# Dynamic Multipath Estimation by Sequential Monte Carlo Methods

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## BIOGRAPHY

Michael Lentmaier was born in Ellwangen, Germany. He received the Dipl.-Ing. degree in electrical engineering from University of Ulm, Germany in 1998, and the Ph.D. degree in telecommunication theory from Lund University, Sweden, in 2003. As a Postdoctoral Research Associate he spent 15 months at University of Notre Dame, Indiana, and four months at University of Ulm. Since January 2005, he has been with the Institute of Communications and Navigation at the German Aerospace Center (DLR). His current research is focused on signal processing algorithms for GNSS receivers.

Bernhard Krach received the Dipl.-Ing. degree in electrical engineering from University of Erlangen-Nuremberg, Germany, in 2005. Since that he has been with the Institute of Communications and Navigation at the German Aerospace Center (DLR).

Patrick Robertson was born in Edinburgh, in 1966. He received the Dipl.-Ing. degree in electrical engineering from the Technical University of Munich, in 1989 and a Ph.D. from the University of the Federal Armed Forces, Munich, in 1995. Since 1990 he has been working at the Institute for Communications Technology at the German Aerospace Centre (DLR) in Oberpfaffenhofen, Germany. From 1990 to 1993 he shared this position with a part time teaching post at the University of the Federal Armed Forces, Munich. He has been active in the fields of mobile packet transmission, digital terrestrial television (DVB-T), multimedia transmission, synchronization, Turbo coding, and broadband wireless indoor communications networks and since January 1999 he is acting as leader of the research group ”Broadband Systems”. His contributions within a number of EU and national R&D projects (DACAR, dTTb, HDTV-T, MINT, DVBird, MCP, Daidalos) have included work on key technical definition and standardization, as well as leading several work packages, acttivities and task forces. Dr. Robertson's current interests include wireless mobile communications, navigation systems, service architectures and Bayesian inference techniques for context awareness within pervasive computing systems. He has published numerous scientific papers and holds several international patents in the areas of communications networks, mobile service discovery, indoor navigation, and systems for travel and tourist applications on wireless information devices.

Thanawat Thiasiriphet was born in Chiang Mai, Thailand, in 1982. He received the B.Sc. and M.Sc. in Communications Engineering from Chaing Mai University in 2004, and University of Ulm in 2007 respectively. He did his master thesis with the Institute of Communications and Navigation, German Aerospace Center (DLR), Munich. Since June 2007, he has been working toward his Ph.D. degree at the Institute of Information Technology, University of Ulm.

## ABSTRACT

A sequential Bayesian estimation algorithm for multipath mitigation is presented, with an underlying movement model that is especially designed for dynamic channel scenarios. In order to facilitate efficient integration into receiver tracking loops it builds upon complexity reduction concepts that previously have been applied within Maximum Likelihood (ML) estimators. To demonstrate its capabilities under different GNSS signal conditions, simulation results are presented for both artificially generated random channels and high resolution channel impulse responses recorded during a measurement campaign.

## 1 INTRODUCTION

A major error source within global navigation satellite systems (GNSS) comes from multipath, the reception of additional signal replica due to reflections, which introduce a bias into the estimate of the delay lock loop (DLL) of a conventional navigation receiver. For efficient removal of this bias it is possible to formulate advanced maximum likelihood (ML) estimators that incorporate the echos into the signal model and are capable of achieving the theoretical limits given by the Cramer Rao bound. For static channels without availability of prior information the ML approach is optimal and performs significantly better than other techniques, especially if the echos have short delay. Various ML approaches have been proposed in the literature, characterized essentially by different efficient maximization strategies over the likelihood function [1], [2], [3], [4], [5]. An estimator based on Sequential Importance Sampling (SIS) methods (particle filtering) for static multipath scenarios has been considered in [6], which has the advantage that prior channel knowledge can be incorporated. The drawback of ML estimator techniques in general is that the parameters are assumed to be constant during the time of observation. Independent estimates are obtained for successive observation intervals, whose length has to be adapted to the dynamics of the channel.

Most challenging for GNSS receivers are slowly changing environments where the assumption that channel parameters stay constant over sufficiently long observation times no longer is satisfied. Consider for example the high resolution channel impulse response received by a car driving in an urban channel environment [7], given in Figure 1. Figure 2 shows the simulated performance of a conventional DLL with narrow correlator [8] for a selected part of the route, which is depicted in Figure 3. The example

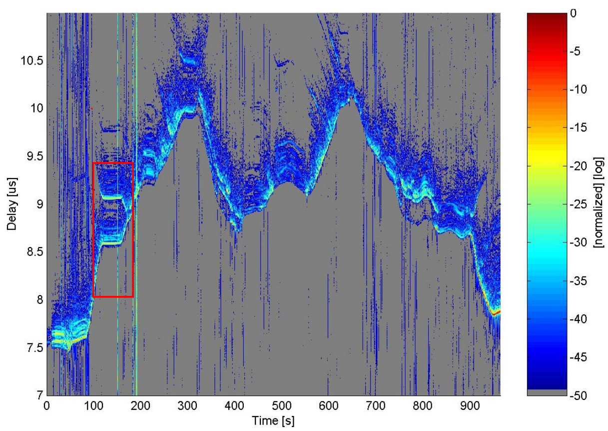



**Figure 1.** Example of urban channel measurements in Munich for 10 degree elevation. The red frame encloses a selected part of the route including a stop at a traffic light.

shows that carrier smoothing can reduce the multipath bias significantly during the period when the car does not move. In a changing environment, however, frequent cycle slips make the smoothed pseudo ranges diverge from the correct solution.

In this paper we consider the important practical case of such dynamic channel scenarios and assess how the time-delay estimation can be improved if information is available about the temporal evolution of the channel parameters, including statistical knowledge about the occurence of multipath replica. Our approach is based on Bayesian filtering, the optimal and well-known framework to address such dynamic state estimation problems. Sequential Monte Carlo (SMC) methods are used for computing the posterior probability density functions (PDFs) of the signal parameters.

