Markovian Channel Modeling for Multipath Mitigation in Navigation Receivers

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Abstract— **Multipath is today still one of the most crucial problems in satellite navigation, in particular in urban environments, where the received navigation signals can be affected by blockage, shadowing and multipath reception. Latest multipath mitigation algorithms are based on the concept of sequential Bayesian estimation and improve the receiver performance by exploiting the temporal constraints of the channel dynamics, which have to be characterized by a first-order Markovian model for this purpose. In this paper such a channel model is introduced. Simulation results show the benefit of the novel model.**

I. Introduction

Within global navigation satellite systems (GNSS), such as the Global Positioning System (GPS) or the future European satellite navigation system Galileo, the user position is determined based upon the code division multiplex access (CDMA) navigation signals received from different satellites using the time-of-arrival (TOA) method [1]. A major error source for positioning comes from multipath, the reception of additional signal replica due to reflections caused by the receiver environment. The reception of multipath introduces a bias into the time delay estimate of the delay lock loop (DLL) of a conventional navigation receiver, which finally leads to a bias in the receiver's position estimate. Future receiver algorithms [2] exploit prior knowledge about the temporal channel statistics through the use of statistical channel models, which allows to improve the multipath performance of the receiver.

In this paper a novel channel model suitable for the use in future mitigation algorithms is introduced. To motivate the model a brief introduction on the concept of sequential Bayesian receiver algorithms is given, in particular focusing the role of the channel model during the algorithm computations. Simulation results conclude the paper.

II. The Sequential Bayesian Approach

For the sequential approach the problem of multipath mitigation becomes one of *sequential estimation of a hidden Markov process*: The unknown channel parameters are estimated based on an evolving sequence of received noisy channel outputs $\mathbf{z}_k$. The reader is referred to [3] which gives a derivation of the general framework for optimal estimation of temporally evolving parameters by means of inference via *sequential Bayesian estimation*. The entire history of observations (over the temporal index k) can be written as

$$\mathbf{Z}_k \widehat{=} \{\mathbf{z}_{k'}, k' = 1, \dots, k\} \ . \qquad (1)$$

The goal is to determine the *posterior* probability density function (PDF) of every possible channel characterization given all channel observations: $p(\mathbf{x}_k|\mathbf{Z}_k)$, whereas $\mathbf{x}_k$ represents the characterization of the hidden channel state. Once this posterior PDF is evaluated either that channel configuration that maximizes it can be determined - the so called maximum a-posteriori (MAP) estimate; or the expectation can be chosen - equivalent to the minimum mean square error (MMSE) estimate.

In the so-called *prediction step* the recursive sequential Bayesian estimation algorithm computes the *prior* PDF $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ from the posterior PDF at time instance $k-1$, $p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})$ via the Chapman-Kolmogorov equation:

$$p(\mathbf{x}_k|\mathbf{Z}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1}) \mathrm{d}\mathbf{x}_{k-1} \ , \qquad (2)$$

with $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ being the state transition PDF of the Markov process. In the *update step* the new posterior PDF for step k is obtained via

$$p(\mathbf{x}_k|\mathbf{Z}_k) = \frac{p(\mathbf{z}_k|\mathbf{x}_k) p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \ . \qquad (3)$$

The likelihood term $p(\mathbf{z}_k|\mathbf{x}_k)$ represents the probability of the measured channel output, conditioned on a certain configuration of channel parameters at the same time step k. To apply (2) and (3) correctly two conditions need no be fulfilled: At first the noise affecting successive channel outputs is independent of the past noise values, so *each channel observation depends only on the present channel state*, and secondly future channel parameters given the present state of the channel and all its past states, depend only on the present channel state (and not on any past states).

III. Channel Model

A. Multipath Channel Signal Model

The complex valued baseband-equivalent received signal in a navigation receiver is assumed to be equal to

$$z(t) = \sum_{i=0}^{N_m} e_i(t) \cdot a_i(t) \cdot s(t - \tau_i(t)) + n(t) \ , \quad (4)$$

