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120

3D ANALYSIS OF COUPLED THIN-WALLED STRUCTURAL ELEMENTS BY MEANS OF STANDARD BOUNDARY-ELEMENT FORMULATIONS

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ABSTRACT

Recently, some improvements of the generic BE/BE coupling algorithm based on Krylov's solvers, proposed by the first author of this paper and outlined in previous papers, were carried out by including discontinuous boundary elements in the BE code. As a consequence of that, traction continuity conditions, necessary for simulating edges and corners at coupling interfaces in models based on continuous boundary elements, must no longer be taken into account. Thus, the respective BE models for complex 3D regions become considerably simpler. In this paper, a general description of the main features of the algorithm is furnished. In addition, some strategies for making the analysis of moderate thin-walled structural elements by means of three-dimensional standard BE formulations possible are presented. A study of the performance of the code is also shown, where mainly computational efficiency parameters such as response precision and CPU time for assembling the system of algebraic equations and solving it are commented. Engineering systems involving thin-walled elements, possibly containing cracks, are simulated. High-precision IMSL routines (available in FORTRAN compilers) are used to study the performance of the code. Strategies for improving the conditioning of the systems are also commented.

KEYWORDS. Thin-walled structural elements, generic BE/BE coupling algorithm, discontinuous boundary elements, Krylov's solvers

INTRODUCTION

As a matter of fact, a tridimensional analysis makes it possible to have a more insightful comprehension of continuum problems. Indeed, one-dimensional or two-dimensional formulations are derived from tridimensional ones by taking into account some simplifying hypotheses concerning e.g. geometry and loading. As consequence of these assumptions, some numerical problems, such as numerical locking or convergence problems, may occur when simplified formulations are applied [1]. Unfortunately computational hardware limitations have restricted the use of 3D formulations in solving practical engineering problems, for which, non-rarely, large models (sometimes containing millions of unknowns) are necessary.

In applications of the Finite Element Method (FEM) for modeling thin-walled domains, structural elements such as plate or shell elements, based on simplified formulations, remain nowadays still very useful, though numerical difficulties may happen. In such cases, 3D continuum elements are in effect avoided, then 3D FE models for solving correctly this kind of problem may be very heavy. Contrary to the FEM, the Boundary Element Method (BEM) has proven to be very efficient to analyze engineering problems via 3D formulations. A reason for that is the dimension reduction of the problem, which naturally implies models with fewer elements, and, additionally, the quality of the results; relatively accurate responses may be obtained with relatively poor BE models. Thus, BE formulations along with the continuously increasing computer capacity may be a very interesting alternative for solving engineering problems by means of 3D formulations. Regarding particularly thin-domain interior problems, it has been proven that, contrary to crack-like problems, the arising system of algebraic equations will not degenerate [2]. It is however worth mentioning, that special care must be taken with the near singular integrals.

In this work, the 3D generic BE–BE coupling strategy published by the first author in previous papers [3-6] is extended to thin-walled problems, possibly containing cracks and reinforced with stiffeners. The subregion technique is used to simulate domain cracks, and the integration procedure based on triangular polar coordinates is adapted for evaluating the near singular integrals.

A characteristic of the coupling algorithm [3-6] is that the subsystems associated with the various subdomains of the model are treated as they were in effect uncoupled. Iterative solvers are used, and as consequence of that the global system must not be explicitly assembled. On the other hand, the use of discontinuous elements avoids imposing additional traction continuity conditions at interface nodes [6-7]. Of course, the coupling algorithm will efficiently work if so will the applied iterative solver. Fortunately, several kinds of Krylov's solvers have been intensively used in real–life industrial applications and proven to be very efficient [7-9]. The Jacobi–preconditioned bi–conjugate gradient solver (J-BiCG) is the only Krylov's solver applied in the analyses here.

In the case studies presented, one shows the efficiency of the coupling algorithm implemented in the NAESY code in terms of its precision, system conditioning, required CPU time, etc. The models analyzed involve the coupling of geometrically complex thin-walled subdomains. Comparisons with results obtained by using a high–precision IMSL solver (available along with FORTRAN compilers) and the ANSYS are shown. The potential of the coupling algorithm developed for solving engineering problems in general is also highlighted.

