.

shown in figure 4. The resulting angular rates errors are less than 0.007 marcsec/sec around the three axes. The process noise matrix Q used for the Kalman filter propagation equations (18) are considered to be

$$Q = diag\left(\left[(10^{-12})^2, (10^{-12})^2, (10^{-12})^2, (10^{-12})^2, (5 \times 10^{-10})^2, (5 \times 10^{-10})^2, (10^{-11})^2\right]\right)$$

The Kalman filter update equations are based on the updated value of the covariance matrix $P(t)$ which is calculated from Eq. 18. On the other hand, the $P(t)$ is a function of the process noise matrix Q, so the KF tuning for the values of the diagonal element of the Q matrix is very important in determining the resulting accuracy of the estimation. Figures 5 and 6 demonstrate the effect of changing the process noise matrix to be much bigger than the measurement noise matrix R or in other word, the KF will rely more on measurements than on the dynamic model. The process noise matrix Q in this case is chosen to be

$$Q = diag\left(\left[(10^{-6})^2, (10^{-6})^2, (10^{-6})^2, (10^{-6})^2, (5 \times 10^{-5})^2, (5 \times 10^{-5})^2, (2 \times 10^{-6})^2\right]\right)$$

In figure 5 the overall along scan and across quaternion errors is better than that illustrated in figure 3 but it is more noisy. This is the more realistic case for Gaia. Moreover, the angular rates estimation illustrated in figure 6 is better than the angular rates errors shown in figure 4. All the estimated attitude and rates shown in the previous figures (3, 4, 5, and 6) are based on ideal (error-less) star catalog.

At the beginning of the IDT process, the OGA will be using an initial coarse star catalog that will have errors of about 50 marcsec until the more accurate star catalog is created. At this stage we suggest to have the OGA dependent more on the system dynamic model or in other words to choose the process noise matrix such that it should be less than the measurement noise matrix. After the Astrometic Global Iterative Solution (AGIS)[5] has become available (after 10 months or more) we can easily change the process noise matrix to let the filter rely more on the measurement which will be more accurate than the system model.

Figures 7 illustrates the effects of using star catalog that has errors of about 50 marcsec in both the along and across scan directions on the estimation of the attitude errors when the system rely more on the system dynamic model. On the other hand, figure 8 shows the resulting attitude errors when the system rely more on the measurements.

THE BATCH LEAST SQUARES

The Batch Least Squares (BLS) estimation is the process that can be used to estimate the state vector at an epoch time after accumulating batch measurement data for a given interval. The BLS solution selects a state estimate *x* that minimizes the sum of the squares of the calculated residuals. Although the BLS is computationally intensive for a large number of measurements, it leads to a good estimation of the state vector.[11] The BLS estimation could be done using one of two approaches; the first approach is the Maximum Likelihood estimates and the other is the Least Squares Trajectory estimates.

Maximum Likelihood Estimates

The basic idea of the maximum likelihood function for parameters estimation is to obtain the best attitude quaternion and angular rates for a given interval of time. In order to have the most accurate

