Degree of Polarization:

Theory and Applications for Weather Radars

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Overview

Fully polarimetric POLDIRAD data are used to evaluate,
by means of unitary transformations,
the degree of polarization corresponding to different transmit states

The work is aimed at answering the following question:

Can the degree of polarization add value to dual-polarization weather radar measurements ?

p_c or p_{45} are available to dual-polarization radars at hybrid

p_h is available to dual-polarization radars transmitting horizontal polarization

DEGREE OF POLARIZATION

$$E(t) = \begin{bmatrix} E_1(t) \\ E_2(t) \end{bmatrix} = \begin{bmatrix} |E_1(t)| e^{i\varphi_1(t)} \\ |E_2(t)| e^{i\varphi_2(t)} \end{bmatrix}$$

Jones vector for a partially polarized wave (one sample)

$$Cov(E) = J = \langle E \cdot E^+ \rangle = \begin{bmatrix} \langle |E_1(t)|^2 \rangle & \langle E_1(t) E_2(t)^* \rangle \\ \langle E_1(t)^* E_2(t) \rangle & \langle |E_2(t)|^2 \rangle \end{bmatrix}$$

Wolf's Coherency Matrix (N samples)

$$U^+ J U = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \qquad p = \sqrt{1 - \frac{4 \cdot \det(J)}{(trace(J))^2}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$

Degree of polarization

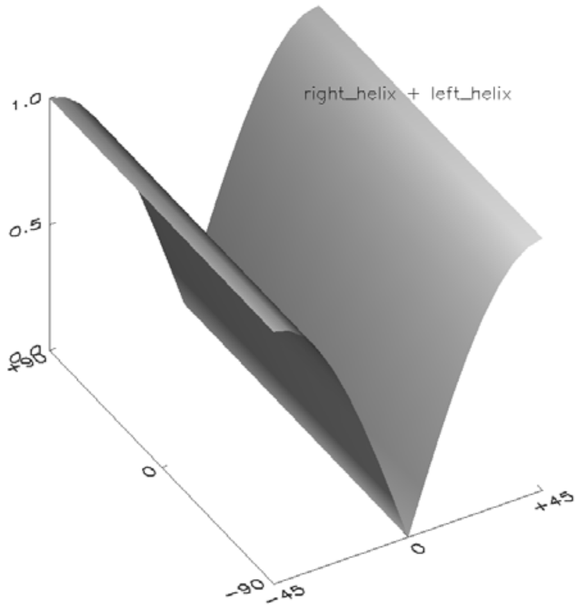
- For a given incoherent target, the degree of polarization of the backscattered wave does, in general, depend on the polarization state of the transmitted wave

- Such a function, which may be named as 'depolarization response', can be plotted either on the Poincare sphere or with the help of surface plots

$$p(\chi,\psi) \quad \begin{cases} p_{+45},\ p_{-45},\ (p_{45}) \\ p_{RHC},\ p_{LHC},\ (p_{C}) \\ p_{H},\ p_{V} \end{cases}$$

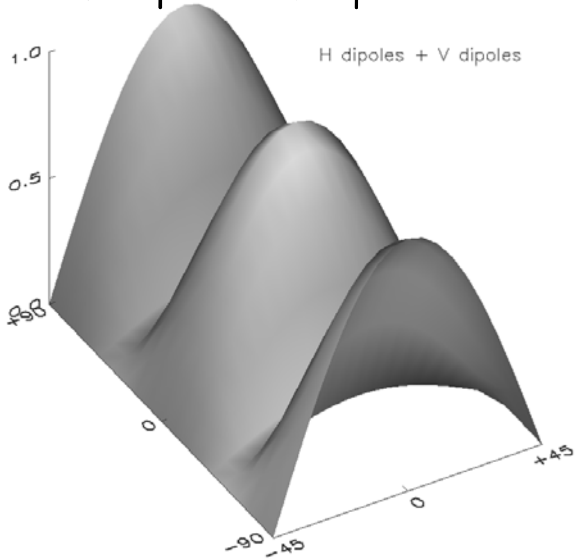
- The depolarization response is, of course, dependent on the illuminated incoherent target.

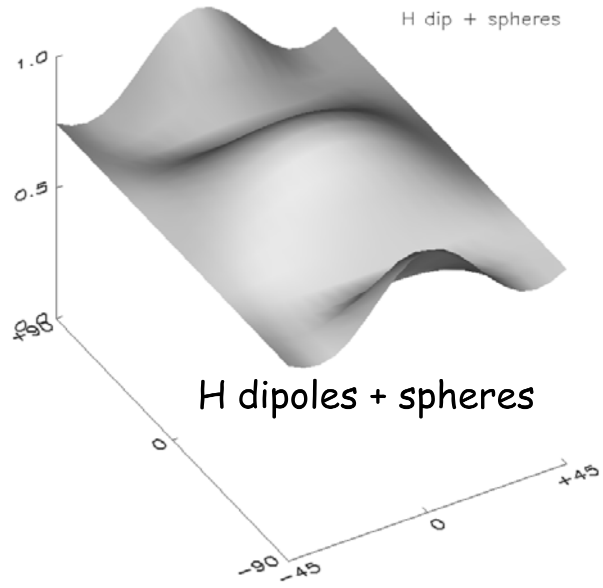
examples



Right helices + left helices

H dipoles + V dipoles





H dipoles + spheres

The full depolarization response is available only from a fully polarimetric system.
Dual polarization radars provide the degree of polarization for the transmit state in use by the system

DEPOLARIZATION IS A MULTI-DIMENSIONAL CONCEPT

Benchmark variables: H and pho_hv

$$\langle[C]\rangle = [U_{T3}][\Lambda][U_{T3}]^{-1} \qquad [\Lambda] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_{\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0}$$

$$P_i = \frac{\lambda_i}{\sum_i \lambda_i}$$

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i) \qquad 0 \leq H \leq 1$$

Scattering entropy accounts for the heterogeneity of scattering matrices that come in the formation of the covariance matrix.
It is the most general indicator of "depolarization effects"

$$\rho_{hv}(0) = \frac{\langle S_{hh} S_{vv}^{*} \rangle}{\sqrt{\langle |S_{hh}|^2 \rangle \langle |S_{vv}|^2 \rangle}}$$

The copolar correlation coefficient is normally used in radar meteorology.
Like the degree of polarization, it is a dual-polarization variable

Data Processing

$$S = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

Scattering matrix: (5+1) degrees of freedom !!!