THE COUPLING ALGORITHM

In general, subregions in BE formulations are used because of either geometrical or physical characteristics, or also aiming at increasing the computational efficiency (e.g. development of parallelized codes). Particularly for the curved panel (thin-walled shell) shown in Fig. 1, which is reinforced with stiffeners and presents a crack, the substructuring technique may be also used to establish a BE model. In this case, an alternative for the domain decomposition could be that shown in Fig. 2. Notice that subdomains 2 and 5 are associated with the same stiffener.

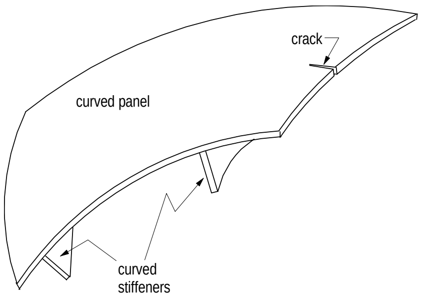

Fig. 1. System panel-stiffener

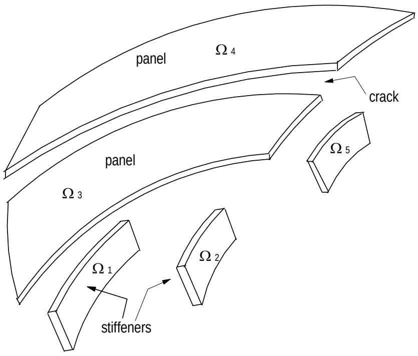

Fig. 2. Domain decomposition

Explicitly, the corresponding global coupled system, after rearranging conveniently the system variables, is given by

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{C}_{13} & \mathbf{C}_{14} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{22} & \mathbf{C}_{23} & \mathbf{0} & \mathbf{C}_{25} \\ \mathbf{C}_{31} & \mathbf{C}_{32} & \mathbf{A}_{33} & \mathbf{C}_{34} & \mathbf{0} \\ \mathbf{C}_{41} & \mathbf{0} & \mathbf{C}_{43} & \mathbf{A}_{44} & \mathbf{C}_{45} \\ \mathbf{0} & \mathbf{C}_{52} & \mathbf{0} & \mathbf{C}_{54} & \mathbf{A}_{55} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \end{Bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & & & & \\ & \mathbf{B}_{22} & & & \\ & & \mathbf{B}_{33} & & \\ & & & \mathbf{B}_{44} & \\ & & & & \mathbf{B}_{55} \end{bmatrix} \begin{Bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \\ \mathbf{y}_5 \end{Bmatrix} + \begin{Bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \\ \mathbf{r}_4 \\ \mathbf{r}_5 \end{Bmatrix}, \quad (1)$$

where $\mathbf{r}_i$ are the volume forces to be considered at the i-th subregion, $\mathbf{A}_{ii}$ and $\mathbf{B}_{ii}$ are its BE matrices (obtained by interchanging the columns of $\mathbf{H}_{ii}$ and $\mathbf{G}_{ii}$ matrices according to the boundary conditions), and $\mathbf{C}_{km}$ is the coupling matrix (containing either $\mathbf{H}_{kk}$ or $\mathbf{G}_{kk}$ terms) related to the interface between subregions k amd m. Of course, $\mathbf{C}_{km} = \mathbf{0}$ if there is no interface between the k–th and m–th subregion. Furthermore, $\mathbf{x}_k$ contains the boundary unknowns and part of the interface values of the k-th subregion, and $\mathbf{y}_k$ contains its prescribed boundary values. Notice that if there is an interface between subregions k and m, part of the interface values are allocated in $\mathbf{x}_k$ and part, in the $\mathbf{x}_m$ vector [7]. But inasmuch as iterative solvers are used, the only operations involved in the solution phase are matrix–vector and vector–vector multiplications, and thereby, the coupled system does not need to be explicitly assembled. In the proposed code particularly, its solution is obtained by operating independently the BE subsystems corresponding to each subregion. Indeed, regarding the numerical treatment (assembling, storing, and processing along the solver), the BE subsystems are treated as the subregions were in effect uncoupled. As matter of fact, based on variables that indicate which nodes are coupled, the interface values are updated so as to assure the imposition of the coupling conditions [4].

