

A Novel Movement Model for Pedestrians Suitable for Personal Navigation

Mohammed Khider, Susanna Kaiser, Patrick Robertson, Michael Angermann, *German Aerospace Center (DLR), Germany*

BIOGRAPHY

All authors are with German Aerospace Center, Institute of Communication and Navigation, 82234 Wessling, Germany, <name>.<lastname>@dlr.de, correspondence Author: Mohammed Khider, Phone ++49 8153 28 2830.

ABSTRACT

In this paper a combination of two movement models, operating at the microscopic level and suitable for pedestrian navigation is developed and tested. The constituents are a Stochastic Behavioral Movement Model to characterize more random motion and a Diffusion Movement Model to characterize a geographic goal a pedestrian might walk towards. A top-level Markov process is used to determine whether to currently use the stochastic behavioral or the diffusion model; therefore, the model switches between motion that is more goal oriented (diffusion model) or stochastic.

Advantages and disadvantages of both individual constituent models are demonstrated and discussed. The combined movement model is demonstrated to achieve the best of both worlds and to avoid the problems associated with using a single model. The properties and the performance of the resulting model will be explained in details.

1. INTRODUCTION

1.1. APPLICATIONS OF MOVEMENT MODELS

Pedestrian movement models are used to quantitatively represent the stochastic nature of pedestrian movement. Research in such movement models has mainly been applied to planning tasks, such as estimating pedestrian flows when planning a train station or airport, or optimization of evacuation procedures and evacuation paths for a large shopping mall or theater [Hel92a][Tek02][OkM93]. Another application area which is becoming increasingly important is that of dynamic indoor positioning and navigation. Within the scope of this paper, the purpose of such movement models is to simulate a single pedestrian's movement as

realistically as possible, so that it can be applied to positioning. The reason that a movement model is needed in these cases lies in the dynamic nature of most pedestrian indoor navigation applications: The user's position will be estimated continuously so as to allow services such as personalized travel assistance or indoor navigation directions. In addition, it can be shown that a dynamic positioning system is more accurate than a "single-shot" static estimator which essentially provides a position estimate based on positioning measurements of a single time instance. To implement mathematically sound dynamic estimators one needs an accurate and realistic movement model (also known as the a-priori state transition model) of the dynamic system: Here the user's stochastic movement (position, velocity, attitude, etc).

In the context of such dynamic estimators we have grounded our work on the formalism of *sequential Bayesian estimation*, of which the well known Kalman Filter is a special case [AMGC02]. Basically, a sequential Bayesian estimator updates an estimate of a system's state over the course of time, given a set of new observations at each time instance. The estimator thus incorporates the new observations with all previous ones, and in order to do so correctly it needs to incorporate the possible changes of the system's state from one time instance to the next. Essentially, the more "predictable" the system state transitions are, the more the measurements can be filtered over time. If measurements happen to be unavailable for one or more time steps, then the movement model allows a prediction of the state estimate. Dead-reckoning essentially builds on this principle whereby the underlying movement model is a very simple one.

Another application of this movement model presented here is in simulation and validation of indoor positioning systems by simulating realistic pedestrian traces, and applying these traces as the controlling parameters of a system that simulates sensors such as indoor GNSS receivers.

1.2. PREREQUISITES FOR A NAVIGATION MOVEMENT MODEL

In sequential Bayesian estimation the choice of algorithm will affect the way in which the movement model is incorporated. For estimation of non-linear and

non-Gaussian processes, and especially in situations where the state estimates are multimodal, one often resorts to sequential Monte Carlo (SMC) techniques, such as Particle Filtering [AMGC02] [GSS93]. In this case, the movement model must fulfill the following conditions:

1. It must be statistically accurate in describing the state transition probabilities $\text{Prob}\{x_k | x_{k-1}\}$; this represents the probability of all possible system states at time k , given a certain known system state at $k-1$. In our case, the model describes the probability distribution of the user's position, velocity, attitude, as well as other parameters, given the knowledge of these parameters one time-step previously.
2. For particle filtering applications with transition probabilities as the importance function one must be able to efficiently draw samples from, and compute, the state transition probabilities $\text{Prob}\{x_k | x_{k-1}\}$ for any given previous state x_{k-1} .
3. The model should take into account individual variances of the user, such as their walking speed.
4. It must take into account the known building layout, such as walls, doors, stairs, as well as other local factors affecting pedestrians' motion.
5. Ideally, both 2D and 3D motion should be represented.
6. It must incorporate at least position, speed and heading.

An example of a very simple movement model that does not fulfill all requirements above is that where the heading and speed follow a (bounded) random walk process with additive white Gaussian process noise. This model cannot incorporate walls or the fact that people generally walk along corridors.