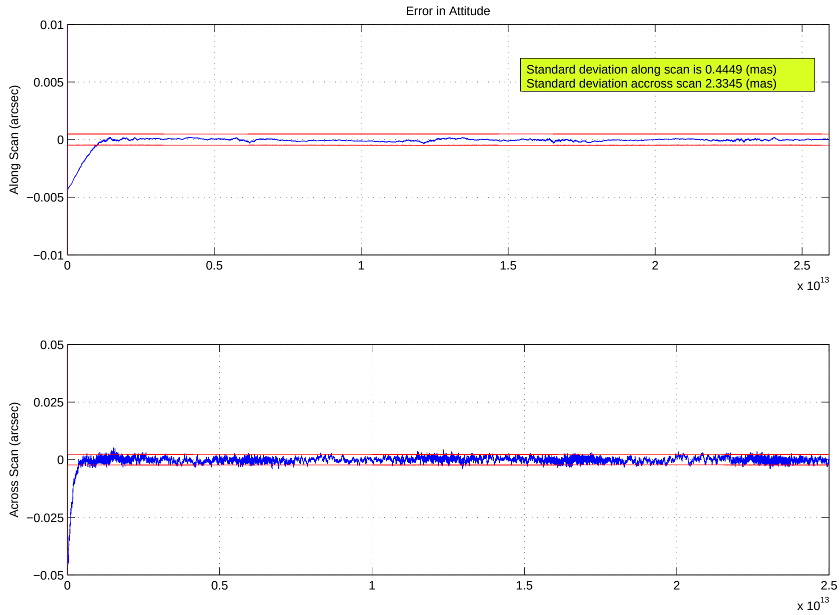


Figure 7 The attitude errors using catalog with 50 marcsec errors.

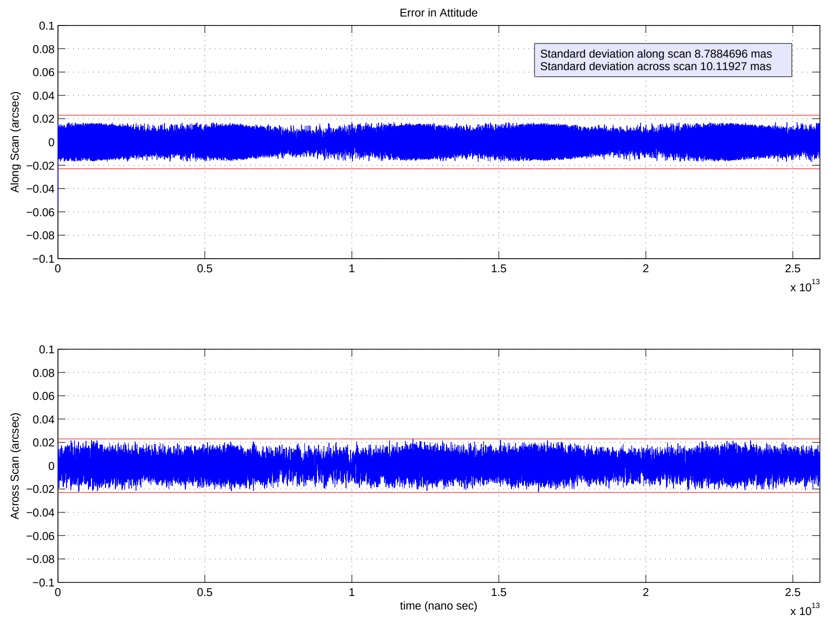


Figure 8 The attitude errors using catalog with 50 marcsec errors and different KF tuning.

estimate of the unknown parameters of the Gaia OGA for each batch of measurements, which include the initial conditions of the attitude quaternions and the initial condition of the angular rates, we have to minimize the following cost function;

$$J(\Theta) = \frac{1}{2}\sum_{k=1}^{N}(\eta^m_{1,k} - \eta^e_{1,k})^2 + (\eta^m_{2,k} - \eta^e_{2,k})^2 + (\zeta^m_{1,k} - \zeta^e_{1,k})^2 + (\zeta^m_{2,k} - \zeta^e_{2,k})^2 \tag{26}$$

where Θ is the unknown parameters vector which includes the initial conditions for quaternions and angular rates for each batch interval and also the constant value for the rate bias for that batch. $\eta^m_{i,k}$ and $\zeta^m_{i,k}$; are the measured field coordinate angles for field of view number i ($i = 1 or 2$) at the observation time t_k, and

$\eta^e_{i,k}$ and $\zeta^e_{i,k}$; are the estimated field coordinate angles for field of view number i at the observation time t_k, which are calculated as follows;

1. For any set of Θ we can use the S/C dynamics equations (Eq. 16) to obtain the quaternion and angular rates for the given interval time (i.e. 1 hour). Then the associated B-Splines is derived for these estimated quaternion by using Eq. 7.

