O. Vaschalde

Calibration of a Four-Hole Probe in Sub-, Trans- and Supersonic Flow

Date: January 2008
Client: -
Prepared for: practical education at DLR from September 1st, 2007 to January 31st, 2008 during 2nd year of studies at ENSIAME, Valenciennes

Head of the institute:
Prof. Dr.-Ing. R. Mönig

Copying and communicating the contents of this document or any other information – even of sections – to third parties only with permission of the client, of DLR
Keywords:
Probe calibration, transonic flow

** Calibration of a Four-Hole Probe in Sub-, Trans- and Supersonic Flow **

**Abstract**

When applying pneumatic probes within aerodynamic research there is a demand of calibrating these probes. Especially at transonic velocities there is a general lack of sensitivity resulting in big errors. To overcome this problem it is necessary to include additional information – the pressure behind the probe – into the calibration process.

In this report the wind tunnel for probe calibration and the probe of interest are described as well as the applied calibration process. Results are compared with and without considering the back pressure in the probe’s wake.

German Aerospace Center e.V.
In der Helmholtz-Gemeinschaft
Site Göttingen
Institute of Propulsion Technology
Turbine Department
Content

Notations IV

Acknowledgement V

I. Introduction 1

I.1. Presentation of DLR ................................................................. 1
I.2. Subject of the training course .................................................. 3

II. SEG wind tunnel 4

II.1. Description of the wind tunnel .................................................. 4
II.2. Operation of the wind tunnel ................................................... 7

III. The probe 9

III.1. General information about pressure probes ............................... 9
III.2. Description of the probe ......................................................... 10

IV. Theoretical study 12

IV.1. Basic equations ...................................................................... 12
IV.2. Calculation of the Mach number in SEG .................................. 13

V. Probe calibration 15

V.1. General procedure ................................................................. 15
V.2. Measurement program and positioning of the probe ................. 15
V.3. Characteristics of the probe in a three-dimensional flow field .... 17
V.4. Incidence angle effects ............................................................ 24
  V.4.a. Calculation of $\alpha$ from $C_\alpha$ ......................................... 24
  V.4.b. Study of the probe at zero incidence .................................. 30
  V.4.c. The transonic problem for the determination of the Mach number .................................. 34
V.5. Error sensitivity ................................................................. 36

VI. Conclusion 39

Bibliography 40

List of figures 41

Appendix 44
Notations:

Wind tunnels:

- **RGG**: Windkanal für Rotierende Gitter Göttingen (rotating cascades)
- **EGG**: Windkanal für Ebene Gitter Göttingen (straight cascades)
- **NGG**: Niedergeschwindigkeits-Gittermessstrecke Göttingen (straight cascades and low velocity)
- **SEG**: Sondeneichkanal Göttingen (probe calibration)

Ma
Re

- **α**: Incidence angle of the probe
- **β**: Radial angle of the probe

γ

- **γ**: Adiabatic index (γ=1.4 for the air)
- **Cp**: Heat capacity
- **T**: Temperature
- **R**: Specific gas constant
- **u**: Flow velocity
- **h**: Enthalpy

p0

- **p₀**: Total pressure
- **p₀s**: Stagnation pressure of the probe (middle hole)
- **psl**: Left tube pressure
- **psr**: Right tube pressure
- **pbu**: Back tube pressure
- **psm**: Mean of psl and psr

Cα

- **Cₐ**: Pressure coefficient relative to α
- **CMa**: Mach number coefficient calculated from p₀ and p₀s
- **CMab**: Mach number calculated from pbu and p₀s
Acknowledgment

I would first like to thank Dr. Friedrich Kost, my supervisor at DLR, for his welcome, for answering to all my questions, and for his precious help during my training course.

I also thank all the employees of the department « Antriebstechnik – Turbine » for their welcome and their kindness.

My acknowledgments go also to M. Bernard Desmet my supervisor in Valenciennes, to M. Alain Lecocq and the employees of the ENSIAME for giving me the chance to work in one of the most famous aerospace centers in Europe.
I. Introduction

I.1. Presentation of DLR

DLR (Deutsches Zentrum für Luft- und Raumfahrt) is the German center for research in aeronautics and aerospace. It is involved in many research projects national as well as international.

It was created in 1969 from the merge of three German centers of research:

- AVA (Aerodynamische Versuchsansalt Göttingen), experimental laboratory of aerodynamics at Göttingen, founded in 1907 by Ludwig Prandtl;
- DVL (Deutsche Versuchsanstalt für Luftfahrt), German experimental laboratory of aeronautics, created in 1912;
- DFL (Deutsche Forschungsanstalt für Luftfahrt), German laboratory of research in aeronautics, founded in 1936.

Today, DLR employs nearly 5000 people at several sites in Germany and in the World: Köln-Porz, Berlin-Adlershof, Bonn-Oberkassel, Braunschweig, Göttingen, Lampoldshausen, Oberpfaffenhoffen, Stuttgart, Bruxelles, Paris and Washington. DLR also established partnerships with other research centers like EREA (Etablissements de Recherche Européens en Aéronautique), NASA, NASDA (Japan) and the Russian research. The different programs studied at the DLR cover a large spectrum of disciplines, such as:

- fluid mechanics
- flight mechanics
- energetics
- materials
- mechatronics and robotics
- telecommunications and high-level electronics
- optics
- biophysics
The site of Göttingen is one of the most famous aerospace research centers in Germany. Research here is focused on aerodynamics, aeroelasticity, turbines and acoustics.

![Figure 1: DLR site of Göttingen](image)

I completed my training course in the turbine department, specialized in the study of turbine blades. This department owns four wind tunnels to study different kinds of flows:

* SEG (Sondeneichkanal Göttingen) to calibrate probes.
* RGG (Windkanal für Rotierende Gitter Göttingen), for the study of turbine stages.
* EGG (Windkanal für Ebene Gitter Göttingen) for the study of straight cascades.
* NGG (Niedergeschwindigkeits-Gittermessstrecke Göttingen) has the same function as EGG but allows for larger models at lower flow velocities.
I.2. Subject of the training course

In order to study turbine cascades, the department « Antriebstechnik-Turbine » needs pressure probes. However, before using these probes, they have to be calibrated. That’s why DLR designed a wind tunnel specifically intended for probe calibration.

The main task of my training course was to calibrate a four-hole probe at the SEG for use in other wind tunnels, like the RGG.