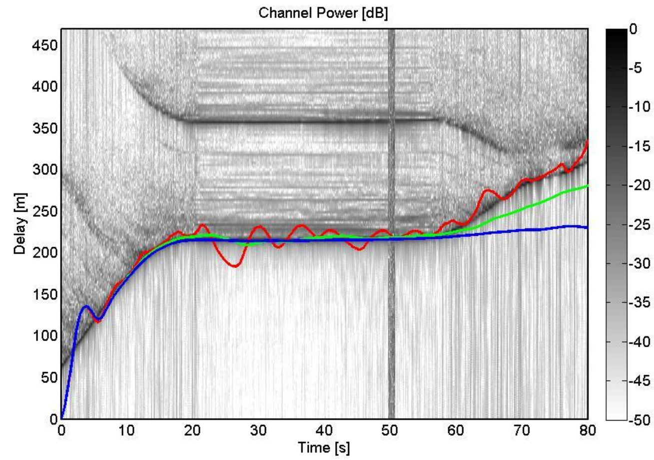



**Figure 2.** Performance of DLL without carrier smooting (red), with 10 s carrier smoothing (green), and with 100 s carrier smoothing (blue), respectively.

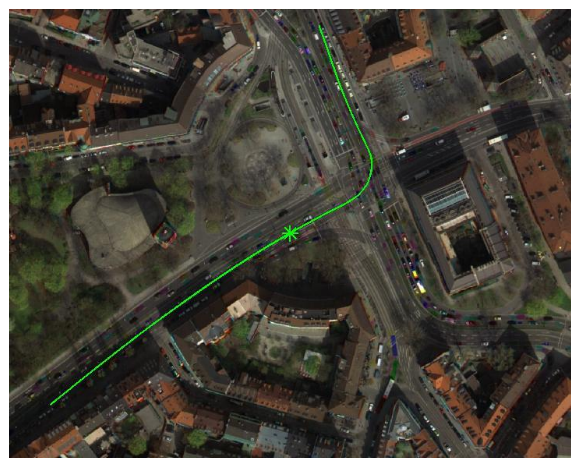


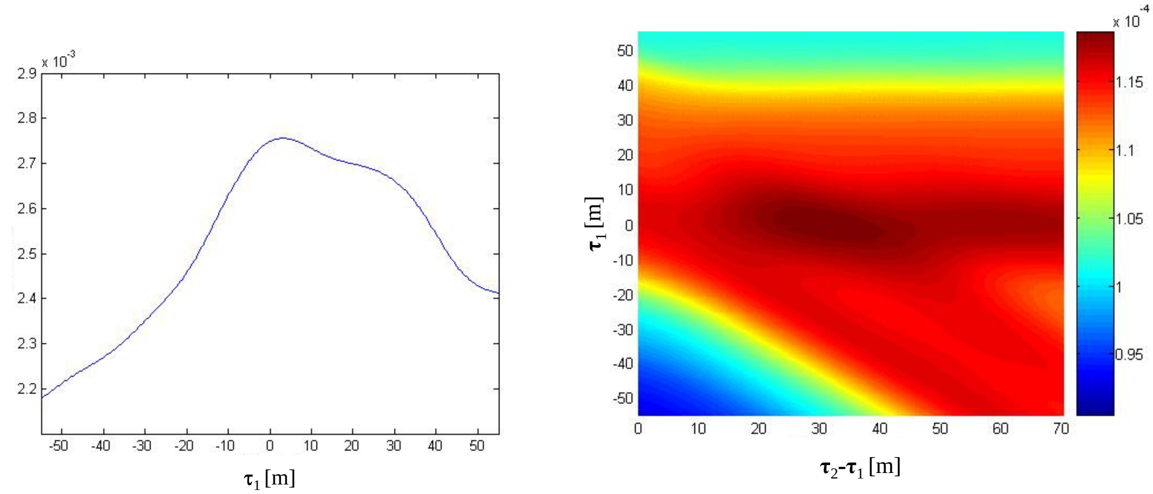
**Figure 3.** Environmental view of the selected part of the route (green line) with the stop marked by a green star.

## 2 SIGNAL MODEL

Assume that the complex valued baseband-equivalent received signal is equal to

$$z(t) = \sum_{i=1}^{N_m} e_i(t) \cdot a_i(t) \cdot [c(t) * g(t - \tau_i(t))] + n(t) \quad ,(1)$$

where $c(t)$ is a delta-train code sequence that is modulated on a pulse $g(t)$, $N_m$ is the total number of allowed paths reaching the receiver (to restrict the modeling complexity), $e_i(t)$ is a binary function that controls the activity of the $i'$th path and $a_i(t)$ and $\tau_i(t)$ are their individual complex amplitudes and time delays, respectively. The signal is disturbed by additive white Gaussian noise $n(t)$. Grouping blocks of $L$ samples at times $(m + kL)T_s$, $m = 0, \ldots, L-1$, together into vectors $\mathbf{z}_k$, $k = 0, 1, \ldots,$ whilst assuming the parameter functions $e_i(t)$, $a_i(t)$ and $\tau_i(t)$ being constant within the corresponding time interval and equal to $e_{i,k}$, $a_{i,k}$ and $\tau_{i,k}$,




(a) Assuming one path

(b) Assuming two paths

**Figure 4.** Example of the likelihood function for a channel with $N_m = 2$, $\tau_1 = 0$ m, and $\tau_2 = 30$ m.

this can be rewritten as

$$\mathbf{z}_k = \underbrace{\mathbf{C}\mathbf{G}(\tau_k)\mathbf{E}_k\mathbf{a}_k}_{\mathbf{s}_k} + \mathbf{n}_k \quad . \tag{2}$$

In the compact form on the right hand side the samples of the delayed pulses $\mathbf{g}(\tau_{i,k})$ are stacked together as columns of the matrix $\mathbf{G}(\tau_k) = [\mathbf{g}(\tau_{1,k}), \ldots, \mathbf{g}(\tau_{N_m,k})]$, $\mathbf{C}$ is a matrix representing the convolution with the code, and the delays and amplitudes are collected in the vectors $\tau_k = [\tau_{1,k}, \ldots, \tau_{N_m,k}]^T$ and $\mathbf{a}_k = [a_{1,k}, \ldots, a_{N_m,k}]^T$ respectively. Furthermore, for concise notation we use $\mathbf{E}_k = \operatorname{diag}[\mathbf{e}_k]$ whilst the elements of the vector $\mathbf{e}_k = [e_{1,k}, \ldots, e_{N_m,k}]^T$, $e_{i,k} \in [0,1]$, determine whether the $i'$th path is active or not by being either $e_{i,k} = 1$ corresponding to an active path or $e_{i,k} = 0$ for a path that is currently not active. The term $\mathbf{s}_k$ denotes the signal hypothesis and is completely determined by the channel parameters $\tau_k$, $\mathbf{a}_k$ and $\mathbf{e}_k$.

