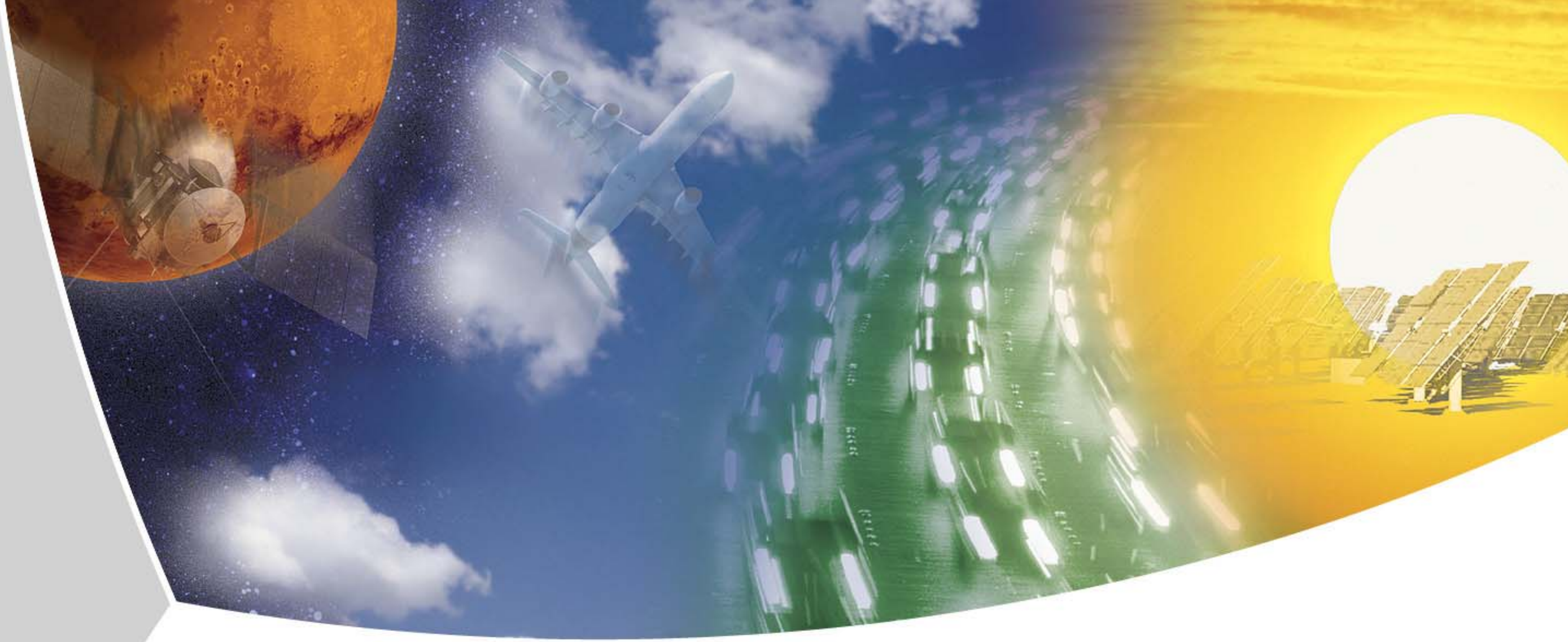


Degree of Polarization for Operational Weather Radars

Michele Galletti	DLR
David H. O. Bebbington	University of Essex
Madhu Chandra	TU-Chemnitz
Thomas Boerner	DLR

DLR Deutsches Zentrum für Luft- und Raumfahrt e.V.
in der Helmholtz-Gemeinschaft

SUMMARY

- Degree of Polarization: Theoretical Aspects
- Entropy: a paradigm for DoP performance
- A look at the Data
- Conclusions

Degree of Polarization DoP

$$E(t)=\begin{bmatrix}E_1(t)\\E_2(t)\end{bmatrix}=\begin{bmatrix}|E_1(t)|\,e^{i\varphi_1(t)}\\|E_2(t)|\,e^{i\varphi_2(t)}\end{bmatrix}$$

Measurements done with a dual-pol coherent receiver can be considered samples of a random Jones vector.

$$Cov(E)=J=\langle E\cdot E^{+}\rangle$$

The covariance of a random Jones vector is Wolf's coherency matrix J

$$p=\sqrt{1-\frac{4\det(J)}{(trace(J))^2}}=\frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}$$

DoP is a basis-invariant quantity and does not depend on the orthogonal pairs of polarimetric channels chosen to sample the backscattered wave...

For a coherent target, the return is totally polarized, regardless of the transmit polarization state.
For an incoherent target, DoP depends on the transmit polarization state.