This task can be divided into three steps: firstly, we will determine the coefficients which will be used along our study. Secondly, we will proceed to several measurements with our probe in the SEG by changing independently the Mach number and the Reynolds number. Thirdly, we will study all the data acquired and we will try to set up equations describing the behaviour of our probe.
II. SEG wind tunnel

Figure 2: SEG wind tunnel

II.1. Description of the wind tunnel

The wind tunnel is a closed circuit where dried air is blown. In order to reduce operating expenses, SEG shares with RGG two compressors of 90kW. Used independently or in parallel, their compression ratio is 3 and they provide an air flow of 43 m$^3$/s. Cooling units are located downstream of the compressors, allowing to maintain the fluid at a desired temperature between 22°C and 42°C.

A compressed air reserve as well as a vacuum pump respectively located upstream and downstream of the test section allows obtaining a total pressure between 30 kPa and 290 kPa.
The test section is located in a large chamber where probes up to 700 mm long can be installed. Motorized systems controlled by a computer allow setting the probe at different positions. The air passes first filters and a grid to be homogenized, then an exchangeable nozzle before arriving at the probe. This system of exchangeable nozzles provides a good flexibility to the SEG and allows choosing precisely the desired Mach number.

Figure 3: Diagram of the SEG circuit

Figure 4: Test chamber of SEG
By choosing the right pressure and the right nozzle, this wind tunnel, gives us the ability to set a Mach number independently from the Reynolds number. Indeed, the Mach number is a function of the ratio between the total pressure, $p_0$, and the pressure in the test chamber, $p$, according to the following equation:

$$Ma = \sqrt[\gamma-1]{\frac{2}{\gamma-1} \left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

for an adiabatic flow and a perfect diatomic gas. We will develop this formula later in this report.

SEG operates at Mach numbers between 0.2 and 1.8 and Reynolds number from $10^5$ to $2.1*10^6$.

*Figure 5: Range of operation*
II.2. **Operation of the wind tunnel**

Two computers allow the control of the SEG.

The first one runs the SIMATIC software (Siemens) which controls in real time the Mach number, the temperature and the desired pressures.

![Figure 6: Screenshot of the wind tunnel control software (SIMATIC main window)](image-url)
The second one runs the LabVIEW software which collects all the data from the measurements applying the DatSEG program. It enables also to control the positioning of the probe during the calibration process.

Figure 7: Screenshot of the probe control software (LabVIEW)
III. The probe

III.1. General information about pressure probes

The design of a probe must obey several criteria. Indeed, due to the intrusive nature of this type of measurements the influence of the probe on the flow characteristics has to be limited as well as the accuracy of the measurements kept. As a result we have to take in account these following criteria:

- **1- Mechanical criterion:** the probe has to resist its conditions of use from the mechanical point of view.
- **2- Criterion of adaptability:** the probe should be applicable for different test campaigns.
- **3- Aerodynamic criterion:** the probe inevitably modifies the behaviour of the flow around it. It will thus be necessary to reduce the disturbances to the minimum and, accordingly, to optimize the shape.
- **4- Criterion of manufacture:** the probe has to be machined, in spite of, generally, its small dimensions.

Moreover, the user of the probe has to limit the measurement errors, which are static or dynamic. This can be carried out during a trial run:

- by minimizing the number of measurements with the probe;
- by choosing the coefficients of calibration adequately.
III.2. Description of the probe

The probe, which was built here at DLR Göttingen, will be called RRT6 probe. It is made of three tubes one-millimetre in diameter welded side by side and folded at 90° at their end. This particular shape (« cobra » probe) allows introducing the probe in very narrow spaces, such as between a stator and a rotor of a turbine. Although this configuration isn’t perfect (the stem of the probe affects the flow), this kind of probe is very easy to manufacture at low cost. The left and right tubes are bevelled to 45° at their end. There is also a thermocouple in the front of the probe and a fourth tube at the back of the probe, which is not folded like the others one, but bevelled backwards 45° at its end.

Figure 8: Sketch of the RRT6 probe
Therefore, we have 4 pressure tappings that were named as follows:

- psl: left tube (when we look directly face to the probe))
- psr: right tube
- p0s: center tube (Pitot pressure)
- pbu: back tube

*Figure 9: Photograph of the probe*
IV. Theoretical study

IV. 1. Basic equations

Calibrating a probe means to study how the probe reacts to a flow the characteristics of which are known. Then we are going to establish the equations of the behaviour of the probe in order to use it for its initial purpose: find the characteristics of the flow from the pressures given by the probe. Nevertheless, before doing this we have to define dimensionless coefficients from the raw data acquired by the probe.

If we have:

$$ p_{sm} = \frac{p_{sl} + p_{sr}}{2} $$

We define:

$$ C_a = \frac{p_{sl} - p_{sr}}{p_{0s} - p_{sm}} \quad (1) $$

$$ C_{Ma} = Ma(p_{sm}, p_{0s}) \quad (2) $$

$$ C_{Ma_b} = Ma(p_{bu}, p_{0s}) \quad (3) $$

where the Mach number is obtained from the following formula:

$$ Ma(p, p_0) = \sqrt{\frac{2}{\gamma - 1} \left[ \left( \frac{p}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]} \quad (4) $$

with $\gamma = 1.4$ for air.
IV. 2. Calculation of Mach number in SEG

Let’s derive the formula which gives the Mach number from the total pressure \( p_0 \) and from the pressure in the test chamber \( p \). We will consider air as a perfect gas and the flow in the SEG as steady and adiabatic.

\[
\frac{h + \frac{u^2}{2}}{2} = \text{const.}
\]

which can also be written:

\[
C_p T + \frac{u^2}{2} = C_p T_0 = \text{const.}
\]

\[
\frac{T_0}{T} = 1 + \frac{u^2}{2 C_p T}
\]

applying:

\[
C_p = \frac{\gamma R}{\gamma - 1} \quad \text{(Mayer formula)}
\]

We find:

\[
\frac{T_0}{T} = 1 + \frac{(\gamma - 1)u^2}{2 \gamma RT}
\]

and as we know that the Mach number is given by the following formula:

\[
Ma = \frac{u}{c} = \frac{u}{\sqrt{\gamma RT}}
\]

\[
\frac{T_0}{T} = 1 + \frac{(\gamma - 1)Ma^2}{2}
\]
Finally, if we consider the flow as isentropic, we can use the Laplace law:

\[
\frac{p_0}{p} = \left( \frac{T_0}{T} \right)^{\frac{\gamma}{\gamma-1}}
\]

so:

\[
\frac{p_0}{p} = \left( 1 + \frac{\gamma-1}{2} Ma^2 \right)^{\frac{\gamma}{\gamma-1}}
\]

And we get the Mach number according to the following formula:

\[
Ma(p,p_0) = \sqrt{\frac{2}{\gamma-1} \left[ \left( \frac{p}{p_0} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right]}
\]
V. Probe calibration

V.1. General procedure

We are going to do several measurements with our probe in the SEG, at different Mach numbers and different Reynolds numbers. We then plot the results applying the software TECPLPOT. We have to find the best interpolation of our plots in order to be able to get the flow values from the pressure coefficients.