Another characteristic of the coupling algorithm is that discontinuous boundary elements are used. Thus, traction discontinuity at interface corners and edges may be simulated by imposing only the following coupling conditions (see Fig. 3):

$$u_k^{(i)} = u_k^{(j)} \text{ and } \quad p_k^{(i)} = -p_k^{(j)} \quad . \quad (2)$$

Additional traction continuity conditions are no longer necessary.

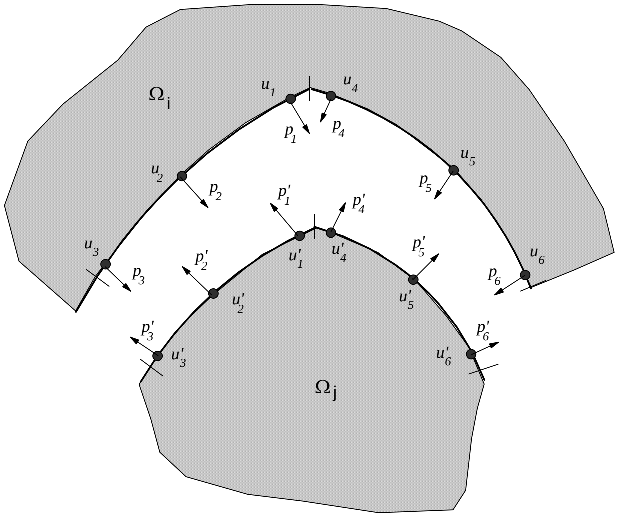

Figure 2. Coupling conditions at interface elements

APPLICATIONS

To observe the performance of the coupling strategy, a thin-cylindrical pipe under internal pressure $p_0 = 0.001\ N/mm^2$ is analyzed (see Fig. 4). The shell is reinforced with internal stiffeners and has a crack, and as a consequence of the symmetry only one eighth of the pipe is discretized (see Fig. 5 and Fig. 6). The physical and geometrical data of the shell are: $E = 205{,}000\ N/mm^2$, $\nu = 0.00$, $h = 0.5a$, and three BE substructuring models, generically described below, are used to analyze the problem. In the model 1, 2 subregions are considered (1 for the stiffener, and 1 for the shell); in model 2, 6 subregions (2x2 for the shell, and 2 for the stiffener), and in model 3, 12 subregions (3x3 for the shell, and 3 for the stiffener) are used. Furthermore, 4 different BE meshes per subregion are generated for each model. The deflections at the cylinder shell along the z-axis are shown in graphs 7 – 9 for the different models. In these graphs, the NAESY responses (obtained with use of the IMSL and J-BiCG solvers) are compared to the ANSYS ones.

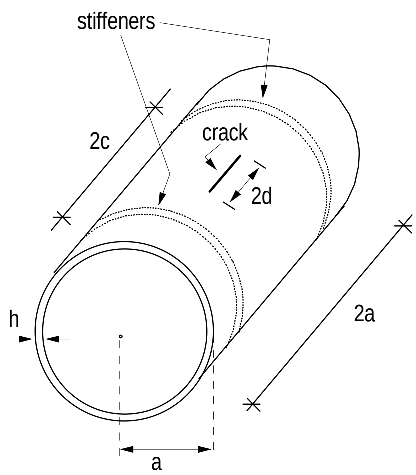

Figure 4. Thin-walled cylindrical pipe

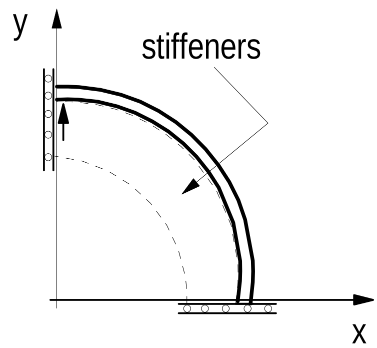

Figure 5. Cross-section of the shell (one-fourth)