Full S matrices measurements
at H/V polarization basis
Alternate pulse scheme

$$[S]_1 , [S]_2 , \ldots\ldots\ldots , [S]_N$$

U must belong to SU(2)*

$$U_C = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$U_{45} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Covariance matrix
at circular polarization basis

$$\langle [C]_C \rangle = \left\langle \begin{bmatrix} |S_{RR}|^2 & S_{RL} S_{RR}^* & S_{LL} S_{RR}^* \\ S_{RR} S_{RL}^* & |S_{RL}|^2 & S_{LL} S_{RL}^* \\ S_{RR} S_{LL}^* & S_{RL} S_{LL}^* & |S_{LL}|^2 \end{bmatrix} \right\rangle$$

ρ_{RHC}, ρ_{LHC}, (ρ_C)
ORTT, CDR, ALD

Covariance matrix
at H/V polarization basis

$$\langle [C]_{H/V} \rangle = \left\langle \begin{bmatrix} |S_{hh}|^2 & S_{hv} S_{hh}^* & S_{vv} S_{hh}^* \\ S_{hh} S_{hv}^* & |S_{hv}|^2 & S_{vv} S_{hv}^* \\ S_{hh} S_{vv}^* & S_{hv} S_{vv}^* & |S_{vv}|^2 \end{bmatrix} \right\rangle$$

ρ_H, ρ_V
ρ_{hv}, KDP
ZHH, ZVV, ZDR, LDR

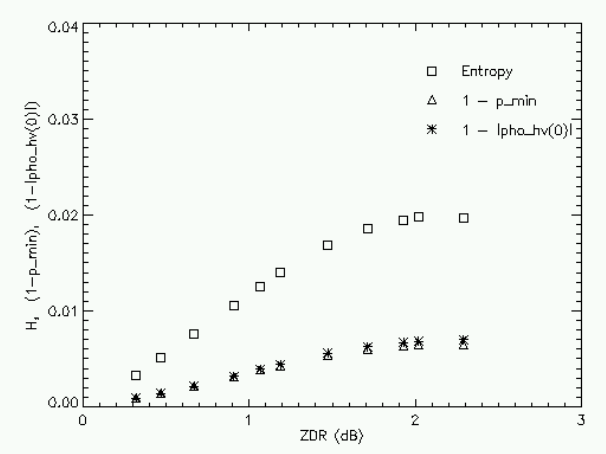
Covariance matrix
at ±45 polarization basis

$$\langle [C]_{45} \rangle = \left\langle \begin{bmatrix} |S_{//}|^2 & S_{\wedge} S_{//}^* & S_{\backslash\backslash} S_{//}^* \\ S_{//} S_{\wedge}^* & |S_{\wedge}|^2 & S_{\backslash\backslash} S_{\wedge}^* \\ S_{//} S_{\backslash\backslash}^* & S_{\wedge} S_{\backslash\backslash}^* & |S_{\backslash\backslash}|^2 \end{bmatrix} \right\rangle$$

ρ_{+45}, ρ_{-45}, (ρ_{45})

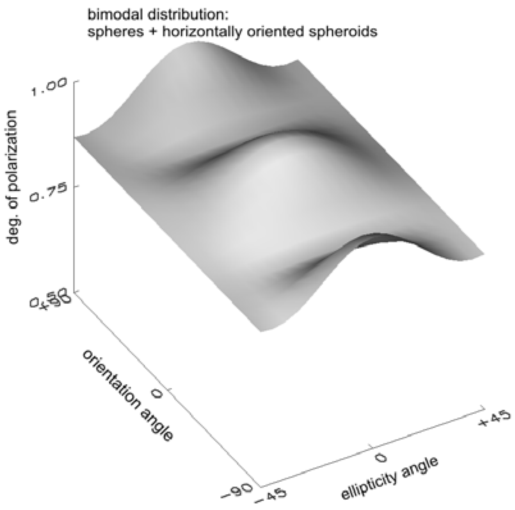
*D.H.O. Bebbington, “Target Vectors: Spinorial Concepts”, Proceedings of the 2nd International Workshop on Radar Polarimetry, IRESTE, Nantes, France, Sept. 1992

Model for rain



$$\left(1-p_{45}^{2}\right)=\left[\frac{2\left|S_{HH}\right|\left|S_{VV}\right|}{\left|S_{HH}\right|^{2}+\left|S_{VV}\right|^{2}}\right]^{2}\left(1-\left|\rho_{hv}(0)\right|^{2}\right)$$

