The Pressure-Strain Correlation of a Turbulent Homogeneous Shear Flow Under Strongly-Stable Stratification

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Results are presented concerning the direct numerical solution of the exact time-dependent three-dimensional Navier-Stokes equations and the first law of thermodynamics for an incompressible turbulent flow possessing uniform gradients of the mean velocity and temperature in the vertical direction. The equations are solved in a cubical domain representing a part of the homogeneous flow and divided into 64^3 grid cells. Boundary conditions are periodic in the horizontal and shear-periodic in the vertical (shear) direction. The model is described in detail in ELGHOBASHI et al. [1] and in SCHUMANN et al. [2].

The behaviour of a homogeneous shear-flow with initially fully turbulent, isotropic velocity and temperature fields under strongly-stable stratification is considered. In flows with Richardson-numbers (Ri) larger than 0.25 the turbulent kinetic energy is expected to decrease with time. The trend of increasing conversion of kinetic energy into potential energy for increasing Ri-numbers inverts if the stratification is very stable ($Ri = 1$, see Figure 1). This effect can be explained in studying the dynamical behaviour of the turbulent vertical momentum flux $\overline{uw}$ and the heat flux $\overline{wT}$ for $Ri = 1$. Figure 2 shows that these quantities, here normalized by their rms-values, change from negative to positive fluxes. These counter-gradient fluxes, also observed by KOMORI et al. [3] in a water-tank experiment and discussed in [1], indicate that the initially turbulent flow changes

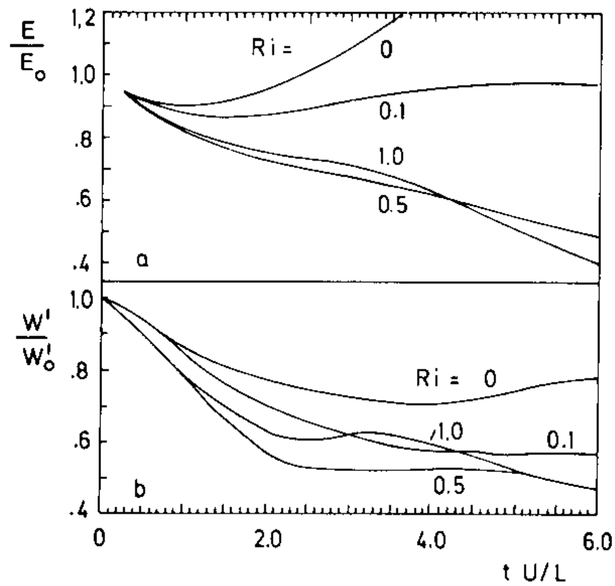

Figure 1. Time variation of the turbulent kinetic energy E (a) and of the rms-value of w (b) normalized by their initial values for different Ri

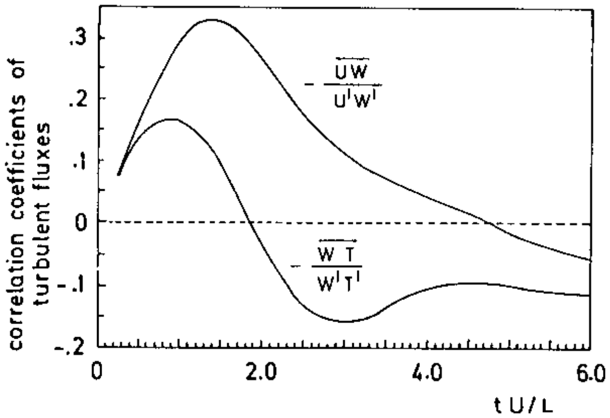

Figure 2. Time variation of the mean one-point correlations $-\overline{uw}/u'w'$ and $-\overline{wT}/w'T'$ normalized by the rms-values for $Ri = 1$

more and more to a wave-like motion. The fluxes no longer damp the turbulent motion rather than enhance the kinetic energy in its vertical component $\overline{ww}$ in the time interval $2.0 < tU/L < 3.5$ (Figure 1).