1.3. SUMMARY OF RELATED WORK

According to the application, the interest might vary among different forms of pedestrian movement models. For example only statistical measures like means and densities might be of interest when modeling pedestrian groups. On the other hand, a detailed model of pedestrian behavior is of interest for other applications such as navigation – the main application in this paper.

Additionally, some special movement behaviors have to be considered for some specific applications. For example a movement model that will be used to model firemen or rescuers has to consider some special behaviors like sliding, climbing, jumping and rolling.

1.3.1. CLASSIFICATION OF MOVEMENT MODELS

According to the above mentioned application areas, models for pedestrian movement are developed at three different levels. They are the mezoscopic, the

macroscopic level and the microscopic level [Hel92a] [Tek02].

A) MEZOSCOPIC LEVEL

According to the application, it is sometimes sufficient to describe the pedestrian movement models using approximate equations for the mean values of velocities as a function of some parameters like the pedestrian's age or activity. This describes the pedestrian behavior at the mezoscopic level. These models are similar to the Boltzmann equations and their interpretation as gas-kinetic [HSBP00] representation of human movement is valid. However in contrast to the ordinary gas description, pedestrian behavior representation with Boltzmann equations takes into consideration the effects of humans' aims and interactions. Mezoscopic modeling was primarily made for traffic simulations, but later applied to pedestrian modeling.

B) MACROSCOPIC LEVEL

It is sometimes the case that in addition quantities describing the velocity probability density (typically the mean velocity and velocity variance) of pedestrians are of interest. In such cases fluid dynamic equations [Hel92b] are sufficient to model the human behavior – this is denoted as macroscopic pedestrian modeling. The origins of these models are still the gas-kinetic equations and they also originate from transportation modeling. The root of these models is the continuum model by Lighthill and Whitham (1955) [Add05] which solves differential flow equations.

An example in which humans are modeled at the macroscopic level is the Al Gadhi et al (2001) model [AMH01]. They constructed a flow model that describes the events during the Hajj (pilgrimage to Mecca).

C) MICROSCOPIC LEVEL

At the microscopic level every pedestrian is treated as an individual and the behavior of pedestrians' interaction is observed.

There exist several analytical models that try to describe the microscopic behavior of a pedestrian. They are based on granular-physics of flow. The mathematical formulation of microscopic models generally leads to complicated equations that are impossible to be solved analytically and hard to be solved with a computer. But they can be approached using Monte Carlo Simulations. These kinds of simulations are called Microscopic Pedestrian Simulation Models (MPSMs). MPSM is a computer simulation model of pedestrian movement where every pedestrian in the model is treated as individual.

Sometimes the term “Agent Based Models” is used in the literature. It still refers to the modeling of human behavior at the microscopic level.

By monitoring individual pedestrian’s behavior, one can obtain some general characteristics regarding group behaviors. Examples of these general characteristics are the behavior in queues and the generation of freely-forming groups [Hel91].

1.3.2. MICROSCOPIC MOVEMENT MODELS

All levels of description of pedestrian movement models take into account pedestrian *intentions* and *interactions*.

A Microscopic description is more of interest in the navigation domain. They also have practical applications in the evacuations plans, the design of pedestrian areas, and as experimental & optimization design tool. Accordingly the work in this paper addresses the microscopic modeling approach.

At the Microscopic level, the pedestrian is affected by two forces. The first pushes the pedestrian toward his goal and the other introduces a repulsive force to push the pedestrian far from other pedestrians and obstacles. Geographical, spatial and geometrical environments are essential parameters that have to be considered for a reasonable pedestrian model.

There are several microscopic pedestrian simulation models:

A) SOCIAL FORCE MODEL

This model assumes that the pedestrian is subjected to social forces that motivate him for any movement that he does [HeM95] [LKF05]. First force is the pedestrian’s motivation to reach his destination. The second force is the interaction away from other pedestrians and the response away from obstacles and boundaries.

B) MAGNETIC FORCE MODEL

This model assumes that the pedestrian movement can be modeled using a combination between the equation of motion and the magnetic fields rules [OkM93].

C) BENEFIT COST MODEL

According to this model, the simulated area is divided into a square grid [Res04] [Tek02]. The pedestrian will decide to move to one of the nine cells in the field that has the highest “benefit” according to some scoring function. The model provides mathematical formulas for calculating the benefit of the cells surrounding the pedestrian.

D) CELLULAR AUTOMATA MODELS

Cellular Automata models have been developed primarily for traffic and used later for pedestrian modeling [YFL+03] [WLF03] [DJT01]. They have two standard steps that run in parallel at every time step. The first step is deciding which path to follow out of the available paths. The second step is giving the pedestrian some speed based on the free space in his vicinity.