In the case of rain, p_c, p_{45} and ρ_{hv} take on the same numerical values

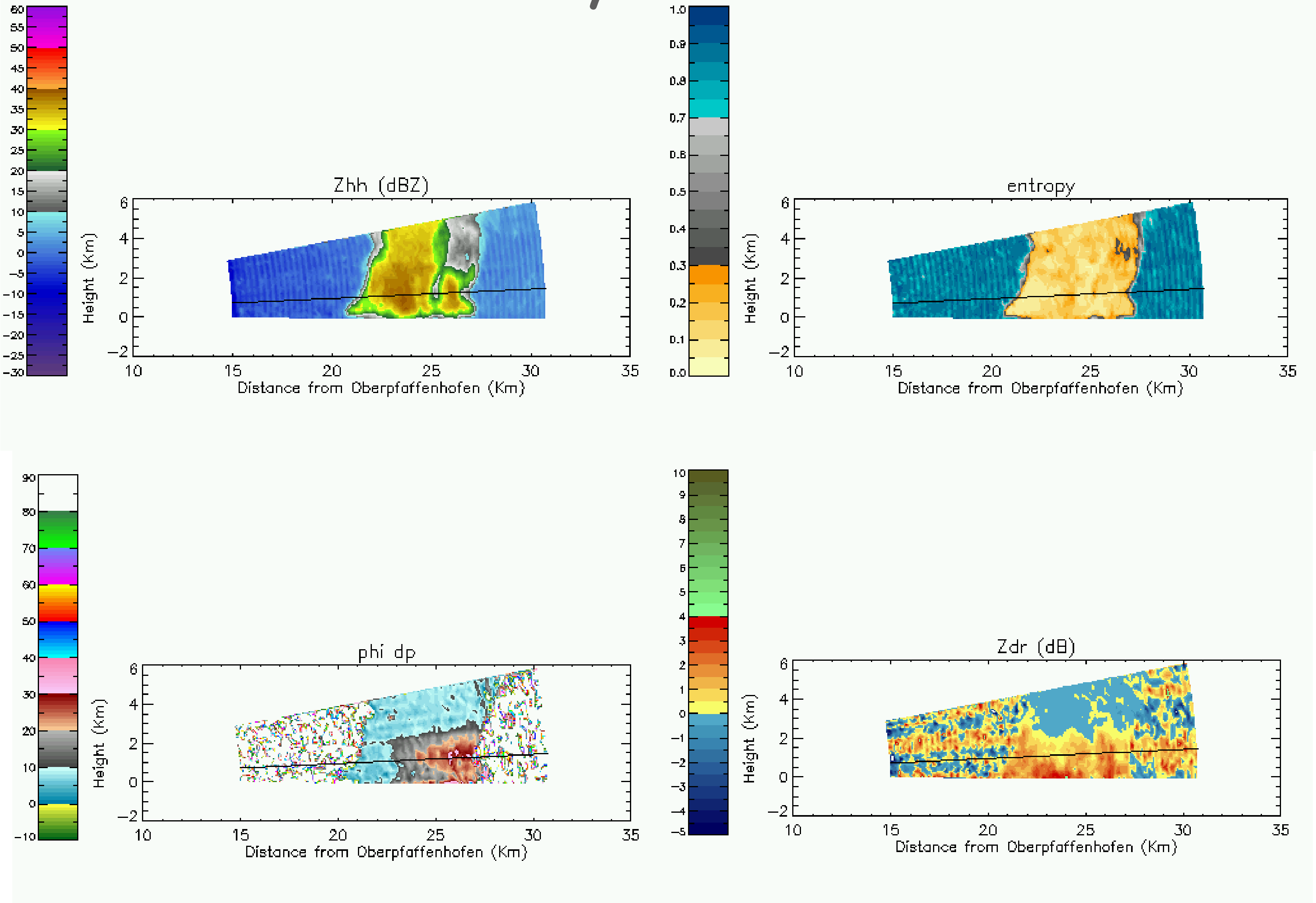


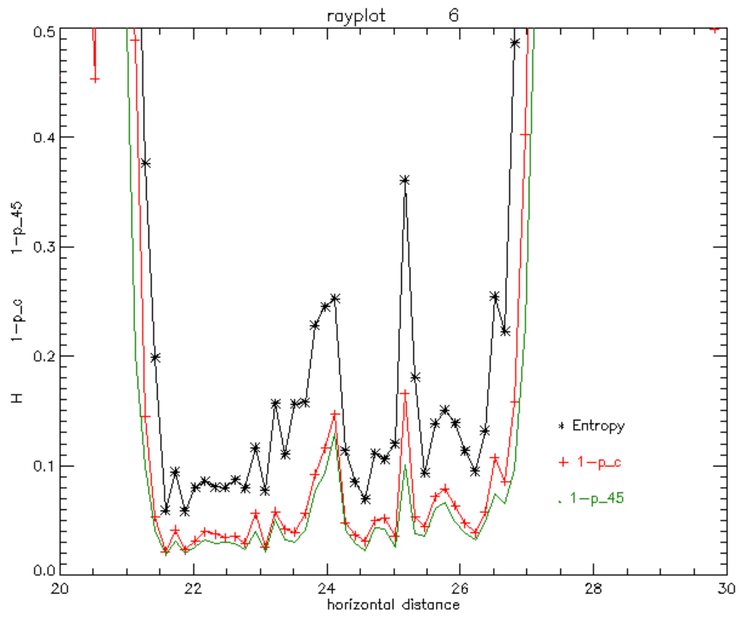
In the case of rain, the depolarization response is minimal on the circular/slant circle and maximal for H and V.

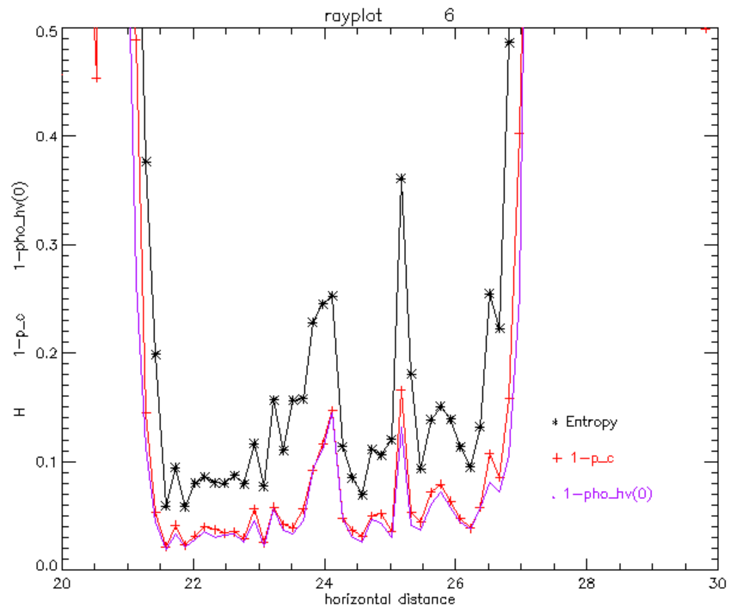
DSD, Mie scattering and drop oscillation affect
p_C, p_{45} and ρ_{hv} ,
but not p_H or p_V

p_{max} offers the best polarimetric contrast!!