where $s(t)$ is the CDMA navigation signal, N_m is the maximum number of considered multipath replica reaching the receiver (to restrict the modeling complexity), $e_i(t)$ is a binary function that controls the activity of the i'th path and $a_i(t)$ and $\tau_i(t)$ are their individual complex amplitudes and time delays, respectively. The signal is disturbed by additive white Gaussian noise $n(t)$. The signal is sampled at times $(m+k)T_s$, $m = 0, \dots, L-1$ and grouped in blocks of L samples together into vectors $\mathbf{z}_k$ and $\mathbf{n}_k$, with the block index $k = 0, 1, \dots$. The parameter functions $e_i(t)$ and $\tau_i(t)$ are assumed to be constant and equal to $a_{i,k}$, $e_{i,k}$ and $\tau_{i,k}$ for the duration of an entire block. Furthermore the vectors $\boldsymbol{\tau}_k = [\tau_{0,k}, \dots, \tau_{N_m,k}]^T$, $\mathbf{a}_k = [a_{0,k}, \dots, a_{N_m,k}]^T$, and $\mathbf{e}_k = [e_{0,k}, \dots, e_{N_m,k}]^T$ are used, with $e_{i,k} \in [0, 1]$ to determine whether the i'th path is active or not by being either $e_{i,k} = 1$ corresponding to an active path or $e_{i,k} = 0$ for a path that is currently not active. In the compact form the samples of the delayed replica $\mathbf{s}(\tau_{i,k})$ are stacked together as columns of the matrix $\mathbf{S}(\boldsymbol{\tau}_k) = [\mathbf{s}(\tau_{0,k}), \dots, \mathbf{s}(\tau_{N_m,k})]$ and we may write

$$\begin{aligned} \mathbf{z}_k &= \mathbf{S}(\boldsymbol{\tau}_k)\mathbf{E}_k\mathbf{a}_k + \mathbf{n}_k \\ &\widehat{=} \mathbf{s}_k + \mathbf{n}_k \ , \end{aligned} \quad (5)$$

with $\mathbf{E}_k = \mathrm{diag}(\mathbf{e}_k)$, and the associated likelihood function becomes

$$p(\mathbf{z}_k|\mathbf{s}_k) = \frac{1}{(2\pi)^L \sigma^{2L}} \cdot \exp\left[-\frac{1}{2\sigma^2} \left(\mathbf{z}_k - \mathbf{s}_k\right)^H \left(\mathbf{z}_k - \mathbf{s}_k\right) \right] \ . \quad (6)$$

The likelihood function plays a central role in this paper; its purpose is to quantify the conditional probability of the received signal conditioned on the unknown signal (specifically the channel parameters).

B. Markovian Process Model

To exploit the advantages of sequential estimation for the task of multipath mitigation/estimation the actual channel characteristics have to be described such that these are captured by $p(\mathbf{x}_k|\mathbf{x}_{k-1})$. In other words, the model must be a first order Markov model and all transition probabilities must be known. Here the channel is approximated as follows:

- The channel is totally characterized by a direct line-of-sight (LOS) path (index $i = 0$) and at most N_m echoes.
- Each path has complex amplitude $a_{i,k}$ and delay $\tau_{i,k}$, where echoes are constrained to have delay $\tau_{i,k} \geq \tau_{0,k}$, $i = 1, \dots, N_m$, to reflect that multipath replica are physically constrained to arrive later at the receiver than the LOS path.
- The delay of each path follows the process
$$\tau_{i,k} = \tau_{i,k-1} + \dot{\tau}_{i,k-1}\Delta t + n_{i,\tau} + n_\tau \ , \quad (7)$$
whereas n_τ is the same value for all indices i.
- Each parameter $\dot{\tau}_{i,k}$ that specifies the rate of the change of the path delay follows its own process:
$$\dot{\tau}_{i,k} = \dot{\tau}_{i,k-1} + n_{i,\dot{\tau}} + n_{\dot{\tau}} \ , \quad (8)$$
whereas $n_{\dot{\tau}}$ is the same value for all indices i.
- Each echo is either "on" or "off", as defined by the channel parameter $e_{i,k} \in \{1 \equiv \text{"on"}, 0 \equiv \text{"off"}\}$, where $e_{i,k}$, $i = 1, \dots, N_m$ follows a simple two-state Markov process with a-symmetric crossover and same-state probabilities:
$$p(e_{i,k} = 0|e_{i,k-1} = 1) = p_{\text{onoff}} \ , \quad (9)$$
$$p(e_{i,k} = 1|e_{i,k-1} = 0) = p_{\text{offon}} \ . \quad (10)$$
- The LOS component is always present and consequently $e_{0,k} = 1$ for all k.
- Appearing echoes ($e_{i,k} = 1$ and $e_{i,k-1} = 0$) are initialized with
$$\tau_{i,k} = \tau_{0,k} + |\tau_m + n_{\tau_0}| \ , \quad (11)$$
$$\dot{\tau}_{i,k} = \dot{\tau}_{0,k} + n_{\dot{\tau}_0} \ , \quad (12)$$
with the characteristic constant τ_m.
- Blockage and shadowing of the LOS signal is considered through variations of the LOS amplitude $a_{0,k}$.
- The complex amplitudes $a_{i,k}$ depend on the previous amplitudes $a_{i,k-1}$ through
$$a_{i,k} = e^{-j2\pi f_0 \Delta t \dot{\tau}_{i,k}} \cdot a_{i,k-1} + n_{i,a_i} \ . \quad (13)$$
Thus the rate of change in the delay affects the evolution of the complex amplitude in a statistical manner in order to consider the physical relations between phase, Doppler-frequency, and time delay adequately.

