ON THE MINIMUM NUMBER OF TRACKS FOR SAR TOMOGRAPHY

Matteo Nannini, Rolf Scheiber and Alberto Moreira

German Aerospace Centre - DLR
Microwaves and Radar Institute
Oberpfaffenhofen, Germany
e-mail: matteo.nannini@dlr.de

ABSTRACT

The main drawback of SAR Tomography (SARTom) is the considerable number of tracks required to achieve the 3-dimensional (3D) representation of a viewed scene. The key point concerns the trade-off between the vertical resolution and the control on ambiguities phenomena. This paper deals with the problem of the determination of the minimum number of required tracks when super-resolution subspace methods are applied. The results are validated on real data acquired in L-band by the E-SAR system of the German Aerospace Centre.

***Index Terms*—** SAR Tomography, super-resolution, system dimension.

1. INTRODUCTION

SAR Tomography (SARTom) is an imaging technique that allows multiple phase centre separation in the vertical (height) direction, leading to a 3D reconstruction of the imaged scene. It is usually performed after standard 2D SAR processing and operates on a stack of coregistered SAR images. In [1] the first demonstration of airborne SAR tomography, using Fourier beamforming techniques, has been carried out and the main constraints in terms of resolution and ambiguity rejection have been analysed. If the number of scatterers to be solved inside a resolution cell is a priori known, it is possible to reduce the number of acquisitions [2], anyhow, for the general case this information is not known and a generic volumetric target has to be assumed. In this case, the ambiguity height V defines the baseline d_{Nyq} between the acquisitions

$$d_{Nyq} \leq \frac{\lambda\, r_0}{2\, V} \tan(\theta_0), \quad (1)$$

where λ is the wavelength, r_0 is the slant range distance and θ_0 is the look angle of the master track.

The desired resolution specifies the length of the tomographic aperture L_{tomo}. The required number of passes is $N = L_{\text{tomo}}/d + 1$.

Considering now the typical acquisition geometry of airborne systems (e.g. the E-SAR system of the German Aerospace Centre - DLR), it has been demonstrated [1] that for $2 - 3m$ resolution in height a number of acquisitions ranging between $N = 13$ and $N = 20$ is required.

This large number of tracks required for SARTom makes it an expensive and, for a large volume thickness, an unfeasible task. A reduction on the number of passes is of fundamental importance in order to exploit tomography for future spaceborne missions.

In the recent years, it has been shown that extending direction of arrivals (DOAs) estimation techniques to SARTom [3, 4], the Fourier resolution can be overcome and, therefore, it is supposed that the length of the synthetic aperture can be reduced without impacting the system capability to solve for targets in the height direction. This reduction will result in less flight tracks to be performed.

With this paper, the minimum system dimension for SARTom, when subspace methods are applied, is investigated.

2. PROBLEM FORMULATION

The information content of a SAR resolution cell is the projection of the 3D scattering contributions into a 2D plane. Assuming that the scattering mechanism verifies the Born approximation, after standard 2D SAR processing, the information content of a resolution cell for the range-azimuth coordinate (r, a) can be written as

$$s(r, a) = \int_{\theta_{min}}^{\theta_{Max}} \gamma(\theta)\ \exp\left(j\frac{4\pi}{\lambda}r\right)\ d\theta, \quad (2)$$

where $\gamma(\theta)$ represents the complex reflectivity function of point scatterers located at a distance r from the sensor in the direction θ from the track position (see Fig.1). Obviously, there is a direct relation between the height of the scatterer and the angle θ once r is defined.