Case study 1: Convective event

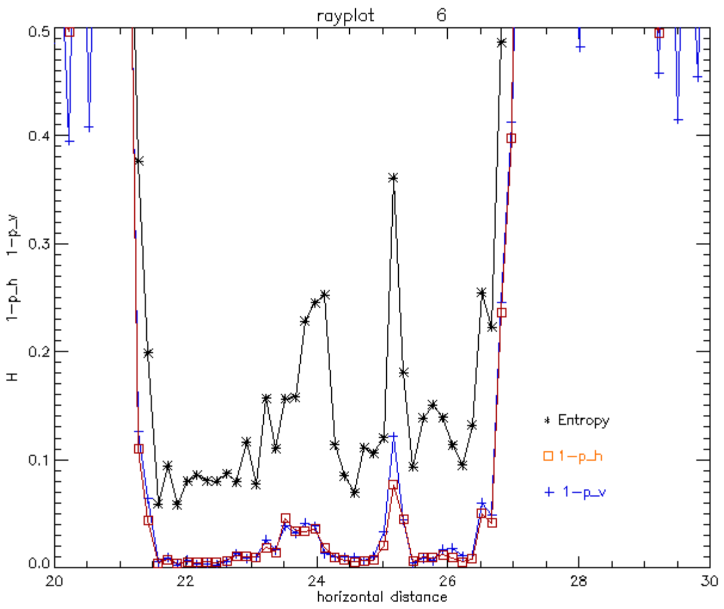


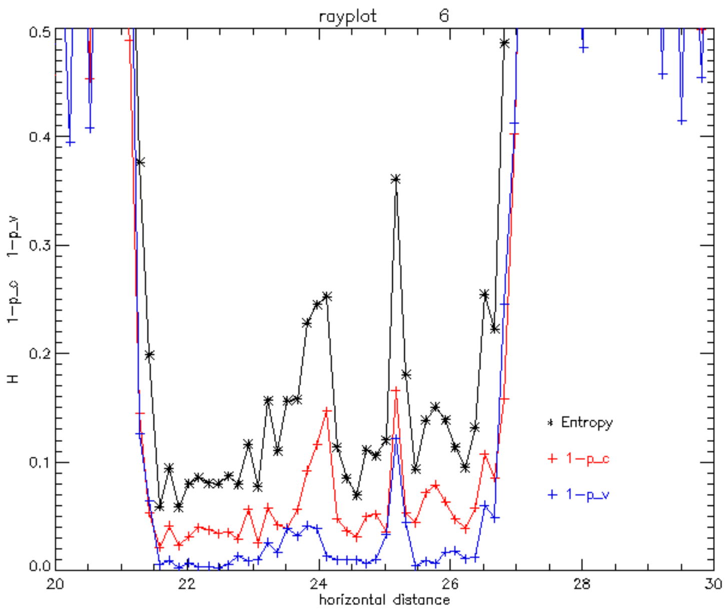




If H is low, p_c, p_{45} and ρ_{hv} take on the same numerical values.
This is the case for rain or rain/small hail mixtures.
In particular, p_c and p_{45} are minimal in this case

Note that the lower bound of these variables is DSD dependent !!



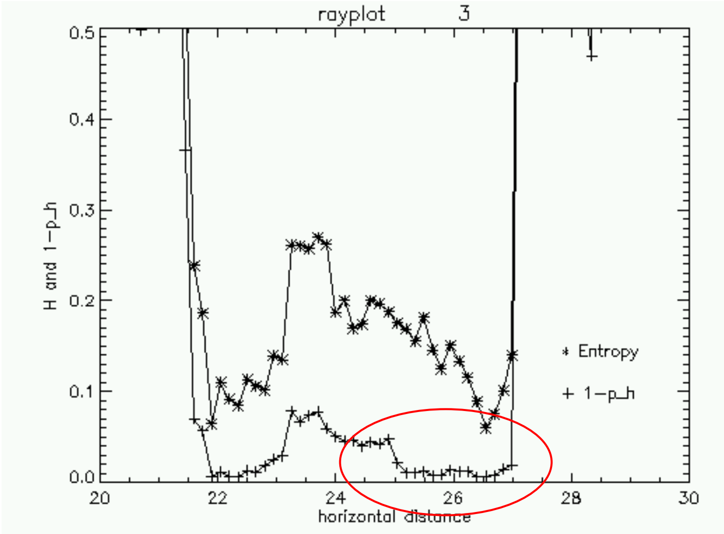


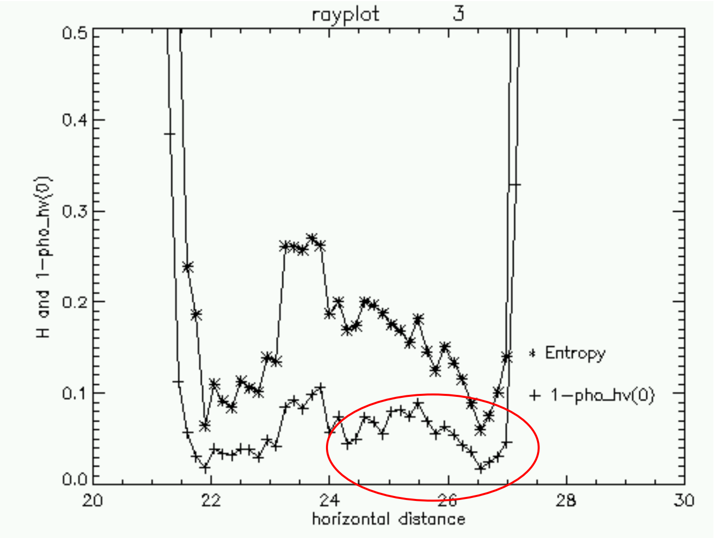
On the other hand, p_H and p_V have different properties when rain is illuminated. The lower bound of these variables is 0, regardless of the DSD !!!

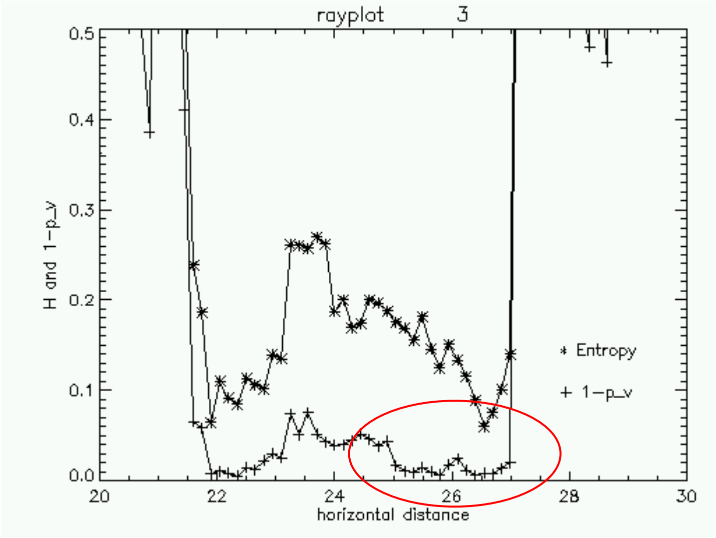
↓

Optimal variables for rain-non rain discrimination due to optimal polarimetric contrast !! Hydrometeor discrimination, clutter detection, biological targets detection.