E) QUEUING NETWORK MODELS

Queuing Network Models [MaS98] [OsB07] were used in pedestrian movement model world to simulate evacuation plans in case of fire and other emergency plans.

2. A COMBINED DIFFUSION AND STOCHASTIC BEHAVIORAL MODEL

A. STOCHASTIC BEHAVIORAL MOVEMENT MODEL

Human movement at the kinematical level is parameterized by physical parameters such as speed, direction of motion and position. However, speed and direction are affected by several “human” states. Examples of these states are pursued activity, emotions, degree of disorientation, age, obstacles and weather. For example, it is more usual for a disoriented pedestrian to walk irregularly compared to somebody who is walking a familiar route. On the other hand, a pedestrian running to catch a train is faster in general compared to another pedestrian who is window shopping. Some of these parameters affect the movement more than others. Building layouts are obviously amongst the main parameters that affect the movement of the pedestrian. For instance, the pedestrian cannot penetrate a wall under any normal circumstances.

Movement constraints that control physical parameters are categorized into two groups. The first category includes parameters that the system can determine accurately like age, weather, time of day and parameters that can be derived from external data such as ground steepness or obstacles at the pedestrian’s position. The other category includes parameters that are varying according to the human behavior. In general it is not simple to determine straightforwardly parameters falling into this category. Examples of these are pursued activity, disorientation, and emotions. In our approach these variables are modeled in a simplified fashion using Markov processes. The idea of using Markov Chains for describing human behaviors could also be found in [PeL99], [ZhN02] and [AdA04]. The transition probabilities of these Markov chains are set according to a-priori assumptions that are rooted in common sense assumptions.

Eleven parameters that affect the human movement are considered during this work. These parameters can be extended or modified according to the scenario and the application. The system is used to specify the new states of the first category parameters such as time of day, weekday, age, obstacles, ground steepness and weather. Markov chains are used to specify the new states of the second category parameters such as emotions, disorientation, activity, activeness and arousal. Some of the first category parameters such as weather, obstacles and steepness depend on the pedestrian's current position.

Our model assumes that given the specific instantiation of our eleven parameters, the physical motion parameters of the pedestrian can be specified with reasonable accuracy. Given the user's current situation with respect to these eleven parameters, the next step our model follows is to stochastically draw the pedestrian's actual movement following a distribution function that is conditioned on these eleven parameters. To do so, a Gaussian distribution was assumed for the main physical random variables; the speed and the direction. The values of the mean and the standard deviation of the user's speed and direction, given the specific values of each of the eleven parameters variables are predetermined. They are based on statistical data [Bek95] [GrG04] as well as common-sense assumptions.

Each of our eleven parameter results in a specific mean and standard deviation of the pedestrian's speed and direction. A weighted average is used to combine these means and standard deviations into a single mean and standard deviation for speed and direction. The combination results are then used to parameterize our Gaussian distributions. New speed and new direction values are then drawn from these Gaussian distributions. Finally, the position at the next time step is calculated as a

function of the drawn speed and direction using the equations of motion and the old position. In this way, we have drawn a new value of the state x_k from $\text{Prob}\{x_k | x_{k-1}\}$ given the old state. The state

x_k comprises all above mentioned eleven parameters that are drawn according to their Markov process or known a-priori (e.g. age, weather), as well as the physical motion states (heading, speed, position).

The states of our eleven parameters evolve over time resulting in variations in pedestrian's speed and direction and accordingly his position. A flow diagram of how the Stochastic Behavioral movement model works is shown in figure (1). Details of this model can be found in [WKAR06] and [Khi05].

Whereas this model is capable of representing movement well in situations without external constraints it is not suited for situations in which walls or roads have a strong influence on the movement. We shall demonstrate in following examples, this model leads to a high probability of getting stuck in a room. This is because the random movement which the model is following does not react to the presence of a door.

B. DIFFUSION MOVEMENT MODEL

To overcome the problem of staying in a room for a longer time with the above described model and for the simulated pedestrian not being able to find a suitable exit, the *diffusion movement model* taken from [KAL03] is applied. This model is derived from the principle of gas diffusion in space studied in thermodynamics and is a standard solution for path finding of robots [ScA93]: The idea is to have a source continuously effusing gas that disperses in free space and which gets absorbed by walls