Interestingly, these dynamics are also reflected in the pressure-strain correlation

$$\phi_{ij} = \overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}. \tag{1}$$

Figure 3 shows the dimensionless ϕ_{11}, ϕ_{22}, ϕ_{33} and ϕ_{13} as a function of tU/L obtained by direct simulation. ϕ_{11} and ϕ_{33} change sign in the time-interval $1.8 < tU/L < 2.7$ when the counter-gradient heat-flux occurs and attains its maximum value (Figure 2). Looking at the dynamic equations of the components of the turbulent kinetic energy the interaction of fluxes and strain can be explained even if the linear parts are considered alone. These equations read in non-dimensional

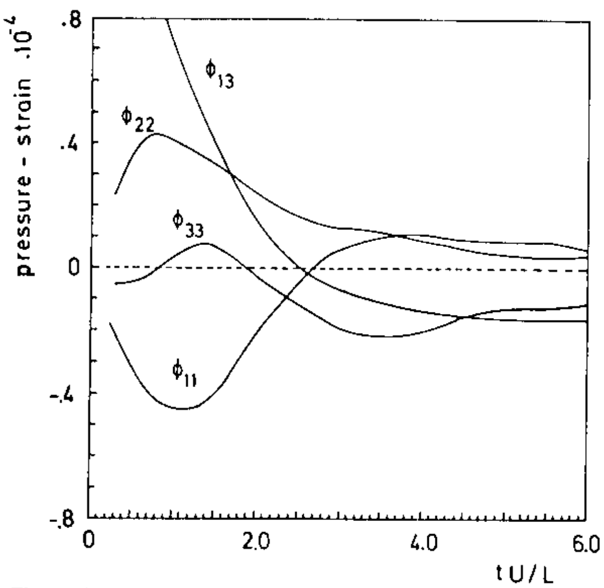

Figure 3. Time variation of the pressure-strain correlations ϕ_{11}, ϕ_{22}, ϕ_{33} and ϕ_{13} obtained by direct simulation for $Ri = 1$

form:

$$\frac{\partial \overline{uu}}{\partial t} = -2S\,\overline{uw} \quad + \phi_{11} - \epsilon_{11} \tag{2}$$

$$\frac{\partial \overline{vv}}{\partial t} = \qquad\qquad + \phi_{22} - \epsilon_{22} \tag{3}$$

$$\frac{\partial \overline{ww}}{\partial t} = 2\,|Ri|\,\overline{wT} + \phi_{33} - \epsilon_{33} \tag{4}$$

where ϵ_{ij} stands for the dissipation rate of momentum fluxes and $S = L/U\, d\overline{U}/dz$ for the non-dimensional shear. L is the scale-length of the domain and U the scale-velocity of the flow. (Note that $\sum \phi_{ii} = 0$.)

Initially, ϕ_{11} is negative and redistributes energy from the streamwise ($\overline{uu}$) component to the lateral ($\overline{vv}$) and the vertical ($\overline{ww}$) parts because $\overline{uu}$ is enhanced strongest due to shear ($\overline{uw}$ is large and negative, (2)) while ϕ_{22} and ϕ_{33}, both positive, act like source terms for $\overline{vv}$ and $\overline{ww}$ (Fig. 2, 3). The former obtains more energy than the latter due to the strong stability of the turbulent flow which prohibits any increase of energy against the buoyancy forces. The heatflux $\overline{wT}$ is negative during this period and thus, acts as a sink term in the $\overline{ww}$-equation (4). The situation, however, is reversed when the turbulent heat-flux changes sign. $\overline{ww}$ is now mostly supported by $\overline{wT}$ ("wave-pumping") and ϕ_{33} as a negative quantity redistributes energy in equal shares to $\overline{uu}$ and $\overline{vv}$ (ϕ_{11} and ϕ_{22} are positive, see Figure 3). At this time $-\,\overline{uw}$ is decreasing but still positive. Later on, also a counter-gradient momentum-flux occurs ($-\,\overline{uw} < 0$, Figure 2) accompanied by a small increase of ϕ_{33} and correspondingly a decrease of ϕ_{11} and ϕ_{22}. The situation has now reached a "quasi-steady" state ([2]).

After the strong decrease of $-\,\overline{wT}$ and its sign-change at $tU/L \approx 1.5$ (Figure 2), the flow is no longer purely turbulent but has clearly wave-like characteristics. This is also indicated by the shift of the phase angle between w and T ([3]). With increasing Ri the angle shifts from $\pm\,\pi$ (i.e. turbulent motion) to $\pm\,\pi/2$ (i.e. wave motion) and further on to zero which describes a flow with a wave-like motion intermittent by "hot eddies" which are responsible for the occurence of the counter-gradient heat-flux.