Ray 3: Lesson learned !!





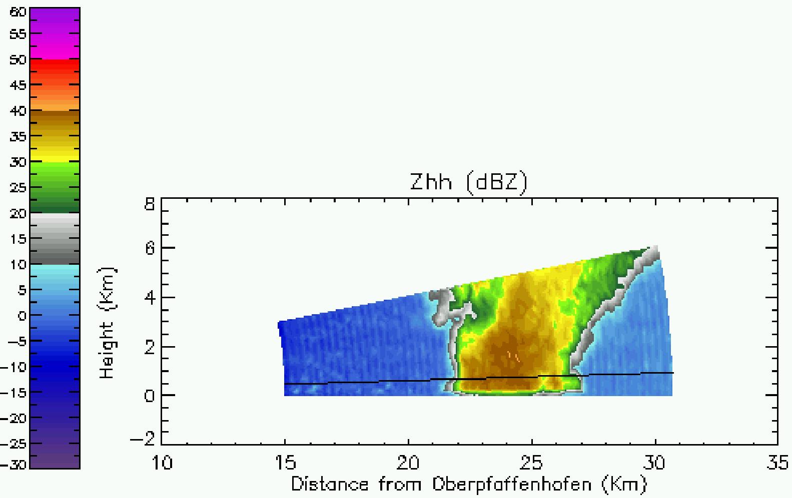


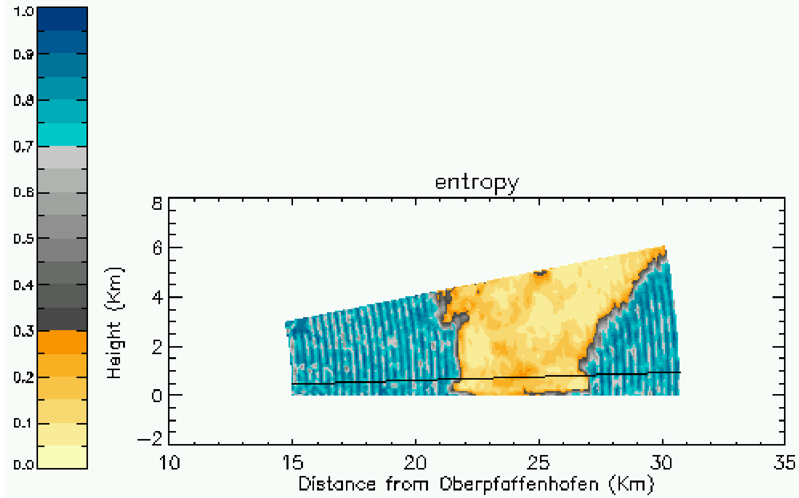
p_H and p_V perform better than ρ_{hv} or H

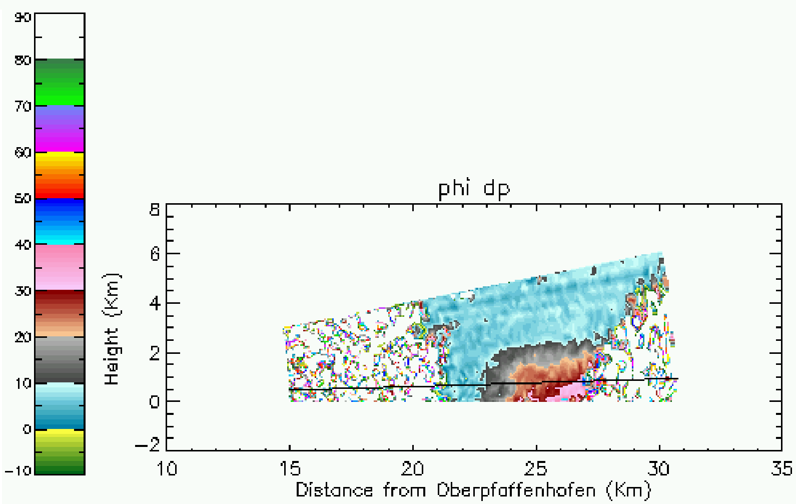
ρ_{hv} and H lower bound is DSD dependent,
p_H and p_V lower bound is zero
(contrast with frozen hydrometeors is enhanced)

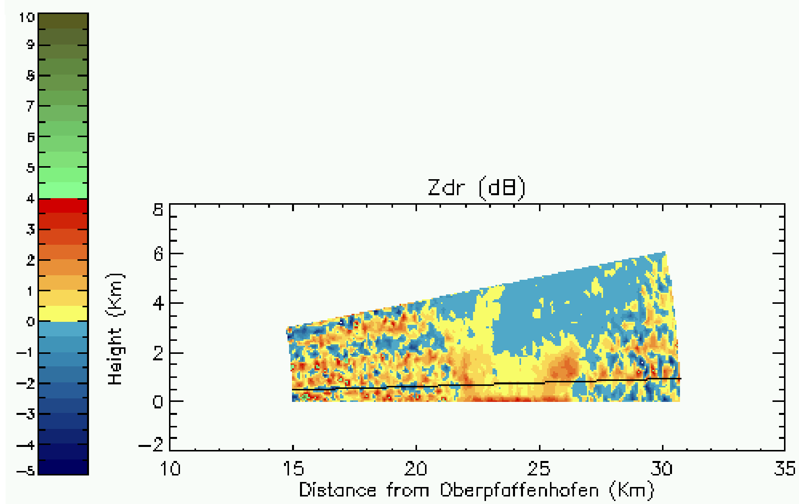
ρ_{hv} and H slightly affected by decorrelation