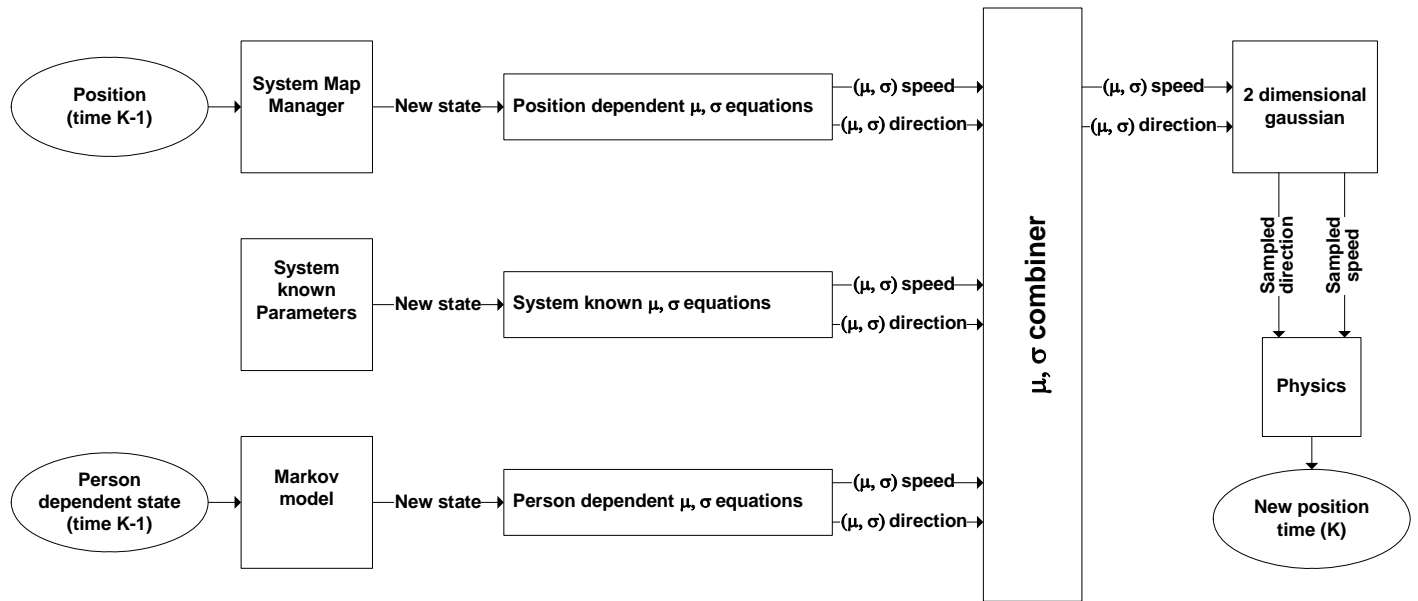


Figure 1: Stochastic Behavioral Movement Model

and other obstacles. To keep the model's complexity low, the diffusion movement model is confined to a rectangular area. The central assumption of our model is that a pedestrian will walk along a "sensible" path from her current location to some destination randomly changing the destination she is walking towards.

For the rectangular area a set \mathcal{W} of N_w destination points $\{(x_1, y_1), \dots, (x_{N_w}, y_{N_w})\}$ has to be specified, where each destination point represents a source effusing gas. For each destination point

$$W_m(x_m, y_m) \in \mathcal{W} \quad (1)$$

a so called diffusion matrix \mathbf{D}_m is pre-computed. The diffusion matrix for a particular destination point contains the values for the gas concentration at each possible waypoint when gas effused from that destination/source point. For this, a filter \mathbf{F} of size $n \times n$ is applied:

$$f_{p,q} = \frac{1}{n^2} \quad \forall p, q: p, q = 0, 1, \dots, n \quad (2)$$

Inaccessible points (e.g. walls and closed areas) are considered in the computation of the diffusion matrix via a discrete layout map matrix. The layout map matrix \mathbf{L} defines the accessible and inaccessible areas for pedestrians with

$$l_{i,j} = \begin{cases} 1 & \text{if } l_{i,j} \text{ is accessible} \\ 0 & \text{if } l_{i,j} \text{ is not accessible} \end{cases} \quad \forall i, j: i = 0, \dots, N_x, j = 0, \dots, N_y \quad (3)$$

where $N_x \times N_y$ is the size of the rectangular area.

The diffusion is expressed by a convolution of the diffusion matrix \mathbf{D}_m with the filter matrix \mathbf{F} element-wise multiplied by the layout map matrix \mathbf{L} :

$$d_{i,j}(k+1) = l_{i,j} \cdot \sum_{p=1}^n \sum_{q=1}^n d_{i+p-1, j+q-1}(k) \cdot f_{p,q} \quad (4)$$