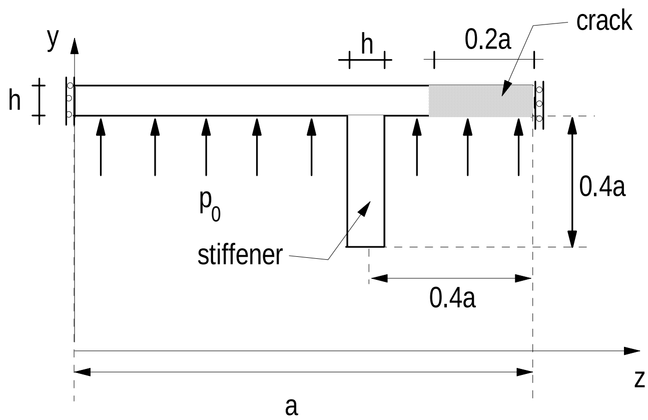

Figure 6. Lateral view of the pipe along the crack

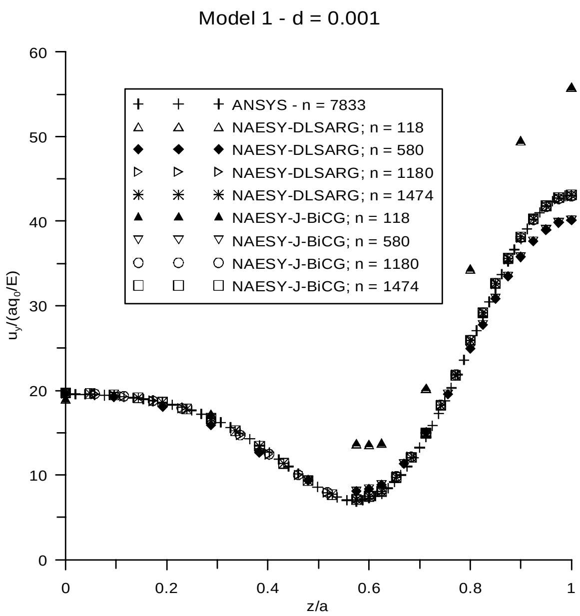

Figure 7. u_y-displacement along the z-axis (model 1)

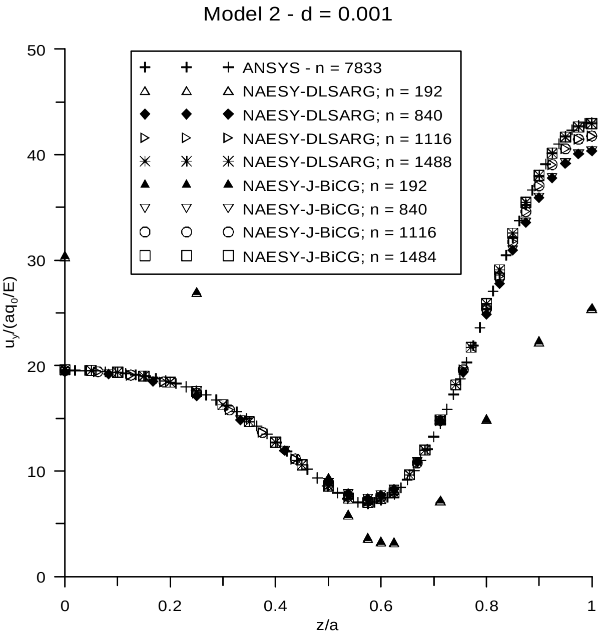

Figure 8. u_y-displacement along the z-axis (model 2)

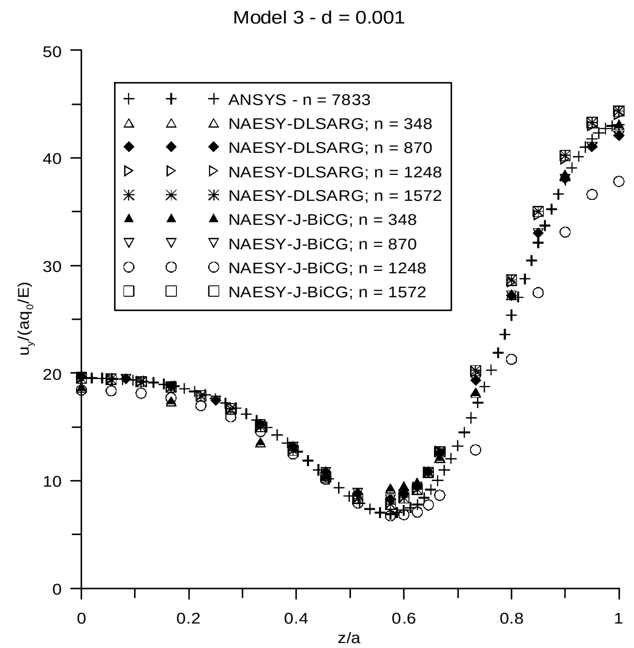

Figure 9. u_y-displacement along the z-axis (model 3)