Case study 2: Convective event

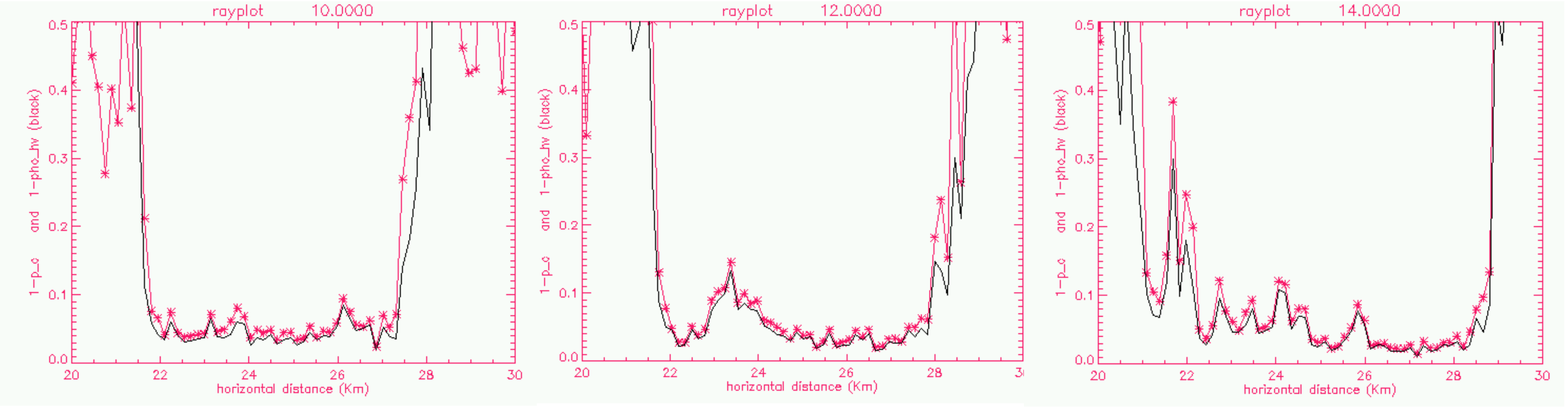








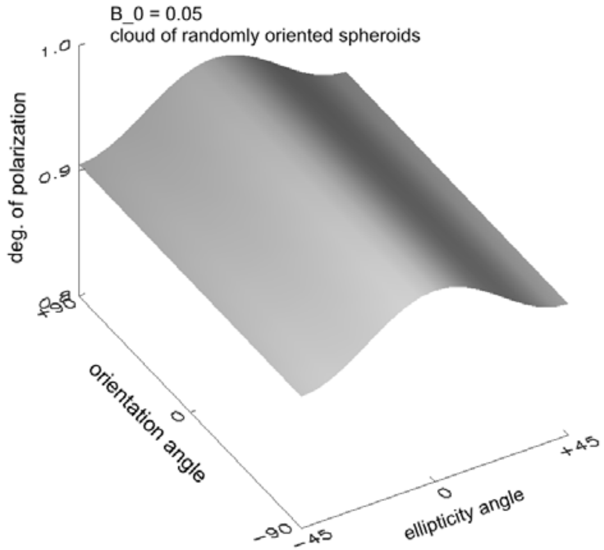
Generally, when entropy is low (rain, dry snow) ρ_{hv} and p_c or p_{45} Take on the same numerical values.

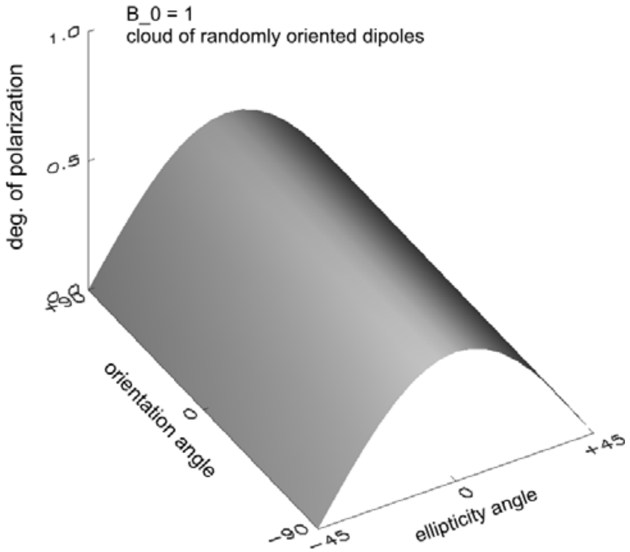


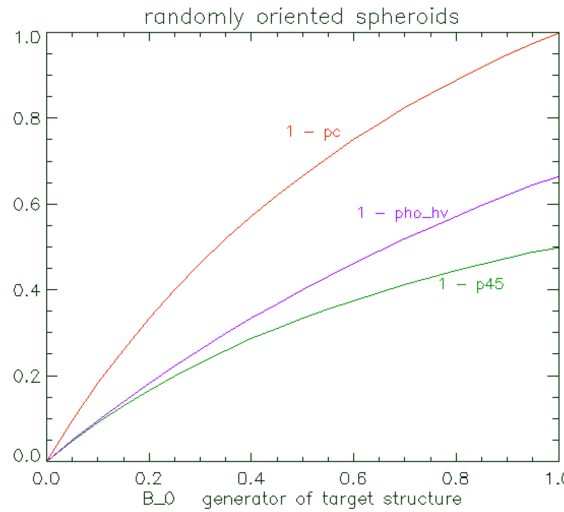
Examples for low entropy weather targets

However, when entropy is higher,
- irregularly shaped hydrometeors, biological targets, clutter -
these variables can differ.

Model for irregularly shaped hydrometeors: a cloud of randomly oriented spheroids