V.2. Measurement program and positioning of the probe

To calibrate the probe, we have to record the pressures given by the four tapping points of the probe at different angle values. The probe can be rotated according to two angles:

- the incidence angle $\alpha$ (from $-30^\circ$ to $30^\circ$)
- the radial angle $\beta$ (from $-10^\circ$ to $10^\circ$)

Figure 10: Angles of rotation of the probe
For each measurement point, the probe stops moving for half a second, which is sufficient for the flow to become steady again around the probe. The total pressure in the inlet will be set either to 100kPa or 30kPa in order to have different Reynolds numbers for the future use of the probe in the RGG. The temperature will be around 22°C.

The calibration of the probe covers a wide range of Mach numbers, beginning at Ma=0.2 and ending at Ma=1.8. We thus obtain measurements for 18 different Mach numbers. To achieve these velocities, we utilize in the SEG a subsonic nozzle (Ma = 0.2 - 0.9), a transonic nozzle (Ma = 0.95 - 1.3) and three supersonic (Laval) nozzles (Ma = 1.4, 1.6, 1.8).

Before starting the probe needs to be aerodynamically centered, because its « geometrical zero » according to \( \alpha \) can differ from its « aerodynamic zero ». Therefore the probe is positioned in the SEG face to the flow, and then when the air runs at a fixed Mach number, the probe rotates according to \( \alpha \) from -30° to +30°.

Finally, the LabVIEW software acquires and saves all the pressure data, we only have to check the incidence angle which is associated with \( \psi_L = \psi_r \). This angle will be the « aerodynamic zero » of our probe and all the further measurements will be referred to this angle.

The raw data from the LabVIEW software aren’t easily processable, so an existing program written in FORTRAN will rearrange the data in order to be able to use them directly in TECPLOT.
During the calibration, we encountered a problem: we could not come down to 30 kPa total pressure with supersonic flows. The inlet pressure of the compressor 2 was below 15 kPa which prompted the automatic control system of the SEG to stop the compressor. We had to choose the lowest total pressures that we could reach, which were:

- $p_0 = 35$ kPa at $Ma = 1.4$
- $p_0 = 40$ kPa at $Ma = 1.6$
- $p_0 = 51$ kPa at $Ma = 1.8$

Nevertheless, it will not be a problem for the future use of the probe. Indeed, once positioned between a stator and a rotor of a turbine, where the flow is supersonic, the total pressure is about 100 kPa.

\[ \text{V.3. Characteristics of the probe in a three-dimensional flow field} \]

As we obtained numerous data (almost 150 plots with TECPLOT) it will be impossible to detail all the results. That's why we will study results for only specific Mach numbers, other results will be shown in the annex.

After importing raw data in TECPLOT, we plotted the evolution of $C_\alpha$ and $p_{0s}$ against $\alpha$ and $\beta$ in order to study the three-dimensional behaviour of the probe. Here are the results:
Subsonic flow:

\[ p_0 = 30 \text{ kPa}: \]

**Figure 12:** Evolution of \( C_\alpha \) against \( \alpha \) and \( \beta \) at \( Ma=0.6 \) and \( p_0=30\text{kPa} \)

**Figure 13:** Evolution of \( p_{0s} \) against \( \alpha \) and \( \beta \) at \( Ma=0.6 \) and \( p_0=30\text{kPa} \)
Figure 14: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=0.6$ and $p_0=100\text{kPa}$

Figure 15: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=0.6$ and $p_0=100\text{kPa}$
Transonic flow:

\[ p_0 = 30 \text{ kPa}: \]

**Figure 16:** Evolution of \( C_{\alpha} \) against \( \alpha \) and \( \beta \) at \( Ma=1.15 \) and \( p_0=30\text{kPa} \)

**Figure 17:** Evolution of \( p_0s \) against \( \alpha \) and \( \beta \) at \( Ma=1.15 \) and \( p_0=30\text{kPa} \)
Figure 18: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $p_0=100\,kPa$

Figure 19: Evolution of $p_0s$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $p_0=100\,kPa$
Supersonic flow:

$p_0 = 40$ kPa:

\[ C_{\alpha} \]

\begin{align*}
\text{Figure 20: Evolution of } C_{\alpha} \text{ against } \alpha \text{ and } \beta \text{ at } Ma=1.6 \text{ and } p_0=40\text{kPa}
\end{align*}

\[ p_0^* \]

\begin{align*}
\text{Figure 21: Evolution of } p_0^* \text{ against } \alpha \text{ and } \beta \text{ at } Ma=1.6 \text{ and } p_0=40\text{kPa}
\end{align*}
Figure 22: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.6$ and $p_0=100kPa$

Figure 23: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.6$ and $p_0=100kPa$
We can notice that for all Mach numbers and all total pressures, the pressure coefficient $C_\alpha$ is independent of the radial angle $\beta$ with the exception of high incidence angles ($|\alpha| > 20^\circ$). This will be useful to determine the incidence angle $\alpha$.

On the other hand, the stagnation pressure, $p_{0s}$, measured by the probe is dependent on both, incidence angle and radial angle.

This behaviour is similar for all Mach numbers and all the Reynolds numbers of the calibration.

