Abstract

The purpose of this paper is to present a possible approach to include capacity constraints in discrete choice models without at least some capacity-related variables missing, like e.g. the price of a commodity. Airport choice models often do not contain air fares because of measurement difficulties as air fares are often not included in passenger surveys and thus essential information regarding ticket price is lost to the researcher. Since air fares vary more across ticket categories at a given airport than a ticket category varies across different airports, air fare related information cannot be reconstructed in many cases by the researcher. However, capacity constraints are becoming increasingly more important and thus including capacity constraints in airport choice models is indicated in some cases.

The model approach is based on individual utility maximisation and thus fits into the discrete choice framework. Furthermore, nonlinear programming techniques are employed to find a feasible solution regarding capacity constraints. Thereby, detailed statements on how limited airport capacity changes individual air traveller behaviour with regard to airport choice are possible. The paper concludes with a simple example to demonstrate the methodology and show the impact of limited airport capacity on airport choice of air travellers.

Keywords: Airport and access mode choice model, Capacity constraints, Discrete choice model, Nonlinear Programming
1. Introduction

Capacity constraints at airports are becoming increasingly more important in recent years. They include e.g. limited runway and terminal capacity, but also restrictions like night curfews, noise & emission budgets or noise & emission limits. In many cases, they aim at reducing the negative effects of air transport on the population surrounding the airport. However, from the point of view of the airport and the air traveller, these constraints reduce the available capacity to handle passenger demand and thus it is necessary to consider capacity constraints for forecasting future airport and access mode choice.

Air fares and capacity constraints are closely interrelated at any airport: In the long run, less available capacity to handle air transport demand at a given airport induces higher air fares until capacity constraints are met. Airport choice models including exact ticket prices allow for the quantification of the value of capacity expansion actions from the point of view of the air traveller (see e.g. Wei 2008).

But, at least over a short time horizon, air fares may not fully reflect the capacity situation at a given airport. Furthermore, some airport choice models (see e.g. Gelhausen 2007a, Innes and Doucet 1990, Moreno and Muller 2003, Ozoka and Ashford 1988, Windle and Dresner 1995) do not include ticket prices as an explanatory variable or employ proxy variables instead, either because exact differences in air ticket prices between different airports are less important for airport choice in an unconstrained airport environment, or they are not available to the researcher due to the survey design. Since air fares vary more across ticket categories at the same airport than a ticket category varies across different airports (Moreno and Muller 2003, p. 19), ticket price related information usually cannot be reconstructed fully by the researcher if it is not already included in the survey. Furthermore, for long-term aggregate airport choice forecasting purposes, it is difficult for the researcher to determine which tickets on which relations increase in price how much in the future, so that capacity constraints are met.

The first choice of an air traveller regarding the departure airport may not necessarily be met in a capacity constraint airport environment. However, this is a major
assumption of airport choice models based on discrete choice theory. Therefore, a possible approach to model airport choice in a constrained airport environment without referring to exact ticket prices for each relation and ticket type is described in this paper. The model is demonstrated by means of a simple yet illustrative example to show the practical consequences of capacity constraints on airport choice.

Figure 1: Impact of capacity constraints on airport choice (Gelhausen 2007b)

Figure 1 illustrates three possible consequences of capacity constraints at airports:

- If travel disutility is high from the point of view of the air traveller and capacity expansion is possible, airport capacity will be enlarged very likely. One example in Germany is the airport Frankfurt Hahn.
- On the other hand, if travel disutility is low and capacity expansion is not possible, the air travel demand surplus will most likely be served by neighbouring airports. One example in Germany is the airport of Düsseldorf.
- However, if both travel disutility is high and capacity expansion is not possible, demand is most likely lost. One example in Germany is the airport Hof Plauen.

Germany has a rather dense network of airports, so the focus of the analysis lies on the second case, where capacity exceeding air travel demand is served by neighbouring airports. Every few years the German Air Traveller Survey is conducted at major German airports. In 2003, more than 200 000 air travellers were interviewed at 19 international airports (e.g. Frankfurt/Main and Munich) and five regional airports (e.g. Frankfurt Hahn). The survey reveals that about 67% choose the nearest airport
for departure; however, so-called spatial planning regions are served by at least three airports, whereas the maximum number is 14. On average, a spatial planning region is served by eight airports (Wilken et al. 2007, p. 172). Therefore, although two thirds of the air travellers choose the nearest airport for departure, there is a considerable degree of competition among airports.

