JOINT BAYESIAN POSITIONING AND MULTIPATH MITIGATION IN GNSS

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ABSTRACT

A sequential Bayesian estimation algorithm for joint positioning and multipath mitigation within satellite navigation receivers is presented. The underlying process model is especially designed for dynamic user scenarios and dynamic channel conditions. To demonstrate its capabilities simulation results are presented.

***Index Terms*—** Navigation, Synchronization, Satellite navigation systems, Multipath channels

1. INTRODUCTION

Within global navigation satellite systems (GNSS) the user position is determined based upon the navigation signals received from different satellites (here index by $j = 1, \dots, M$) using the time-of-arrival (TOA) method [1]. Multipath, the reception of additional signal replica due to reflections caused by the receiver environment, is a major source of positioning errors in GNSS, as it introduces a bias into the time delay estimate of the delay lock loop (DLL) of a conventional navigation receiver. Conventional approaches try to mitigate the effect of multipath for each received satellite separately per channel, either by modification of the traditional DLL detector slope [1] or by explicit estimation of the multipath channel parameters [2]. Other approaches exploit the advantageous properties of the position domain likelihood [3]. However, none of these make explicit use of the user's and channel's temporal or spatial dynamics. To address this drawback we suggest a joint positioning and multipath estimation approach based on Bayesian filtering, the optimal and well-known framework to address dynamic state estimation problems.

2. SIGNAL MODEL

Assume that the complex valued baseband-equivalent received signal for the receiver processing channel associated to satellite j is equal to

$$z_j(t) = \sum_{i=1}^{N_m} e_{i,j}(t) \cdot a_{i,j}(t) \cdot [c_j(t) * g(t - \tau_{i,j}(t))] + n_j(t), \quad (1)$$

where $c_j(t)$ is a delta-train code sequence that is modulated on a pulse $g(t)$, N_m is the total number of paths reaching the receiver, $e_{i,j}(t)$ is a binary function that controls the activity of the i'th path and $a_{i,j}(t)$ and $\tau_{i,j}(t)$ are their individual complex amplitudes and time delays, respectively. The signal is disturbed by additive white Gaussian noise $n_j(t)$. Grouping blocks of L samples at times $(m + kL)T_s$, $m = 0, \dots, L-1$, together into vectors $\mathbf{z}_{j,k}$, $k = 0, 1, \dots$, whilst assuming the parameter functions $e_{i,j}(t)$, $a_{i,j}(t)$ and $\tau_{i,j}(t)$ to be constant within the corresponding time interval and equal to $e_{i,j,k}$, $a_{i,j,k}$ and $\tau_{i,j,k}$, the signal for block k can be rewritten as

$$\mathbf{z}_{j,k} = \underbrace{\mathbf{C}_j \mathbf{G}(\boldsymbol{\tau}_{j,k}) \mathbf{E}_{j,k} \mathbf{a}_{j,k}}_{\mathbf{s}_{j,k}} + \mathbf{n}_{j,k} \quad . \quad (2)$$

In the compact form the samples of the delayed pulses $\mathbf{g}(\tau_{i,j,k})$ are stacked together as columns of the matrix $\mathbf{G}(\boldsymbol{\tau}_{j,k}) = [\mathbf{g}(\tau_{1,j,k}), \dots, \mathbf{g}(\tau_{N_m,j,k})]$, $\mathbf{C}_j$ is a matrix representing the convolution with the code, and the delays and amplitudes are collected in the vectors $\boldsymbol{\tau}_{j,k} = [\tau_{1,j,k}, \dots, \tau_{N_m,j,k}]^T$ and $\mathbf{a}_k = [a_{1,j,k}, \dots, a_{N_m,j,k}]^T$, respectively. For concise notation we use $\mathbf{E}_{j,k} = \operatorname{diag}[\mathbf{e}_{j,k}]$ whilst the elements of the vector $\mathbf{e}_{j,k} = [e_{1,j,k}, \dots, e_{N_m,j,k}]^T$ fulfill $e_{i,j,k} \in [0, 1]$. The term $\mathbf{s}_{j,k}$ denotes the signal hypothesis and is completely determined by the channel parameters $\boldsymbol{\tau}_{j,k}$, $\mathbf{a}_{j,k}$ and $\mathbf{e}_{j,k}$. Using (2) we can write the associated *channel likelihood function* as

$$p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k}) = \frac{1}{(2\pi)^L \sigma_j^{2L}} \cdot \exp\left[-\frac{|\mathbf{z}_{j,k} - \mathbf{s}_{j,k}|^2}{2\sigma_j^2}\right]. \quad (3)$$