**V.4. Incidence angle effects**

**V.4.a. Determination of $\alpha$ from $C_\alpha$**

If we plot the evolution of the pressure coefficient, $C_\alpha$, against the incidence angle $\alpha$, we notice that, for all Mach numbers, we obtain a linear relation. This relation is visualized by a linear fit for incidence angles between $-16^\circ$ and $+16^\circ$. 
Subsonic flow:

Figure 24: Evolution of $C_\alpha$ against $\alpha$ for subsonic flow and $p_0=30\text{kPa}$

Figure 25: Evolution of $C_\alpha$ against $\alpha$ for subsonic flow and $p_0=100\text{kPa}$
Transonic flow:

Figure 26: Evolution of $C_\alpha$ against $\alpha$ for transonic flow and $p_0 = 30$ kPa

Figure 27: Evolution of $C_\alpha$ against $\alpha$ for transonic flow and $p_0 = 100$ kPa
Supersonic flow (only for $p_0 = 100\text{kPa}$):

![Graph](image)

*Figure 28: Evolution of $C_\alpha$ against $\alpha$ for supersonic flow and $p_0 = 100\text{kPa}$*

Once again, these results remain valid for all Mach numbers and all Reynolds numbers.

The linear behaviour of the pressure coefficient allows to write:

$$C_\alpha = a_1 \cdot \alpha + a_0$$

We observed that $a_0$ could be neglected (values around $10^{-2}$ and $10^{-3}$), so the final equation is:

$$C_\alpha = a_1 \cdot \alpha \quad (5)$$
The different $a_1$ coefficients for $-16^\circ < \alpha < +16^\circ$ are listed below:

**Subsonic flow:**

$p_0 = 30\text{kPa}$

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1092</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1040</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1039</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1030</td>
</tr>
<tr>
<td>0.6</td>
<td>0.1026</td>
</tr>
<tr>
<td>0.7</td>
<td>0.1026</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1026</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1027</td>
</tr>
</tbody>
</table>

$p_0 = 100\text{kPa}$

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1027</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1010</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1001</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1015</td>
</tr>
<tr>
<td>0.6</td>
<td>0.1019</td>
</tr>
<tr>
<td>0.7</td>
<td>0.1023</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1025</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1024</td>
</tr>
</tbody>
</table>

**Transonic flow:**

$p_0 = 30\text{kPa}$

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0.1029</td>
</tr>
<tr>
<td>1.00</td>
<td>0.1026</td>
</tr>
<tr>
<td>1.05</td>
<td>0.1021</td>
</tr>
<tr>
<td>1.10</td>
<td>0.1017</td>
</tr>
<tr>
<td>1.15</td>
<td>0.1017</td>
</tr>
<tr>
<td>1.20</td>
<td>0.1017</td>
</tr>
<tr>
<td>1.25</td>
<td>0.1015</td>
</tr>
<tr>
<td>1.30</td>
<td>0.1014</td>
</tr>
</tbody>
</table>
**p₀= 100kPa**

<table>
<thead>
<tr>
<th>Ma</th>
<th>a₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0.1023</td>
</tr>
<tr>
<td>1.00</td>
<td>0.1021</td>
</tr>
<tr>
<td>1.05</td>
<td>0.1019</td>
</tr>
<tr>
<td>1.10</td>
<td>0.1019</td>
</tr>
<tr>
<td>1.15</td>
<td>0.1016</td>
</tr>
<tr>
<td>1.20</td>
<td>0.1015</td>
</tr>
<tr>
<td>1.25</td>
<td>0.1015</td>
</tr>
<tr>
<td>1.30</td>
<td>0.1016</td>
</tr>
</tbody>
</table>

**Supersonic flow:**

Only for p₀ = 100kPa because of the limitations of the compressor.

<table>
<thead>
<tr>
<th>Ma</th>
<th>a₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.1013</td>
</tr>
<tr>
<td>1.6</td>
<td>0.1010</td>
</tr>
<tr>
<td>1.8</td>
<td>0.09812</td>
</tr>
</tbody>
</table>

Thus, we get a mean value of the coefficient a₁ = 0.10215 ± 0.54 E-3

We can now easily determine the incidence of the air flow on the probe by calculating the pressure coefficient Cᵢ thanks to equation (1) and to obtain α from the equation (5). Besides, this method will allows us to find the « aerodynamic zero » very quickly.
V.4.b. Study of the probe at zero incidence

The figures 29, 30 and 31 show the evolution of the coefficient $C_{Ma}$ against Mach number for the total pressure $p_0 = 100$ kPa with $\alpha = 0$ and $\beta = 0$.

**Figure 29:** Evolution of $C_{Ma}$ against $Ma$ for subsonic flow and $p_0 = 100$ kPa

**Figure 30:** Evolution of $C_{Ma}$ against $Ma$ for transonic flow and $p_0 = 100$ kPa
Once the probe is calibrated, we are able, thanks to these plots, to find the velocity of the flow from the coefficient $C_{Ma}$. Subsonic and supersonic speeds allow finding $Ma$ from $C_{Ma}$ easily. But for the transonic speeds, we can see on the Figure 30 that the curve has locally a negative gradient. This phenomenon is due to a shock wave in front of the probe which causes subsonic conditions at the pressure tappings; this wave is getting closer to the probe when the Mach number increases, but the conditions at the pressure tappings remain subsonic. As a result, there is a lack of information at transonic speeds.

This transonic « zero gradient » phenomenon affects the determination of the Mach number and creates errors because a small variation of $C_{Ma}$ covers a wide range of Mach numbers.
Here is a schlieren picture of the probe during the calibration process at Mach 1.4. Schlieren pictures allow highlighting areas where the gas density suddenly changes thanks to the deviation of the light. The larger the density gradient, the more the light is deviated. On this picture we can see the shock wave in front of the probe and the separation of the flow at the back of the probe.

Figure 32: Schlieren picture of the probe
We can observe this problem on the *figures 33* and *34* which represent now the evolution of the Mach number against $C_{Ma}$.

*Figure 33: Evolution of $Ma$ against $C_{Ma}$ for $p_0$=30 to 51kPa*

*Figure 34: Evolution of $Ma$ against $C_{Ma}$ for $p_0$ = 100 kPa*

We will talk about these fits further in this report.
V.4.c. The transonic problem for the determination of the Mach number

To get rid of this problem, we will take advantage of the fourth pressure tapping at the back of the probe measuring the pressure $p_{bu}$. This tube is located in an area where the flow velocity is still subsonic (downstream of the shock wave and in the wake of the probe), so the probe will be far more sensitive to a pressure variation. If we consider equation (3), and if we plot the evolution of the Mach number against $C_{Mab}$, the « zero gradient » disappears for transonic flows, and the determination of the Mach number is more precise.