## 3 MAXIMUM LIKELIHOOD ESTIMATION

Using (2) we can write the associated *likelihood function* as

$$p(\mathbf{z}_k|\mathbf{s}_k) = \frac{1}{(2\pi)^L \sigma^{2L}} \cdot \exp\left[-\frac{1}{2\sigma^2}\left(\mathbf{z}_k - \mathbf{s}_k\right)^H \left(\mathbf{z}_k - \mathbf{s}_k\right)\right] \tag{3}$$

The likelihood function will play a central role in the algorithms discussed in this paper; its purpose is to quantify the conditional probability of the received signal conditioned on the unknown signal (specifically the channel parameters). The concept of ML multipath estimation has drawn substantial research interest since the first approach was proposed in [1]. Despite being treated differently in various publications the objective is the same for all ML approaches, namely to find the signal parameters that maximize (3) for a given observation $\mathbf{z}_k$:

$$\hat{\mathbf{s}}_k = \arg\max_{\mathbf{s}_k} \{p(\mathbf{z}_k|\mathbf{s}_k)\} \quad . \tag{4}$$

Figure 4 shows an example of the likelihood function for a static two path channel with delays $\tau_1 = 0$ m and $\tau_2 = 30$ m. According to (3) the shape of this function depends on the assumed number of active paths in the received signal $\mathbf{z}_k$. If a single path is assumed, the ML estimator is closely related to the conventional DLL. The echo at 30 m relative delay leads to a distortion of the likelihood function shown in Figure 4(a), resulting in a translation of its maximum and hence a biased estimate. With a correct number of paths, on the other hand, the second path is included in the maximization problem, now given by the two-dimensional likelihood function depicted in Figure 4(b). For practical implementation of the ML estimator different maximization strategies exist, which basically characterize the different approaches. Despite offering great advantages for theoretical analysis the practical advantage of the generic ML concept is questionable due to a number of serious drawbacks:

- The ML estimator assumes that the channel is static for the observation period and is not able to exploit its temporal correlation throughout the sequence $k = 1, \ldots$. Measured channel scenarios have shown significant temporal correlation [9].
- Despite being of great interest in practice the estimation of the number of received paths is often not addressed. The crucial problem here is to correctly estimate the current number of paths to avoid over-determination, since an over-determined estimator will tend to use the additional degrees of freedom to match the noise by introducing erroneous paths. Various

complex heuristics based on model selection are employed to estimate the number of paths, but they suffer from the problem of having to dynamically adjust the decision thresholds. Typically only a single hypothesis is tracked, which in practice causes error event propagation.

- The ML estimator does only provide the most likely parameter set for the given observation. No reliability information about the estimates is provided. Consequently ambiguities and multiple modes of the likelihood are not preserved by the estimator.

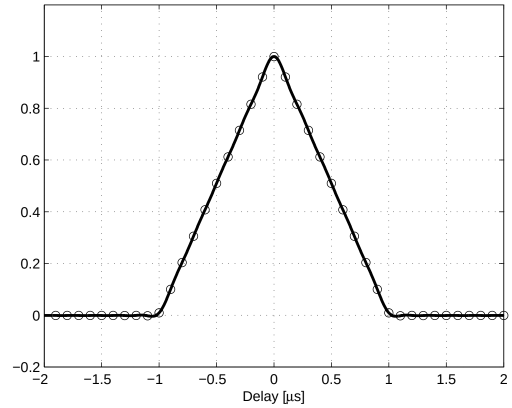
## 4 EFFICIENT LIKELIHOOD COMPUTATION

In [3] a general concept for the efficient representation of the likelihood (3) was presented, which is applicable to many of the exisiting ML mutlipath mitigation methods. The key idea of this concept is to formulate (3) through a vector $\mathbf{z}_{c,k}$ resulting from an orthonormal projection of the oberserved signal $\mathbf{z}_k$ onto a smaller vector space, so that $\mathbf{z}_{c,k}$ is a sufficient statistic according to the Neyman-Fisher factorization [10] and hence suitable for estimating $\mathbf{s}_k$. In other words the reduced signal comprises the same information as the original signal itself. In practice this concept becomes relevant as the projection can be achieved by processing the received signal (2) with a bank of correlators and a subsequent decorrelation of the correlator outputs. A variant of this very general concept, applied in [5], has also been referred to as the *Signal Compression Theorem* in [11] for a set of special projections that do not require the step of decorrelation due to the structure of the used correlators. For instance, unlike the correlation technique used in [1], the one suggested in [5] already projects onto an orthogonal and thus uncorrelated subspace, similar to the code matched correlator technique proposed in [3]. Due to complexity reasons all practically relevant realizations of ML estimators [1] [5] operate in a projected space, namely after correlation. The corresponding mathematical background will be discussed below, including also interpolation of the likelihood and elimination of complex amplitudes as further methods for complexity reduction.

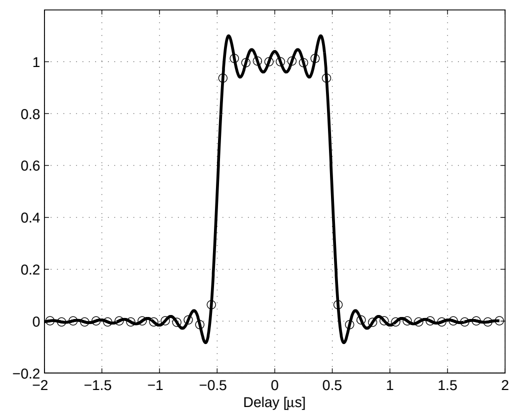
### 4.1 Data Compression

As explained above the large vector containing the received signal samples $\mathbf{z}_k$ is linearly transformed into a vector $\mathbf{z}_{c,k}$ of much smaller size. Following this approach the likelihood according to (2) can be rewritten as

$$\begin{aligned} p(\mathbf{z}_k|\mathbf{s}_k) &= \frac{1}{(2\pi)^L\sigma^{2L}} \exp\left[-\frac{\mathbf{z}_k^H\mathbf{z}_k}{2\sigma^2}\right] \\ &\quad\cdot \exp\left[\frac{\Re\{\mathbf{z}_k^H\mathbf{Q}_c\mathbf{Q}_c^H\mathbf{s}_k\}}{\sigma^2} - \frac{\mathbf{s}_k^H\mathbf{Q}_c\mathbf{Q}_c^H\mathbf{s}_k}{2\sigma^2}\right] \\ &= \frac{1}{(2\pi)^L\sigma^{2L}} \exp\left[-\frac{\mathbf{z}_k^H\mathbf{z}_k}{2\sigma^2}\right] \\ &\quad\cdot \exp\left[\frac{\Re\{\mathbf{z}_{c,k}^H\mathbf{s}_{c,k}\}}{\sigma^2} - \frac{\mathbf{s}_{c,k}^H\mathbf{s}_{c,k}}{2\sigma^2}\right], \end{aligned} \tag{5}$$