Considering now a finite number of scatterers N_s within the range of $\theta \in [\theta_{min}, \theta_{Max}]$, it is possible, by means of a matrix formulation, to represent the ensemble of the signals acquired by the tomographic constellation as

$$\boldsymbol{x} = \boldsymbol{A}\,\boldsymbol{\gamma} + \boldsymbol{n}, \quad (3)$$

where $\boldsymbol{x} \in \mathbb{C}^N$ represents the set of the N focused signals for a position (r, a). $\boldsymbol{A} \in \mathbb{C}^{N \times N_s}$ contains the so-called steering vectors $\boldsymbol{a}(\boldsymbol{\theta}) \in \mathbb{C}^N$

$$\boldsymbol{a}(\boldsymbol{\theta}) = \exp\left(j\frac{4\,\pi}{\lambda}\boldsymbol{r}(\boldsymbol{\theta})\right), \tag{4}$$

with $\boldsymbol{r}(\theta_i) = [r_1(\theta_i), \cdots, r_N(\theta_i)]$ denoting the distances between the scatterer located at $\theta = \theta_i$ and the array of SAR sensors; $\boldsymbol{n}$ represents additive noise components.

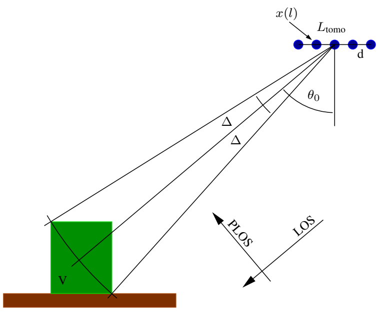


Fig. 1. Tomographic constellation and angular dispersion parameters. Line-of-sight (LOS) and perpendicular line-of-sight (PLOS) directions.

It is well known that in order to estimate the DOAs of the N_s scatterers, the MUSIC algorithm [5] represents one of the most important tools. The basic idea behind the algorithm is to define two subspaces: *signal* and *noise* subspace. The MUSIC response is defined as

$$P_{MU}(\theta_i) = \frac{1}{\boldsymbol{a}^H(\theta_i)\,\boldsymbol{E_N}\,\boldsymbol{E}_N^H\,\boldsymbol{a}(\theta_i)}, \tag{5}$$

where H stands for conjugate transpose and $\boldsymbol{E_N}$ is an $N \times (N - N_s)$ matrix whose columns are the eigenvectors of the sample covariance matrix that span the noise subspace.

Since in reality, point-like responses are not common and, especially for SARTom, the great interest is given by volumetric structures, the application of the MUSIC algorithm allows a first estimation of the mean phase centre of the volumetric target and its width.

For this reason once the number of phase centers N_s is defined, only the first N_s peaks of the response have relevance. In order to design a tomographic constellation, when such subspace methods are used, it is of fundamental importance to evaluate the expected behaviour of the eigenvalues of the covariance matrix as we shall see in the next sections.

3. DISTRIBUTED SOURCE MODEL

In order to assign a dimension to the signal subspace, the number of sources has to be defined. This parameter can be estimated from the rank of the sample covariance matrix

$$\boldsymbol{R} = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{x}(k)\,\boldsymbol{x}^H(k), \tag{6}$$

where K is the number of the snapshots. In the case of distributed scatterers this matrix will have full rank, therefore the noise subspace will have dimension equal to zero and MUSIC-like algorithms cannot be used [6]. Despite that, the greater part of the energy is often concentrated in few eigenvalues. The identification of this number of *dominant* eigenvalues will represent the effective signal space dimension [6, 7], that will be called N_{efs}.

In [6] it has been shown that, if a uniform distribution is assumed, the widest spread of the signal energy among the dominant eigenvalues is achieved. This implies that a uniform distribution represents the worst case scenario. For this reason, two uniformly distributed (u.d.) scatterers representing the canopy and the ground respectively, can be assumed.

It can be demonstrated that a single u.d. source represents an upper bound in terms of dominant eigenvalues of the related covariance matrix representing two u.d. sources contained in the same angular sector. Therefore a single u.d. source will be considered.

4. MINIMUM NUMBER OF TRACKS

To determine the number of dominant eigenvalues of a distributed source, a continuous acquisition along the tomographic aperture is first assumed. The concept will be then generalized to a discrete acquisition geometry.