2. At any observation time (t_{obs}) the measured quaternion vector ($\mathbf{q}_m$) is calculated using Eq. 4.

3. The position on the sky of the observed star (α_i, δ_i) for the associated FOV is used to calculate the proper direction of the observed star ($\mathbf{u}$),

4. The estimated field coordinate angles are then computed from

$$\begin{bmatrix} \cos\zeta\cos\eta \\ \cos\zeta\sin\eta \\ \sin\zeta \end{bmatrix} = \mathbf{A}(\mathbf{q})\mathbf{u} \tag{27}$$

   where, $\mathbf{A}(\mathbf{q})$ is the attitude matrix associated with the measured quaternion at t_{obs}.

5. The cost function $J(\Theta)$ is then computed from Eq. 26, and then another set of Θ is chosen so that the cost function is minimized.

The objective 'cost' function in Eq. 26 is an *implicit function* of the initial conditions vector Θ. Therefore, any optimization methods (e.g. differential evolution or gradient based) can be applied to obtain the best estimated values for the vector of the unknown parameters.

In other words, our optimization problem is then; Find the best values of the vector of variables Θ that minimize the objective function $\{J(\Theta)\}$ while satisfying the following constraint

$$\Theta_{obt} \in [\Theta_o - \delta\Theta_o, \Theta_o + \delta\Theta_o] \tag{28}$$

where δ is a very small scalar value that could be chosen based on the accuracy of the measurements, and Θ_o is the initial guess for the unknown parameters, in this case we could start with the on-board initial quaternion and rates for each batch.

Least Squares Trajectory Estimates

The least squares trajectory estimates is based on finding the trajectory and the model parameters for which the square of the difference between the modeled observations and the actual measurements becomes as small as possible.[13] In other words, find a trajectory which best fits the observations in a least-squares of the residuals sense, as illustrated in figure (9).

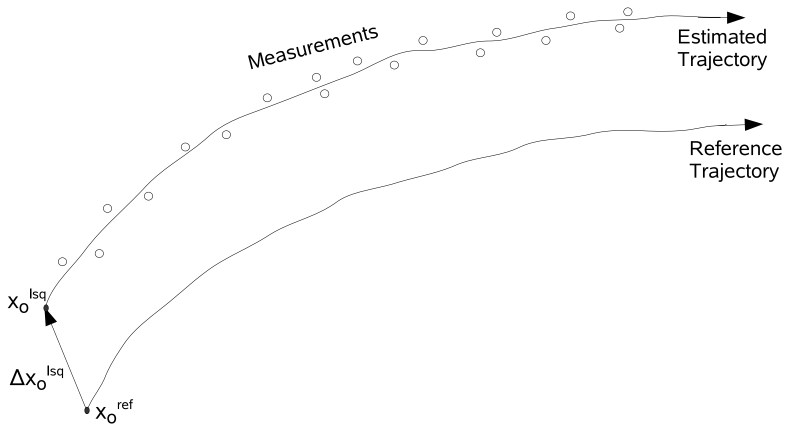


Figure 9 The parameters of a reference trajectory are corrected using the measurements and the least squares fit.

The dynamic equations for the quaternion and angular rates are given in Eq. 16 with an initial value $\mathbf{x}_o = \mathbf{x}(t_o)$ at epoch t_o, where $\mathbf{x}(t_i)$ is the state vector defined on Eq. 12.