(a) Signal matched




(b) Code matched

**Figure 5.** Output of two types of canoncical component type correlator banks usable for data size reduction according to (9)

with the compressed received vector $\mathbf{z}_{c,k}$ and the compressed signal hypothesis $\mathbf{s}_{c,k}$:

$$\mathbf{z}_{c,k} = \mathbf{Q}_c^H\mathbf{z}_k, \quad \mathbf{s}_{c,k} = \mathbf{Q}_c^H\mathbf{s}_k \ , \tag{6}$$

and the orthonormal compression matrix $\mathbf{Q}_c$, which needs to fulfill

$$\mathbf{Q}_c\mathbf{Q}_c^H \approx \mathbf{I}, \quad \mathbf{Q}_c^H\mathbf{Q}_c \approx \mathbf{I} \ , \tag{7}$$

to minimize the compression loss. According to [3] the compression can be two-fold so that we can factorize

$$\mathbf{Q}_c = \mathbf{Q}_{cc}\mathbf{Q}_{pc} \tag{8}$$

into a *canonical component decomposition* $\mathbf{Q}_{cc}$ and a *principal component decomposition* $\mathbf{Q}_{cc}$. In [3] two choices for $\mathbf{Q}_{cc}$ are proposed

$$\mathbf{Q}_{cc} = \begin{cases} \mathbf{C}\mathbf{G}(\tau^b)\mathbf{R}_{cc}^{-1} & \textit{Signal matched} \\ \mathbf{C}(\tau^b)\mathbf{R}_{cc}^{-1} & \textit{Code matched} \end{cases} \ , \tag{9}$$

where the elements of the vector $\tau^b$ define the positions of the individual correlators. The noise-free outputs of the corresponding correlator banks are illustrated in Figure 5. To decorrelate the correlator outputs as mentioned above the whitening matrix $\mathbf{R}_{cc}$ can be obtained from a QR decomposition of $\mathbf{CG}(\tau^b)$ and $\mathbf{C}(\tau^b)$ respectively. Apart from practical implementation issues both correlation methods given by (9) are equivalent from a conceptual point of view. For details on the compression through $\mathbf{Q}_{pc}$ the reader is referred to [3].

### 4.2 Interpolation

In order to compute (5) independently of the sampling grid advantage can be made of of interpolation techniques. Using the discrete Fourier transformation (DFT), with $\Psi$ being the DFT matrix and $\Psi^{-1}$ being its inverse counterpart (IDFT), we get:

$$\mathbf{s}_{c,k} = \underbrace{\mathbf{Q}_c^H \mathbf{C}\Psi^{-1}\mathrm{diag}\left[\Psi\mathbf{g}(0)\right]}_{\mathbf{M}_{s_c}=\text{const.}} \Omega(\tau_k)\mathbf{E}_k\mathbf{a}_k \quad , \qquad (10)$$

with $\Omega(\tau_k)$ being a matrix of stacked vectors with Vandermonde structure [3].

### 4.3 Amplitude Elimination

In a further step we reduce the number of parameters by optimizing (5) for a given set of $\tau_k$ and $\mathbf{e}_k$ with respect to the complex amplitudes $\mathbf{a}_k$, which can be achieved through a closed form solution. Using

$$\mathbf{S}_{c,k} = \mathbf{M}_{s_c}\Omega(\tau_k)\mathbf{E}_k \qquad (11)$$

and obtaining $\mathbf{S}_{c,k}^+$ by removing zero columns from $\mathbf{S}_{c,k}$ one yields the corresponding amplitude values of the active paths:

$$\hat{\mathbf{a}}_k^+ = \left(\mathbf{S}_{c,k}^{+H}\mathbf{S}_{c,k}^+\right)^{-1}\mathbf{S}_{c,k}^{+H}\bar{\mathbf{z}}_{c,k} \quad . \qquad (12)$$

As we have introduced a potential source of performance loss by eliminating the amplitudes and thus practically are disregarding their temporal correlation, we propose to optimize (5) using

$$\bar{\mathbf{z}}_{c,k} = \frac{1}{Q}\cdot\sum_{l=0}^{Q-1}\mathbf{z}_{c,k-l} \qquad (13)$$

with the adjustable averaging coefficient $Q$. Please note that $\mathbf{a}_k^+$ is equal to the ML amplitudes for $Q = 1$ . When evaluating (5) we use

$$\mathbf{s}_{c,k} = \mathbf{S}_{c,k}\hat{\mathbf{a}}_k \quad , \qquad (14)$$

whereas the elements of the vector $\hat{\mathbf{a}}_k$ that are indicated to have an active path $(a_{k,i} : i \rightarrow e_{k,i} = 1)$ are set equal to the corresponding elements of $\hat{\mathbf{a}}_k^+$. All other elements $(a_{k,i} : i \rightarrow e_{k,i} = 0)$ can be set arbitrarily as their influence is masked by the zero elements of $\mathbf{e}_k$.

## 5 SEQUENTIAL ESTIMATION

### 5.1 Optimal Solution

In Section 2 we have established the models of the underlying time variant processes. The problem of multipath mitigation now becomes one of *sequential estimation of a hidden Markov process*: We want to estimate the unknown channel parameters based on an evolving sequence of received noisy channel outputs $\mathbf{z}_k$. The channel process for each range of a satellite navigation system can be modelled as a first-order Markov process if future channel parameters given the present state of the channel and all its past states, depend only on the present channel state (and not on any past states). We also assume that the noise affecting successive channel outputs is independent of the past noise values; so *each channel observation depends only on the present channel state*.

Intuitively we are exploiting not only the channel observations to estimate the hidden channel parameters (via the likelihood function), but we are also exploiting our prior knowledge about the statistical dependencies between successive sets of channel parameters. We know from channel measurements that channel parameters are time varying but not independent from one time instance to the next; for example, an echo usually experiences a "life-cycle" from its first occurrence, then a more or less gradual change in its delay and phase over time, until it disappears [9].