Defining Δ as the half of the extension of the volumetric structure (see Fig.1) it is possible to observe that the cross-correlation between the received signals of two elements ($x(l)$ and $x(l')$) of the aperture has the following expression

$$E\left[x(l)\,x^*(l')\right] = \exp\left(j\frac{4\pi}{\lambda}(l-l')\sin(\theta_0)\right)\ \text{sinc}\left(\frac{4}{\lambda}(l-l')cos(\theta_0)\Delta\right), \tag{7}$$

with Δ small (true for typical SAR acquisition geometry).

In [1] the band limited nature of the tomographic signal has been demonstrated. Under this assumption, the eigenproblem can be faced by means of the prolate spheroidal wave functions if a sufficient oversampling is guaranteed.

It is possible to demonstrate that the eigenvalues depend on the radial prolate spheroidal functions $R_{0n}^{(1)}(c, 1)$ [8] and on the parameter c, the so-called space-bandwidth product, given by

$$c = \frac{2\pi\ \Delta\ cos(\theta_0)}{\lambda}\ L_{\text{tomo}}. \tag{8}$$

Most of the energy is concentrated in the first $\lceil 2c/\pi \rceil$ eigenvalues [8] ($\lceil\cdot\rceil$ represents the next integer greater than the argument). Therefore, it can be concluded that for the SARTom case the number of effective scatterers is

$$N_{efs} = \left\lceil \frac{2c}{\pi} \right\rceil = \left\lceil \frac{4\ \Delta\ cos(\theta_0)}{\lambda}\ L_{\text{tomo}} \right\rceil. \qquad (9)$$

In order to allow the system to describe the volume under consideration, no underestimation of the effective signal subspace is allowed. Indeed, no dominant eigenvalues have to be excluded. Hence, it has to be guaranteed that through the eigenvalues decomposition the following conditions for N holds

$$N = N_{efs} + M \qquad (10)$$

$$\min_N \frac{\mu_{N-1}}{\mu_0} \simeq 0 \qquad (11)$$

where M is an integer value greater than one, μ_0 and μ_{N-1} are the strongest and the weakest eigenvalues, respectively. The tables in [9] will allow to identify these parameters. Condition (11) means that we are considering the smallest system dimension because only one eigenvalue will be related to the noise subspace.

Until now, the length of the tomographic aperture has been assumed as known. In the next section, boundary conditions in order to define it are described.

5. MINIMUM TOMOGRAPHIC APERTURE LENGTH

Since the minimum number of dominant eigenvalues that one wants to retrieve is $N_{efs} = 2$ (ground, canopy components), this implies that L_{tomo}

$$L_{\text{tomo}} > \frac{\lambda r_0}{2\ h_{min}}\ tan(\theta_0), \qquad (12)$$

where h_{min} is the minimum height where the two sources are located.

One can observe that (12) has a similar expression to the equivalent representation of L_{tomo} described in [1] with the exception that h_{min} takes the place of the Fourier tomographic resolution ρ_F. It is important to remark that for subspace methods it is not possible to find a direct expression for the resolution, because it depends on the data itself. At the same time it is useful to find a link between the Fourier resolution and the minimum height h_{min} in order to evaluate the impact of the system reduction. Without loss of generality, it can be said that within a volume height of $2\rho_F$ two main contributions whose distance corresponds to ρ_F have to be identified. Therefore, once a reference resolution has been defined, the correspondent h_{min} and the minimum length L_{tomo} can be identified.

6. AIRBORNE CASE SIMULATION: L-BAND

The scene to be viewed consists of a volume of a maximum height in the perpendicular line-of-sight (PLOS) direction of $30m$. The master track of the tomographic constellation is located in the center of it and its height above ground is $H = 3200m$, the slant range coordinate related to the centre of the scene is $r_0 = 4500m$ (refer to Fig.1). The frequency used is L-band corresponding to a wavelength $\lambda = 24cm$. With this geometry, the minimum allowed baseline is around $d_{Nyq} = 25m$ (1).