2. Modelling capacity constraints in airport choice

2.1 Methodological background

The methodological basis of airport and access mode choice analysis in this paper is given by the concept of discrete choice theory. The central building block in analysing choice behaviour is the assumption of individual utility maximisation. Utility represents an abstract measure of the subjective attractiveness of an alternative computed by a function of the alternative attributes of each alternative, like e.g. access cost, access time and supply of non-stop and low-cost flights to the chosen destination in the case of airport and access mode choice. In many cases this function is a weighted sum of the alternative attributes, with the weights depending on subjective preferences of the decision maker, i.e. here the air traveller. The decision maker is assumed to choose the one with the highest utility, but from an external point of view, this individual utility maximisation process is not fully measurable and thus represents a random variable. Therefore, from an external point of view, the utility function is decomposable into a deterministic component composed of the aforementioned decision-relevant alternative attributes and an additive stochastic component with expectation zero and a given variance and stochastic distribution. As a result, only evidence in form of choice probabilities relating to the alternative with the highest utility can be given. However, summed up over homogenous market segments, these choice probabilities equal market shares by alternative and market segment. Figure 2 illustrates the idea of discrete choice models (Gelhausen et al. 2008). For a detailed introduction into discrete choice models see e.g. Ben-Akiva and Lerman (1985).
Traveller: „Which alternative is the best for me?“

Evaluation of alternatives by means of utility

Lack of observability, measurement errors, …

Forecaster: „Which alternative is most likely the best for him?“

Choice probabilities

Summing up over homogenous populations

Market segment specific market shares of all alternatives

Figure 2: Concept of discrete choice models (Gelhausen et al. 2008)

Figure 3 displays the choice probability \( P_i \) of alternative \( i \) subject to the utility difference between itself and the next best alternative \( j \), which is denoted as \( V_i - \max(V_j) \), and the variance \( \sigma \) of the utility function. The choice probability of alternative \( i \) rises with increasing utility difference to the next best alternative and vice versa. The steepness of the choice probability curve depends on the variance of the stochastic component of the utility function. Choice probabilities tend to an equal distribution with increasing variance of the stochastic component of the utility functions, whereas they tend to be more distinct with decreasing variance. In the case of an infinite variance the choice probability \( P_i \) in figure 3 is represented by a straight line parallel to the abscissa, whereas a step function describes \( P_i \) in the case of no variance.

Figure 3: Choice probabilities in the logit-model (Maier and Weiss 1990, p. 140)
The scale parameter $\mu$ of the logit-model and the variance of the stochastic component of the utility function are inversely related, i.e. a high variance equals a small scale parameter (Ben-Akiva and Lerman 1985, p. 104f).

2.2 Discrete choice and capacity constraints

The principle of individual utility maximisation is employed to allow for capacity constraints within an airport and access mode choice model based on discrete choice analysis. The main idea is to minimise the loss of personal welfare of an air traveller caused by limited airport capacity to handle air travel demand. The central assumption of the model is as follows: The more unequal an air traveller prefers the alternatives in his choice set the greater are his efforts to depart from his favourite airport. This relation is described by the utility differences $(V_i - \max(V_j), i \neq j)$ in figure 3. These efforts include e.g. early booking or paying higher ticket prices.