**Figure 35:** Evolution of $Ma$ against $C_{Mab}$ for $p_0 = 30$ to $51$ kPa

**Figure 36:** Evolution of $Ma$ against $C_{Mab}$ for $p_0 = 100$ kPa
These curves were fitted by the following functions:

\[ Ma = b_0 + b_1 \times \frac{(C_{Ma} - 0.527)}{|C_{Ma} - 0.527|^{0.5}} + b_2 \times \frac{(C_{Ma} - 0.527)}{|C_{Ma} - 0.527|^{0.7}} \]

Where:

<table>
<thead>
<tr>
<th>Probe RRT6</th>
<th>30 à 51 kPa</th>
<th>100 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient (b_0)</td>
<td>1.124711223017</td>
<td>1.15510659458</td>
</tr>
<tr>
<td>Coefficient (b_1)</td>
<td>1.443185445788</td>
<td>1.4861884779</td>
</tr>
<tr>
<td>Coefficient (b_2)</td>
<td>0.0222934922878</td>
<td>0.040941033949</td>
</tr>
</tbody>
</table>

Polynomial function of \(C_{Mab}\):

\[ Ma = g_0 + g_1 \times (C_{Mab}) + g_2 \times (C_{Mab})^2 + g_3 \times (C_{Mab})^3 + g_4 \times (C_{Mab})^4 + g_5 \times (C_{Mab})^5 + g_6 \times (C_{Mab})^6 + g_7 \times (C_{Mab})^7 \]

where:

<table>
<thead>
<tr>
<th>Probe RRT6</th>
<th>30 à 51 kPa</th>
<th>100 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient (g_0)</td>
<td>-0.3225466461649</td>
<td>-0.373381399514</td>
</tr>
<tr>
<td>Coefficient (g_1)</td>
<td>3.869297067202</td>
<td>3.940303837422</td>
</tr>
<tr>
<td>Coefficient (g_2)</td>
<td>-10.47696194858</td>
<td>-9.37331497695</td>
</tr>
<tr>
<td>Coefficient (g_3)</td>
<td>16.27384862194</td>
<td>12.08893627133</td>
</tr>
<tr>
<td>Coefficient (g_4)</td>
<td>-12.33385166483</td>
<td>-6.504600173734</td>
</tr>
<tr>
<td>Coefficient (g_5)</td>
<td>4.418654255879</td>
<td>0.5919247647549</td>
</tr>
<tr>
<td>Coefficient (g_6)</td>
<td>-0.5982357629218</td>
<td>0.5915794394819</td>
</tr>
<tr>
<td>Coefficient (g_7)</td>
<td>---------------</td>
<td>-0.1410152394165</td>
</tr>
</tbody>
</table>

Thanks to these coefficients and these fit functions, we are now able to get with a quite good precision the flow velocity from the pressures given by the probe.
V.5. **Error sensitivity**

We wanted to know how an error on the stagnation pressure $p_{0s}$ could affect our calculation of the Mach number. So we added to our values of $p_{0s}$ an error of 0.1%, which spread in our calculations: new $p_{0s}$, new $C_{Ma}$ coefficient which, once injected in our fit function, gives a new Mach number. The final error is described as:

\[
\text{error} = \text{old Mach number} - \text{new Mach number}
\]

Results obtained for $p_{0} = 30$ to $51$ kPa:

<table>
<thead>
<tr>
<th>Ma</th>
<th>Error computed from $C_{Ma}$</th>
<th>Error computed from $C_{Mab}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.006279612</td>
<td>0.004475113</td>
</tr>
<tr>
<td>0.3</td>
<td>0.01260087</td>
<td>0.009600603</td>
</tr>
<tr>
<td>0.4</td>
<td>0.018604019</td>
<td>0.010916716</td>
</tr>
<tr>
<td>0.5</td>
<td>0.019198992</td>
<td>0.011753944</td>
</tr>
<tr>
<td>0.6</td>
<td>0.016031805</td>
<td>0.003648256</td>
</tr>
<tr>
<td>0.7</td>
<td>0.006758402</td>
<td>0.02462669</td>
</tr>
<tr>
<td>0.8</td>
<td>0.004710999</td>
<td>0.002833135</td>
</tr>
<tr>
<td>0.9</td>
<td>0.012600779</td>
<td>0.012657851</td>
</tr>
<tr>
<td>0.95</td>
<td>0.016711748</td>
<td>0.006116905</td>
</tr>
<tr>
<td>1</td>
<td>0.007109954</td>
<td>0.007990864</td>
</tr>
<tr>
<td>1.05</td>
<td>0.013231746</td>
<td>0.005839854</td>
</tr>
<tr>
<td>1.1</td>
<td>0.048200076</td>
<td>0.021081748</td>
</tr>
<tr>
<td>1.15</td>
<td>0.004920327</td>
<td>0.006969407</td>
</tr>
<tr>
<td>1.2</td>
<td>0.000413756</td>
<td>0.008051434</td>
</tr>
<tr>
<td>1.25</td>
<td>0.021793126</td>
<td>0.005304657</td>
</tr>
<tr>
<td>1.3</td>
<td>0.018508777</td>
<td>0.000219402</td>
</tr>
<tr>
<td>1.4</td>
<td>0.029352356</td>
<td>0.011559107</td>
</tr>
<tr>
<td>1.6</td>
<td>0.045011247</td>
<td>0.01304997</td>
</tr>
<tr>
<td>1.8</td>
<td>0.030635246</td>
<td>0.006049075</td>
</tr>
</tbody>
</table>
This was plotted against the Mach number:

When the Mach number is computed from $C_{Ma}$ (blue curve), we can see that the error on $p_0$s begins to have a significant impact on the calculation of the Mach number for transonic and supersonic flows. On the other hand, when the Mach number is computed from $C_{Mab}$ (pink curve), an error is still present but less important (between 0.0002 and 0.025).
Same procedure, for $p_0 = 100$ kPa:

<table>
<thead>
<tr>
<th>Ma</th>
<th>Error computed from $C_{Ma}$</th>
<th>Error computed from $C_{Mab}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.003455555</td>
<td>0.006042992</td>
</tr>
<tr>
<td>0.3</td>
<td>0.011615299</td>
<td>0.009367573</td>
</tr>
<tr>
<td>0.4</td>
<td>0.014035767</td>
<td>0.005198751</td>
</tr>
<tr>
<td>0.5</td>
<td>0.01483303</td>
<td>0.013579558</td>
</tr>
<tr>
<td>0.6</td>
<td>0.010583185</td>
<td>0.004838935</td>
</tr>
<tr>
<td>0.7</td>
<td>0.003831044</td>
<td>0.018673784</td>
</tr>
<tr>
<td>0.8</td>
<td>0.001984011</td>
<td>0.010519003</td>
</tr>
<tr>
<td>0.9</td>
<td>0.007903073</td>
<td>0.00845876</td>
</tr>
<tr>
<td>0.95</td>
<td>0.016304456</td>
<td>0.003397662</td>
</tr>
<tr>
<td>1</td>
<td>0.020683636</td>
<td>0.008144105</td>
</tr>
<tr>
<td>1.05</td>
<td>0.041661297</td>
<td>0.006994561</td>
</tr>
<tr>
<td>1.1</td>
<td>0.002013759</td>
<td>0.003790201</td>
</tr>
<tr>
<td>1.15</td>
<td>0.07850483</td>
<td>0.018174007</td>
</tr>
<tr>
<td>1.2</td>
<td>0.115074279</td>
<td>0.002895968</td>
</tr>
<tr>
<td>1.25</td>
<td>0.138908574</td>
<td>0.002683272</td>
</tr>
<tr>
<td>1.3</td>
<td>0.046023817</td>
<td>0.011165046</td>
</tr>
<tr>
<td>1.4</td>
<td>0.05066408</td>
<td>0.033129569</td>
</tr>
<tr>
<td>1.6</td>
<td>0.034662282</td>
<td>0.019333817</td>
</tr>
<tr>
<td>1.8</td>
<td>0.008390101</td>
<td>0.008844278</td>
</tr>
</tbody>
</table>

The results were plotted:

![Figure 38: Evolution of the error against $Ma$ for $p_0=100$ kPa](image)
We can see as before that for transonic and supersonic flows the error on $p_{0s}$ seriously affects the determination of the Mach number when it is computed from $C_{Ma}$, whereas the calculation from $C_{Ma}_b$ remains relatively stable.

Once more the shock wave ahead of the probe is responsible for this error sensitivity in transonic and supersonic flows. The fourth pressure tapping works very effective in such cases to reduce the error.

**VI. Conclusion**

All the data acquired thanks to the SEG allowed us to understand the behaviour of our probe. We have defined coefficients useful for the calibration of the probe and fit functions which will give us the Mach number of the flow. We encountered a technical problem which prevented us to maintain a 30kPa total pressure for supersonic flows during the calibration process. Nevertheless, in the future and for supersonic flows, the probe will be used with a 100 kPa total pressure only, a domain which was successfully calibrated.

What this training course highlighted, is the good effectiveness of the fourth tube located at the back of the probe. On one hand this pressure tapping is less sensitive to an error due to a shock, and on the other hand it avoids the transonic « zero gradient » problem and provides a more precise Mach number for transonic speeds. But, as every intrusive measurement system, this probe will always have an impact on the flow and will always modify it, which will bring errors. But the use of non-intrusive optical measurement systems (thanks to lasers) is still too expensive and too complicated compared with the use of this probe. DLR has now a calibrated probe at low cost, which is relatively effective and which can be used for a wide range of flow speeds.
Bibliography:

A New Test Facility for Probe Calibration Offering Independent Variation of Mach number and Reynolds Number.

[2] Kost F.
Calibration of Combined Pressure, Temperature Probes for Application in the Rotating Cascade Tunnel (RGG).

[3] Steinville R.
Calibration of a three-hole cylindrical pressure probe in sub- and supersonic flow
DLR Internal Report IB 225 – 2005 A 01, January 2005

[4] Leost Y.
Eichung einer zylindrischen 4-Lochsonde in Unter- und Überschallströmung
DLR Internal Report IB 225 – 2006 A 08, December 2006

[5] Délery J.
Méthodes de mesures en aérodynamique
ONERA

Tecplot 9.0 User’s Manual
2001
List of figures:

Figure 1: DLR site of Göttingen  2
Figure 2: SEG wind tunnel  4
Figure 3: Diagram of the SEG circuit  5
Figure 4: Test chamber of SEG  5
Figure 5: Range of use of SEG  6
Figure 6: Screenshot of the wind tunnel control software (SIMATIC)  7
Figure 7: Screenshot of the probe control software (LabVIEW)  8
Figure 8: Plan of the RRT6 probe  10
Figure 9: Photograph of the probe  11
Figure 10: Angles of rotation of the probe  15
Figure 11: Example of a data files  16

Subsonic

Figure 12: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=0$, $6$ and $p_0=30kPa$  18
Figure 13: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=0.6$ and $p_0=30kPa$  18
Figure 14: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=0.6$ and $p_0=100kPa$  19
Figure 15: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=0.6$ and $p_0=100kPa$  19

Transonic

Figure 16: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $p_0=30kPa$  20
Figure 17: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $p_0=30kPa$  20
Figure 18: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $p_0=100kPa$  21
Figure 19: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $p_0=100kPa$  21

Supersonic

Figure 20: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.6$ and $p_0=40kPa$  22
Figure 21: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.6$ and $p_0=40kPa$  22
Figure 22: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.6$ and $p_0=100kPa$  23
Figure 23: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.6$ and $p_0=100kPa$  23
Figure 24: Evolution of $C_\alpha$ against $\alpha$ for a subsonic flow and for $p_0=30kPa$  25
Figure 25: Evolution of $C_\alpha$ against $\alpha$ for a subsonic flow and for $p_0=100kPa$  25
Figure 26: Evolution of $C_\alpha$ against $\alpha$ for a transonic flow and for $p_0=30kPa$  26
Figure 27: Evolution of $C_\alpha$ against $\alpha$ for a transonic flow and for $p_0=100kPa$  26
Figure 28: Evolution of $C_\alpha$ against $\alpha$ for a supersonic flow and $p_0=100kPa$  27
Figure 29: Evolution of $C_{Ma}$ against $Ma$ for a subsonic flow and for $p_0=100kPa$  30
Figure 30: Evolution of $C_{Ma}$ against $Ma$ for a transonic flow and for $p_0=100kPa$  30
Figure 31: Evolution of $C_{Ma}$ against $Ma$ for a supersonic flow and for $p_0=100kPa$  31

