A Direct Comparison of SAR Processing as Non-Orthogonal Transform to both Fourier and Wavelet Transform

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***Abstract*— The term SAR processing is widely used to denote the process of producing fully focused SAR images from SAR raw data. Due to the variety of different approximations which can be assumed for real operating SAR instruments (e.g. low squint angles, short synthetic aperture, etc.) but also due to the complexity of todays SAR instruments (stripmap, ScanSAR, spotlight mode, etc.) many SAR processing algorithms have been proposed so far. Due to the richness of details, it appears difficult to describe SAR processing in general.**

However, in a previous work we have modeled SAR processing as transform. The forward SAR transform is inherently performed during SAR data acquisition. Any recorded data are defocused due to the SAR imaging technique. The task of SAR processing is to focus recorded data which reverses the SAR data acquisition process. By investigating SAR processing as transform, similarities to other transforms can be found comparing the integral transform kernels and the way a function basis is generated. Especially, Fourier and Wavelet transform appear in close relationship to SAR imaging and SAR Processing. Fast Fourier Transforms (FFTs) are embedded in SAR processing for the execution of fast convolutions. On the other hand, the wavelet transform performs faster convolutions than FFTs but is no general convolution tool. Furthermore, chirp signals fulfill the formal conditions for being wavelets and might be interesting for the design of a wavelet-like fast SAR processing algorithm. In this paper, we study the closely related transforms for finding a direct fast transition from SAR raw data to SAR images without Fourier domain techniques. A sliding in-place transition from SAR raw data to SAR images should be possible in that way.

I. Introduction

SAR processing is a challenge. It is a complicated task depending on many variables and parameters - even on wind and weather conditions as in the airborne case. However, SAR processing can be modeled in general as two-dimensional convolution with range-variant convolution kernel [3]. After correcting the effect of Range Migration (RM), SAR processing can be modeled even simpler: The transform is then separable and it remains applying only one-dimensional convolutions to SAR raw data in both, azimuth and range direction. So, having in mind that SAR processing is mainly performing convolutions there is a special need of fast convolution algorithms. As the Fast Fourier Transform (FFT) is the fastest known technique for performing convolutions, many SAR processing algorithms employ FFTs and there seems to be no alternative to using FFTs if computational efficiency is required.

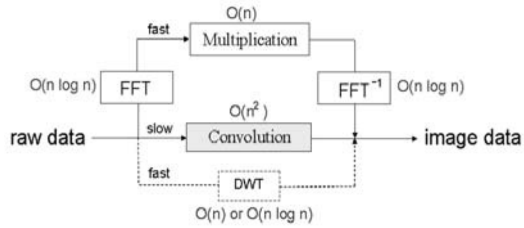

Fig. 1. SAR processing mainly requires performing convolutions. State-of-the-art SAR processing algorithms use the Fast Fourier Transform (FFT) for executing fast convolutions. In contrast, it should be possible to design a Discrete Wavelet Transform (DWT) like algorithm dedicated to SAR processing needs which operates directly towards image data.

However, the Discrete Wavelet Transform (DWT) allows performing faster convolutions compared to the FFT but is no *general* convolution tool. It is inherently adapted to convolutions with versions of its basic wavelet. Basic wavelets generate the entire function basis and, herewith, also the entire signal space. In case of Synthetic Aperture Radar, the entire SAR raw data signal space is often modeled such that it is generated by chirp signals in both, azimuth and range direction. Furthermore, chirp signals fulfill the admissibility condition required for Integral Wavelet Transforms (IWTs). They herewith qualify as wavelet in the IWT sense which encourages hope that a DWT like algorithm might be possible to design for fast SAR processing [4]. Such transform is desirable for several reasons:

- SAR processing will be faster.
- It will not require external transforms (FFTs).
- It enables time-domain in-place calculations.
- It allows sliding transitions from raw to image data.

Hence, the general idea is to replace FFT convolutions by faster time-domain sliding convolutions. Towards this goal, we compare in this paper Fourier, Wavelet and SAR transform, investigate their common properties and point out differences. Especially investigated is the way Fourier and Wavelet transform gain computational speed and how this can be adopted to SAR processing.

Section II introduces the Fourier transform, Section III models SAR processing as transform and deduces the Point

Scatterer Response (PSR) function which generates SAR raw data and plays the role of a basic wavelet known in wavelet theory. The Wavelet transform is accordingly introduced in Section IV. Section V compares Fourier, Wavelet and SAR transform as integral transforms defined via Hilbert space scalar products. Special attention is contributed to the integral transform kernels and the way they generate their function basis. Finally, we summarize the results and give an outlook to future work.

II. Fourier Transform

The Fourier transform plays a central role in signal analysis. It is defined as the integral

$$\mathcal{F}_\psi s(f) := \int_{-\infty}^{+\infty} s(t)\,\psi(ft)\,dt \tag{1}$$

using $\psi(t) = e^{-2\pi i\,t}$ as integral transform kernel, $i = \sqrt{-1}$. This kernel is closely related to chirp signals (7) present in SAR raw data. In contrast to chirps, the Fourier kernel has no quadratic term $k\,t^2$ and also no time localization $rect(t)$. The Fourier transform basis functions $e^{-2\pi i\,f\,t}$ are compressed and dilated versions of kernel $\psi(t) = e^{-2\pi i\,t}$ generated with frequency parameter $f \in \mathbb{R}$ which can be seen as time scaling parameter. The concept of scaling time for the generation of frequencies is also used in wavelet transforms, see parameter a in (10).