The reaction of the pressure-strain correlation to these dynamics cannot be described by the second-order parametrization for ϕ_{ij} proposed by LAUNDER [4,5], although all the contributing terms describing the influence of turbulent interaction, shear and buoyancy are included. The equations read in dimensionless form:

$$\phi_{ij} = \phi_{ij_1} + \phi_{ij_2} + \phi_{ij_3} \tag{5}$$

where

$$\phi_{ij_1} = -c_1 \frac{\epsilon}{E} \Big(\overline{u_i u_j} - \frac{2}{3} E\, \delta_{ij} \Big) \tag{6}$$

$$\phi_{ij_2} = -c_2 \Big(-\overline{u_i u_k}\, S_{j,k} - \overline{u_j u_k}\, S_{i,k} + [\frac{2}{3}\, \overline{u_i u_k}\, S_{i,k}]\, \delta_{ij} \Big) \tag{7}$$

$$\phi_{ij_3} = -c_3 \Big(\,|Ri_j|\, \overline{u_i T} + |Ri_i|\, \overline{u_j T} - [\frac{2}{3}\, |Ri_i|\, \overline{u_i T}]\, \delta_{ij} \Big) \tag{8}$$

describe the turbulent interaction, the production due to shear and the production due to buoyancy, respectively.

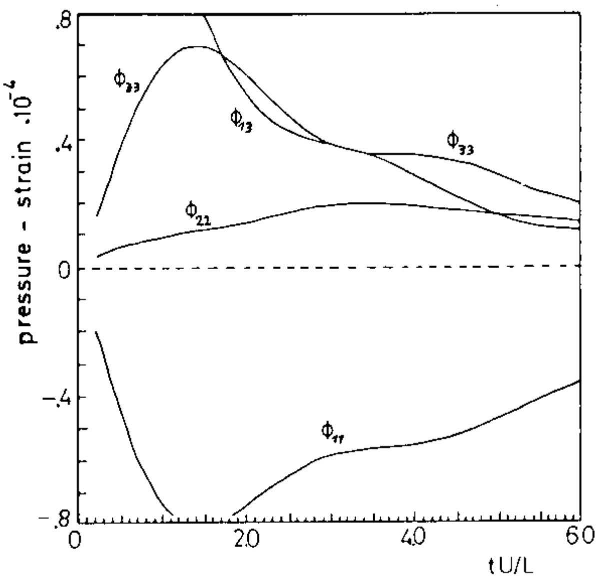

Figure 4. Time variation of the pressure-strain correlations obtained by Launders's model (1975,1976) with the constants $c_1 = 2.0$, $c_2 = 0.6$, $c_3 = 0.3$ for $Ri = 1$

$\epsilon = \nu \overline{(\partial u_i / \partial x_k)^2}$, $E = 1/2\ \overline{u_i u_i}$, $Ri_i = -\alpha g_i L \Delta T / U^2$ $(g_i = (0,0,-g))$, $S_{i,k} = L/U\ \partial \overline{U}_i / \partial x_k = (0,0,S)$ stand for the total rate of dissipation, the kinetic energy, the Richardson-number and the non-dimensional shear, respectively.

Figure 4 shows the time variation of ϕ_{11}, ϕ_{22}, ϕ_{33} and ϕ_{13} obtained by LAUNDER's model for the same flow described above ($Ri = 1$) for the coefficients $c_1 = 2.0$, $c_2 = 0.6$ and $c_3 = 0.3$. Although $-\phi_{11}$ demonstrates the correct development at the beginning qualitatively and decreases when the counter-gradient heat-flux is established, the parametrization with the coefficients mentioned above does not account for the change in the redistribution among the turbulent intensities. The redistribution of energy to the lateral and vertical components is described incorrectly. ϕ_{22} is underestimated and ϕ_{33} is overestimated in comparison with the results of the direct simulation (Figure 3).

The terms ϕ_{ii_1}, ϕ_{ii_2} and ϕ_{ii_3} are shown separately in Figure 5 with the same coefficients as in Figure 4. It can be seen that the turbulent interaction term (6) (dashed lines) dominates and thus, leads to a wrong interpretation of the dynamics of the flow. The value of the coefficient c_1 assumed by LAUNDER which gives good results for a flow under non-buoyant conditions ([5]) is too large for a flow under strongly-stable stratification. On the other hand, the coefficient c_3 which modifies the influence of the buoyancy term (8) seems to be too small for the considered flow.