Furthermore, we could rewrite the measurements ($\eta(t)$ and $\zeta(t)$) in a vector form as

$$\mathbf{z} = \begin{pmatrix} z_1 & . & . & . & . & z_{2n} \end{pmatrix}^T = \begin{pmatrix} \eta_1 & \zeta_1 & . & . & . & . & \eta_n & \zeta_n \end{pmatrix}^T \tag{29}$$

$\mathbf{z}$ is an *2n*-dimensional vector of measurements taken at times $t_1, ..., t_n$. The observations are described by

$$z_i(t_i) = g_i(t_i, \mathbf{x}(t_i)) + \epsilon_i = h_i(t_i, \mathbf{x}_o) + \epsilon_i \tag{30}$$

or briefly

$$\mathbf{z} = \mathbf{h}_i(\mathbf{x}_o) + \epsilon \tag{31}$$

where, g_i denotes the model value of the i^{th} observation as a function of time t_i and the instantaneous state $\mathbf{x}(t_i)$, whereas h_i denotes the same value as a function of the state $\mathbf{x}_o$ at the reference epoch t_o. The quantities ϵ_i account for the difference between the actual and the modeled observations due to measurement errors, which are usually assumed to be randomly distributed with zero mean value.

The least squares trajectory estimates problem may be defined as finding the state $\mathbf{x}_o^{lsq}$, that minimizes the cost function

$$J(\mathbf{x}_o) = \epsilon^T \epsilon = (\mathbf{z} - \mathbf{h}_i(\mathbf{x}_o))^T (\mathbf{z} - \mathbf{h}_i(\mathbf{x}_o)). \tag{32}$$

The linearized version of Eq. 16 around $\mathbf{x}_o^{ref}$, which is initially given from the on-board coarse attitude data, could be written in the form

$$\begin{array}{l} \mathbf{q}(i+1) = [I_{4x4} + 0.5\Omega(\omega(\mathbf{t}))\delta t_{obs}]\,\mathbf{q}(i) = \chi(\omega(\mathbf{t}))\mathbf{q}(i) \\ \omega(i+1) = I_{sc}^{-1}(T_e - \omega(\mathbf{i}) \times I_{sc}\omega(\mathbf{i}))\delta t_{obs} + \omega(i) \end{array} \tag{33}$$

where $\delta t_{obs} = t_{obs}(i+1) - t_{obs}(i)$.

So, from the above equation, we can write the instantaneous attitude quaternion $\mathbf{q}(t_i)$ as a function of the initial attitude quaternion $\mathbf{q}(t_o)$

$$\mathbf{q}(t_i) = \chi(\omega(\mathbf{t_{i-1}}))\chi(\omega(\mathbf{t_{i-2}})).....\chi(\omega(\mathbf{t_1}))\chi(\omega(\mathbf{t_o}))\mathbf{q}(t_o) \tag{34}$$

The solution of the cost function $J(\mathbf{x}_o)$ is given by taking the derivatives w.r.t. $\mathbf{x}_o$ which leads to

$$\Delta\mathbf{x}_o^{lsq} = (H^T H)^{-1}(H^T \Delta\mathbf{z}) \tag{35}$$

where, H is the partial derivatives of the modeled observations ($\eta(t)$ and $\zeta(t)$) with respect to the state vector at the reference epoch t_o. However, because the observations are only dependent on the attitude quaternions and not on the angular rates then the H could be written as follows

$$H = \frac{\partial h(\mathbf{q}_o)}{\partial \mathbf{q}_o} = \begin{bmatrix} \dfrac{\partial\eta(t)}{\partial\mathbf{q}(\mathbf{t})}\dfrac{\partial\mathbf{q}(\mathbf{t})}{\partial\mathbf{q_o}} \\ \dfrac{\partial\zeta(t)}{\partial\mathbf{q}(\mathbf{t})}\dfrac{\partial\mathbf{q}(\mathbf{t})}{\partial\mathbf{q_o}} \end{bmatrix} \tag{36}$$

where

$$\frac{\partial\mathbf{q}(\mathbf{t})}{\partial\mathbf{q_o}} = \chi(\omega(\mathbf{t_{i-1}}))\chi(\omega(\mathbf{t_{i-2}})).....\chi(\omega(\mathbf{t_1}))\chi(\omega(\mathbf{t_o})). \tag{37}$$

The new updated values of the initial quaternion for each batch are determined from

$$\mathbf{q}_o^{lsq} = \mathbf{q}_o^{ref} + \Delta\mathbf{q}_o^{lsq}. \tag{38}$$

Also, the associated angular rates for this new attitude quaternion batch may be calculated from equation 6.