Constantly refreshing the source is represented by forcing

$$d_{x_m, y_m} := 1 \quad (5)$$

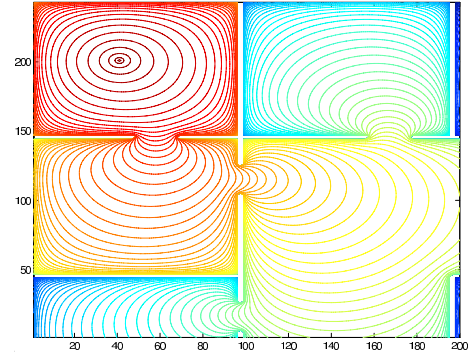
at the destination point. Equation (4) is evaluated repeatedly until the entire matrix is filled with values that are greater than zero (except for walls and closed areas):

$$d_{i,j} > 0 \quad \forall i, j: i = 0, \dots, N_x, j = 0, \dots, N_y \quad (6)$$

The path is computed by backtracking from the destination point W_m towards lower values of the diffusion matrix until the current waypoint CP is reached (see Figure 1a).

0.05	0.1	0.2	0.3	0.4	0.3	0.2
0.1	CP	0.4	0.5	0.6	0.5	0.3
0.2	0.4	0.5	0.6	0.7	0.6	0.4
0.3	0.5	0.6	0.7	0.8	0.7	0.5
0.4	0.6	0.7	0.8	0.9	0.8	0.6
0.5	0.7	0.8	0.9	W_m	0.9	0.7
0.4	0.5	0.6	0.7	0.9	0.7	0.5

a



b

Figure 1: Path finding (a) and contour (b) of [KAL03]

In [KAL03], the path finding with the diffusion model is compared to the Lee algorithm [Lee61]. It is shown that the diffusion provides a more realistic path than the one generated by using Lee's algorithm, since the path keeps a distance from obstacles. However, the computational effort of the diffusion algorithm is higher than the other.

We prefer to use the more realistic path and keep the computation effort during running the movement model low by pre-computing for all destination points the diffusion matrix \mathbf{D}_m and storing the angle of the direction of the path in an angle matrix \mathbf{A}_m . The angle matrix \mathbf{A}_m contains for each position in the rectangular area the value of the angle of the direction towards destination point W_m . Since the angle for the path part between start point and the next point of the path always results in angles of values 0° , 90° , 180° and 270° , we took the fifth point of the path (a reasonable value for middle speed, other values could be considered) to compute the angle for that start point. Taking the angle does not necessarily result in staying exactly on the path. Since it depends on the speed how far the pedestrian is moving in the direction of the angle, the pedestrian may leave the path. During the next step a new angle is taken from the new position, so that the deviation from the diffusion path is corrected again. For our movement model this deviation is negligible, if not actually desired, because the

true destination points are anyhow not known and may change, and the direct path may also not be the one followed by a real person in the real world (for instance to avoid bumping into someone else, or other random or intentional deviations).

Since the destination point for the pedestrian is not known, the destination points are chosen randomly (assuming here a uniform distribution). The destination point is stored until the destination is reached, changed or until the other movement model is used (see later). The destination is changed with some assumed probability to take into account the individual variances of the user, such as changing her destination. The speed of the pedestrian is predicted with the stochastic behavioral movement model.

We show that with the diffusion movement model the pedestrian find the exit of a room faster than with the stochastic behavioral movement model, especially when the door opening is small. The disadvantage of using the diffusion movement model is that if we assume that the destination point is not known then this true destination point may not be in or close to our set of N_w destinations, so that the model is not able to capture the actually observed motion particularly towards the end of the true trace that is walked. Furthermore, it does not model local random motion very well, such as when a person is not walking to some target – for example whilst walking around in an office talking to somebody. Therefore, a combination of both approaches is proposed in our new model. We found that a combination of both models is particularly advantageous and will describe the properties and performance of the resulting model in detail.

C. COMBINED MOVEMENT MOEL

The behavioral and the diffusion movement model are combined via an extended Markov model (see Figure 2). The combined model switches between motion that is more stochastic (section 2.A) or goal oriented (section 2.B).

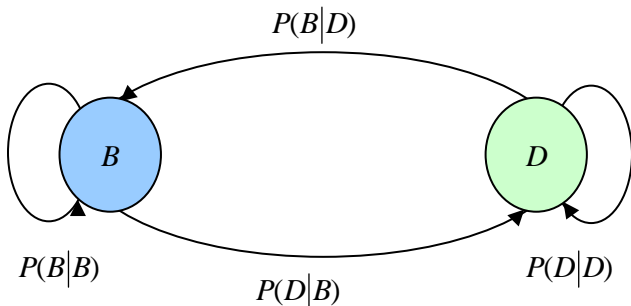


Figure 2: Markov Model

The Markov model contains two states: state B where the behavioral movement model is used and state D where the

diffusion movement model is used. In addition, we have the following probabilities:

- $P(B|D)$ is the transition probability that model B is used under the condition that the previous used model was model D. Similarly, $P(D|B)$ is the transition probability that model D is used under the condition that the previous used model was model B.
- $P(B|B)$ is the state probability that model B is used under the condition that model B is used also in the previous step. Similarly, $P(D|D)$ is the state probability that model D is used under the condition that model D is used also in the previous step.