We name this function DoP = DoP(transmit state)
the DEPOLARISATION RESPONSE OF AN INCOHERENT TARGET

DoP – Isotropic targets (Z_{DR}=0)

A simple model for isotropic weather targets can be thought of as a cloud of randomly oriented spheroids.

$$K_{iso} = \begin{bmatrix} 1+B_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1+B_0 \end{bmatrix}$$

B_0 is the generator of target structure.
K is the Kennaugh matrix of an isotropic target.

$$p = \frac{\sqrt{\cos^2(2\chi) + (1-B_0)^2 \sin^2(2\chi)}}{1+B_0}$$

Analitical expression of DoP for isotropic targets

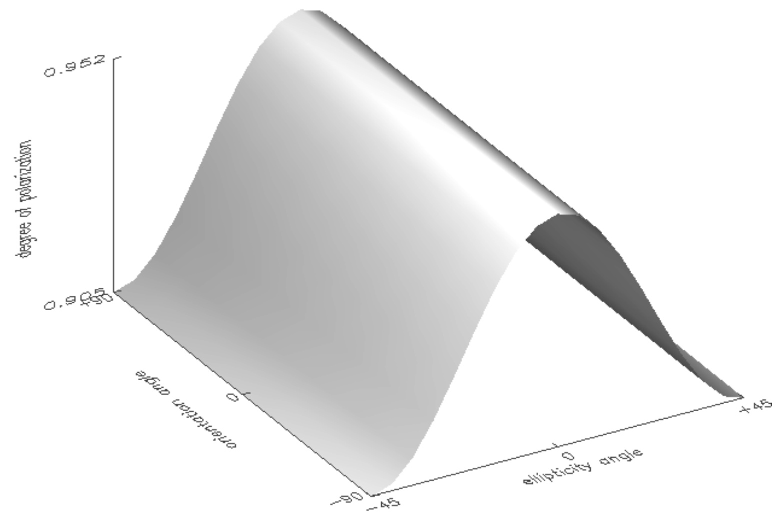
$$1 - p_{\min} = \frac{2B_0}{1+B_0} \qquad 1 - p_{\max} = \frac{B_0}{1+B_0}$$

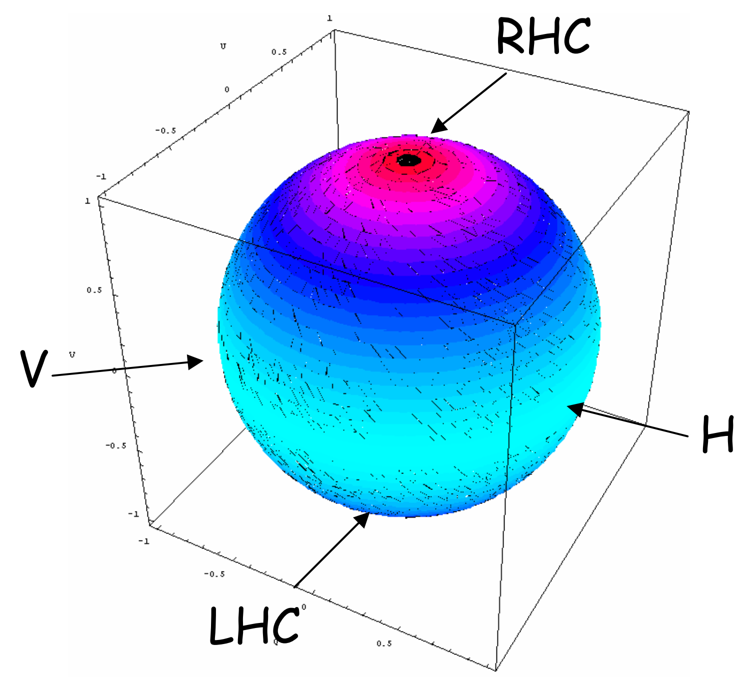
Between the minimum and the maximum there is a simple 3 dB difference!!

DoP – Isotropic targets (Z_{DR}=0)

$$K_{iso} = \begin{bmatrix} 1+B_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1+B_0 \end{bmatrix}$$