Subsonic $Ma=0.3-0.6-0.9$ $p_0=30kPa$

Figure 32: Schlieren picture of the probe  32
Figure 33: Evolution of $Ma$ against $C_{Ma}$ for $p_0=30$ to $51kPa$  33
Figure 34: Evolution of $Ma$ against $C_{Ma}$ for $p_0=100kPa$  33
Figure 35: Evolution of $Ma$ against $C_{Ma}$ for $p_0=30$ to $51kPa$  34
Figure 36: Evolution of $Ma$ against $C_{Ma}$ for $p_0=100kPa$  34
Figure 37: Evolution of the error against $Ma$ for $p_0=30$ to $51kPa$  37
Figure 38: Evolution of the error against $Ma$ for $p_0=100kPa$  38

Subsonic $Ma=0.3-0.6-0.9$ $p_0=30kPa$

Figure 39: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  45
Figure 40: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  46
Figure 41: Evolution of $C_{Ma}$ against $\beta$  47
Figure 42: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  48
Figure 43: Evolution of $C_{Ma}$ against $\beta$  49
Figure 44: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  50
Figure 45: Evolution of $p_{0s}$ against $\beta$  51
Figure 46: Evolution of $C_\alpha$ against $\alpha$  51
Figure 47: Evolution of $C_{Ma}$ against $Ma$  52
Figure 48: Evolution of $C_{Ma}$ against $Ma$  52
Subsonic $Ma=0.3-0.6-0.9 \quad p_0=100\text{kPa}$

Figure 49: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  \hspace{1cm} 53
Figure 50: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  \hspace{1cm} 54
Figure 51: Evolution of $C_{Ma}$ against $\beta$  \hspace{1cm} 55
Figure 52: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  \hspace{1cm} 56
Figure 53: Evolution of $C_{Ma b}$ against $\beta$  \hspace{1cm} 57
Figure 54: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  \hspace{1cm} 58
Figure 55: Evolution of $p_{0s}$ against $\beta$  \hspace{1cm} 59
Figure 56: Evolution of $C_\alpha$ against $\alpha$  \hspace{1cm} 60
Figure 57: Evolution of $C_{Ma}$ against $Ma$  \hspace{1cm} 60
Figure 58: Evolution of $C_{Ma b}$ against $Ma$  \hspace{1cm} 60

Transonic $Ma=1-1.15-1.3 \quad p_0=30\text{kPa}$

Figure 59: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  \hspace{1cm} 61
Figure 60: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  \hspace{1cm} 62
Figure 61: Evolution of $C_{Ma}$ against $\beta$  \hspace{1cm} 63
Figure 62: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  \hspace{1cm} 64
Figure 63: Evolution of $C_{Ma b}$ against $\beta$  \hspace{1cm} 65
Figure 64: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  \hspace{1cm} 66
Figure 65: Evolution of $p_{0s}$ against $\beta$  \hspace{1cm} 67
Figure 66: Evolution of $C_\alpha$ against $\alpha$  \hspace{1cm} 68
Figure 67: Evolution of $C_{Ma}$ against $Ma$  \hspace{1cm} 68
Figure 68: Evolution of $C_{Ma b}$ against $Ma$  \hspace{1cm} 68

Transonic $Ma=1-1.15-1.3 \quad p_0=100\text{kPa}$

Figure 69: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  \hspace{1cm} 69
Figure 70: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  \hspace{1cm} 70
Figure 71: Evolution of $C_{Ma}$ against $\beta$  \hspace{1cm} 71
Figure 72: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  \hspace{1cm} 72
Figure 73: Evolution of $C_{Ma b}$ against $\beta$  \hspace{1cm} 73
Figure 74: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  \hspace{1cm} 74
Figure 75: Evolution of $p_{0s}$ against $\beta$  \hspace{1cm} 75
Figure 76: Evolution of $C_\alpha$ against $\alpha$  \hspace{1cm} 76
Figure 77: Evolution of $C_{Ma}$ against $Ma$  \hspace{1cm} 76
Figure 78: Evolution of $C_{Ma b}$ against $Ma$  \hspace{1cm} 76

Supersonic $Ma=1.4 \quad p_0=35\text{kPa}$

Figure 79: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  \hspace{1cm} 77
Figure 80: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  \hspace{1cm} 77
Figure 81: Evolution of $C_{Ma}$ against $\beta$  \hspace{1cm} 77
Figure 82: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  \hspace{1cm} 78
Figure 83: Evolution of $C_{Ma b}$ against $\beta$  \hspace{1cm} 78
Figure 84: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  \hspace{1cm} 78
Figure 85: Evolution of $p_{0s}$ against $\beta$  \hspace{1cm} 78

Supersonic $Ma=1.6 \quad p_0=40\text{kPa}$

Figure 86: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  \hspace{1cm} 79
Figure 87: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  \hspace{1cm} 79
Figure 88: Evolution of $C_{Ma}$ against $\beta$  \hspace{1cm} 79
Figure 89: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  \hspace{1cm} 80
Figure 90: Evolution of $C_{Ma b}$ against $\beta$  \hspace{1cm} 80
Figure 91: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  \hspace{1cm} 80
Figure 92: Evolution of $p_{0s}$ against $\beta$  \hspace{1cm} 80
\textbf{Supersonic $Ma=1.8$ $p_0=51kPa$}

Figure 93: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  
Figure 94: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  
Figure 95: Evolution of $C_\alpha$ against $\alpha$  
Figure 96: Evolution of $C_{Ma}$ against $\beta$  
Figure 97: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  
Figure 98: Evolution of $C_{Ma b}$ against $\beta$  
Figure 99: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  
Figure 100: Evolution of $p_{0s}$ against $\beta$  

\textbf{Supersonic $Ma=1.4-1.6-1.8$ $p_0=100kPa$}

Figure 101: Evolution of $C_\alpha$ against $\alpha$ and $\beta$  
Figure 102: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$  
Figure 103: Evolution of $C_{Ma}$ against $\beta$  
Figure 104: Evolution of $C_{Ma b}$ against $\alpha$ and $\beta$  
Figure 105: Evolution of $C_{Ma b}$ against $\beta$  
Figure 106: Evolution of $p_{0s}$ against $\alpha$ and $\beta$  
Figure 107: Evolution of $p_{0s}$ against $\beta$  
Figure 108: Evolution of $C_\alpha$ against $\alpha$  
Figure 109: Evolution of $C_{Ma}$ against $Ma$  
Figure 110: Evolution of $C_{Ma b}$ against $Ma$
Appendix
Subsonic, \( p_0 = 30 \text{kPa} \)