Note that for the probabilities the following equations hold:

$$\begin{aligned} P(B|B) &= 1 - P(D|B) \\ P(D|D) &= 1 - P(B|D) \end{aligned} \quad (7)$$

In our case, the same-state probabilities are large and the transition probabilities are very small. Therefore, if the behavioral movement model is used, it will be applied several times and changing the movement model happens only occasionally. With this, the combined model switches between both models randomly with a small transition probability, where each model may be successively used for several time steps.

3. SYSTEM DESIGN AND IMPLEMENTATION

Bayesian Filters are widely used in estimation problems that are related to some noisy sensors. They provide two main benefits: First, sensor outputs are represented using probability distribution instead of hard decisions which helps in combining several heterogeneous sensors. Second, it allows including the system dynamics (the movement models) in the estimation process. Accordingly, an accurate movement model is beneficial in the estimation process.

Qualitative and qualitative analysis of the designed movement model were required. The designed movement model was tested in an already available distributed simulation and demonstration indoor/outdoor environment for mobility, location and context applications. The environment allows plugging-in several types of sensors, Bayesian Filters and movement models. Testing the implemented movement model using the above environment can help us in obtaining a quantitative indication on how much performance improvements different movement models will add to the overall estimation process in navigation applications.

Additionally, a quantitative comparison between the Stochastic Behavioral Movement Model, the Diffusion Movement Model and the Combined Movement Model were needed. Accordingly, we built a test scenario where

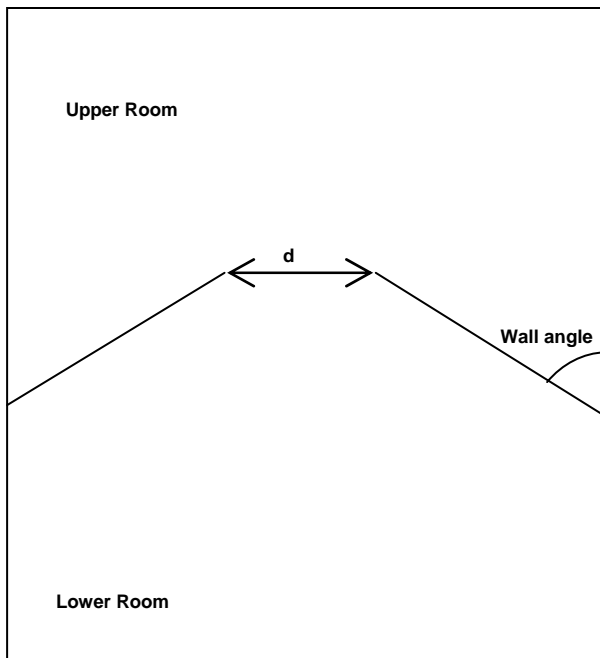


Figure 4: Simulated scenario

one pedestrian was simulated in an area of two rooms that are connected with one door. Figure (4) shows a drawing of the simulated scenario. The time needed for the pedestrian to escape a room (dwell time) was measured. The width of the door and the angle of the wall between the two rooms were varied. To make sure of fair measurements, the areas of the two rooms were maintained equal for the different cases. That was done by moving the location of the wall between the two rooms.

Performance comparisons between three movement models in some specific floor plans scenarios were obtained. The effect of various floor plans on the dwell time was studied and indications on which movement model performs better as a function of the floor plan were obtained. The results of these simulations are discussed in the next section.

4. SIMULATION RESULTS

From Figure (5) we can see that for the Stochastic Behavioral Movement Model, the dwell time in the lower room increases as the angle of the wall increases. On the other hand, for the Diffusion Movement Model, the dwell time is approximately constant with the change of the wall angle. From the figure, we can see that the combined movement model curve lies in the middle between the two models as expected.

From figure (6) we can see that for the Stochastic Behavioral Movement Model, the dwell time decreases as the size of the door between the two room increases. On the other hand, for the Diffusion Movement Model, the dwell time is approximately constant with the change of the door size – this is much more realistic, since people will be able to walk through both a narrow door and a

wider one, and their probability to do so is largely independent on the door's width.