The performance of the code in terms of the solver-CPU time is shown in the graphs 10–12. In these graphs the J-BiC solver is scaled by the high-precision IMSL-DLSARG solver. The curves are drawn for the different *d*-values (used to generate discontinuous boundary elements) considered, and notice that the IMSL solver works with the explicit coupled system.

Finally, to show the conditioning of the system, the estimated condition number (calculated with the IMSL-DLFCRG routine) is plotted in function of the system order, *n*, for the different models (graphs 13-15).

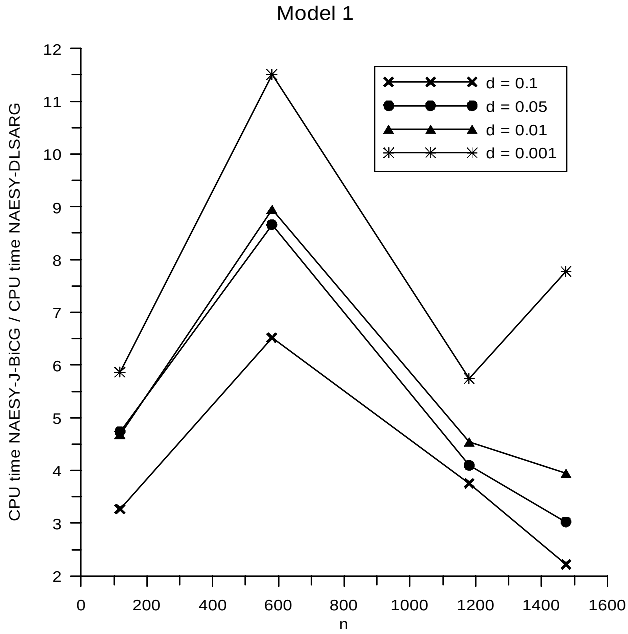

Figure 10. CPU-time curves (model 1)

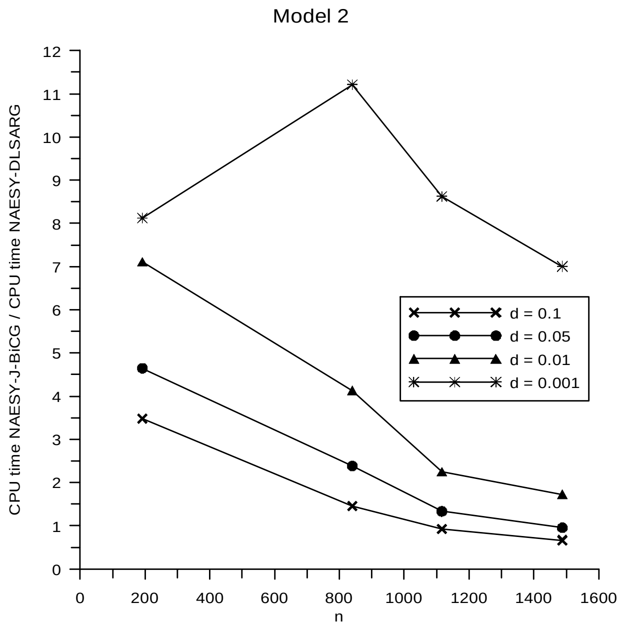

Figure 11. CPU-time curves (model 2)

CONCLUSIONS

The results demonstrate that tridimensional BE formulations are an interesting alternative to solve thin-walled structural elements. One observes that, compared to the FE model adopted (with 3855 finite elements), precise results (Figs. 7-9) were obtained with a relatively small number of boundary elements ($ne \leq 516$).