2.1. Optimization

To reduce the number of signal parameters to be estimated we optimize (3) for a given set of $\boldsymbol{\tau}_{j,k}$ and $\mathbf{e}_{j,k}$ with respect to the complex amplitudes $\mathbf{a}_{j,k}$, which can be achieved through a closed form solution. Using

$$\mathbf{S}_{j,k} = \mathbf{C}_j \mathbf{G}(\boldsymbol{\tau}_{j,k}) \mathbf{E}_{j,k} \quad (4)$$

and obtaining $\mathbf{S}_{j,k}^{+}$ by removing zero columns from $\mathbf{S}_{j,k}$ we compute the corresponding ML amplitude values of the active paths:

$$\mathbf{a}_{j,k}^{\mathrm{ML}} = \left(\mathbf{S}_{j,k}^{+H} \mathbf{S}_{j,k}^{+}\right)^{-1} \mathbf{S}_{j,k}^{+H} \mathbf{z}_{j,k} \quad . \quad (5)$$

When evaluating (3) we use

$$\mathbf{s}_{j,k} = \mathbf{S}_{j,k}\hat{\mathbf{a}}_{j,k} \quad , \tag{6}$$

where the elements of the vector $\hat{\mathbf{a}}_{j,k}$ that are indicated to have an active path are set equal to the corresponding elements of $\mathbf{a}_{j,k}^{\mathrm{ML}}$.

2.2. Parameter Transformation

The signal paramters $\tau_{1,j,k}$ associated to different ranging sources j are mutually dependent because of the common receiver position and clock offset. To exploit this fact we replace the line-of-sight delays by their navigation parameter equivalents using the TOA equation [1]

$$\tau_{1,j,k} = \left|\mathbf{p}_{j,k}^t - \mathbf{p}_k^r\right| c^{-1} + \tau_k^r + \varepsilon_{j,k} \tag{7}$$

with the known position of the transmitting satellite $\mathbf{p}_{j,k}^t$, the receiver position $\mathbf{p}_k^r$, the receiver clock bias τ_k^r and the speed of light c. The error term $\varepsilon_{j,k}$ includes atmospheric propagation errors and transmitter clock offsets and is assumed to be known.

3. PROCESS AND SYSTEM MODEL

The purpose of the process model is to characterize the temporal evolution of the parameters introduced in Section 2 in a probabilistic fashion.

3.1. User Model

The temporal evolution of the receiver position used in (7) can be characterized by a physical movement model of the user or vehicle that carries the receiver. Here we use

$$\mathbf{p}_k^r = \mathbf{p}_{k-1}^r + \dot{\mathbf{p}}_{k-1}^r \cdot T_s + \mathbf{n}_p \tag{8}$$
$$\dot{\mathbf{p}}_k^r = \dot{\mathbf{p}}_{k-1}^r + \mathbf{n}_{\dot{p}} \tag{9}$$

with $\dot{\mathbf{p}}_k^r$ being the temporal derivative of $\mathbf{p}_k^r$, and $\mathbf{n}_p$, $\mathbf{n}_{\dot{p}}$ being vectors of element-wise uncorrelated zero-mean white Gaussian noise, whose elements have a given variance of σ_x^2, σ_y^2, σ_z^2 and $\sigma_{\dot{x}}^2$, $\sigma_{\dot{y}}^2$, $\sigma_{\dot{z}}^2$, respectively.

3.2. Clock Model

The clock model is used to characterize the local receiver clock offset τ_k^r and its drift $\dot{\tau}_k^r$. We use this simple model:

$$\tau_k^r = \tau_{k-1}^r + \dot{\tau}_{k-1}^r \cdot T_s + n_\tau \quad , \tag{10}$$
$$\dot{\tau}_k^r = \dot{\tau}_{k-1}^r + n_{\dot{\tau}}. \tag{11}$$

The noise terms n_τ and $n_{\dot{\tau}}$ are realizations of a zero-mean white Gaussian noise process of variance σ_τ^2 and $\sigma_{\dot{\tau}}^2$, respectively.