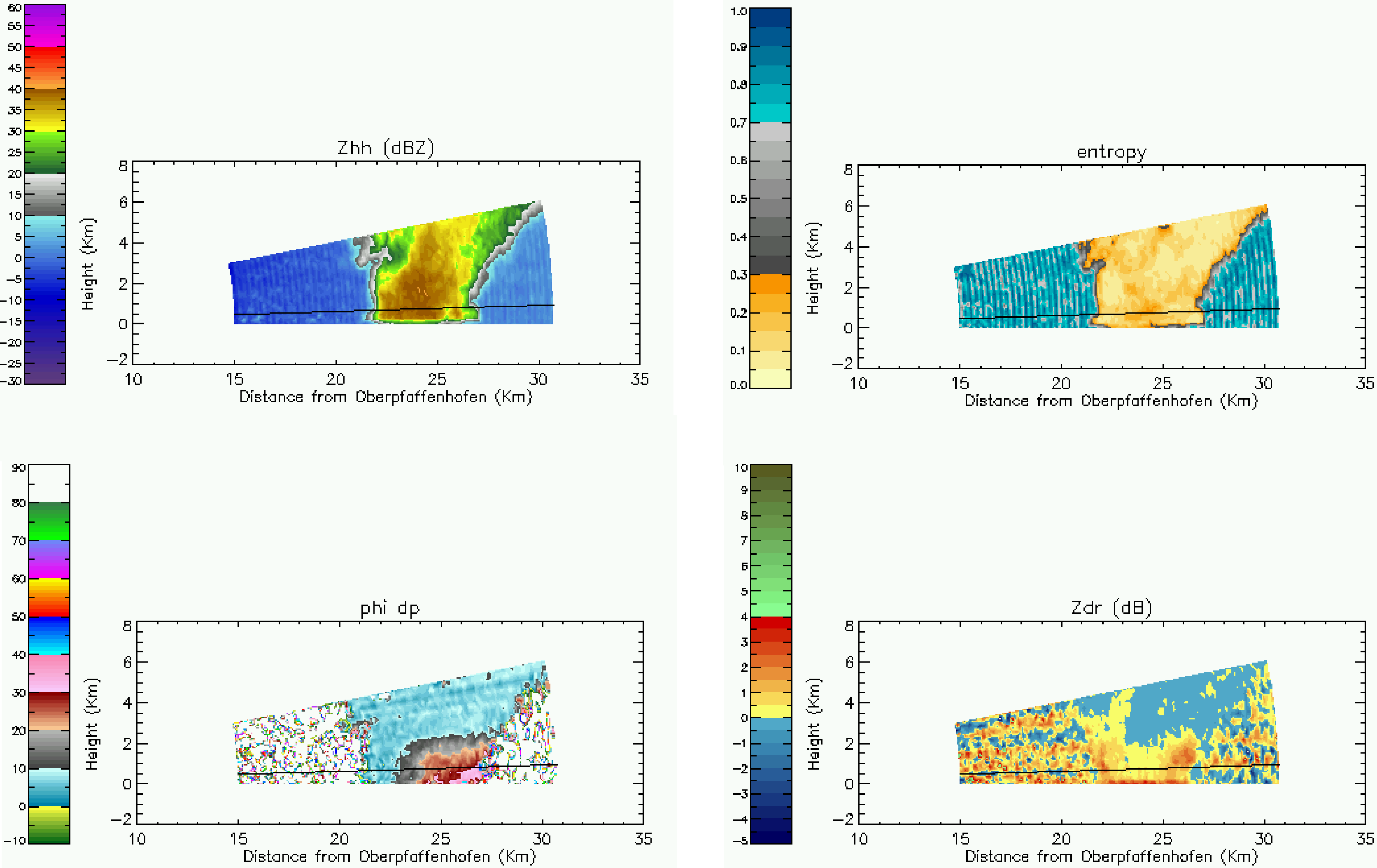




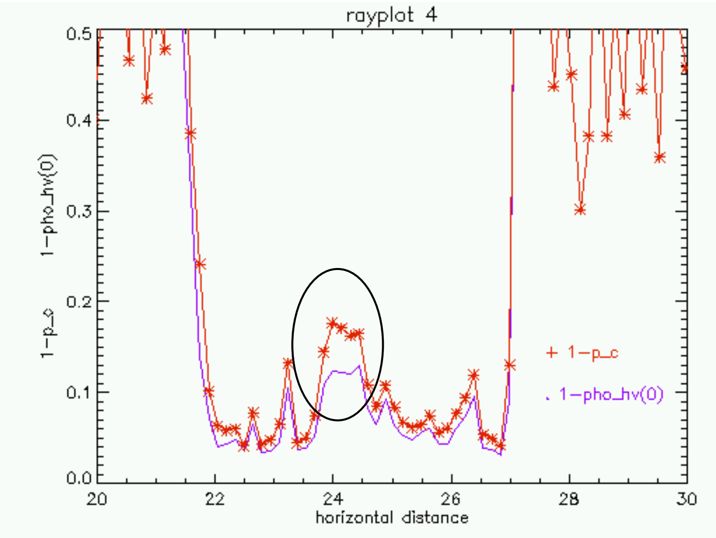
$$1 - p_{MAX} = \frac{B_0}{1 + B_0} \qquad 1 - p_{MIN} = \frac{2B_0}{1 + B_0}$$

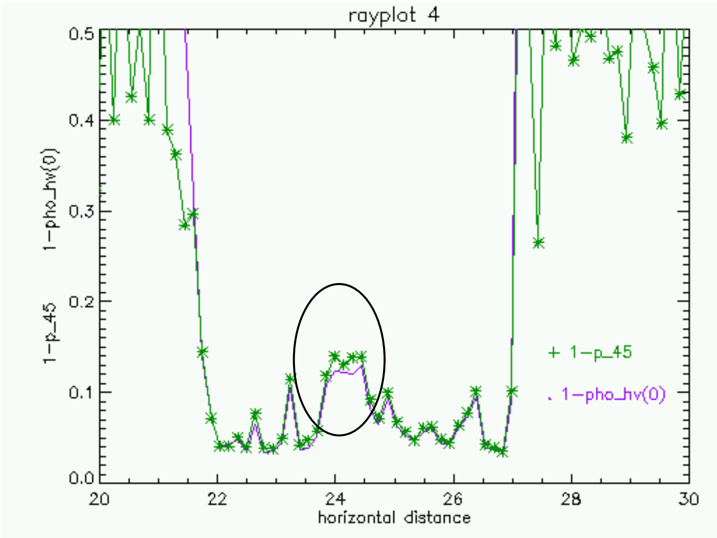
$$K_{iso} = \begin{bmatrix} 1 + B_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 + B_0 \end{bmatrix}$$

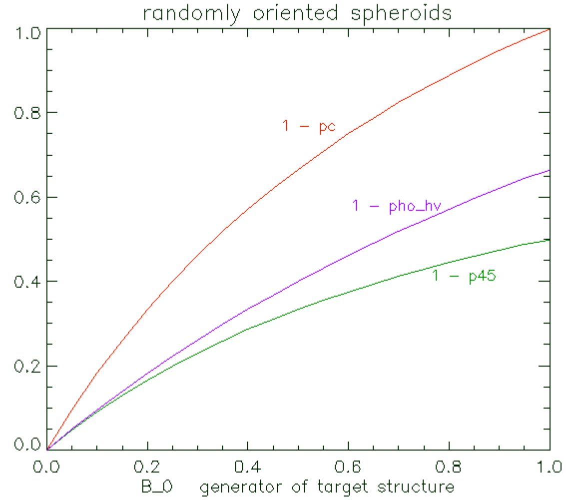
Case study 2: Convective event



Ray 4: Isotropic weather targets







Depolarization sensitive variables react in different ways, depending on the target: they are independent after all !!