The model implicitly incorporates nine i.i.d. noise sources: Gaussian $n_{i,\tau} \sim \mathcal{N}(0, \sigma^2_{i,\tau})$, $n_{i,\dot{\tau}} \sim \mathcal{N}(0, \sigma^2_{i,\dot{\tau}})$, $n_\tau \sim \mathcal{N}(0, \sigma^2_\tau)$, $n_{\dot{\tau}} \sim \mathcal{N}(0, \sigma^2_{\dot{\tau}})$, $n_{\tau_0} \sim \mathcal{N}(0, \sigma^2_{\tau_0})$, $n_{\dot{\tau}_0} \sim \mathcal{N}(0, \sigma^2_{\dot{\tau}_0})$, and complex Gaussian $n_{i,a_i} \sim \mathcal{N}(0, \sigma^2_{i,a_i})$, as well as the noise process driving the state changes for $e_{i,k}$. These sources provide the randomness of the model. The noise sources n_τ and $n_{\dot{\tau}}$ are included to model the impact of the receiver clock on the individual delays and delay rates, since they are actually affected simultaneously by the same random process. Finally, $\Delta t = LT_s$ is the time between instances $k-1$ and k. It is assumed that all model parameters (i.e. Δt, noise variances, and the "on"/"off" Markov model) are independent of k. Note that the model implicitly represents the number of paths through the time variant parameter

$$N_{m,k} = \sum_{i=0}^{N_m} e_{i,k} \ . \quad (14)$$

Using $\dot{\boldsymbol{\tau}}_k = [\dot{\tau}_{0,k}, \dots, \dot{\tau}_{N_m,k}]^T$, the hidden channel state vector $\mathbf{x}_k$ is thus represented as:

$$\mathbf{x}_k \widehat{=} [\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k] \ . \quad (15)$$

The channel state model used here is motivated by channel modeling work for multipath prone environments such as the urban satellite navigation channel [4], [5]. In fact the process of constructing a channel model in order to characterize the channel for signal level simulations and receiver evaluation comes close to our task of building a first order Markov process for sequential estimation. It is important to point out that the Bayesian estimator is only as good as its system models matches the real world situation. The state model needs to capture *all* relevant hidden states with memory and needs to correctly model their dependencies, while adhering to the first order Markov condition. Furthermore, any memory of the measurement noise affecting the likelihood function $p(\mathbf{z}_k|\mathbf{x}_k)$ must be explicitly contained as additional states of the model $\mathbf{x}$, so that the measurement noise is i.i.d.. Practically there will be always a mismatch between the assumptions taken in the model and the real world situation.

IV. Estimator Implementation

Various algorithms are known to implements the Bayesian recursion (2) and (3), including the Kalman filter, the grid-based filter and the family of particle filtering algorithms [3]. Certain restrictions are imposed on the use of these algorithms. The objective here is to estimate the channel parameters (15) using the likelihood (6) and the process defined in III-B, which makes the estimation complex: The amplitude parameters $a_{i,k}$ are continuous and the measurement depends linearly on them like the activity parameters $e_{i,k}$, which are discrete and thus follow a discrete evolution. In difference the observations depend nonlinearly on the continuous delays $\tau_{i,k}$, which are also nonlinear with respect to their dynamics. A straightforward way would be to implement the estimation algorithm completely with a particle filter, which is the most general method with respect to system nonlinearities, but depending on the considered number of paths N_m the state space in such a filter becomes large and it becomes difficult to cover the entire space with a reasonable number of particles. To consider the nonlinearities while keeping the state space to be covered by the particles as small as possible, it was proposed to reduce the computational complexity of the filter by means of marginalization over the linear state variables [6], a technique also known as Rao-Blackwellization [7]. In a marginalized filter, particles are still used to estimate the non-linear states, while for each of the particles the linear states can be estimated analytically. The marginalized estimator used here factorizes the posterior density two-fold according to

$$p(\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_k) = \underbrace{p(\mathbf{a}_k|\mathbf{Z}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}_{\text{Kalman filter}}\underbrace{p(\mathbf{e}_k|\mathbf{Z}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}_{\text{Grid-based filter}}\underbrace{p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_k)}_{\text{Particle filter}} \ . \tag{16}$$