3.3. Multipath Channel Model

The multipath channel is determined by the parameters $e_{i,j,k}$ and $\tau_{i,j,k}$ with $i > 0$. According to [4] their temporal evolution is modeled by the following statistical processes:

3.3.1. Multipath Activity

According to (2) each path is either "on" or "off", as defined by channel parameter $e_{i,j,k} \in \{1 \equiv \text{"on"}, 0 \equiv \text{"off"}\}$, where $e_{i,j,k}$ is assumed to follow a simple two-state Markov process with a-symmetric crossover and same-state probabilities:

$$p(e_{i,j,k} = 0 | e_{i,j,k-1} = 1) = p_{\mathrm{onoff}} \quad , \tag{12}$$
$$p(e_{i,j,k} = 1 | e_{i,j,k-1} = 0) = p_{\mathrm{offon}} \quad . \tag{13}$$

3.3.2. Multipath Delay

The associated delays of the multipath replica are characterized by

$$\boldsymbol{\tau}_k^{\mathrm{mp}} = \boldsymbol{\tau}_{k-1}^{\mathrm{mp}} + \dot{\boldsymbol{\tau}}_{k-1}^{\mathrm{mp}} \cdot T_s + \mathbf{n}_{\mathrm{mp}} \quad , \tag{14}$$
$$\dot{\boldsymbol{\tau}}_k^{\mathrm{mp}} = \dot{\boldsymbol{\tau}}_{k-1}^{\mathrm{mp}} + \mathbf{n}_{\dot{\mathrm{m}}\mathrm{p}} \quad , \tag{15}$$

where for concise notation we have used $\boldsymbol{\tau}_k^{\mathrm{mp}} \widehat{=} \{\boldsymbol{\tau}_{j,k}^{\mathrm{mp}}, j = 1, \ldots, M\}$ with $\boldsymbol{\tau}_{j,k}^{\mathrm{mp}} = [\tau_{2,j,k}, \ldots, \tau_{N_m,j,k}]^T$. M is the total number of received satellites. The temporal derivative of $\boldsymbol{\tau}_k^{\mathrm{mp}}$ is denoted by $\dot{\boldsymbol{\tau}}_k^{\mathrm{mp}}$ and $\mathbf{n}_{\mathrm{mp}}$, $\mathbf{n}_{\dot{\mathrm{m}}\mathrm{p}}$ are vectors of element-wise uncorrelated zero-mean white Gaussian noise of variance σ_{mp}^2 and $\sigma_{\dot{\mathrm{m}}\mathrm{p}}^2$, respectively.

3.4. State Vector

Considering the signal model and the process model we collect the remaining unknowns using $\mathbf{e}_k \widehat{=} \{\mathbf{e}_{j,k}, j = 1, \ldots, M\}$ into the state vector

$$\mathbf{x}_k \widehat{=} \{\mathbf{p}_k^r, \dot{\mathbf{p}}_k^r, \tau_k^r, \dot{\tau}_k^r, \boldsymbol{\tau}_k^{\mathrm{mp}}, \dot{\boldsymbol{\tau}}_k^{\mathrm{mp}}, \mathbf{e}_k\} \quad . \tag{16}$$

3.5. Likelihood Factorization

So far we have introduced the channel likelihood (3) associated to the receiver processing channel j. Given (3) we are now able to calculate the overall likelihood, namely $p(\mathbf{z}_k|\mathbf{x}_k)$ with $\mathbf{z}_k \widehat{=} \{\mathbf{z}_{j,k}, j = 1, \ldots, M\}$. Assuming independent noise realization for the channels this function can be written in product form as

$$p(\mathbf{z}_k|\mathbf{x}_k) = C \cdot \prod_{j=1}^{M} p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k}) \tag{17}$$

with C being a normalizing constant. Please note that according to (4), (5), (6) and (7) the signal hypothesis $\mathbf{s}_{j,k}$ is determined completely by $\mathbf{x}_k$ and $\mathbf{z}_{j,k}$ and it can be shown that $p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k}) \approx p(\mathbf{z}_{j,k}|\mathbf{x}_k)$.

4. SEQUENTIAL ESTIMATION

To overcome the drawback of the conventional approaches mentioned in Section 1 our objective here is to address the introduced estimation problem with a sequential estimator, which is able to exploit not only a single set of observations $\mathbf{z}_k$ to estimate the hidden parameters $\mathbf{x}_k$ (via the likelihood function), but is also able to exploit our knowledge about the statistical dependencies between successive sets of position, clock and multipath channel parameters, in order to improve the performance of the estimator.

