Steering of the Reference Timescale for German Galileo Test Environment

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BIOGRAPHY

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INTRODUCTION

The steering of timescales involves a wide range of different algorithms which each deals with different timescale aspects. For instance producing highly accurate time and frequency offset estimations for different prediction intervals or computing reasonable steering values which on the one hand hold time and frequency nearby zero and on the other hand few affect Allan Deviation. In the following, two different steering algorithms are shortly introduced: the method of the National Physical Laboratory of Israel (INPL) [1] and the Least Quadratic Gaussian Control approach [2]. In contrast to the INPL method which corresponds to a parameterized ad-hoc solution of the timescale steering problem, the LQG Control approach which uses Kalman filtered measurements and the solution of the regulator problem as steering implementation is a considerable more complex method. To come up with the answer to the question if the in-deep approach of the LQC method is justified the performance of both steering techniques is tested in the situation of a simulated steering against the public frequency standard of the Physikalische Technische Bundesanstalt (PTB).

TWO TIMESCALE STEERING TECHNIQUES

The steering or controlling of the offset between two (atomic) clocks states a fundamental task which occurs often in timing laboratories. Mostly, the controlling methods are used to link hardware clocks against external public frequency standards like UTC(PTB) or UTC(BIPM) and to backup crucial time service clocks. Before introducing two different controlling algorithms, the problem of offset steering is generally specified. Two configuration parameters τ and φ are used to simulate different steering configurations. τ is equal to the time interval between offset measurements and φ characterizes the steering and prediction interval. For example, offset measurements referenced to UTC(PTB) are published every day. Each publication contains offset measurements of the past day in 15 minute interval. To simulate this situation, the measurement and steering interval are set to τ=15 min and φ =1 d. Since at steering time only offsets of the last day are available, time and frequency offset are predicted. That means besides computing reasonable control values, a steering algorithm also deals with (optimal) offset prediction.
In the following descriptions we explicitly note random processes with capital letters (e.g. X(t)) and its corresponding sample with small letters (e.g. x(t)).

Method of the National Physical Laboratory of Israel (INPL)

The INPL technique was developed and used at National Physical Laboratory of Israel (INPL) [1] and is a reasonable starting point to outline the general workflow of steering algorithms. The technique was applied to steer a software clock which was deduced of a clock ensemble against UTC(BIPM). The method can easily be adopted to steer the offset between two individual hardware clocks.

The steering value u(t) is deduced in the following way. First, frequency offset Y(t-φ) is approximated by the use of older measurements and second, the last steering value u(t-φ) is known. So frequency Y(t) is estimated by

$$\hat{y}_m(t) := \frac{x(t-\varphi)-x(t-2\varphi+1)}{\varphi} + mu(t-\varphi)$$

The parameter m is used to handle the impact of the last steering action. The corresponding offset X(t) is now estimated by

$$\hat{x}(t) := lx(t-\varphi)+\varphi\hat{y}_m(t-\varphi) = lx(t-\varphi)+\varphi\left(\frac{x(t-\varphi)-x(t-2\varphi+1)}{\varphi}+mu(t-\varphi)\right)$$

Parameter l is added to control the steering effects. It is obvious that adding $\hat{x}(t)$ would set the timescale roughly against zeros. Since the target is to steer time and frequency offset the steering value is completed by:

$$u(t) = -\hat{y}_m(t) - l\frac{\hat{x}(t)}{\varphi}$$

The parameters m and l allow varying the behaviour of the algorithm. Its optimal chose depends on the timescale properties and the steering configurations.
The algorithm utilizes especially the linear timescale model and enables to control the steering by two parameters. Frequency is estimated by using only two former offset measurements although generally more measurements are available.

Least Quadratic Gaussian (LQG) Control

The Least Quadratic Gaussian Control (LQG) is a more extensive method to steer timescales. The control system includes besides assuming the 2-state timescale model a detailed stochastic model to handle both non-deterministic effects of timescales and random measurements. In LQG control, Kalman filter is used to process the random offset measurements and the solution of the regulator problem is applied to obtain appropriate steering values.
Kalman filter is a common and modern method to deal with timescales. The first step in its usage is the approximation of the timescale process by the well known stochastic differential equation

$$\frac{d}{dt}\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} + W(t) \quad (1)$$

where W(t) is a 2-state White Gaussian Noise Process with time invariant covariance