Now that our major assumptions have been established we may apply the concept of *sequential Bayesian estimation*. The reader is referred to [12] which gives a derivation of the general framework for optimal estimation of temporally evolving (Markovian) parameters by means of inference; and we have chosen similar notation. The entire history of observations (over the temporal index $k$) can be written as

$$\mathbf{Z}_k \widehat{=} \{\mathbf{z}_i, i = 1,\ldots,k\} \quad , \qquad (15)$$

similarly we denote the sequence of parameters of our hidden Markovian process by

$$\mathbf{X}_k \widehat{=} \{\mathbf{x}_i, i = 1,\ldots,k\} \quad . \qquad (16)$$

So $x_i$ represents the characterization of the hidden channel state, including the parameters that specify the signal hypothesis $\mathbf{s}_i$ given in (2). Our goal is to determine the *posterior* probability density function (PDF) of every possible channel characterization given all channel observations: $p(\mathbf{x}_k|\mathbf{Z}_k)$ (see Figure 7). Once we have evaluated this posterior PDF we can either determine that channel configuration that maximizes it - the so called maximum a-posteriori (MAP) estimate; or we can choose the expectation - equivalent to the minimum mean square error (MMSE) estimate. In addition, the posterior distribution itself contains all uncertainty about the current range and is thus the optimal measure to perform sensor data fusion in an overall positioning system.

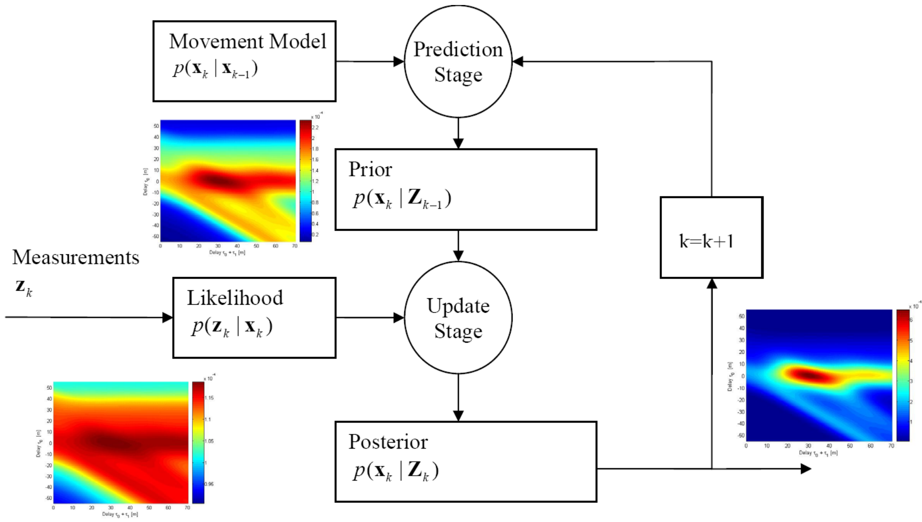



**Figure 6.** Illustration of the recursive Bayesian estimator.

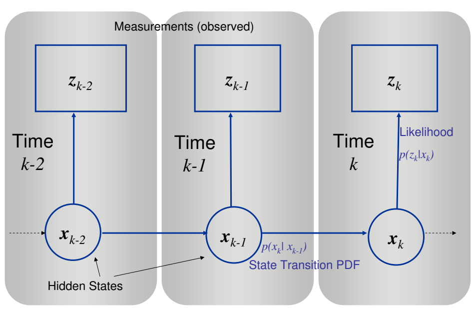



**Figure 7.** Illustration of the hidden Markov estimation process for three time instances. Our channel measurements are the sequence $\mathbf{z}_i, i = 1, \ldots, k\}$, and the channel parameters to be estimated are $\mathbf{x}_i, i = 1, \ldots, k\}$

It can be shown that the sequential estimation algorithm is recursive, as it uses the posterior PDF computed for time instance $k-1$ to compute the posterior PDF for instance $k$ (see Figure 6). For a given posterior PDF at time instance $k-1$, $p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})$, the *prior* PDF $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ is calculated in the so-called *prediction step* by applying the Chapman-Kolmogorov equation:

$$p(\mathbf{x}_k|\mathbf{Z}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1}) \mathrm{d}\mathbf{x}_{k-1} \quad , \qquad (17)$$

with $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ being the state transition PDF of the Markov process. In the *update step* the new posterior PDF for step $k$ is obtained by applying Bayes' rule to $p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}_{k-1})$ yielding the normalized product of the likelihood $p(\mathbf{z}_k|\mathbf{x}_k)$ and the prior PDF:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{Z}_k) &= p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}_{k-1}) \\ &= \frac{p(\mathbf{z}_k|\mathbf{x}_k, \mathbf{Z}_{k-1}) p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \\ &= \frac{p(\mathbf{z}_k|\mathbf{x}_k) p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \quad . \end{aligned} \qquad (18)$$

The term $p(\mathbf{z}_k|\mathbf{x}_k) = p(\mathbf{z}_k|\mathbf{s}_{c,k})$ follows from (5) and represents the probability of the measured channel output (often referred to as the likelihood value), conditioned on a certain configuration of channel parameters at the same time step $k$. The denominator of (18) does not depend on $\mathbf{x}_k$ and so it can be computed by integrating the numerator of (18) over the entire range of $\mathbf{x}_k$ (normalization).

To summarize so far, the entire process of prediction and update can be carried out recursively to calculate the posterior PDF (18) sequentially, based on an initial value of $p(\mathbf{x}_0|\mathbf{z}_0) = p(\mathbf{x}_0)$. The evaluation of the likelihood function $p(\mathbf{z}_k|\mathbf{x}_k)$ is the essence of the update step. Similarly, maximizing this likelihood function (i.e. ML estimation) would be equivalent to maximizing $p(\mathbf{x}_k|\mathbf{Z}_k)$ only in the case that the prior PDF $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ does not depend on $\mathbf{Z}_{k-1}$ and when all values of $x_k$ are a-priori equally likely. Since these conditions are not met, evaluation of $p(\mathbf{x}_k|\mathbf{Z}_k)$ entails all the above steps.

### 5.2 Sequential Estimation using Particle Filters

The optimal estimation algorithm relies on evaluating the integral (17), which is usually a very difficult task, except for certain additional restrictions imposed on the model and the noise process. So very often a suboptimal realization of a Bayesian estimator has to be chosen for implementation.

In this paper we use a Sequential Monte Carlo (SMC) filter, in particular a Sampling Importance Resampling Particle Filter SIR-PF according to [12]. In this algorithm the posterior density at step $k$ is represented as a sum, and is specified by a set of $N_p$ particles:

$$p(\mathbf{x}_k|\mathbf{Z}_k) \approx \sum_{j=1}^{N_p} w_k^j \cdot \delta(\mathbf{x}_k - \mathbf{x}_k^j) \quad , \tag{19}$$

where each particle with index $j$ has a state $\mathbf{x}_k^j$ and has a *weight* $w_k^j$. The sum over all particles' weights is one. In SIR-PF, the weights are computed according to the principle of *Importance Sampling* where the so-called proposal density is chosen to be $p(\mathbf{x}_k|\mathbf{x}_{k-1} = \mathbf{x}_{k-1}^j)$, and with *resampling* at every time step. For $N_p \to \infty$ the approximate posterior approaches the true PDF.