B_0=0.05
Randomly oriented, slightly oblate spheroids
Graupel



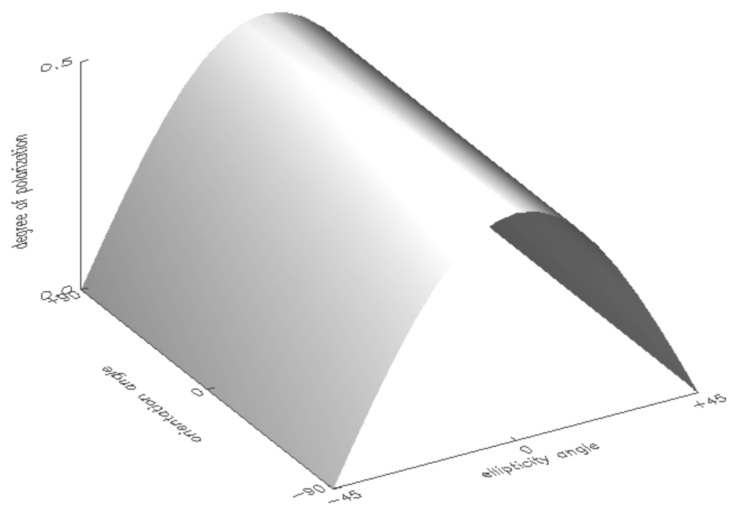


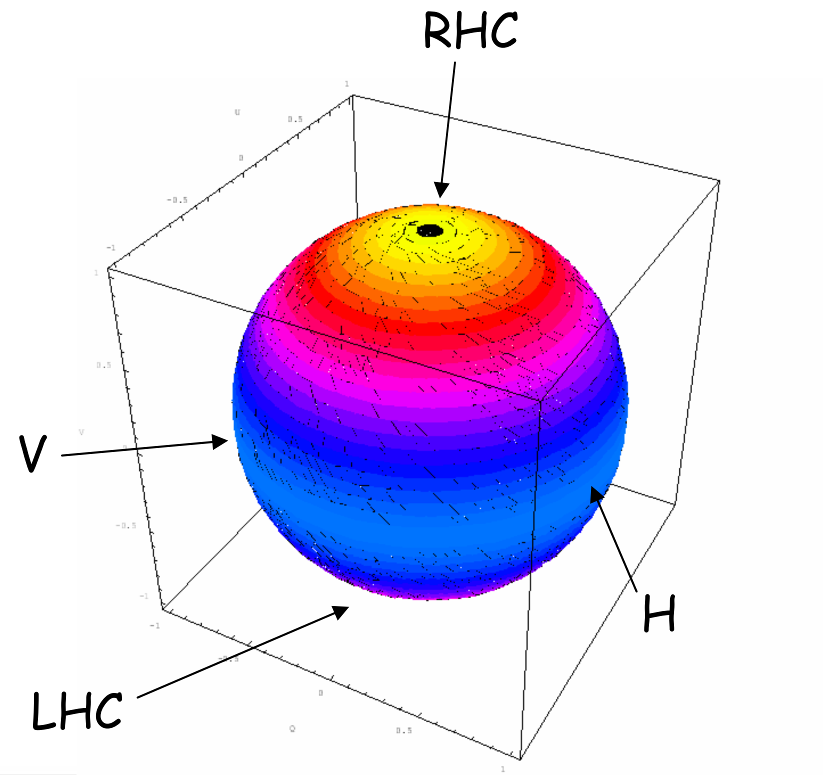
DoP – Isotropic targets (Z_{DR}=0)

$$K_{iso} = \begin{bmatrix} 1+B_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1+B_0 \end{bmatrix}$$

B_0=1

Randomly oriented dipoles

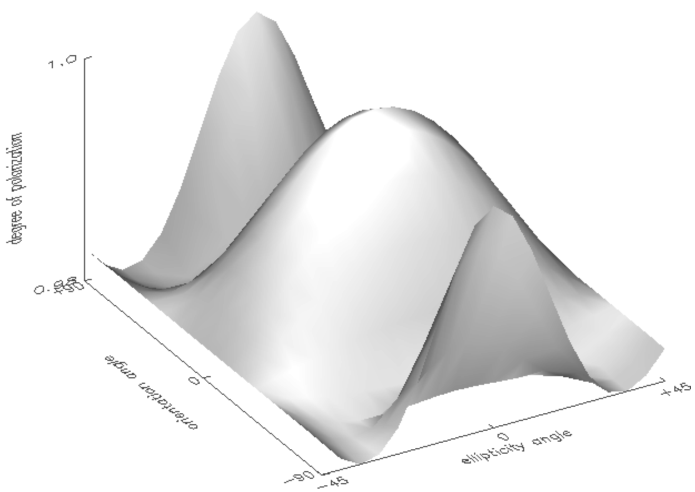


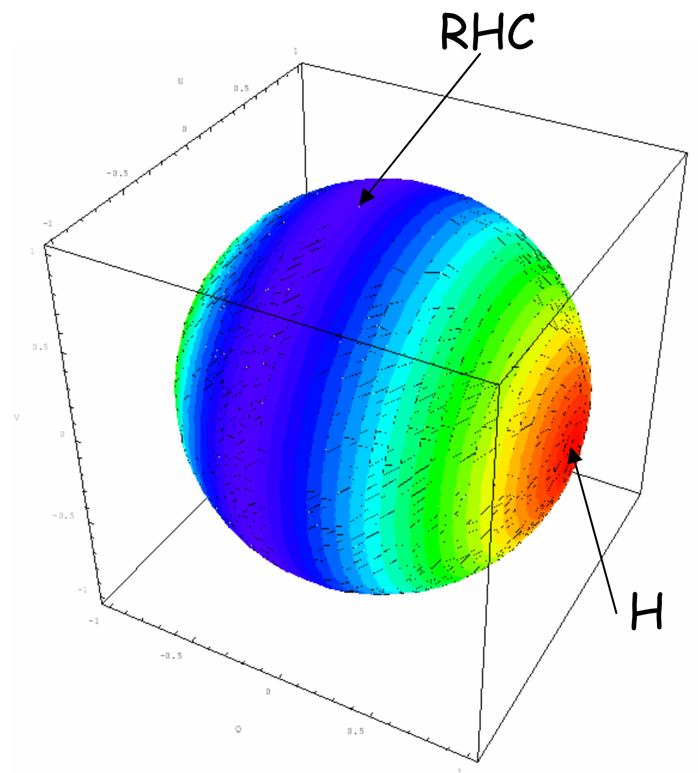


DoP – Anisotropic targets (Z_{DR}>0) (Rain)

$$K_{rain} = \begin{bmatrix} 1.0394 & 0.397 & 0 & 0 \\ 0.397 & 1.0394 & 0 & 0 \\ 0 & 0 & 0.9605 & 0 \\ 0 & 0 & 0 & -0.9605 \end{bmatrix}$$