4.1. Optimal Solution

Given the models introduced in Section 2 and 3 the problem of positioning and multipath mitigation now becomes one of *sequential estimation of a hidden Markov process*: We want to estimate the unknown position, clock and multipath channel parameters, namely the hidden state $\mathbf{x}_k$ based on an evolving sequence of received noisy observations $\mathbf{z}_k$. To achieve this we may apply the concept of *sequential Bayesian estimation*. The reader is referred to [5] which gives a derivation of the general framework for optimal estimation of temporally evolving (Markovian) parameters by means of inference. We have chosen similar notation. The entire history of observations (over the temporal index k) can be written as

$$\mathbf{Z}_k \widehat{=} \{\mathbf{z}_q, q = 0, \ldots, k\} \ . \quad (18)$$

As $\mathbf{x}_k$ represents the characterization of the hidden state our goal is to determine the *posterior* probability density function (PDF) of every possible state characterization given all observations: $p(\mathbf{x}_k|\mathbf{Z}_k)$.

The sequential estimation algorithm is recursive as it uses the posterior PDF computed for time instance $k-1$ to compute the posterior PDF for instance k. For a given posterior PDF at time instance $k-1$, $p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})$, the *prior* PDF $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ is calculated in the so-called *prediction step* by applying the Chapman-Kolmogorov equation:

$$p(\mathbf{x}_k|\mathbf{Z}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1}) \mathrm{d}\mathbf{x}_{k-1} \ , \quad (19)$$

with $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ being the state transition PDF of the Markov process. In the *update step* the new posterior PDF for step k is obtained by applying Bayes' rule to $p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}_{k-1})$ yielding the normalized product of the likelihood $p(\mathbf{z}_k|\mathbf{x}_k)$ and the prior PDF:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{Z}_k) &= p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}_{k-1}) \\ &= \frac{p(\mathbf{z}_k|\mathbf{x}_k, \mathbf{Z}_{k-1}) p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \\ &= \frac{p(\mathbf{z}_k|\mathbf{x}_k) p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \ . \end{aligned} \quad (20)$$

The denominator of (20) does not depend on $\mathbf{x}_k$ and so it can be computed by integrating the numerator of (20) over the entire range of $\mathbf{x}_k$ (normalization).

4.2. Sequential Estimation using Particle Filters

The optimal estimation algorithm relies on evaluating the integral (19), which is usually a very difficult task, except for certain additional restrictions imposed on the model and the noise process. Hence very often a suboptimal realization of a Bayesian estimator has to be chosen for implementation. In the work presented here we use a Sequential Monte Carlo (SMC) filter, in particular a Sampling Importance Resampling Particle Filter SIR PF according to [5]. In this algorithm the posterior density at step k is represented by a set of N_p particles, where each particle with index μ has a state $\mathbf{x}_k^\mu$ and has a *weight* w_k^μ. The key step in which the *measurement* for instance k is incorporated, is in the calculation of the weight w_k^μ which for the SIR PF can be shown to be the likelihood function: $p(\mathbf{z}_k|\mathbf{x}_k^\mu)$. The characterization of the *process* enters in the algorithm when at each time instance k, the state of each particle $\mathbf{x}_k^\mu$ is drawn randomly from $p(\mathbf{x}_k|\mathbf{x}_{k-1}^\mu)$.

5. PERFORMANCE EVALUATION

To demonstrate the capabilities of the proposed estimator simulations were carried out. The employed navigation signal is a BPSK modulated GPS C/A code signal having a two-sided bandwidth of 20 MHz. In the simulations it is assumed that four satellites are received with a C/N_0 of 50 dB-Hz respectively. The geometry of the four transmitting satellites is 58, 65, 135 and 195 degrees for the azimuth values and 67, 27, 51 and 39 degrees for the elevation values. The SIR PF runs with a observation period of 10 ms. As reference the SIR PF results are shown together with results obtained based upon conventional signal tracking and least squares (LS) position estimation [1] with a non-coherent delay lock loop with 0.15 chip early/late correlator spacing and 2 Hz tracking loop bandwidth.

5.1. Static Multipath Channel

In Figure 1 the performance of the SIR PF is shown by means of the root mean square error (RMSE) of the minimum mean square error (MMSE) position estimates obtained from the posterior as a function of the multipath delay for a static multipath on the signal associated to the satellite channel $j = 1$ at a signal to multipath ratio of $6dB$. It can be observed that the SIR PF performs significantly better than the conventional DLL+LS approach even without the estimator modeling the multipath ($N_m = 1$). Further improvement is possible, if the multipath is taken into account by the SIR PF ($N_m = 2$).