Therefore, capacity at airports is filled up with air travel demand simultaneously in decreasing order of the utility differences $(V_i - \max(V_j), i \neq j)$ for each air traveller until the capacity limit of a given airport is reached. This individual utility maximisation process is modelled by means of a so-called synthetic price, which takes the same value for all air travellers at a given airport; however, the synthetic price may vary among different airports. In particular, the synthetic price takes the value zero at unconstrained airports. The more air travel demand exceeds available capacity at a given airport, the higher the value of the synthetic price is for this airport. In equilibrium between air travel demand and air travel supply, which is represented by airport capacities, airport attractiveness of constrained airports is artificially reduced by means of the airport-specific synthetic price and thereby capacity exceeding demand is reassigned to airports with free capacity according to individual utility maximisation. Thus, all capacity constraints are met with a minimum loss of personal welfare from the point of view of the individual air traveller. Figure 4 summarises the algorithm.
The synthetic price variable is included in the deterministic component of the utility function and its coefficient is equal for all market segments. The coefficient value and the variable value are not identified in equilibrium between air travel demand and airport capacities, i.e. the value of the variable has to be doubled if the value of the coefficient is halved to achieve the same result. Thus, the coefficient is fixed arbitrarily to a value of minus one. Equation (1) shows the deterministic component of a linear utility function of a logit-model including the synthetic price, which is multiplied by the scale parameter $\mu$:

\[
\mu * V_i^{sp} = \mu * \left( \sum_k b_k * x_{k,i} - x_{sp,i} \right)
\]

with

- $b_k$: Coefficient of attribute $k$ including alternative-specific coefficients
- $x_{k,i}$: Value of attribute $k$ for alternative $i$
- $x_{sp,i}$: Value of the synthetic price for alternative $i$
- $\mu$: Scale parameter

The values of the scale parameters have to be fixed to an arbitrary value, in most cases they take a value of one, to enable identification of the model parameters to estimate. However, differences in the variance of the stochastic component of the utility function across market segments represented by the scale parameter in (1) are thereby included in the coefficient estimates. Therefore, it is not possible to fix the coefficient of the synthetic price to a value of one for all market segments, if the scale
parameter is arbitrarily fixed to the same value for all market segments; otherwise, the dependence between choice probabilities, differences in the variance of the stochastic component of the utility function and the coefficient of the synthetic price variable is neglected. To account for differences in the variance of the stochastic component of the utility function the coefficient of the synthetic price variable has to be divided by the actual scale parameter; however, it is not possible to identify both the model coefficients and the scale parameter without fixing the latter to an arbitrary value. However, as already described earlier, the coefficient of the synthetic price variable can take any value, if there is only one single market segment to model. The synthetic price coefficients are only identifiable relative to each other in the case of more than one market segment since the logit-model is translation invariant. Therefore, the synthetic price coefficient of one market segment has to be fixed arbitrarily with the remaining coefficients set appropriately, taking into account the relative proportions of the market-specific variances. One way to achieve this is to weight the synthetic price variable of each market segment according to length of the coefficient vector excluding the synthetic price multiplied by minus one:

\[
\text{b}_{\text{sp,MSi}} = -\sqrt{\sum_k \left| b_{k,MSi} \right|^2}
\]

- \( b_{k,MSi} \): Coefficient of attribute \( k \) (including alternative-specific coefficients) of market segment \( MSi \)
- \( b_{sp,MSi} \): Synthetic price coefficient for market segment \( MSi \)

Figure 5 illustrates the relationship between choice probability, the value of the scale parameter and the synthetic price variable. A larger scale parameter \( \mu_{MSi} \) results in a larger impact of the synthetic price variable on the choice probability \( P_i \) of alternative \( i \). In the extreme case of the scale parameter \( \mu_{MSi} \) going towards zero the effect of a given value of the synthetic price variable diminishes. However, the effect of a given value of the synthetic price depends as well on the relative attractiveness of alternative \( i \) for any non-zero scale parameter. A higher relative attractiveness leads to a smaller effect of a given value for the synthetic price variable on the choice probability \( P_i \) of alternative \( i \), as the choice probability curve runs flatter the higher \( P_i \) is.
The full model is given by:

\[
\max \sum_{i, MS_i} \left( \sum_{k} b_{k, MS_i} x_{k, i} + b_{sp, MS_i} x_{sp, i} \right) \rightarrow \max
\]

Subject to:

\[
\sum_{OD, MS_i} y_{OD, MS_i} P_{i, OD, MS_i} \leq \text{Capacity}_i \quad \forall i
\]

\[
x_{sp, i} \geq 0
\]

\(y_{OD, MS_i}\): Number of O-D air travellers from market segment MSi

\(P_{i, OD, MS_i}\): Probability of an O-D air traveller from market segment MSi to depart from airport i

\(\text{Capacity}_i\): Maximum capacity of airport i to handle O-D air passengers (e.g. per year)