Rain, bimodal distribution, (Z_{DR} = 1.7dB)





DoP theory: Overview

- DoP at H or V send is always maximal
- DoP at circular is always minimal
- DoP at slant send is minimal for anisotropic and maximal for isotropic targets
- A way to measure DoP capability to capture information is a confrontation with Entropy!!
- As Entropy captures the full polarimetric diversity,
  the minimal DoP should mirror its behavior quite faithfully…

Entropy (from Cloude decomposition)

$$\begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \rightarrow \begin{bmatrix} S_{HH} \\ \sqrt{2} S_{HV} \\ S_{VV} \end{bmatrix} = \underline{\Omega}_i$$

The elements of the S matrix can be arranged in a target feature vector....

$$[C] = \sum_i \underline{\Omega}_i \cdot \underline{\Omega}_i^+ = \begin{bmatrix} \langle |S_{HH}|^2 \rangle & \sqrt{2}\langle S_{HH} S_{HV}^* \rangle & \langle S_{HH} S_{VV}^* \rangle \\ \sqrt{2}\langle S_{HV} S_{HH}^* \rangle & 2\langle |S_{HV}|^2 \rangle & \sqrt{2}\langle S_{HV} S_{VV}^* \rangle \\ \langle S_{VV} S_{HH}^* \rangle & \sqrt{2}\langle S_{VV} S_{HV}^* \rangle & \langle |S_{VV}|^2 \rangle \end{bmatrix}$$

Averaging yields the Covariance matrix....

$$[C] = [U_3] \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} [U_3]^+$$

Diagonalization decomposes the covariance matrix into its principal components....

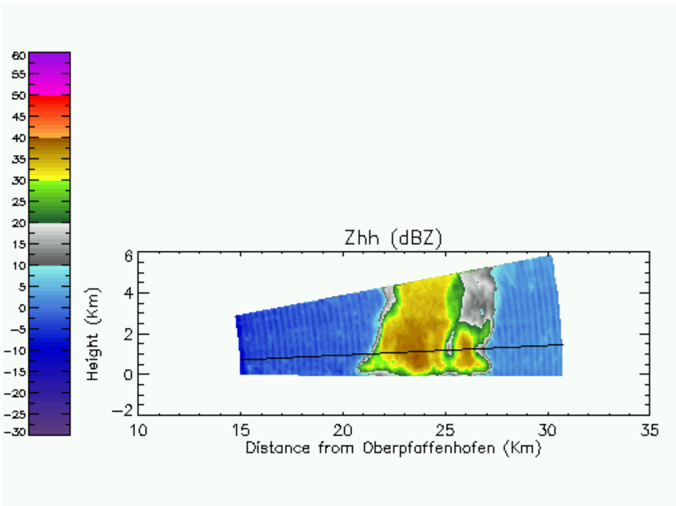
$$P_i = \frac{\lambda_i}{\sum \lambda_i} \qquad 0 \le P \le 1$$

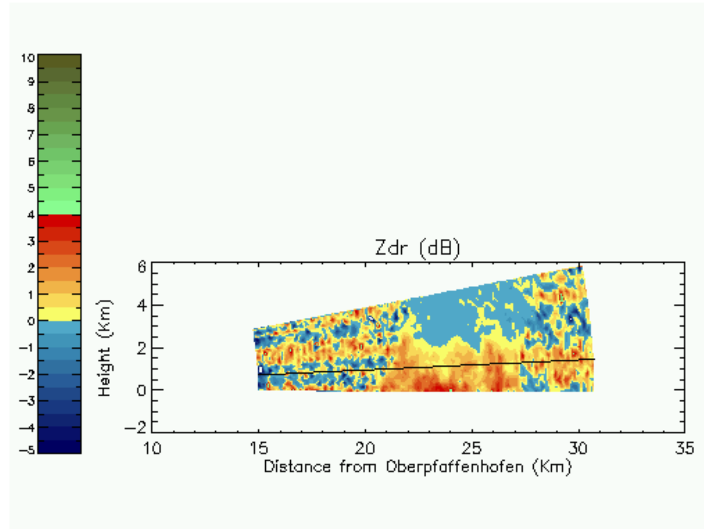
$$H = -\sum_{i=1}^{3} P_i \log_3(P_i) \qquad 0 \le H \le 1$$