The results of the least squares trajectory estimates are illustrated in figure 10 for 2 hours batch length and the initial conditions are set by using the on-board attitude. The resulting attitude determination errors (3 σ) is less than 4 marcsec along scan and less than 60 marcsec across scan. These results are not better than the estimated by Kalman filter with good tuning but the least squares estimates algorithm is easier to implement than the Kalman filter algorithm and also it is not sensitive to initial conditions and the choice of the measurement and system noise covariance matrices as in KF case.

Furthermore, figure 11 demonstrates the effect of using different batch interval on the attitude estimation errors. Three batch intervals are investigated; 0.5 hour, 1 hour and 3.5 hours, in which we can see that increasing the batch interval to 3.5 hours has more attitude errors than the other two. On the other hand, decreasing the interval less than one hour has no significant effect on the total attitude errors.

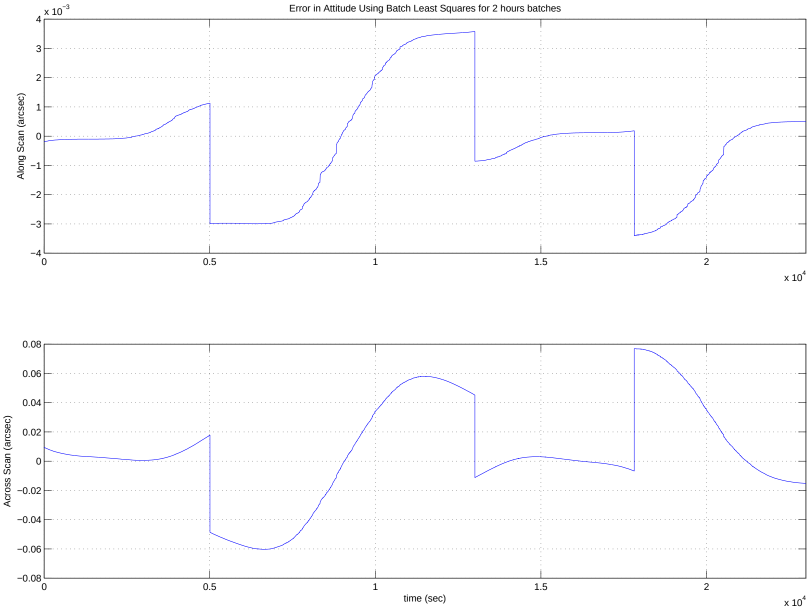


Figure 10 The attitude errors using least squares trajectory for 2 hours batch.