**Figure 39: Evolution of \( C_\alpha \) against \( \alpha \) and \( \beta \) at \( Ma=0.3 \), \( 0.6 \), \( 0.9 \)**
Figure 40: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=0.3$

0.6

0.9
Figure 41: Evolution of $C_{Ma}$ against $\beta$ at $Ma=0.3$, 0.6, 0.9
Figure 42: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=0.3$, $0.6$, $0.9$.
Figure 43: Evolution of \( C_{\text{Mab}} \) against \( \beta \) at \( Ma=0.3 \ 0.6 \ 0.9 \)}
Figure 44: Evolution of $p_{os}$ against $\alpha$ and $\beta$ at Ma=0.3

0.6

0.9
Figure 45: Evolution of $p_0s$ against $\beta$ at $Ma=0.3$, $0.6$, and $0.9$. 
Figure 46: Evolution of $C_\alpha$ against $\alpha$

Figure 47: Evolution of $C_{Ma}$ against $Ma$

Figure 48: Evolution of $C_{Mab}$ against $Ma$
Subsonic, $p_0 = 100$ kPa

Figure 49: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=0.3$

$0.6$

$0.9$
Figure 50: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at Ma=0.3, 0.6, 0.9.
Figure 51: Evolution of $C_{Ma}$ against $\beta$ at $Ma=0.3$  
$0.6$  
$0.9$
Figure 52: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at Ma=0.3, 0.6, 0.9
Figure 53: Evolution of $C_{Mab}$ against $\beta$ at $Ma=0.3$  
0.6  
0.9
Figure 54: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=0.3$, $0.6$, $0.9$
Figure 55: Evolution of $p_0 s$ against $\beta$ at $Ma=0.3$, $0.6$, $0.9$
Figure 56: Evolution of $C_\alpha$ against $\alpha$

Figure 57: Evolution of $C_{Ma}$ against $Ma$

Figure 58: Evolution of $C_{Ma_b}$ against $Ma$
Figure 59: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $Ma=1.3$. 

Transonic, $p_0=30$ kPa
Figure 60: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=1.15$

1.15

1.3
Figure 61: Evolution of $C_{Ma}$ against $\beta$ at $Ma=1.15$ and $Ma=1.3$.
Figure 62: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=1.15$ and $Ma=1.3$. 
Figure 63: Evolution of $C_{Mab}$ against $\beta$ at $Ma=1$

1.15
1.3
Figure 64: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.15$, $1.3$
Figure 65: Evolution of $p_{0s}$ against $\beta$ at $Ma=1$

1.15

1.3
Figure 66: Evolution of $C_\alpha$ against $\alpha$

Figure 67: Evolution of $C_{Ma}$ against $Ma$

Figure 68: Evolution of $C_{Mab}$ against $Ma$
Transonic, $p_0 = 100\text{kPa}$

Figure 69: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.15$

$Ma=1.15$

$Ma=1.3$
Figure 70: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=1$

1.15

1.3
Figure 71: Evolution of $C_{Ma}$ against $\beta$ at $Ma=1$

- $Ma=1$
- $Ma=1.15$
- $Ma=1.3$
Figure 72: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=1$.
Figure 73: Evolution of $C_{Ma b}$ against $\beta$ at $Ma=1$

$1.15$

$1.3$
Figure 74: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1$

1.15

1.3
Figure 75: Evolution of $p_0$s against $\beta$ at $Ma=1$

- 75 -
Figure 76: Evolution of $C_\alpha$ against $\alpha$

Figure 77: Evolution of $C_{Ma}$ against $Ma$

Figure 78: Evolution of $C_{Mab}$ against $Ma$
Supersonic, $p_0=35\text{kPa}$

**Figure 79:** Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.4$

**Figure 80:** Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=1.4$

**Figure 81:** Evolution of $C_{Ma}$ against $\beta$ at $Ma=1.4$
Figure 82: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=1.4$

Figure 83: Evolution of $C_{Mab}$ against $\beta$ at $Ma=1.4$

Figure 84: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.4$

Figure 85: Evolution of $p_{0s}$ against $\beta$ at $Ma=1.4$
Supersonic, $p_0 = 40$ kPa

**Figure 86:** Evolution of $C_{\alpha}$ against $\alpha$ and $\beta$ at $Ma=1.6$

**Figure 87:** Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=1.6$

**Figure 88:** Evolution of $C_{Ma}$ against $\beta$ at $Ma=1.6$
Figure 89: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=1.6$

Figure 90: Evolution of $C_{Mab}$ against $\beta$ at $Ma=1.6$

Figure 91: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.6$

Figure 92: Evolution of $p_{0s}$ against $\beta$ at $Ma=1.6$
Supersonic, $p_0 = 51\text{kPa}$

Figure 93: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.8$

Figure 94: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=1.8$

Figure 95: Evolution of $C_\alpha$ against $\alpha$ at $Ma=1.8$

Figure 96: Evolution of $C_{Ma}$ against $\beta$ at $Ma=1.8$
Figure 97: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=1.8$

Figure 98: Evolution of $C_{Mab}$ against $\beta$ at $Ma=1.8$

Figure 99: Evolution of $p_{0s}$ against $\alpha$ and $\beta$ at $Ma=1.8$

Figure 100: Evolution of $p_{0s}$ against $\beta$ at $Ma=1.8$
Supersonic, $p_0 = 100\text{kPa}$

**Figure 101**: Evolution of $C_\alpha$ against $\alpha$ and $\beta$ at $Ma=1.4$

1.6
1.8
Figure 102: Evolution of $C_{Ma}$ against $\alpha$ and $\beta$ at $Ma=1.4$

1.6

1.8
Figure 103: Evolution of $C_{Ma}$ against $\beta$ at $Ma=1.4$, $1.6$, $1.8$.
Figure 104: Evolution of $C_{Mab}$ against $\alpha$ and $\beta$ at $Ma=1.4$, $Ma=1.6$, $Ma=1.8$
Figure 105: Evolution of $C_{Mab}$ against $\beta$ at $Ma=1.4$, 1.6, 1.8
Figure 106: Evolution of $p_{05}$ against $\alpha$ and $\beta$ at $Ma=1.4$

1.6

1.8
Figure 107: Evolution of $p_{0s}$ against $\beta$ at $Ma=1.4$

1.6

1.8
Figure 108: Evolution of $C_\alpha$ against $\alpha$

Figure 109: Evolution of $C_{Ma}$ against $Ma$

Figure 110: Evolution of $C_{Ma_b}$ against $Ma$