$$VAR(W(t)) = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}$$

[3]. Analyzing the 2-state equation of (1) it is evident that frequency offset Y(t) is modelled by combining two white Gaussian noise processes with variance q_1 and q_2. The process with parameter q_1 is applied in the first state of equation (1) and disturbs frequency directly with white noise. The second process with parameter q2 is used in the second state equation, thus, disturbs the first derivate of Y(t). This process corresponds to model frequency as Random Walk Process. Both processes white and random walk are overlaid to model the non-deterministic behaviour of timescales. In ADEV figures, each noise type is expressed by a distinct slope. White frequency noise has a slope equal to -0.5 and random walk frequency is expressed by a slope equal to 0.5. Notice, timescales are often disturbed by additional noise types like flicker frequency noise (ADEV slope 0) or white phase noise (ADEV slope 1). These types are not covered by the two parameters of the introduced 2-state timescale model.
The discrete solution of equation (1) for time step τ

$$\begin{pmatrix} X(t+\tau) \\ Y(t+\tau) \end{pmatrix} = \begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix}\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} + V(t) \text{ with } VAR(V(t)) = \tau\begin{pmatrix} q_1 + \frac{q_2\tau^2}{3} & \frac{q_2\tau}{2} \\ \frac{q_2\tau}{2} & q_2 \end{pmatrix}$$

is applied as 2-state-space model (V(t) White Gaussian Noise). The Allan Deviation of X(t) is known and equal to

$$ADEV(X(t),\tau) = q_1\frac{1}{\tau} + q_2\frac{1}{3}\tau^2$$

where q_1 and q_2 parameterizes white frequency and random walk noise.
Both parameters are chosen in the way that ADEV(X(t),τ) approximates the empirical Allan Deviation of the observed timescale which includes in general more noise types in a reasonable way. Although the 2-state timescale model is in general only an approximation of the real dynamics of a timescale it is enough to improve the measurements and to reduce measurement noise.
The second step in applying Kalman filter is the specification of the measurement process. Measurements are conventionally modelled by

$$Z(t) = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} + N(t) \text{ with } VAR(N(t)) = r$$

(N(t) white Gaussian noise).
The solution of the introduced Kalman filter model is applied to process the offset measurements and to return more accurate state estimates. The Kalman filtered time and frequency offsets are labelled with $\tilde{x}(t)$ and $\tilde{y}(t)$.

The outputs are now used to compute steering values. Since at steering time t only filter outputs older than t-φ are available the actual state (time and frequency offset) are predicted. The simulations showed that the timescale model

$$\begin{pmatrix} \hat{x}(t) \\ \hat{y}(t) \end{pmatrix} = \begin{pmatrix} 1 & \varphi \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x}(t-\varphi) \\ \tilde{y}(t-\varphi) \end{pmatrix} + \begin{pmatrix} \varphi \\ 1 \end{pmatrix} u(t-\varphi) \qquad (2)$$

is not sufficient to produce accurate predictions for longer steering intervals φ. One reason may be that frequency $\tilde{y}(t-\varphi)$ corresponds to the shorter measurement interval τ. To overcome this situation we separately compute frequency similar to the INPL case by fractional difference building of two former Kalman outputs:

$$\bar{y}(t-\varphi) = \frac{\tilde{x}(t-\varphi) - \tilde{x}(t-2\varphi+1)}{\varphi}$$

So (2) modifies as

$$\begin{pmatrix} \hat{x}(t) \\ \hat{y}(t) \end{pmatrix} = \begin{pmatrix} 1 & \varphi \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x}(t-\varphi) \\ \bar{y}(t-\varphi) \end{pmatrix} + \begin{pmatrix} \varphi \\ 1 \end{pmatrix} u(t-\varphi) \qquad (3)$$

The task is now to find a sequence of frequency control values u(t) which hold time $\hat{x}(t)$ and frequency $\hat{y}(t)$ nearby zero. This situation corresponds to the solution of the noise-free regulator problem. The relevant quadratic cost function is

$$\sum_{i=1}^{\infty} w_x \hat{x}^2(i) + w_y \hat{y}^2(i) + w_u u^2(i) \qquad (4)$$