The objective function (3) causes O-D air travellers to be assigned to airports with a minimum overall loss of welfare due to limited airport capacities. Side condition (4) ensures that capacity restrictions at every airport i are met. The capacity is measured in terms of O-D passengers; however, airport capacity depends on the number and
types of aircrafts arriving and departing. Therefore, a given aircraft mix at an airport has to be transformed into O-D passenger numbers for the model to be applied to real situations. The maximum numbers of arriving, departing and transit passengers of an airport are input to the model. Side condition (5) means that the synthetic price cannot take negative values. Figure 6 illustrates a possible solution procedure.

If the capacity constraint at a given airport is violated, the synthetic price at this airport is just raised enough so that its capacity constraint is met. This procedure is reiterated until all capacity constraints are met or violated sufficiently small.

3. An illustrative example

The impact of capacity constraints on airport choice is illustrated by means of airport choice of air travellers in the Cologne region. Air travel demand in the Cologne region is mainly served by the three airports of Cologne/Bonn, Düsseldorf and Frankfurt/Main; however, Frankfurt/Main mainly serves travel demand from the Cologne region to intercontinental destinations. The model is subdivided into seven market segment-specific sub models according to trip destination and trip purpose. These are domestic, European and intercontinental destinations, whereas trip purpose is subdivided into private and business; however European travel for private reasons is further subdivided into short stay (up to four days) and holiday (five days and longer). The model employed (Gelhausen and Wilken 2006) is airport-specific; therefore Berlin is chosen as an example for a domestic destination, Barcelona as an
example for a European destination and Dallas as an example for an intercontinental destination (see figure 7).

![Figure 7: Airport choice in the Cologne region (Gelhausen 2007b)](image)

Table 1 shows the synthetic price coefficients per market segment for the airport and access mode choice model in Gelhausen and Wilken (2006). The scale parameters of the three level nested logit-model are set to a value of one on the lowest level of the nesting structure to enable model parameter identification. The synthetic price coefficients are computed according to (2). A lower synthetic price coefficient corresponds to an air traveller making greater efforts to depart from his most favourite airport.

<table>
<thead>
<tr>
<th>Market segment</th>
<th>Synthetic price coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRD Private</td>
<td>-190,4206827</td>
</tr>
<tr>
<td>BRD Business</td>
<td>-30,00161228</td>
</tr>
<tr>
<td>EUR Short stay</td>
<td>-216,1687109</td>
</tr>
<tr>
<td>EUR Holiday</td>
<td>-235,8279302</td>
</tr>
<tr>
<td>EUR Business</td>
<td>-10,94517837</td>
</tr>
<tr>
<td>INT Private</td>
<td>-58,95990278</td>
</tr>
<tr>
<td>INT Business</td>
<td>-48,73136221</td>
</tr>
</tbody>
</table>

Table 1: Synthetic price coefficients per market segment

Passengers travelling to domestic (BRD) and European (EUR) destinations are generally more sensitive to capacity constraints than intercontinental (INT)
passengers, i.e. intercontinental air travellers undertake greater efforts to depart from their favourite airport, e.g. because there are fewer attractive options. Business travellers make greater efforts to depart from their favourite airport than passengers travelling for private reasons.

Figure 8: Impact of capacity constraints on airport choice in the Cologne region (Gelhausen 2007)

Figure 8 shows both the base scenario with no capacity constraints as well as the capacity constrained scenario. The scenario is only for illustration purposes and thus fully hypothetical regarding O-D demand and airport capacities: Air travel demand is set to a value of 100 PAX per market segment and capacity is constrained to a value of 100 PAX for Cologne/Bonn and Düsseldorf, whereas Frankfurt/Main can take up to 300 originating PAX. In the base case, air travel demand is served mainly by the airports of Cologne/Bonn, Düsseldorf and Frankfurt/Main, which handle almost 100% of the air travel demand of the Cologne region. There are some other airports, which serve a negligible share of the demand. However, Cologne/Bonn and Düsseldorf have more demand than they potentially can handle, therefore some air travellers have to depart from different airports. There is a huge amount of air travellers choosing Frankfurt/Main instead, which in turn becomes capacity constrained itself. Furthermore, air travel demand handled at some remote airports increases too. Therefore, air travel demand is spread among more airports: Only 96.76% of the air travel demand of the Cologne region is served by the top seven airports in this
example, against what 99.58% of the demand is served by these seven airports in the absence of capacity constraints. Airport choice significantly changes in the light of airport constraints, thus it seems sensible to account for such effects in choice models, especially for aggregate long-term analysis.