#### 5.2.1 Incorporation of Channel Observations and Characterization

The key step in which the *measurement* for instance $k$ is incorporated, is in the calculation of the weight $w_k^j$ which for the SIR-PF can be shown to be the likelihood function: $p(\mathbf{z}_k|\mathbf{x}_k^j)$. The characterization of the *channel process* enters in the algorithm when at each time instance $k$, the state of each particle $\mathbf{x}_k^i$ is drawn randomly from the proposal distribution; i.e. from $p(\mathbf{x}_k|\mathbf{x}_{k-1}^j)$.

### 5.3 Choice of Appropriate Channel Process

To exploit the advantages of sequential estimation for our task of multipath mitigation/estimation we must be able to describe the actual channel characteristics (channel parameters) so that these are captured by $p(\mathbf{x}_k|\mathbf{x}_{k-1})$. In other words, the model must be a first order Markov model and all transition probabilities must be known. In our approach we approximate the channel as follows:

- The channel is totally characterized by a direct path (index $i = 1$) and at most $N_m - 1$ echos.
- Each path has complex amplitude $a_{i,k}$ and relative delay and $\Delta\tau_{i,k} = \tau_{i,k} - \tau_{1,k}$; where echos are constrained to have delay $\tau_{i,k} \geq \tau_{1,k}$; i.e. $\Delta\tau_{i,k} \geq 0$.
- The different path delays follow the process: (see Figure 8)

$$\tau_{1,k} = \tau_{1,k-1} + \alpha_{1,k-1} \cdot \Delta t + n_\tau \quad , \tag{20}$$

$$\Delta\tau_{i,k} = \Delta\tau_{i,k-1} + \alpha_{i,k-1} \cdot \Delta t + n_\tau \quad . \tag{21}$$

- Each parameter $\alpha_{i,k}$ that specifies the speed of the change of the path delay follows its own process:

$$\alpha_{i,k} = \left(1 - \frac{1}{K}\right) \cdot \alpha_{i,k-1} + n_\alpha \quad . \tag{22}$$

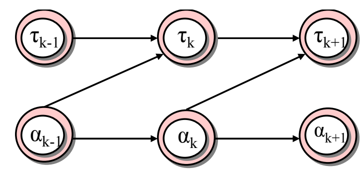



**Figure 8.** Markov model for the path delays.

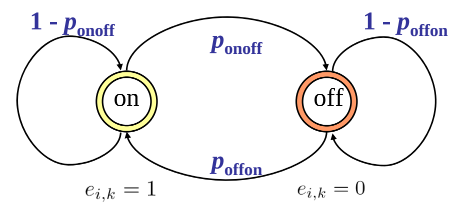



**Figure 9.** Markov model for the multipath activity.

- The magnitudes and phases of the individual paths, represented by the complex amplitudes $a_{i,k}$, are eliminated according to (12) and (14) for the computation of the likelihood (5).
- Each path is either "on" or "off", as defined by channel parameter $e_{i,k} \in \{1 \equiv \text{"on"}, 0 \equiv \text{"off"}\}$,
- where $e_{i,k}$ follows a simple two-state Markov process with a-symmetric crossover and same-state probabilities: (see Figure 9)

$$p(e_{i,k} = 0|e_{i,k-1} = 1) = p_{\text{onoff}} \quad , \tag{23}$$

$$p(e_{i,k} = 1|e_{i,k-1} = 0) = p_{\text{offon}} \quad . \tag{24}$$

The model implicitly incorporates three i.i.d. noise sources: Gaussian $n_\tau$ and $n_\alpha$, as well as the noise process driving the state changes for $e_{i,k}$. These sources provide the randomness of the model. The parameter $K$ defines how quickly the speed of path delays can change (for a given variance of $n_\alpha$). Finally, $\Delta t$ is the time between instances $k-1$ and $k$. We assume all *model parameters* (i.e. $K$, $\Delta t$, noise variances, and the "on"/"off" Markov model) to be independent of $k$. Note that the model directly represents the number of paths as a time variant parameter that is equal to $\sum_{i=1}^{N_m} e_{i,k}$. The hidden channel state vector $\mathbf{x}_k$ can therefore be represented as:

$$[\tau_{1,k}, \Delta\tau_{2,k}, \ldots, \Delta\tau_{N_m,k}, \alpha_{1,k}, \ldots, \alpha_{N_m,k}, e_{i,k}, \ldots, e_{N_m,k}]^T \quad . \tag{25}$$

When applied to our particle filtering algorithm, drawing from the proposal density is simple. Each particle stores the above channel parameters of the model and then the new state of each particle is drawn randomly from $p(\mathbf{x}_k|\mathbf{x}_{k-1}^j)$ which corresponds to drawing values for $n_\alpha$ and $n_\tau$ as well as propagating the "on"/"off" Markov model, and then updating the channel parameters for $k$ according to (20)-(24).

### 5.4 Practical Issues

#### 5.4.1 Model Matching

It is important to point out that a sequential estimator is only as good as its state transition model matches the real world situation. The state model needs to capture *all* relevant hidden states with memory and needs to correctly model their dependencies, while adhering to the first order Markov condition. Furthermore, any memory of the measurement noise affecting the likelihood function $p(\mathbf{z}_k|\mathbf{x}_k)$ must be explicitly contained as additional states of the model $\mathbf{x}$, so that the measurement noise is i.i.d.

The channel state model is motivated by channel modelling work for multipath prone environments such as the urban satellite navigation channel [9] [7]. In fact the process of constructing a channel model in order to characterize the channel for signal level simulations and receiver evaluation comes close to our task of building a first order Markov process for sequential estimation. For particle filtering, the model needs to satisfy the condition that one can draw states with relatively low computational complexity. Adapting the model structure and the model parameters to the real channel environment is a task for current and future work. It may even be possible to envisage hierarchical models in which the selection of the current model itself follows a process. In this case, a sequential estimator will automatically choose the correct weighting of these models according to their ability to fit the received signal.