A Kalman filter is used to estimate the amplitudes $\mathbf{a}_k$ analytically conditional on the parameters $\mathbf{e}_k$, $\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k$. The discrete path activity parameters are in turn estimated conditionally on the delays $\boldsymbol{\tau}_k$ and the delay rates $\dot{\boldsymbol{\tau}}_k$ using a grid based method [3], which is appropriate to optimally estimate a discrete state space. Finally the delays $\boldsymbol{\tau}_k$ and the delay rates $\dot{\boldsymbol{\tau}}_k$ are the only remaining parameters that are estimated by the particle filtering algorithm. The update step (3) of the marginalized filter can be expressed as

$$\begin{aligned} p(\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_k) &= \\ &\frac{p(\mathbf{z}_k|\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \cdot p(\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_{k-1}) \\ =& \underbrace{\frac{p(\mathbf{z}_k|\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)} \cdot p(\mathbf{a}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}_{\text{Amplitude estimator: Kalman filter}} \\ &\underbrace{\cdot\frac{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)} \cdot p(\mathbf{e}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}_{\text{Path activity estimator: Grid-based filter}} \\ &\underbrace{\cdot\frac{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \cdot p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_{k-1})}_{\text{Delay and delay rate estimator: Particle filter}} \\ =& \ p(\mathbf{a}_k|\mathbf{Z}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)p(\mathbf{e}_k|\mathbf{Z}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_k) \ . \end{aligned} \tag{17}$$

A. Estimation of Amplitudes

From (17) follows the implementation of the conditional amplitude filter. The conditional posterior density with respect to the complex amplitudes is thus given by

$$p(\mathbf{a}_k|\mathbf{Z}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \frac{p(\mathbf{z}_k|\mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)} \cdot p(\mathbf{a}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) \ . \tag{18}$$

Recalling the structure of the amplitude system model, i.e. (6) and (13), the observed signal $\mathbf{z}_k$ depends linearly on the amplitudes $\mathbf{a}_k$ and the amplitude dynamics are linear conditional on the delay rates. Hence the Rao-Blackwellization can be applied directly and the prior PDF for the amplitudes is given by the Gaussian

$$p(\mathbf{a}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \mathcal{N}\left(\hat{\mathbf{a}}_k^-, \tilde{\mathbf{P}}_k^-\right) \ , \tag{19}$$

whereas mean and covariance are obtained in the prediction step from the previous time instance $k-1$ through the framework of the Kalman filter equations

$$\hat{\mathbf{a}}_k^- = \tilde{\mathbf{F}}_k \hat{\mathbf{a}}_{k-1} \ , \tag{20}$$

$$\tilde{\mathbf{P}}_k^- = \tilde{\mathbf{F}}_k \tilde{\mathbf{P}}_{k-1} \tilde{\mathbf{F}}_k^T + \tilde{\mathbf{Q}} \ . \tag{21}$$

The matrices $\mathbf{F}_k$ and $\mathbf{Q}$ follow directly from (13) and are computed with

$$\mathbf{F}_k = \operatorname{diag}\left([e^{-j2\pi f_0 \Delta t \dot{\tau}_{0,k}}, \ldots, e^{-j2\pi f_0 \Delta t \dot{\tau}_{N_m,k}}]\right), \tag{22}$$

$$\mathbf{Q} = \operatorname{diag}\left([\sigma_{0,a_i}^2, \ldots, \sigma_{N_m,a_i}^2]\right) \ . \tag{23}$$