Now, the minimum length of the tomographic aperture has to be determined. Proceeding as in section 5, defining an equivalent Fourier resolution of $\rho = 3m$ in the PLOS direction (and $\rho_F = \rho \sin(\theta_0)$ in the vertical direction), the minimum volume height corresponds to $h_{min} = 2\rho \sin(\theta_0) = 4.3m$. The minimum value of the tomographic aperture is $L_{\text{tomo}} = 130m$. This aperture will allow to view the two components when their distance is $3m$ in PLOS. The system dimension at this point is determined with the help of the maximum volume height, which is related to the maximum number of effective sources that the system will need to represent.

For the maximum PLOS volume height of $30m$, the space-bandwidth product is $c = 8$. Now, referring to the tables [9] and to the conditions (10) and (11) a minimum number of acquisition of $N = 8$ results. Therefore, the required baseline will be approximately $d = 18m$.

Comparing now this geometry with the one obtained by means of Fourier based techniques we obtain for the later one, a minimum tomographic aperture of $250m$ with at minimum $N = 11$ tracks, because of d_{Nyq}. It is worth to mention that, due to the limited resolution inherent to the Fourier beamformer when compared with subspace methods, a few tracks are added for reducing the baseline value in order to ensure stable ambiguity rejection. In fact, in [1] $N = 13$ passes have been flown in order to allow the system to perform as in this example.

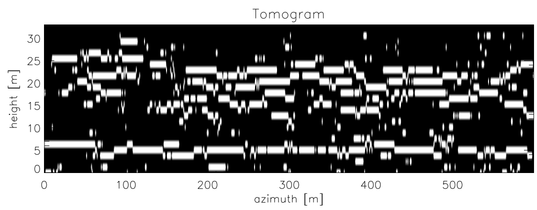
7. EXPERIMENTAL RESULTS

The theory described in the previous section is validated on real data. The E-SAR data set has been acquired in L-band in September 2006 over Dornstetten (Germany) for target detection purposes [10]. The area where the experiment took place is relatively flat and half of the region is covered by non-homogeneous forest stands related to different species. The tree height ranges between $10 - 25m$. The acquisition geometry is nominally a regular horizontal grid of 21 tracks with an average baseline of 20m, resulting in a tomographic horizontal aperture of $400m$. In order to maintain the requirements of the numerical example (minimum PLOS volume height of $6m$ for the mid-range), the minimum suitable aperture length is $L_{\text{tomo}} = 140m$ acquired with $N = 8$ passes.

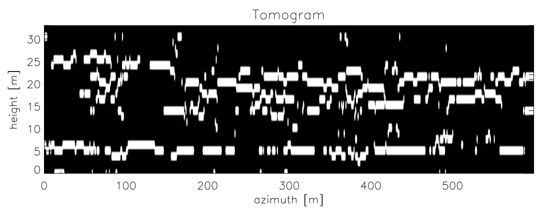
The MUSIC algorithm has been first applied with all the 21 passes Fig.2(a). One can observe that this full aperture MUSIC response allows to determine the position and the extension of the two scattering components (ground, canopy).

For the reduced geometry $N = 8$ the tomographic processing results are presented in Fig.2(b). It is possible to observe that also with the reduced system, the two main contributions of the ground and the canopy are well represented.

Comparing Fig.2(a) with Fig.2(b) it is interesting to observe how the number of dominant components decreases proportionally with the tomographic aperture length as expected from (9). For this reason, the full system is capable to represent the forest structure with higher precision compared with the reduced one. In addition, the reduced aperture dimension increases the minimum angular separation of two close sources.



(a)



(b)

Fig. 2. Tomograms of a forested area obtained by means of the MUSIC algorithm (a) using the full tomographic aperture obtained with $N = 21$ tracks (b) using the reduced aperture with $N = 8$ tracks.

8. CONCLUSION

In this paper a study on the dimension of the tomographic acquisition scenario has been carried out.

The main parameters: minimum number of tracks, distance between the tracks and minimum tomographic aperture length have been determined. It has been shown that the identification of the effective signal and noise subspaces allows to perform an analysis that leads to the estimation of the equivalent number of sources that have to be solved by the system when a distributed scatterer is viewed.

The analysis carried out by means of the prolate spheroidal wave functions consists in the identification of the dominant components of the tomographic band limited signal.

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9. REFERENCES

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