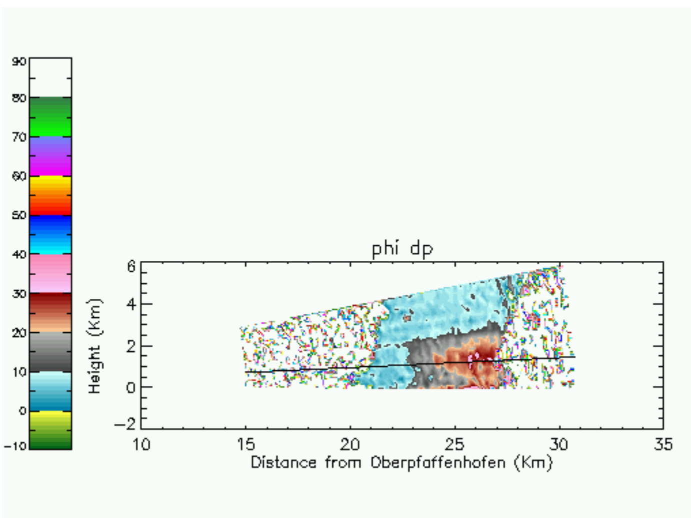
Entropy is a scalar quantity relating to the heterogeneity of statistically independent degrees of freedom existing in the scattering population

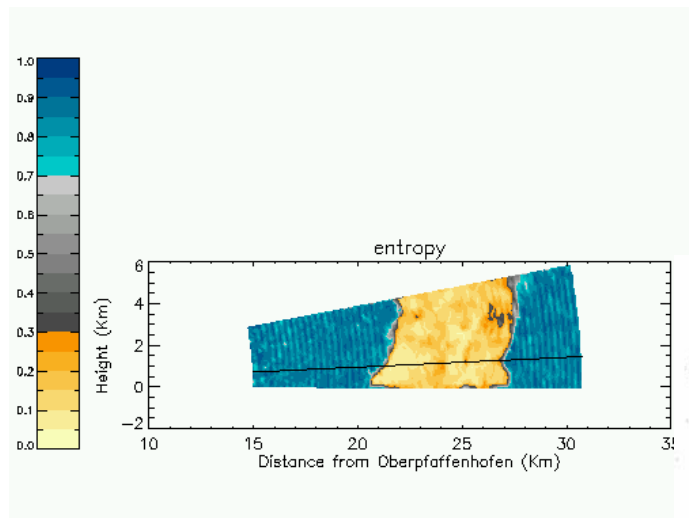
$$C^{'} = P\ C\ P^+$$

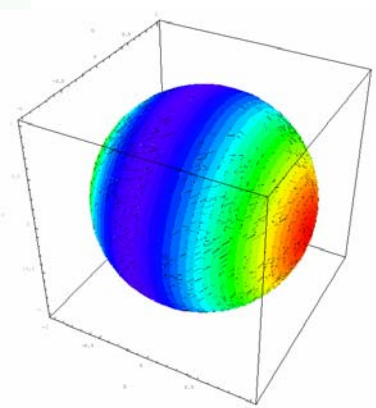
Hydrometeor Discrimination

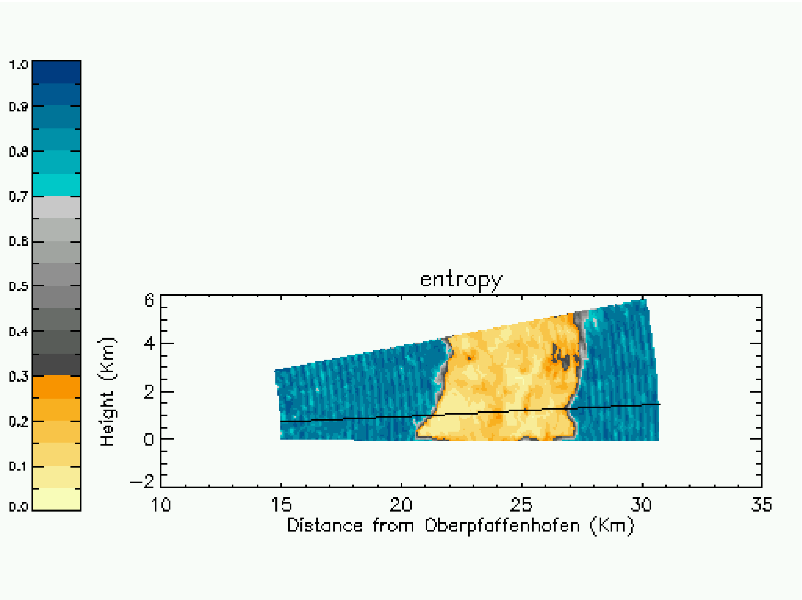

Zhh (dBZ)
Height (Km)
Distance from Oberpfaffenhofen (Km)