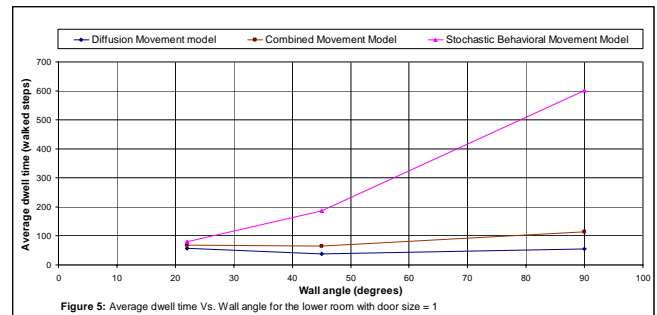


Figure 5: Average dwell time Vs. Wall angle for the lower room with door size = 1

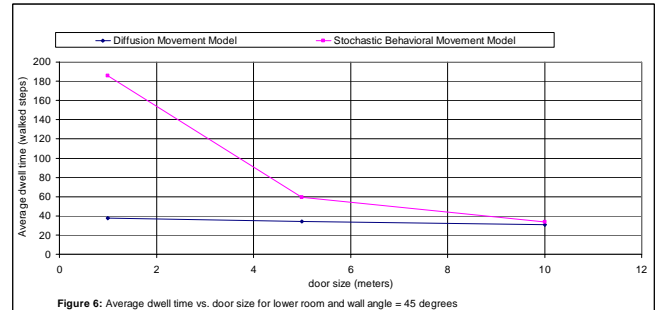


Figure 6: Average dwell time vs. door size for lower room and wall angle = 45 degrees

5. CONCLUSIONS AND OUTLOOK

As far as our simulations and up to present usage of the novel model tell us, we have achieved our original objectives: The combined model works very well in the prediction step of sequential Bayesian estimation and does not show artifacts, such as dwell times in rooms being overly and unrealistically influenced by wall geometries or door sizes. In previous models these artifacts were a significant cause of estimation errors and numerical problems, in particular resulting in excessive numbers of required particles in sequential Bayesian estimation. Furthermore, the novel model is of good use in the (Monte Carlo) simulation of systems that involve mobility, such as mobile radio communication systems or pedestrian navigation, due to its capability of generating probabilistic, yet realistic movement paths by taking into account geometric constraints such as walls or doors.

For the purpose of predicting a real person's movement, be it within a sequential Bayesian estimator for determining the person's location or for proactively performing any service or allocating resources based on the likely future path of the person, any parameter of context information can be treated as a condition in the probabilistic movement process. It remains an exciting challenge to select and measure the most relevant of these parameters in order to improve not only technical aspects such as position accuracy but also our understanding of the degree of randomness, or more philosophically the amount of free will in a person's movement.

6. ACKNOWLEDGMENTS

We would like to extend our thanks to Thomas Jost, Jens Kammann and Kai Wendlandt for their fruitful discussions and help. Special thanks to all the Broadband System Group members for their encouragement and support.