5.2. Dynamic Multipath Channel

Furthermore we have carried out simulations under a dynamic multipath scenario. Results for a randomly chosen dynamic channel are depicted in Figure 2 for two kinds of SIR PFs,

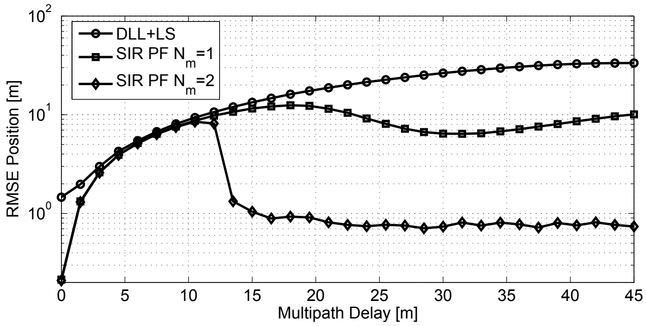

Fig. 1. Static multipath scenario on range 1: Performance of DLL+LS approach, SIR PF with single path model and SIR PF with path activity tracking as function of multipath delay.

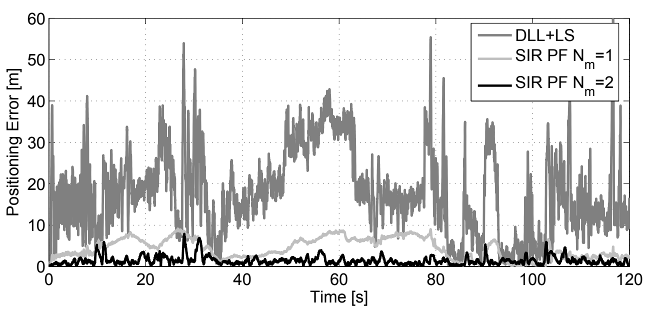

Fig. 2. Performance of DLL+LS approach, SIR PF with single path model and SIR PF with path activity tracking.

one using $N_m = 1$ and the other using $N_m = 2$, both running with 20 000 particles, respectively. The SIR PF results show the magnitude of the error of the MMSE position estimate. Figure 3 shows the multipath channel affecting the fourth channel including the MMSE estimates of the path delays as example. To consider two different types of echoes the amplitude of the echoes in the simulation is either picked randomly from 0.1 up to 0.2 times the amplitude of the direct path (weak echo) or picked randomly from 0.6 up to 0.8 times the amplitude of the direct path (strong echo). The DLL performance suffers significantly from the multipath reception (RMSE = 17.97 m) and the SIR PF using $N_m = 1$ (RMSE = 4.31 m) is able to outperform it, as it exploits the properties of the position domain likelihood as well as the position and clock parameter movement models. Further improvement is achieved with the SIR PF with $N_m = 2$ (RMSE = 1.42 m). Despite up to three echoes being active simultaneously and the estimators restriction to two paths it can be observed that the SIR PF tracks predominantly the strong multipath signals.

6. CONCLUSIONS

We have demonstrated how sequential Bayesian estimation techniques can be applied to the joint positioning and multipath mitigation problem in a navigation receiver. The proposed approach is characterized by a particle filter realization of the prediction and update recursion. The considered movement model has been adapted to dynamic user and multipath channel scenarios and incorporates the number of echoes as a time varying hidden channel state variable that is tracked together with the position and clock parameters in a probabilistic fashion. A promising advantage compared to existing ML estimation approaches is that the posterior PDF at the output of the estimator represents reliability information about the desired parameters and preserves the ambiguities and multiple modes that may occur within the likelihood function. Simulation results for a GPS-like positioning scenario show that the proposed sequential estimator can achieve significant improvements compared to the conventional tracking and positioning approach.

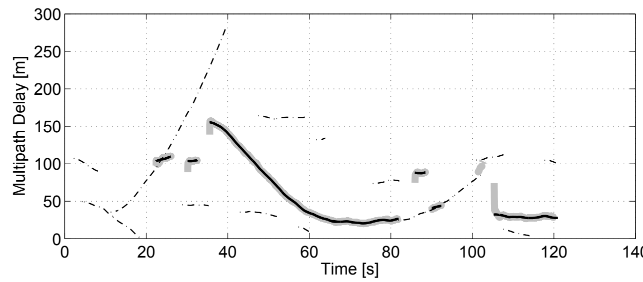

Fig. 3. Example of multipath channel on range 4 with weak (dash-dotted) and strong (bold) echoes. Estimated echo tracks (grey) shown if $p(e_{2,4,k}|\mathbf{Z}_k) > 0.8$.

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