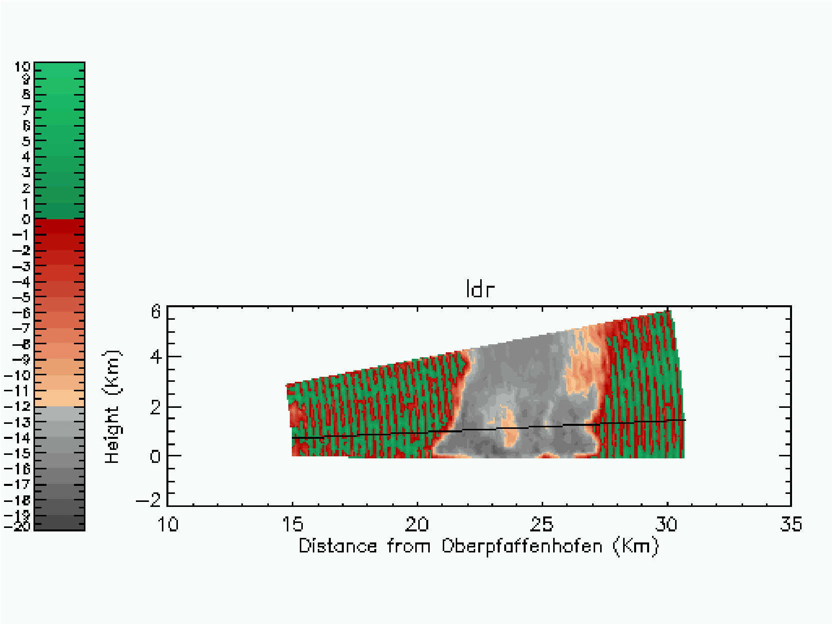

Zdr (dB)
Height (Km)
Distance from Oberpfaffenhofen (Km)

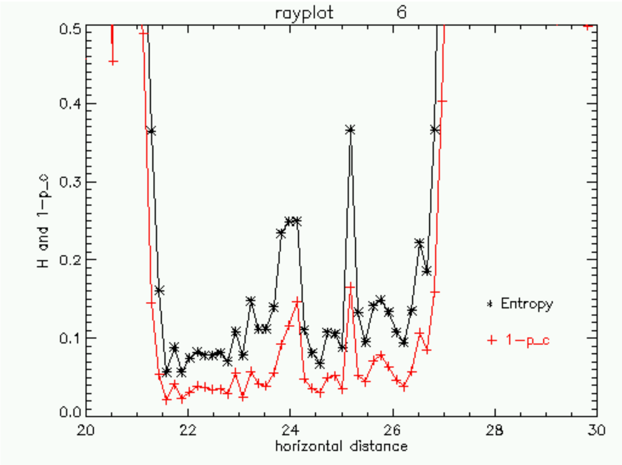

phi dp
Height (Km)
Distance from Oberpfaffenhofen (Km)

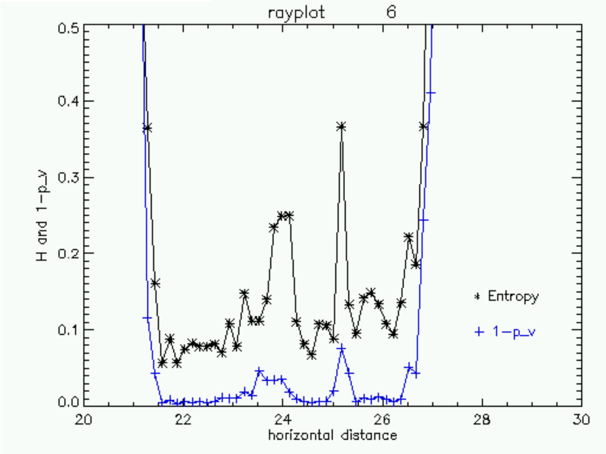

entropy
Height (Km)
Distance from Oberpfaffenhofen (Km)