•For "high" entropy targets, pc and ρ_{hv} do not necessarily take on the same values.

model and data match quite satisfactorily ($B_0 \approx 0.1$):

•p_{45} behaves like ρ_{hv}

•p_c has a larger dynamic range.

Conclusions (I)

Depolarization is a multi-dimensional concept.

In the case of rain, p_c and ρ_{hv} take on the same numerical values.
However, this does not hold in every case:
an example with graupel/hail was investigated, showing the complementary information content of p_c with respect to ρ_{hv}.

In general, for distrubuted targets, dual-polarization variables can differ.
Targets that could be better characterized by the degree of polarization might be (besides frozen hydrometeors, like grupel, hail and ice crystals) clutter, biological targets (insects, birds), volcanic ashes where unconventional shapes might come into play

Use of the degree of polarization at circular/slant send might improve discrimination/ segmentation capabilities, especially for weather radars at hybrid mode:

$$\rho_{hv}^{hy} = \frac{\left\langle (S_{HH} + S_{HV})(S_{VV} + S_{VH})^* \right\rangle}{\sqrt{\left|S_{HH} + S_{HV}\right|^2 \cdot \left|S_{VV} + S_{VH}\right|^2}}$$

$$P = \sqrt{1 - \frac{4 \cdot \det(J)}{(trace(J))^2}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$

Conclusion (II)

For radars transmitting horizontal polarization, the corresponding degree of polarization has unique discrimination capabilities, being maximal for rain.

Polarimetric contrast between rain and non-rain (both meteorological and non-meteorological) targets is enhanced.

References

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"Fully polarimetric analysis of weather radar signatures"
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M. Galletti, D. H. O. Bebbington, M. Chandra, T. Boerner,
"Measurement and Characterization of Entropy and Degree of Polarization of weather radar targets",
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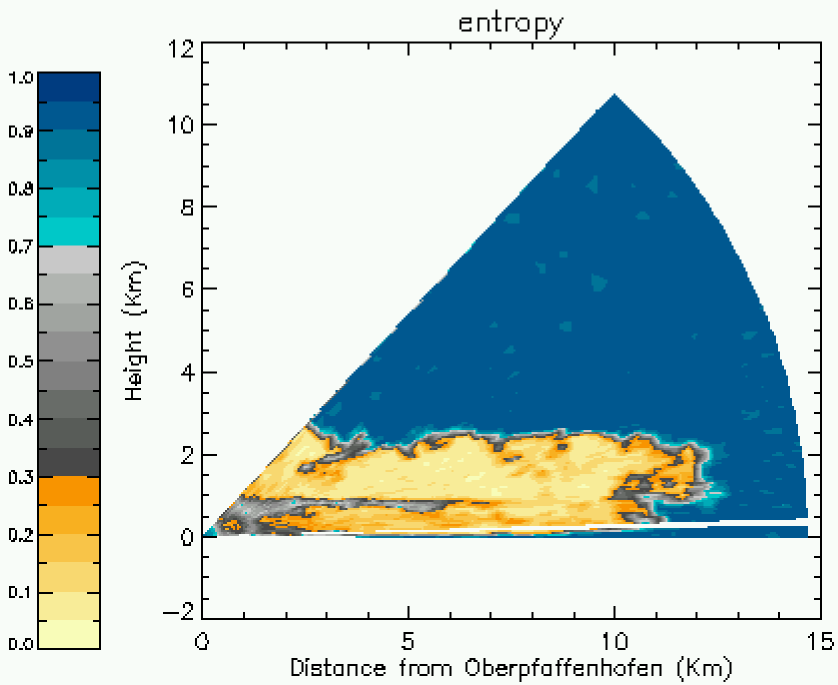
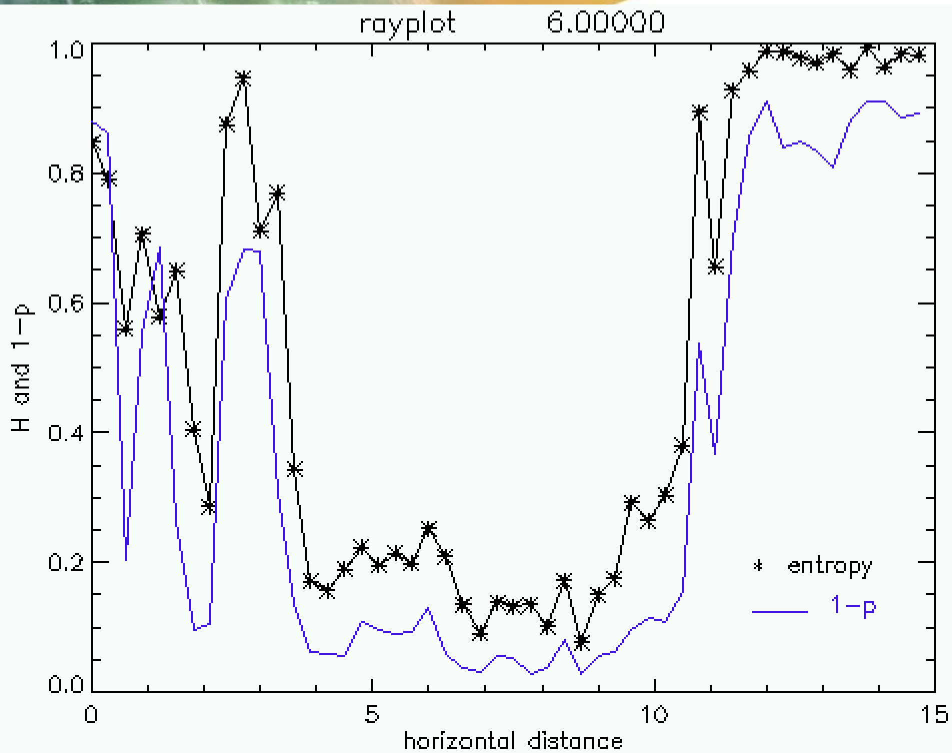
THANK YOU


entropy
Height (Km)
Distance from Oberpfaffenhofen (Km)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
12
10
8
6
4
2
0
−2
0
5
10
15


rayplot 6.00000
H and 1−p
horizontal distance
* entropy
1−p
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15