Three parameters w_x, w_y and w_u are used to individually control the impact of each quantity $\hat{x}(t)$, $\hat{y}(t)$ and u(t). For a detailed analysis and solution of the regulator problem refer to [4], [5] and [2].
We only go back to cite the most important part of the solution which is the gain matrix

$$G(\infty) = \left(\begin{pmatrix} \varphi & 1 \end{pmatrix} K(\infty) \begin{pmatrix} \varphi \\ 1 \end{pmatrix} + w_u \right)^{-1} \begin{pmatrix} \varphi & 1 \end{pmatrix} K(\infty) \begin{pmatrix} 1 & \varphi \\ 0 & 1 \end{pmatrix}$$

where K(∞) is the solution of the corresponding Ricatti equation in steady-state [2]. The gain matrix G(∞) is used in combination with the state prediction (3) to compute the actual steering value

$$u(t) = -G(\infty) \begin{pmatrix} \hat{x}(t) \\ \hat{y}(t) \end{pmatrix}$$

Notice, the two vector entries of G(∞) are time invariant and only depend on w_x, w_y and w_u which are the weights of the cost function (4).

STEERING SIMULATION OF UTC(DLR) AGAINST UTC(PTB)

The performance and parameter impact of both steering techniques are investigated in the situation of steering UTC(DLR) against UTC(PTB). It is subject to choose the individual parameters in the way to minimize Standard Deviation of the simulated timescale.
The steering configuration is approximated by simulating the conventionally participating clock types. Conversely, to model a more authentic measurement environment time measurements are disturbed by flicker noise and not by conventional white noise which is assumed in the Kalman filter model. Measurement noise will not be truly uncorrelated as demanded by a white process. Quite the contrary there will be time intervals in which measurements are disturbed by similar noise values, and, thus are correlated. Flicker processes have a spectral density which is proportional to 1/f that means the noise process contains a large amount of periodic signals with small frequencies and for that reason is correlated over longer time periods.
In the INPL steering method there is no need to know the properties of the measurement noise and in the Kalman Filter technique the flicker noise is treated as white noise process. The standard deviation is estimated by the empiric standard deviation of the flicker process which is in our simulations equals to

$$\sqrt{VAR(N(t))} = 3.3ns$$

We assume that both time laboratories operate Caesium clocks with different noise specifications (Table 1).

	White Frequency Noise at 1 s	**Flicker Frequency Noise at 1 s**
UTC(DLR)	$8*10^{-12}$	$1*10^{-14}$
UTC(PTB)	$2.9*10^{-12}$	$5*10^{-16}$

Table 1: clock noise parameters

The timescale or offset between both Caesium clocks is simulated over one year and with a sampling rate of 15 min. The steering configuration is adapted on the PTB measurement situation. That means measurements are published once a day containing measurements of the last day in 15 min interval. As a result, steering and prediction intervals are equal to one day and the measurement interval is set to 15 min.
The performance test starts with the parameter optimization of the INPL technique. INPL is controlled by two parameters m and l which weight the last steering value and the last time measurement. The steering simulations are computed for m ranging from 0.05 to 1.5 with 0.05 steps and l ranging from 0.05 to 1.5 with 0.05 steps. Simulations showed that smaller step sizes do not significantly improve performances. The optimization task returns that minimal Standard Deviation of the steered and measured timescale is equal to 7.63 ns and yielded by setting m=1.1 and l=0.3. Since the steering is simulated we are also able to evaluate the timescale performance without measurement noise. Using the previous m and l values the Standard Deviation of the steered timescale without measurement noise is equal to 7.25 ns.
The INPL performance is now faced with the results of the LGQ steering technique. In order to apply the steering technique we first have to determine the two Kalman parameters q_1 and q_2 which describe the amount of white and random walk frequency. q_1 and q_2 are chosen in the way that its ADEV(X(t),τ) reasonable approximates the empiric Allan Deviation of the simulated timescale. Figure 1 illustrates the empiric Allan Deviation of the simulation and the (theoretical) Allan Deviation of its approximating process X(t) where $q_1=7.2*10^{-23}$ and $q_2=5.0*10^{-34}$.
After fixing the Kalman parameters optimal control values w_x, w_y and w_u are determined. The steering simulations are processed for w_x in $\{10^i \,|\, i=-6,\ldots,-2\}$, w_y in $\{10^i \,|\, i=3,\ldots,7\}$ and w_u in $\{10^i \,|\, i=3,\ldots,7\}$. The optimization task results that the minimal Standard Deviation of the Kalman filtered and steered timescale is equal to 6.64 ns and yielded setting $w_x=10^{-6}$, $w_y=10^6$ and $w_u=10^4$. The corresponding Standard Deviation of the steered timescale without measurement noise is equal to 6.52 ns.