#### 5.4.2 Integration into a Receiver

For receiver integration the computational complexity of the filtering algorithm is crucial. From a theoretical point of view it is desirable to run the sequential filter clocked corresponding to the coherent integration period of the receiver and with a very large number of particles. From the practical point of view, however, it is desirable to reduce the sequential filter rate to the navigation rate and to minimize the number of particles. Existing ML approaches can help here to achieve a flexible complexity/performance trade-off, as strategies already developed to extend the observation periods of ML estimators can be used directly to reduce the rate of the sequential filtering algorithm.

## 6 SIMULATIONS

For performance assessment computer simulations with the proposed sequential estimator have been carried out. The simulated signal corresponds to a GPS L1 signal with $c(t)$ being a Gold code of length 1023 that is modulated on a bandlimited rectangular pulse. The chip rate is 1.023 MChips/s so that the duration of the codeword is 1ms. The one-sided bandwith of the resulting navigation signal is 5 MHz. The signal's carrier to noise density ratio is selected to be $C/N_0$=45dB-Hz and the multipath power is 6dB lower than that of the direct path. The Bayesian estimator uses a time interval of 1 ms corresponding to the

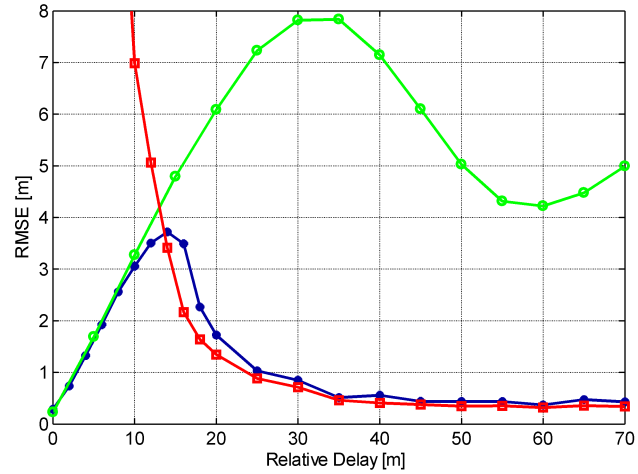



**Figure 10.** Static multipath scenario: Performance of SIR PF as function of relative multipath delay for different path models.

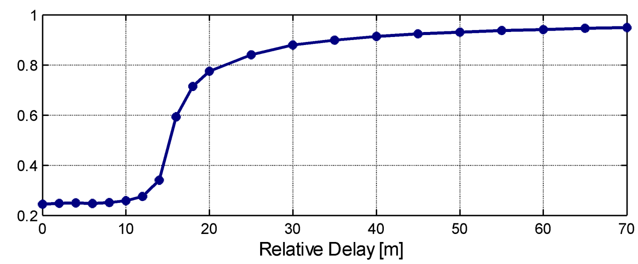



**Figure 11.** Static multipath scenario: Average probability of a two path model for the estimator with path activity tracking.

duration of a codeword. The amplitude averaging coefficient is set to $Q = 10$. The channel parameters $\sigma^2_{i,\tau}$, $\sigma^2_{i,\alpha}$, $K$, and $p(e_{i,k}|e_{i,k-1})$ are selected to fit the statistics of a real channel according to [9]. The SIR PF uses the minimum mean square error (MMSE) criterion to estimate the parameters $\mathbf{x}_k$ from the posterior.

### 6.1 Static Channel

The capability of multipath mitigation techniques is commonly assessed by showing the systematic error due to a single multipath replica plotted as a function of the relative multipath delay in a static channel scenario. In Fig. 10 the root mean square error (RMSE) is shown for the proposed sequential estimator, implemented as a SIR PF with 2000 particles. Estimators with fixed two path model or fixed single path model are also shown for comparison with the implicit path activity tracking. The performance of a single path estimator is comparable to that of a DLL with infinitesimal correlator spacing and shows a considerable bias over a large delay range. The estimator with fixed two path model successfully mitigates the multipath bias for delays greater than 30m. However, as indicated by the Cramer-Rao bound [3], for smaller delays it shows an in-

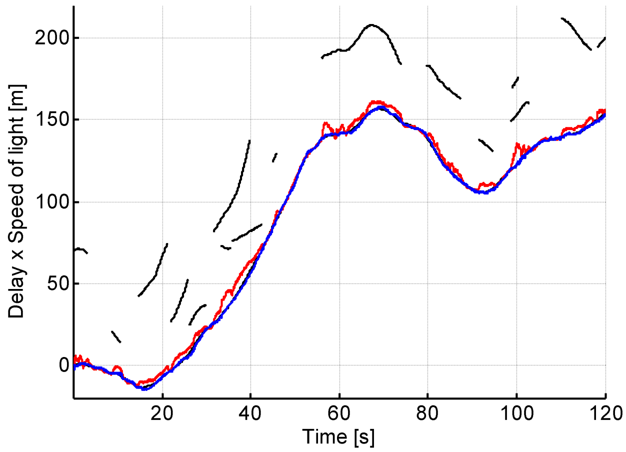



**Figure 12.** Dynamic multipath scenario: Tracking performance of SIR PF and DLL.

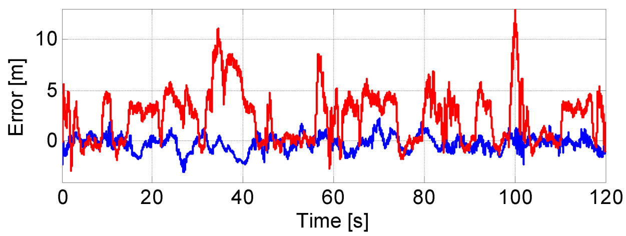



**Figure 13.** Dynamic multipath scenario: Direct path tracking error of SIR PF and DLL for the scenario that is depicted in Fig. 12

creasing variance and is outperformed by the single path estimator. The estimator with path activity tracking is capable of combining the advantages of both models. From the posterior it is possible to calculate the estimated average probability $P(N_{m,k} = 2|\mathbf{Z}_k)$ of a two path model, which is shown in Fig. 11 and indicates the transition between the models: for small delays the two paths essentially merge to a single one. Note that in these simulations the model parameters of the sequential estimator are still the ones designed for the dynamic channel and not optimal for this static scenario.

