Solution of the Boltzmann-neutron transport-equation in plane geometry with anisotropic scattering by the double-P_N-method

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The monoenergetic transport-equation for plane geometry can be solved by the Double-P_N-method. This method is especially suited to problems having discontinuities in the angular distribution of the neutron density $n(z, \mu)$ [1, 2, 7] as the neutron density is expanded in two distinct orthogonal sets:

$$n(z, \mu) = \begin{cases} \sum_{k=0}^{N} (2k+1)\, n_k^+(z)\, P_k(2\mu - 1)\,; & 0 < \mu \leq 1 \\ \sum_{k=0}^{N} (2k+1)\, n_k^-(z)\, P_k(2\mu + 1)\,; & -1 \leq \mu < 0 \end{cases} \tag{1}$$

with

$$n_k^+ = \int_0^1 n(z, \mu)\, P_k(2\mu - 1)\, d\mu$$

$$n_k^- = \int_{-1}^0 n(z, \mu)\, P_k(2\mu + 1)\, d\mu\,.$$

For constant material properties the homogeneous solution is:

$$n_k^{+}(z) = \sum_{l=1}^{2(N+1)} A_l\, G_k^{\pm}(\nu_l)\, e^{\nu_l z}\,;\; k = 0, 1, \ldots N \tag{2}$$

The A_l are to be determined by the boundary conditions. The characteristic roots ν_l and the factors $G_k^{\pm}(\nu_l)$ had been evaluated for $N = 0, 1, 2$ by *Ziering* and *Schiff* [3] for isotropic scattering. *Kellner* [4] published generalized expressions for each $N \geqq 0$ but only for isotropic scattering too.

For linear anisotropic scattering it has been found [7]:

$$G_0 = 1$$

$$G_k^+(x) = (-1)^k \Big\{ [1 - (1+\alpha)\, u(x)]\, P_k(x+1) - (1+\alpha)\, w(x)\, W_{k-1}(x+1) \Big\}\,;\; k = 1, 2, \ldots N \tag{3}$$

$$G_k^-(x) = (-1)^k \Big\{ [\alpha - (1+\alpha)\, u(x)]\, P_k(x-1) - (1+\alpha)\, w(x)\, W_{k-1}(x-1) \Big\}\,;\; k = 0, 1, 2, \ldots N \tag{4}$$

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$$x = \frac{2}{\nu_l}$$

$$u(x) = \frac{3}{8}\, x^2\, c\, f_1\, (c\, f_0 - 1)$$

$$w(x) = x \left[\frac{1}{2}\, c\, f_0 - x^2\, (c\, f_0 - 1) \right]$$

$$\alpha = \frac{u(x)\, P_{N+1}(x-1) + w(x)\, W_N(x-1)}{(1 - u(x))\, P_{N+1}(x-1) - w(x)\, W_N(x-1)}\,.$$

The $2\,(N+1)$ characteristic roots ν_l are the roots of the equation:

$$(1 - 2u(x))\, P_{N+1}(x+1)\, P_{N+1}(x-1) - w(x) \left[P_{N+1}(x+1)\, W_N(x-1) + P_{N+1}(x-1)\, W_N(x+1) \right] = 0 \tag{5}$$

The $P_j(z)$ are the Legendre-Polynomials of the first kind. The $W_j(z)$ are defined using the Legendre Polynomials of the second kind $Q_j(z)$ [5, 6]:

$$W_{j-1}(z) = Q_0(z) \cdot P_j(z) - Q_j(z) \tag{6}$$

c is the number of secondaries [2]. f_l are the coefficients of the expansion of the kernel $f(\mu_0)$:

$$f_l = \int_{-1}^{+1} f(\mu_0)\, P_l(\mu_0)\, d\mu_0 \tag{7}$$

For linear anisotropic scattering $f_l = 0$ if $l > 1$. In the isotropic case $f_1 = 0$ as well.

For the proof, discussion and comparison with the P_N-method refer to [7].

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References

[1] J. Yvon: La diffusion macroscopique des neutrons, une méthode d'approximation. J. Nucl. Energy 1, 4, S. 305—316 (1957).

[2] B. Davison: Neutron transport theory; Oxford 1958.

[3] S. Ziering, D. Schiff: Yvon's method for slabs. Nucl. Sci. Eng 3, S. 635—647 (1958).

[4] H. Kellner: Über die Yvon'sche Approximationsmethode. Atomkernenergie 6, S. 441—443 (1961).

[5] E. T. Whittaker, G. N. Watson: A course of modern analysis. Cambridge 1958.

[6] Jahnke-Emde: Tafeln höherer Funktionen. Leipzig 1948.

[7] U. Schumann, G. Bartsch: Theorie und Rechner-Programme zur Lösung der Neutronen-Transport-Gleichung nach der P_N- und der Doppel-P_N-Methode für Plattengeometrie. TUBIK-16, Technische Universität Berlin (in preparation).