	INPL	LQC	Free
σ including Noise in ns	7.63	6.64	125
σ without Noise in ns	7.25	6.52	125

Table 2: Standard Deviation of Steered Timescales

Table 2 summarizes the results of both steering techniques and outlines that LQC control outperforms INPL in the case with and without measurement noise.

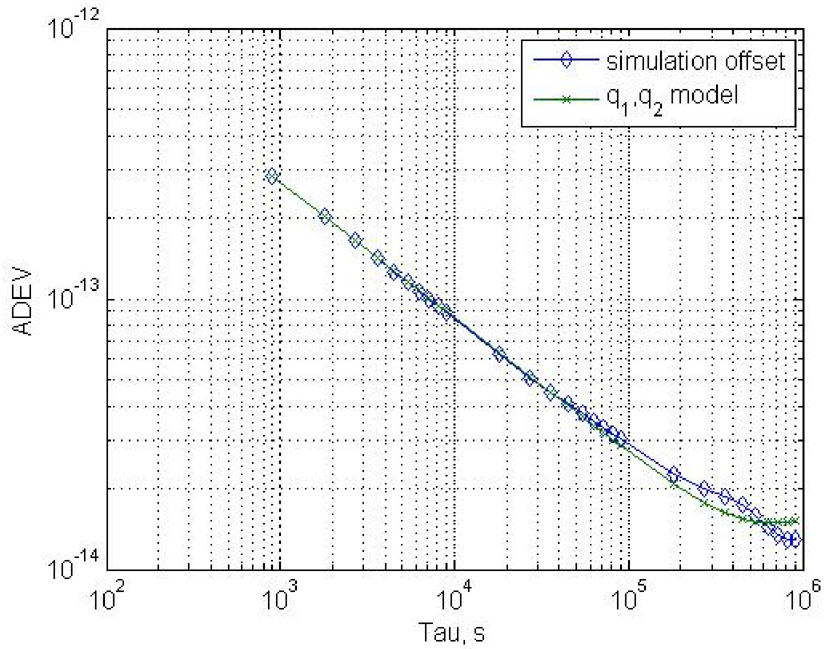

Figure 1: timescale approximation by two model parameters q_1 and q_2

The disadvantage of minimizing the offset between two clocks is that steering affects Allan Deviation. Figure 2 shows the corresponding Allan Deviations without measurement noise. It is obvious that both steering methods increase Allan Deviation for averaging time τ between 10^4 s and 10^6 s. However for these τ values the Allan Deviation of the LQC timescale is smaller than that of the INPL technique. For $\tau > 10^6$ s values the Allan Deviation of LQG and INPL are almost the same and the steered timescales have no more an amount of flicker frequency noise.