<table>
<thead>
<tr>
<th>Market segment</th>
<th>Euro/minute</th>
<th>Frankfurt/Main +64 min</th>
<th>Dortmund +70 min</th>
<th>Niederrhein/Weeze +81 min</th>
<th>Frankfurt Hahn +82 min</th>
<th>Münster/Osnabrück +131 min</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRD Private</td>
<td>0.32</td>
<td>20.48</td>
<td>22.40</td>
<td>25.92</td>
<td>26.24</td>
<td>41.92</td>
</tr>
<tr>
<td>BRD Business</td>
<td>0.75</td>
<td>48.00</td>
<td>52.50</td>
<td>60.75</td>
<td>61.50</td>
<td>98.25</td>
</tr>
<tr>
<td>EUR Short stay</td>
<td>0.31</td>
<td>19.54</td>
<td>21.70</td>
<td>25.11</td>
<td>25.42</td>
<td>40.61</td>
</tr>
<tr>
<td>EUR Holiday</td>
<td>0.49</td>
<td>31.36</td>
<td>34.30</td>
<td>39.69</td>
<td>40.18</td>
<td>64.19</td>
</tr>
<tr>
<td>EUR Business</td>
<td>0.37</td>
<td>23.68</td>
<td>25.90</td>
<td>29.97</td>
<td>30.34</td>
<td>48.47</td>
</tr>
<tr>
<td>INT Private</td>
<td>0.39</td>
<td>24.96</td>
<td>27.30</td>
<td>31.59</td>
<td>31.98</td>
<td>51.09</td>
</tr>
<tr>
<td>INT Business</td>
<td>0.57</td>
<td>36.48</td>
<td>39.90</td>
<td>46.17</td>
<td>46.74</td>
<td>74.67</td>
</tr>
</tbody>
</table>

Value of travel time in Euro

Further negative effects:
- Increase of travel cost, flight plan, etc.

Figure 9: Reduction of personal welfare in Euro due to an increase in travel time (Gelhausen 2007)

There are four major consequences of capacity constraints at airports:

- First, capacity constraints lead to a reduction of personal welfare from the point of view of the air traveller, e.g. because of increased travel time and travel cost (see figure 9 for the value of travel time).
- Capacity constraints may lead to spill-over effects, thus leading to capacity constraints at further airports.
- Thereby, air travel demand is distributed among more airports, benefiting remote and less attractive airports.
- Thus, competition among airports is reduced, leading to higher prices and less innovation, which in turn reduces the personal welfare of the air traveller.

4. Summary and conclusion

This paper shows a possible approach to allow for capacity constraints in airport choice models based on discrete choice theory. The chosen approach combines
discrete choice with nonlinear programming techniques to implement capacity constraints.

Limited capacity to handle air travel demand at an airport may have a significant effect on the choice behaviour of air travellers and thus change airport choice, especially in a decentralised airport environment like Germany. Figure 10 summarises the main consequences of limited capacity to handle air travel demand at airports from the point of view of the air traveller.

**Figure 10: Consequences of capacity constraints at airports on the choice behaviour of air travellers** (Gelhausen 2007)

Limited capacity at some airports leads to demand being distributed among more airports unlike the case of sufficient capacity at every airport. Thereby, competition is decreased at congested airports; however, small remote airports are the beneficiaries, as their market share increases. Spill-over effects may lead to capacity constraints at even more airports, thus intensifying the effects of limited airport capacity to handle air travel demand. From the point of view of the air travellers, personal welfare is reduced, as e.g. travel time and travel cost increase.

Airport capacity to handle O-D demand serves as input for the presented model; however, it may be extended to allow for an endogenous assignment of airport
capacity to arriving, departing and transit passengers as well as the aircraft mix at a given airport.

References


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