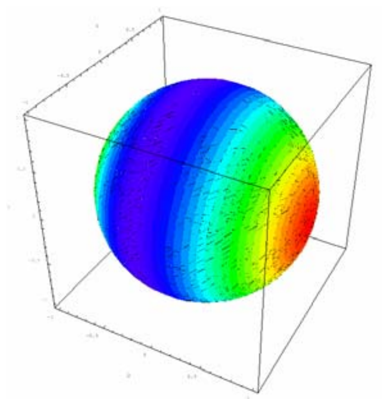








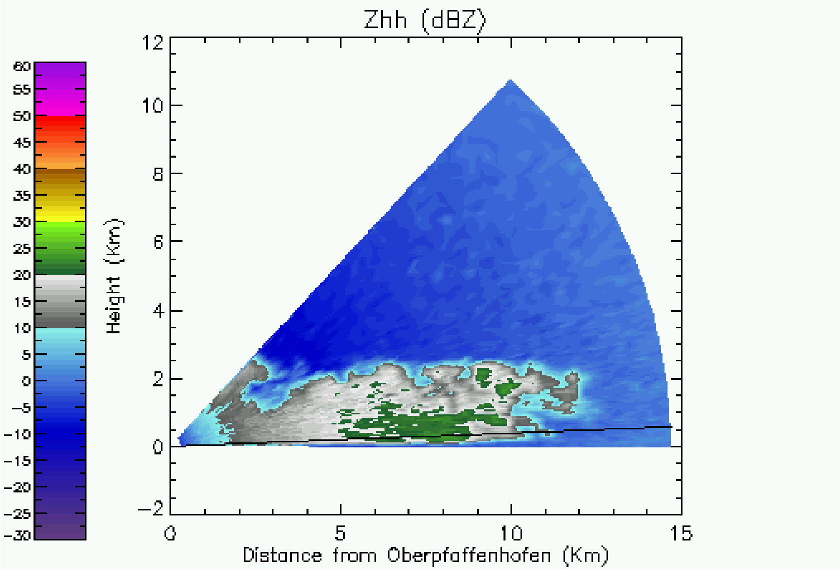


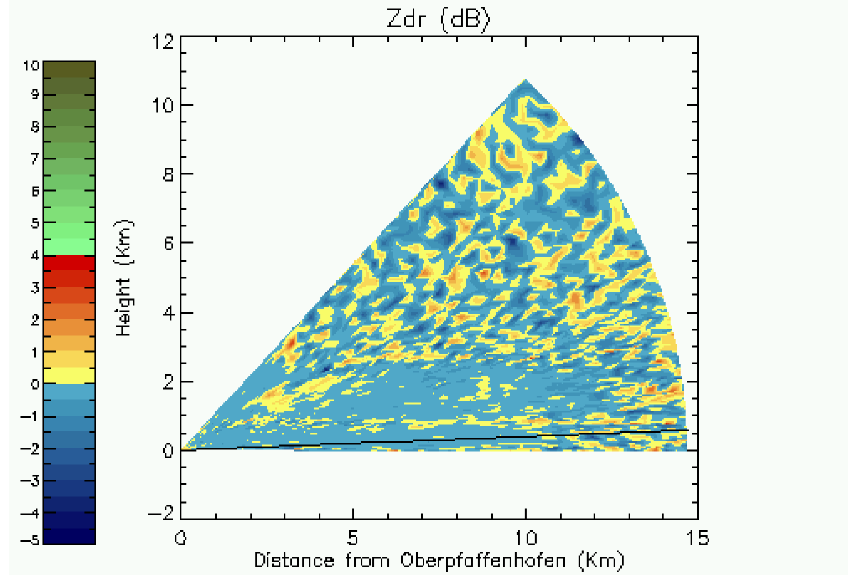


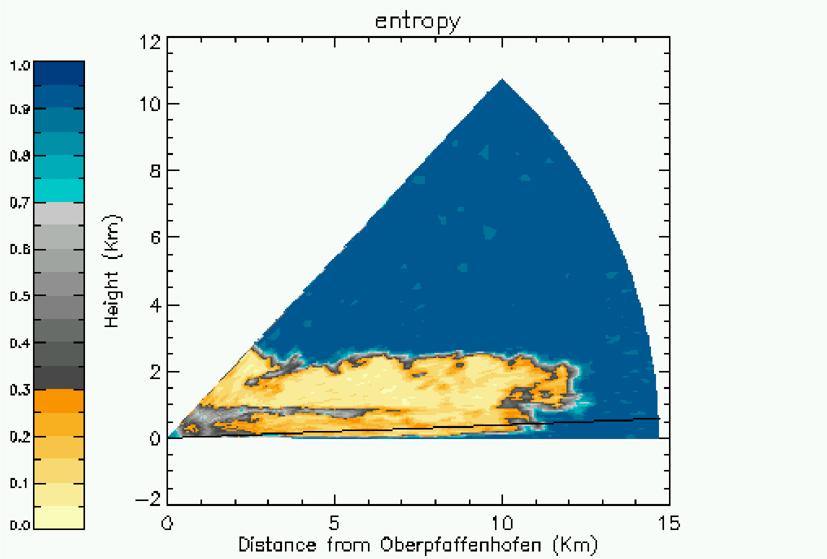
RHC/LHC/+45/-45

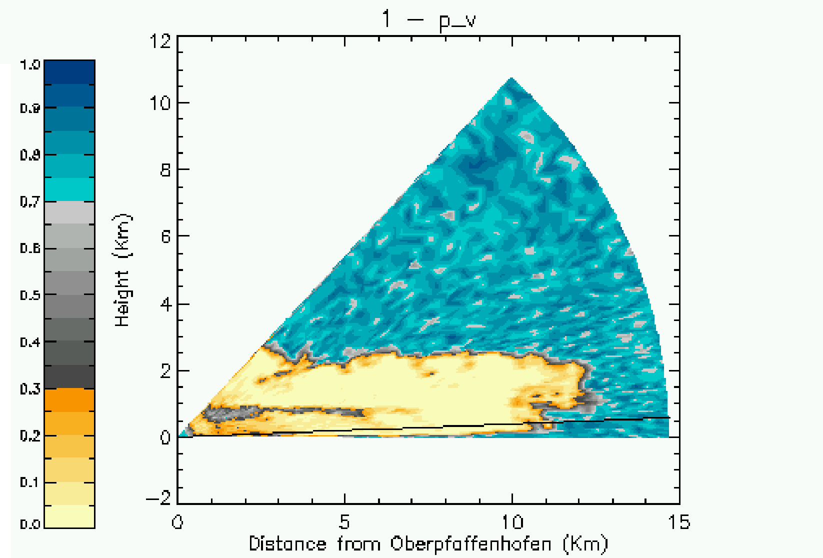
H/V

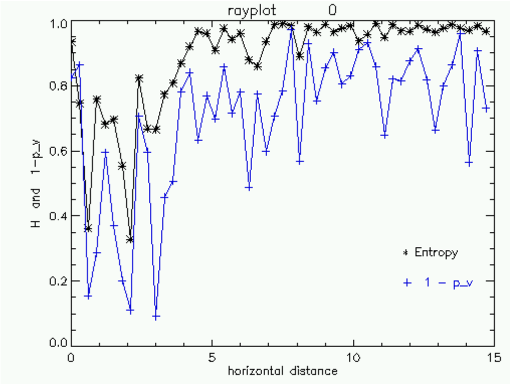
Clutter

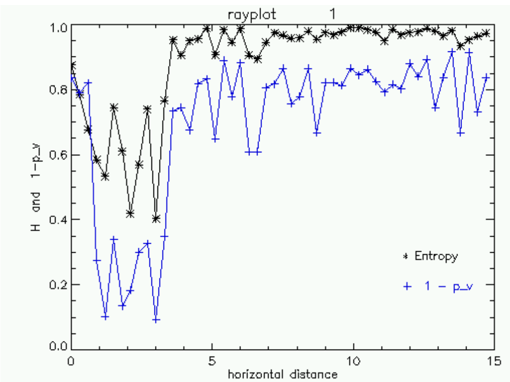

Zhh (dBZ)
Height (Km)
Distance from Oberpfaffenhofen (Km)

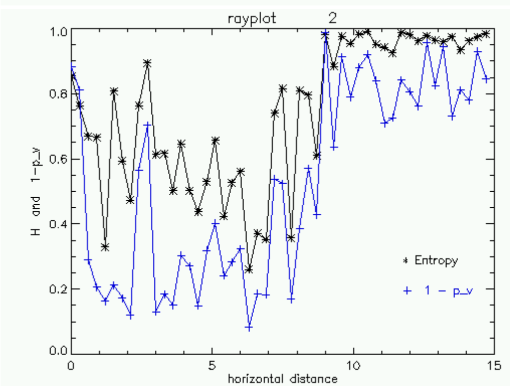

Zdr (dB)
Height (Km)
Distance from Oberpfaffenhofen (Km)


entropy
Height (Km)
Distance from Oberpfaffenhofen (Km)


1 − p_v
Height (Km)
Distance from Oberpfaffenhofen (Km)





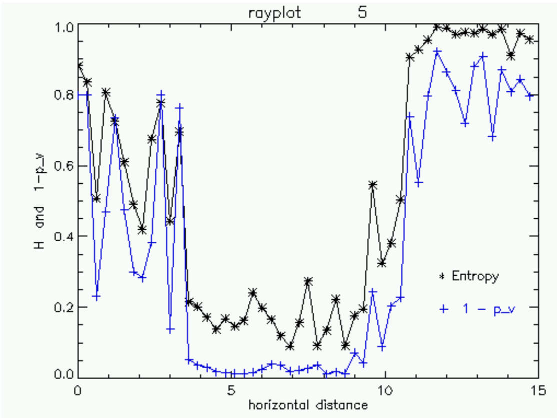