No convergence problem concerning the iterative solver (J-BiCG), which forms the basis of the coupling algorithm, has

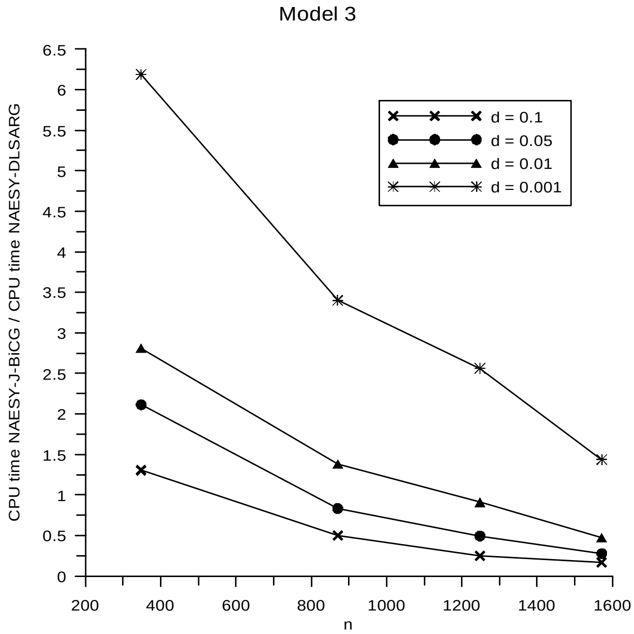

Figure 12. CPU-time curves (model 3)

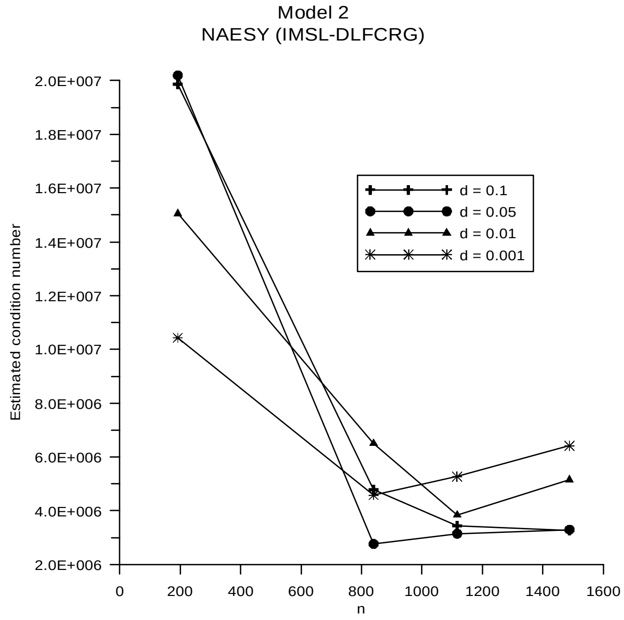

Figure 14. Estimated condition number (model 2)

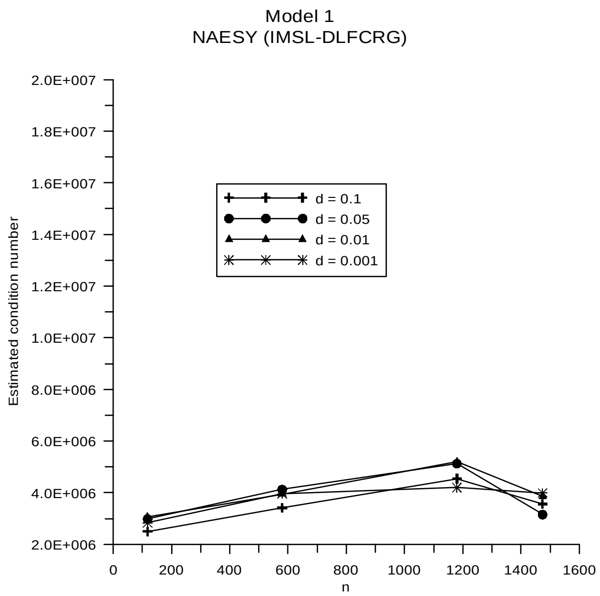

Figure 13. Estimated condition number (model 1)