7. REFERENCES

- [AMGC02] S. Arulampalam, S. Maskell, N. Gordon and T. Clapp: A Tutorial on Particle Filters for On-line Non-linear/Non-Gaussian Bayesian Tracking, IEEE Transactions on Signal Processing, Vol. 50, No. 2, Feb. 2002
- [GSS93] N. Gordon, D. Salmond, and A. Smith, "Novel approach to nonlinear and non-Gaussian Bayesian state estimation," vol. 140. Inst. Elect. Eng., 1993
- [Hel92a] Dirk Helbing: Models for Pedestrian Behavior, Natural Structures. Principles, Strategies, and Models in Architecture and Nature, Part II (Sonderforschungsbereich 230, Stuttgart, 1992)
- [Hel91] Dirk Helbing: A Mathematical Model for the Behavior of Pedestrians, Behavioral Science 36, 298-310, 1991
- [Tek02] Kardi Teknomo: Microscopic Pedestrian Flow Characteristics: Development of an Image Processing Data Collection and Simulation Model, PhD Dissertation, 2002.
- [Add05] J. D. Addison: Flow on Links: Yesterday, Today and Tomorrow, IMA 2005.
- [HeM95] Dirk Helbing, Peter Molnar: Social Force Model for Pedestrian Dynamics, Physical Review E 51, 4282-4286, 1995.
- [LKF05] Taras I. Lakoba, D. J. Kaup, Neal M. Finkelstein: Modifications of the Helbing-Molnar-Farkas-Vicsek Social Force Model for Pedestrian Evolution, SIMULATION, Vol. 81, No. 5, 339-352, 2005.
- [OkM93] Shigeyuki Okazaki and Satoshi Matsushita: A Study of Simulation Model for Pedestrian Movement with Evacuation and Queuing, Proceedings of the International Conference on Engineering for crowd safety, 1993.
- [Res04] Wolfram Ressel: Modeling and Simulation of Mobility, 1st International Workshop on Intelligent Transportation (WIT 2004); Hamburg, Germany, 2004.
- [YFL⁺03] L. Yang, W. Fang, L. Li, R. Huang, and W. Fan: Cellular automata pedestrian movement model considering human behavior, CHINESE SCIENCE BULLETIN -ENGLISH EDITION-2003, VOL 48; PART 16, pages 1695-1699.
- [WLF03] Fang Weifeng, Yang Lizhong, W. Fan: Simulation of bi-direction pedestrian movement using a cellular automata model, Science Direct, Physica A 321 (2003) 633 – 640, 2003.
- [DJT01] J. Dijkstra, A.J. Jessurun and H.J.P. Timmermans: A Multi-Agent Cellular Automata Model of Pedestrian Movement, Pedestrian and Evacuation Dynamics. Berlin, D: Springer-Verlag. 173-181, 2001.
- [OsB07] Carolina Osorio and Michel Bierlaire: An analytic finite capacity queuing network capturing congestion and spillbacks, TRISTAN VI, EPFL, 2007.
- [MaS98] J. MacGregor Smith: Evacuation Networks, Encyclopedia of Optimization, 1998.
- [DeD00] Jake Desyllas and Elspeth Duxbury: Visibility Graph Analysis, (VGA), Royal Institute of Chartered Surveyors conference, 2000.
- [Hel92b] Dirk Helbing: A Fluid Dynamic Model for the Movement of Pedestrians, Complex Systems 6, 391-415, 1992.
- [HSBP00] Serge P. Hoogendoorn, Phil Bovy, Gas-Kinematic Modeling and Simulation of Pedestrian Flows, Transportation Research Record No. 1710, Traffic Flow Theory and Highway Capacity, 2000.
- [ChN05a] David Charypar, Kai Nagel, Q-learning for flexible learning of daily activity plans, Journal of the Transportation Research Board, 2005.
- [ABW04] Gianluca Antonini, Michel Bierlaire and Mats Weber: Simulation of Pedestrian Behavior using a Discrete Choice Model Calibrated on Actual Motion Data, 4th Swiss Transport Research Conference, 2004.
- [CGI⁺05] D. Cavens, C. Gloor, J. Illenberger, E. Lange, K. Nagel, W. A. Schmid: Distributed intelligence in pedestrian simulations, Pedestrian and Evacuation Dynamics 2005, Part 3, Pages 201-212

[GCN05] Christian Gloor, Duncan Cavens, and Kai Nagel: A Message-Based Framework for Real-World Mobility Simulations, Applications of Agent Technology in Traffic and Transportation, 2005.

[ChN05b] David Charypar, Kai Nagel, Generating complete all-day activity plans with genetic algorithms, Transportation, Volume 32, Number 4, July 2005.

[AMH01] S. A. H. Al Gadhi, H. S. Mahmassani and R. Herman, "A speed-concentration relation for bi-directional crowd movements with strong interaction", in Pedestrian and Evacuation Dynamics, Springer- Heidelberg, Pages 3 - 20, 2001

[KAL03] J. Kammann, M. Angermann and B. Lami, "A new mobility model based on Maps", in VTC 2003.

[ScA93] G. K. Schmidt and K. Azam, "Mobile robot path planning and execution based on a diffusion equation strategy", in Advanced Robotics, Vol. 7, No. 5, pp. 479-490, 1993.

[Lee61] C. Lee, "An algorithm for path connections and its applications," in IRE Transactions on Electronic Computing EC-10, pp. 346-365, 1961.

[WKAR06] Wendlandt, Khider, Angermann, Robertson: Continuous location and direction estimation with multiple sensors using particle filtering, IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems, 2006.

[Khi05] M. Khider, Implementation of a Simulator/Demonstrator for the SoftLocation Concept using Bayesian Filters, Master Thesis, University of Ulm, 2005

[PeL99] Pentland and A. Liu, "Modeling and Prediction of Human Behavior," Neural Computation, vol. 11, no. 1, pp. 229-242, 1999.

[ZhN02] T. Zhao and R. Nevatia, "3D Tracking of Human Locomotion: A Tracking as Recognition Approach," in 16th International Conference on Pattern Recognition, 2002, December 2002.

[AdA04] A. Adam and S. Amershi, "Identifying Humans by Their Walk and Generating New Motions Using Hidden Markov Models," The University of British Columbia, Topics in AI: Graphical Models and Computer Animation, Tech. Rep., December 2004.

[Bek95] G. A. Bekey, "Walking," The Handbook of Brain Theory and Neural Networks, MIT press, pp. 1045-1049, 1995.

[GrG04] R.D. Green, and L. Guan, "Quantifying and Recognizing Human Movement Patterns from Monocular Video Images - part I: A new Framework for Modeling Human Motion," IEEE Transactions on Circuits and Systems for Video Technology, pp. 179-190, 2004.