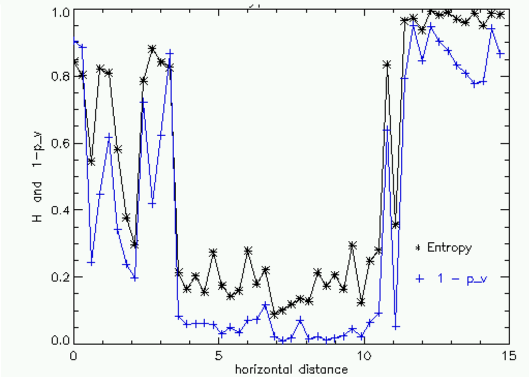
By increasing the elevation,

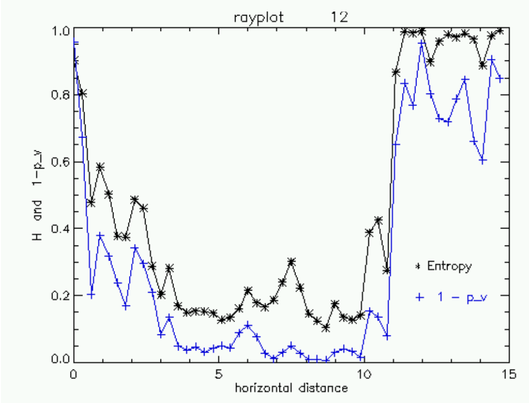
we see a transition from

clutter signatures

to rain signatures







Conclusions

- We started setting the theoretical framework for the behavior of the degree of polarization for incoherent targets, by means of the depolarization response.
- We tested the theory against data with good agreement
- For operational purposes, it is worth noting that the degree of polarization at slant send carries valuable information, especially to planned operational weather radars implementing hybrid polarization, where the linear depolarization ratio is not available

Thanks