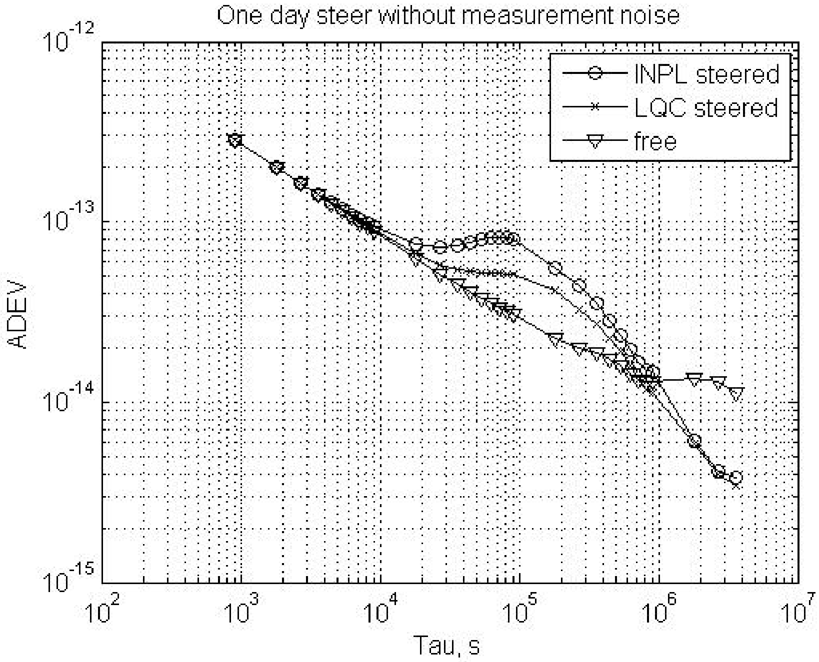

Figure 2: impact of steering on Allan Deviations without measurement noise

The corresponding Allan Deviations including measurement noise are illustrated in Figure 3. For τ values between 10^3 s and $4*10^4$ s Allan Deviation of INPL and free timescale are almost identical. The steering impact first manifests for τ values bigger than $4*10^4$ s and enters earlier than steering with LQC which first increase Allan Deviation for τ values bigger than $6*10^4$ s. The difference between both steering techniques mostly expresses for τ values between 10^3 and $2*10^4$. Here, the ADEV of LQC is distinct lower than both free and INPL steered. This well behaviour is a result of the Kalman filter model which produces accurate offset estimations in the noise environment.

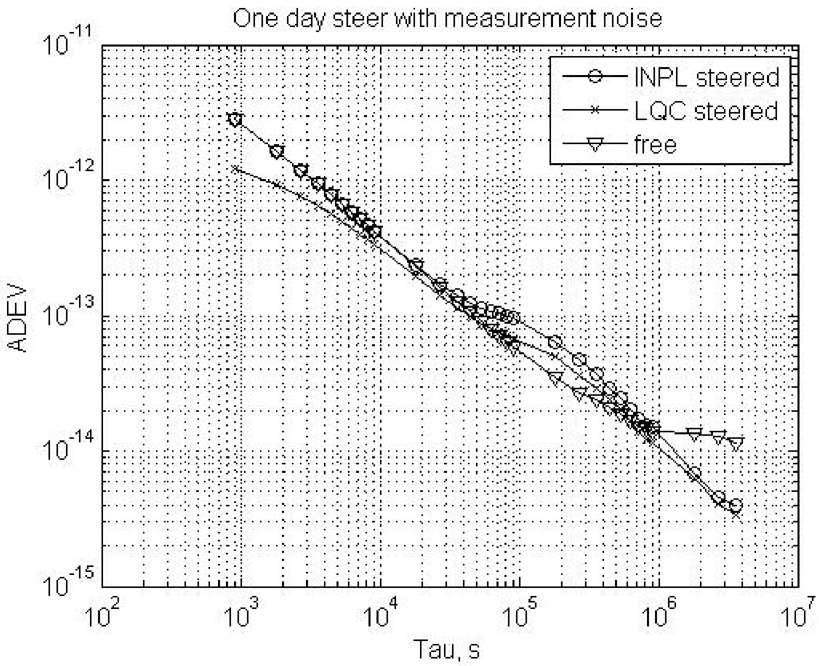

Figure 3: impact of steering on Allan Deviations with measurement noise

The evaluation of the UTC(PTB) steering simulations arise that LQC outperforms INPL in its functionality reducing standard deviation of the timescale offset and also minimal affecting the Allan Deviation. The advancement of processing random measurements with Kalman filter mostly expresses in the evaluation of the Allan Deviation with measurement noise (Figure 3). Here, Allan Deviation of LQC shows the best results especially in short and medium term.

CONCLUSION

The performance difference between both techniques is not as significant as one may expect considering the noticeable difference in algorithmic know-how involved. It mainly results on the usage of Kalman filtered measurements and not only from the usage of Least Quadratic Gaussian Control that LQC outperforms INPL. However, the theory to Least Quadratic Gaussian control is well-known and for this reason it is preferred to INPL.
A second important point to mention is that the simulations clearly point out that the performance in both techniques strongly depends on the accuracy of the computed time and frequency predictions. In particular it is important to focus on advanced algorithms to estimate accurate frequency values at steering time and not only on algorithms producing steering values.
Both techniques assure its functionality to set time and frequency nearby zero. The Standard Deviation of the LQC steered time offset with filtered measurements is equal to 6.6 ns and, thus, in the order of magnitude of the time prediction error of 4 ns of the free and filtered timescale.
Future work will investigate the performance of LQC steering for different frequency offset estimators. The focus will be on the linear regression and moving average method for frequency prediction. The algorithms will be tested for the situation of steering against UTC(BIPM).

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