The notation $\tilde{\bullet}$ and $\hat{\bullet}$ indicates thereby that dimension and values of the respective matrices and vectors correspond to the active paths as given by $\mathbf{e}_k$. Due to the conditional linear Gaussian model the evaluation of (18) is feasible through the application of the Kalman filter update equations and the posterior PDF of the amplitude filter becomes

$$p(\mathbf{a}_k|\mathbf{Z}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \mathcal{N}\left(\hat{\mathbf{a}}_k, \tilde{\mathbf{P}}_k\right) \ , \tag{24}$$

whereas mean and covariance are given by

$$\hat{\mathbf{a}}_k = \hat{\mathbf{a}}_k^- + \tilde{\mathbf{K}}_k \left(\mathbf{z}_k - \tilde{\mathbf{S}}_k \hat{\mathbf{a}}_k^- \right) , \quad (25)$$

$$\tilde{\mathbf{P}}_k = \left(\mathbf{I} - \tilde{\mathbf{K}}_k \tilde{\mathbf{S}}_k \right) \tilde{\mathbf{P}}_k^- , \quad (26)$$

with $\mathbf{S}_k = \mathbf{S}_k(\boldsymbol{\tau}_k)$ and the Kalman gain

$$\tilde{\mathbf{K}}_k = \tilde{\mathbf{P}}_k^- \tilde{\mathbf{S}}_k^T \left(\tilde{\mathbf{S}}_k \tilde{\mathbf{P}}_k^- \tilde{\mathbf{S}}_k^T + \mathbf{R} \right)^{-1} . \quad (27)$$

The value of $\mathbf{R} = \sigma^2 \cdot \mathbf{I}$ follows directly from (6).

B. Estimation of Path Activity

The estimation of the path activity $\mathbf{e}_k$ follows (17) and thus the posterior PDF with respect to the path activity is given by

$$p(\mathbf{e}_k|\mathbf{Z}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \frac{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)} \cdot p(\mathbf{e}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) . \quad (28)$$

The activity state space is discrete and thus can be estimated optimally using a grid-based filter [3]. In this case the prediction (2) simplifies to the evaluation of the sum

$$p(\mathbf{e}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \sum_{\mathbf{e}_{k-1}} p(\mathbf{e}_k|\mathbf{e}_{k-1}, \mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) p(\mathbf{e}_{k-1}|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) . \quad (29)$$

The transition density with respect to the activity states is given by (9) and (10) and depends therefore on the realization of the path transition according to

$$p(\mathbf{e}_k = \bar{\mathbf{e}}_k|\mathbf{e}_{k-1} = \bar{\mathbf{e}}_{k-1}, \mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = N_{\text{offon}} p_{\text{offon}} + N_{\text{onoff}} p_{\text{onoff}} + N_{\text{offoff}}(1 - p_{\text{offon}}) + N_{\text{onon}}(1 - p_{\text{onoff}}) , \quad (30)$$

where N_{offon} is the number of paths switching from "off" to "on", N_{onoff} is the number of paths switching from "on" to "off", N_{offoff} is the number of paths remaining "off", and N_{onon} is the number of paths remaining "on" during the transition from $\bar{\mathbf{e}}_{k-1}$ to $\bar{\mathbf{e}}_k$. Note that there are 2^{N_m} discrete states and 2^{2N_m} transitions to be covered by the grid based filter. The marginal likelihood value used in the update step is given by the solution of the integral

$$p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \int_{\mathbf{a}_k} p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{a}_k, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) p(\mathbf{a}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) \mathrm{d}\mathbf{a}_k, \quad (31)$$

which equals the Gaussian

$$p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \mathcal{N}\left(\tilde{\mathbf{S}}_k \hat{\mathbf{a}}_k^-, \tilde{\mathbf{S}}_k \tilde{\mathbf{P}}_k^- \tilde{\mathbf{S}}_k^T + \mathbf{R} \right) . \quad (32)$$

A proof for (32) can be found in [8].

C. Estimation of Path Delays

Due to the non-linearity in the system model the remaining parts of the state vector, namely the delays and the delay rates, are to be estimated by a particle filter. According to (17) the posterior density with respect to the path delays and delay rates computes with

$$p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_k) = \frac{p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k)}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \cdot p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_{k-1}) . \quad (33)$$