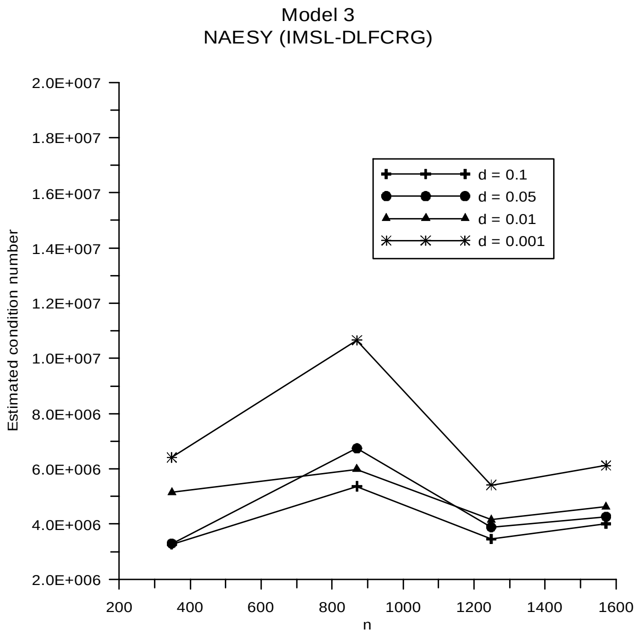

Figure 15. Estimated condition number (model 3)

been also identified, though ill-conditioned systems (condition number $\kappa \geq 3.0 \times 10^6$) have been treated (Figs. 13-15). Indeed, the systems associated with *d*=0.001 are quasi-singular. On the other hand, one sees that the solver-CPU-time curves present a decay with increasing system order *n*. This fact indicates that for larger problems more efficiency should be expected. It is also worth mentioning that a standard BE formulation with no special regularization technique was used. This means that by improving the evaluation of the quasi-singular integrals, the system conditioning may be increased as well. Thus, the CPU time for the convergence of the iterative solver will be considerably reduced.

Interesting future applications of the coupling algorithm proposed may be the modeling of composites and the development of codes for operating in parallel computing platforms.

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REFERENCES
[1] Bernadou, M., 1996, *Finite Element Method for Thin Shell Problems*, Wiley, Chichester.

[2] Liu, Y., 1998, "Analysis of Shell-like Structures by the Boundary Element Method based on 3-D Elasticity: Formulation and Verification", "International Journal for Numerical Methods in Engineering", **41**, pp. 541-558.

[3] Araújo, F. C., Martins, C. J., and Mansur, W. J., 2001, "An Efficient BE Iterative–Solver–Based Substructuring Algorithm for 3D Time-Harmonic Problems in Elastodynamics", "Engineering Analysis with Boundary Elements", **25**, pp. 795–803.

[4] Araújo, F. C., and Silva, K. I., 2004, "The Use of Discontinuous Boundary Elements in the Generic BE/BE Coupling Algorithm - Applications to 3D Potential Problems", *Proc., 5th International Conference on Boundary Element Techniques*, Lisbon, F. Aliabadi, and V. Leitão, eds., EC Ltd., London, UK, Vol. 1, pp. 237-242.

[5] Araújo, F. C., Dors, C., Martins, C. J., and Mansur, W. J., 2002, "Analysis of 3D Time–Harmonic Soil–Foundation Interaction Problems by Using Efficient BE Substructuring Algorithms", *Proc., European Conference on Dynamics*, Munich, A. Balkema Publishers, Rotterdam–Netherlands, Vol. 2, pp. 1267-1272.

[6] Araújo, F. C., Dors, C., Martins, C. J., and Mansur, W. J., 2003, "Dynamic Cross–Interaction between Footings – 3D Frequency–Domain Parallelized Analysis via the BEM", *Proc., International Conference on Boundary Element Techniques IV*, Granada, F. Aliabadi and Gallego, eds., pp. 253–258.

[7] Araújo, F. C., Dors, C., Martins, C. J., and Mansur, W. J., 2004 "New Developments on BE/BE Multi-Zone Algorithms based on Krylov Solvers - Applications to 3D Frequency-Dependent Problems", "Journal of the Brazilian Society of Mechanical Science & Engineering" **26**, pp. 231-248.

[8] Saad, Y., 1996, *Iterative Methods for Sparse Linear Systems*, PWS Publishing, New York.

[9] Saad, Y., and van der Vorst, H.A., 2000, "Iterative solution of linear systems in the 20th century", "Journal of Computational and Applied Mathematics", **123**, pp. 1–33.