Looking at the solid and dotted lines of Figure 5 which represent the shear term and the gravitational term the superposition of both describes the time variation of the pressure-strain correlations already in a qualitatively correct manner. Figure 6 now, shows the results of the parametrization obtained after increasing c_3 to a value of 0.6 which was already suggested by LAUNDER [4] and neglecting the share of the turbulent interaction (i.e. $c_1 = 0$). The dynamical behaviour of ϕ_{ij} experiences at least the sign-changes of ϕ_{11}, ϕ_{33} and ϕ_{13} which are predicted by the direct simulation. The situation is fairly good reproduced also quantitatively when the flow has reached a "quasi-steady" state at $tU/L \approx 4.7$.

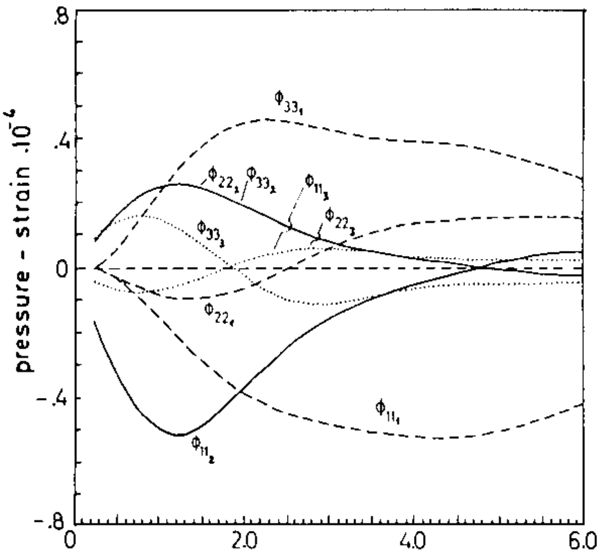

Figure 5. Time variation of the components of the pressure-strain correlations ϕ_{11}, ϕ_{22} and ϕ_{33}; dashed lines: turbulent interaction, solid lines: production due to shear, dotted lines: production due to buoyancy. The coefficients are the same as in Figure 4.

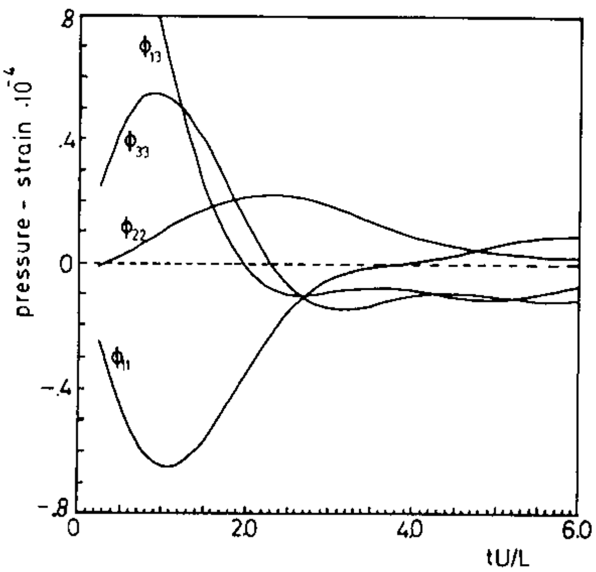

Figure 6. Same as Figure 4 but for the coefficients $c_1 = 0.$, $c_2 = 0.6$, $c_3 = 0.6$.

Not satisfactory, however, is the behaviour of the parametrized quantities ϕ_{22} and ϕ_{33} at the beginning when the vertical momentum flux $\overline{uw}$ dominates the physics of the flow (see Figure 2). A reason might be that almost all energy production due to shear goes from the streamwise to the lateral component because the stratification is too stable to allow an increase of energy in the direction against the buoyancy force. Thus, the energy source of ϕ_{22} due to shear might be larger than the parametrization of ϕ_{22_2} ((7) for $i = j = 2$) suggests. To consider this effect one should examine the redistribution among the flow components for the shear term (7) itself.

In comparison with the results of a direct simulation it could be shown that for flows under strongly-stable stratification ($Ri = 1$) LAUNDER's parametrization of the pressure-strain correlations yields good results when the flow has reached a "quasi-steady" state and if the coefficients are modified to account for the strength of the stability. In the future the dependency of these coefficients on the strength of the stratification will be examined.

Acknowledgement. This work is supported by the Deutsche Forschungsgemeinschaft.

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