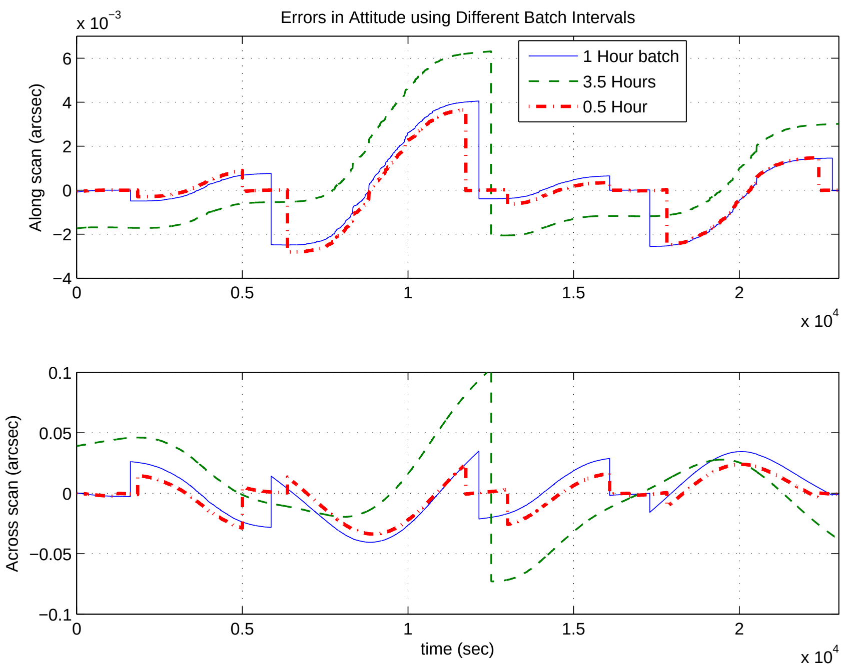


Figure 11 The attitude errors using batch intervals.

CONCLUSION AND DECISION

An algorithm for the Gaia On-Ground Attitude (OGA) determination is designed and discussed in this paper. The Extended Kalman Filter (EKF) is used to produce an accurate on-ground attitude estimation. It utilizes the on-board star tracker measurements for coarse spacecraft quaternion which have errors of about several arcsec and combines it with the payload scientific telescopes measurements which have accuracy of less than milli-arcsec.

The obtained results, using the prescribed algorithm in this work, meet the expected OGA determination requirements. The attitude determination errors and the rate measurement errors (along scan and across scan) have been validated using the proposed EKF algorithm. Moreover, in this study, we design and establish a Batch Least Square (BLS) estimation algorithm to investigate the differences between several estimation techniques in terms of accuracy, speed and robustness. The accuracy obtained using the BLS is less by a factor of two than the accuracy obtained using the EKF. Moreover, the required processing time using the EKF is less by factor of three than using the BLS. On the other hand, the robustness for both methods are almost the same.

As a conclusion of all the simulation results obtained in this study we consider the EKF to be the best OGA algorithm as it meets all the expected requirements with less processing time than the BLS.

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REFERENCES

[1] Frederic Safa, “A Summary of the Gaia Spacecraft Design Document”, 15 March 2006.
[2] Samaan M. A., Theil, S., “Accurate On-Ground Attitude Determination for the Gaia Mission Using Kalman Smoother”, AAS/AIAA Space Flight Mechanics Meeting, Sendona, AZ, Paper 07-100, Jan. 2007.
[3] Astrium EADS, “GAIA AOCS Preliminary Design and Performance Analysis”, GAIA-ASU-TCN-ESM-00027, Feb. 2007.
[4] Lindegren L., “Attitude Parameterization for GAIA”, SAG-LL-30, 1 July 2000.
[5] L. Lindegren, “ Proposed Prototype Processes for the GAIA Global Iterative Solution”, GAIA-LL-34-V.2, 9 March 2001.
[6] C. de Boor, “ A Practical Guide to Splines”, Springer 1978.
[7] Astrium, “Autonomous Star Tracker Requirements Specification”, Issue 1, 2006.
[8] Wertz, J. R., “Spacecraft Attitude Determination and Control”, Kluwer Academic Publishers, the Netherlands, 1990.
[9] Gelb, A. , “ Applied Optimal Estimation”, MIT Press, Cambridge, MA, 1974.
[10] Rauch, H.E., Tung, F., and Striebel, C.T. , “ Maximum Likelihood Estimation of Dynamic Systems”,AIAA Journal, Vol.3, No. 8, August 1965.
[11] Crassidis, and J.L. , Junkins, J.L., “Optimal Estimation of Dynamic Systems”, Chapman and Hall/CRC, 2004.
[12] Ravindra V. Jategaonkar, “Flight Vehicle System Identification”, American Institute for Aeronautics and Astronautics, Vol. 216, 2006.
[13] Montenbruck O., and Gill E. “Satellite Orbits - Models, Methods, and Applications”, Springer-Berlin, 2000.