Here a simple sampling importance resampling particle filter (SIR-PF) according to [9] is used to implement the marginalized delay estimator. In the SIR-PF algorithm the posterior density at step k is represented as a sum, and is specified by a set of N_p particles:

$$p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\mathbf{Z}_k) \approx \sum_{\mu=1}^{N_p} w_k^\mu \cdot \delta(\boldsymbol{\tau}_k - \boldsymbol{\tau}_k^\mu, \dot{\boldsymbol{\tau}}_k - \dot{\boldsymbol{\tau}}_k^\mu) , \quad (34)$$

where each particle with index μ has a state $\boldsymbol{\tau}_k^\mu, \dot{\boldsymbol{\tau}}_k^\mu$ and has a weight w_k^μ. Due to the marginalization each particle carries in addition a grid-based filter, whereas for each of the discrete states a Kalman filter is associated to the particle, resulting thus in 2^{N_m} Kalman filters per particle. The key step in which the measurement for instance k is incorporated, is in the calculation of the weight w_k^μ, which for the SIR-PF used here is the marginalized likelihood function: $p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k^\mu, \dot{\boldsymbol{\tau}}_k^\mu)$. The characterization of the channel process enters in the algorithm when at each time instance k, the state of each particle $\boldsymbol{\tau}_k^\mu, \dot{\boldsymbol{\tau}}_k^\mu$ is drawn randomly from the proposal distribution; i.e. from $p(\boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k|\boldsymbol{\tau}_{k-1}^\mu, \dot{\boldsymbol{\tau}}_{k-1}^\mu)$, which corresponds to drawing values for $n_{i,\tau}$, $n_{i,\dot{\tau}}$, n_τ, $n_{\dot{\tau}}$, n_{τ_0} and $n_{\dot{\tau}_0}$.

The marginal likelihood value, which is required to update the marginal particle filter, is given by summing up the marginal likelihoods over all path activity hypotheses

$$p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) = \sum_{\mathbf{e}_k} p(\mathbf{z}_k|\mathbf{Z}_{k-1}, \mathbf{e}_k, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) p(\mathbf{e}_k|\mathbf{Z}_{k-1}, \boldsymbol{\tau}_k, \dot{\boldsymbol{\tau}}_k) . \quad (35)$$

V. RESULTS

Figure 1 shows the performance of a conventional DLL and a sequential Bayesian estimator, which is implemented according to the two-fold Rao-Blackwellized estimator that was introduced in Section IV. The channel scenario is generated from the urban channel model introduced in [4]. It comprises periods of LOS reception as well as periods of shadowed or even blocked LOS. The scenario resembles a dynamic user within an urban environment, where static periods alternate with periods of movement. The Bayesian estimator uses $N_p = 1000$ particles and takes into account up to $N_m = 1$ multipath replica. The navigation signal used in the simulation corresponds to a GPS C/A signal. As illustrated in Figure 2 the Bayesian algorithm is superior, since it makes use of the prior knowledge, which is given by the proposed Markovian channel model. In particular during periods of blocked LOS, where the errors for the DLL become large, the benefit of the Bayesian approach is revealed.

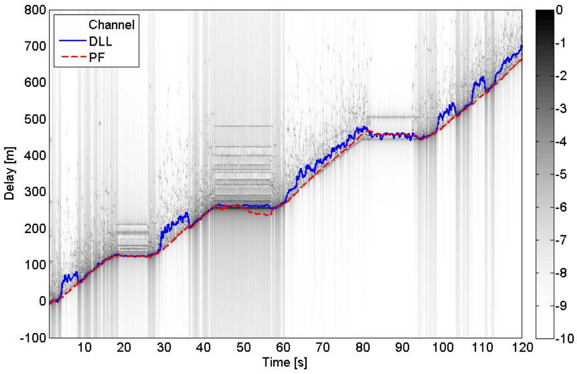

Fig. 1. Channel and LOS track with conventional DLL and marginalized particle filter

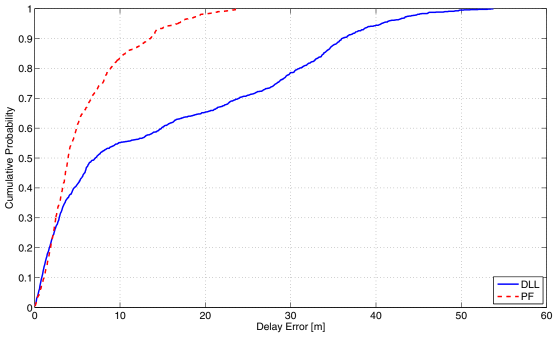

Fig. 2. Cumulative normalized histogram of LOS estimation error

VI. Conclusions

In this paper we have introduced a first order Markovian channel model that can be used for multipath mitigation in navigation receivers. It was shown how a sequential Bayesian channel estimator can take benefit of the introduced channel model, whereas we have proposed an efficient implementation of the estimator by applying the concept of Rao-Blackwellization. The simulation results for an urban environment confirm the benefit of the introduced model.

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