### 6.2 Dynamic Channel

Results for a randomly chosen dynamic channel with up to $N_m = 3$ paths, which matches to the model parameters assumed in the estimator, are depicted in Fig. 12 for a SIR PF with 20 000 particles. The corresponding error of the direct path tracking is shown in Fig. 13, together with that of a conventional non-coherent DLL with 0.1 chip early/late correlator spacing and 1 Hz tracking loop bandwidth. This loop bandwidth was found to result in the smallest RMSE of the DLL for the considered dynamic scenario in the absence of multipath. The DLL performance suffers significantly from the multipath reception. The SIR PF (RMSE = 0.77 m) is less distorted than the DLL (RMSE = 3.49 m)

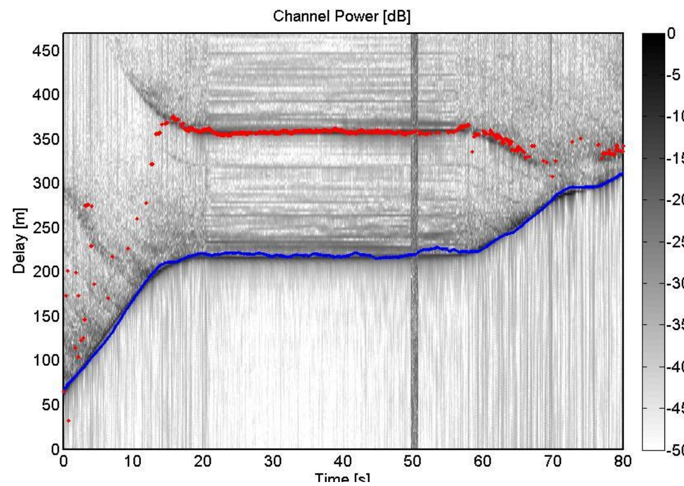



**Figure 14.** Performance of the SIR PF with up to $N_m = 2$ paths for the measured urban channel environment (compare Figure 1. Direct path estimates are shown in blue, second path estimates in red.

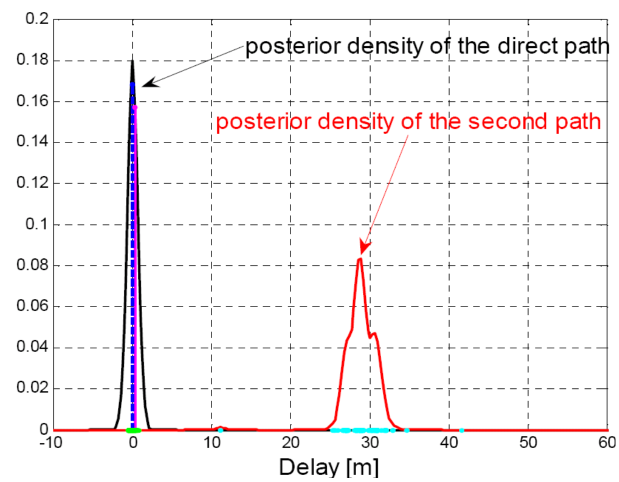



**Figure 15.** Illustration of particles in the delay space at different time steps, and the posterior density of the direct path delay (black) and the second path relative delay (red) given by the particles in different dimension (green and cyan).

and able to detect the activity of multipath implicitly.

We have also applied the SIR PF to the measured channel described in Section 1. The results, given in Figure 14, show again a clear advantage over the DLL with narrow correlator.

A further advantage compared to ML estimation is that the posterior PDF at the output of the estimator represents reliability information about the desired parameters and preserves the ambiguities and multiple modes that may occur within the likelihood function. An example of the posterior obtained from the set of particles is illustrated in Figure 15.

## 7 CONCLUSION

In this paper we have shown that a Bayesian filter is capable of reducing the errors caused by multipath successfully by exploiting the strong temporal correlations of the channel parameters. Our approach is characterized by an improved state transition model that allows us to introduce a Markov model to determine the lifecycle of each individual path, such as temporarily turning a path on and off as well as creating and destroying it. This approach allows to track the signal delays as well as the number of paths implicitly in a probablistic fashion. As all hypotheses are tracked simultaneously the problem of error propagation is avoided.

We have demonstrated how sequential Bayesian estimation techniques can be applied to the multipath mitigation problem in a navigation receiver. The proposed approach is characterized by code matched, correlator based signal compression together with interpolation techniques for efficient likelihood computation in combination with a particle filter realization of the prediction and update recursion. The considered movement model has been adapted to dynamic multipath scenarios and incorporates the number of echos as a time variant hidden channel state variable that is tracked together with the other parameters in a probabilistic fashion.

## REFERENCES


[1] D. van Nee, J. Siereveld, P. Fenton, and B. Townsend, "The multipath estimating delay lock loop: Approaching theoretical accuracy limits," in *Proceedings of the IEEE Position, Location and Navigation Symposium 1994*, Las Vegas, Nevada, USA, 1994.

[2] L. Weill, "Achieving theoretical accuracy limits for pseudoranging in the presence of multipath," in *Proceedings of the ION GPS 1995*, Palm Springs, California, USA, 1995.

[3] J. Selva Vera, "Efficient multipath mitigation in navigation systems," Ph.D. dissertation, Universitat Politecnica de Catalunya, Feb. 2004.

[4] F. Antreich, O. Esbri-Rodriguez, J. Nossek, and W. Utschick, "Estimation of synchronization parameters using SAGE in a GNSS receiver," in *Proceedings of the ION GNSS 2005*, Long Beach, California, USA, 2005.

[5] P. Fenton and J. Jones, "The theory and performance of novatel inc.s vision correlator," in *Proceedings of the ION GNSS 2005*, Long Beach, California, USA, Sept. 2005.

[6] P. Closas, C. Fernandez-Prades, J. Fernandez-Rubio, and A. Ramirez-Gonzalez, "Multipath mitigation using particle filtering," in *Proceedings of the ION GNSS 2006*, Fort Worth, Texas, USA, Sept. 2006.

[7] A. Steingass and A. Lehner, "Land mobile satellite navigation - characteristics of the multipath channel," in *Proceedings of the ION GNSS 2003*, Portland, Oregon, USA, Sept. 2003.

[8] A. van Dierendonck, P. Fenton, and T. Ford, "Theory and performance of narrow correlator spacing in a gps receiver," in *Proceedings of the ION National Technical Meeting 1992*, San Diego, California, USA, 1992.

[9] A. Steingass and A. Lehner, "Measuring the navigation multipath channel - a statistical analysis," in *Proceedings of the ION GNSS 2004*, Long Beach, California, USA, Sept. 2004.

[10] S. M. Kay, *Fundamentals of Statistical Signal Processing - Estimation Theory*. New Jersey: Prentice Hall, 1993.

[11] L. Weill, "Achieving theoretical bounds for receiver-based multipath mitigation using galileo os signals," in *Proceedings of the ION GNSS 2006*, Fort Worth, Texas, USA, Sept. 2006.

[12] S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking," *IEEE Trans. Signal Processing*, vol. 50, no. 2, pp. 174–188, Feb. 2002.