III. SAR Transform

A detailed description of the Synthetic Aperture Radar Point Scatterer Response function (PSR) and the corresponding SAR raw data model can be found for example in [3]. SAR raw data can be, moreover, modeled in three shades of simplification [4]: Assuming that the two-dimensional PSR function is

- *invariant* with range *and* azimuth (simplified model),
- variant with range (standard model) and
- variant with range *and* azimuth (complete model).

The first model does not adapt to the range-varying nature of PSRs and is therefore only a rough model. But it allows to model SAR processing simple: as two-dimensional convolution which fully corresponds to the multiplication of polynomials in two variables. In Figure 2, this model is used to explain the generation of SAR raw data from radar reflectivity[1] and its re-focusing. The simulated reflectivity is real-valued, chirp and raw data simulation are complex but depicted in real part. The simulation here is assumed noise free such that an Inverse Filter can be used to find back to the original reflectivity map without any image degradation. In real cases, the Inverse Filter technique cannot be used because it acts as noise amplifier of those noise outside the nominal SAR system bandwidth and hence, a Matched Filter is used instead [4].

[1]The used image for radar reflectivity simulation is part of 'yosemite.jpg' distributed with Microsoft WindowsME, courtesy of Microsoft Corporation.

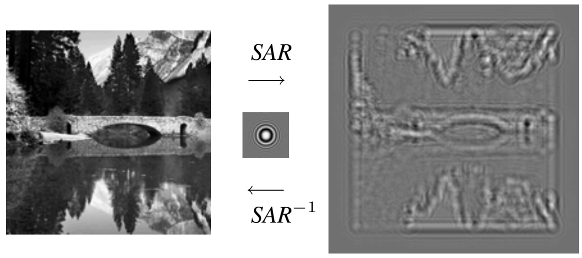

Fig. 2. The SAR transform modeled as 2D convolution: Using an *in*variant two-dimensional reference chirp, a simulated radar reflectivity map (left) is convolved to simulated SAR raw data (right). This forward transform step corresponds to SAR Imaging (defocused reflectivity), inverse convolution corresponds to SAR Processing (focused reflectivity).

A. Simplified Raw Data Model

In the standard SAR case, the Point Scatterer Response is a waveform that changes its 'shape' from near to far range due to the SAR imaging geometry. This range dependence can be expressed by writing ψ_{n_2} instead of ψ in the raw data model below. But for demonstrating the principle of SAR image formation let us assume an invariant ψ for simplicity here. The simplified SAR raw data model is a superposition

$$raw(t) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \sigma(nT)\,\psi(t - nT) \tag{2}$$

of two-dimensional Point Scatterer Responses $\psi(t)$ located at position nT on ground and weighted with radar reflectivity $\sigma(nT)$. The radar reflectivity $\sigma(nT)$ at location nT is the entity actually to measure. For simplicity, let σ also include here any antenna pattern weighting and signal noise. Furthermore, let $t = [t_1, t_2]^\mathsf{T} \in \mathbb{R}^2$ be time with t_1 slow time (azimuth) and t_2 fast time (range), $T = [T_1, T_2]^\mathsf{T}$ the sampling interval, $T_1 = 1/PRF$, PRF the Pulse Repetition Frequency, $n = [n_1, n_2]^\mathsf{T}$ passes through all resolution cells $n_1 = 0, \ldots, N_1 - 1$ and $n_2 = 0, \ldots, N_2 - 1$, $N = [N_1, N_2]^\mathsf{T}$ is the number of raw data pixels in azimuth and range. The product nT is declared via component wise multiplication. A detailed derivation of (2) can be found in [4].

B. Image Formation

SAR processing reverses the convolution $\psi * \cdot$ inherently performed with SAR imaging. Let $\psi^{-\mathbf{1}}$ be the convolution inverse of ψ, i.e. $\psi^{-\mathbf{1}} * \psi = \delta$ is the Dirac impulse, then the image focuses in each pixel according to

$$\begin{aligned} image(t) &= \psi^{-\mathbf{1}}(t) \,*\, raw(t) \\ &= \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \sigma(nT)\,(\psi^{-\mathbf{1}} * \psi)(t - nT) \\ &= \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \sigma(nT)\,\delta(t - nT). \end{aligned} \tag{3}$$

The operation $\psi^{-\mathbf{1}} * \psi$ is often called Pulse Compression. In practice, a convolution inverse (*Inverse Filter*) cannot be used for image formation due to noise but a *Matched Filter*

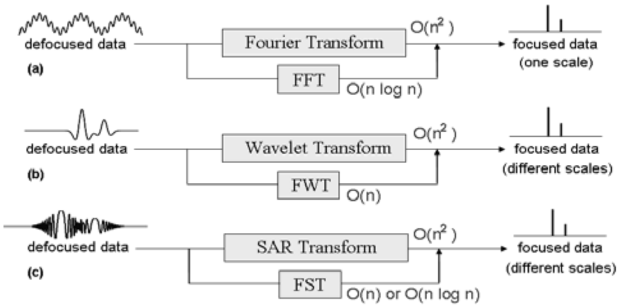

Fig. 3. Three signal models and suitable focusing transforms. With matrix multiplication, a computational effort of at least $O(n^2)$ is required for each transform but fast versions exist for Fourier and Wavelet transform. Similarly, SAR processing should lie in the order of only $O(n)$ or $O(n\,log\,n)$ operations.

is used instead [4]. A third impulse compression technique is the *SPECtral ANalysis Approach* (SPECAN) [5]. Here, a phase deramping function is applied first to the superpositioned PSRs ψ such that the signal becomes a superposition of windowed Fourier transform basis functions

$$\psi(t_1, t_2) = \; rect(t_1, t_2) \cdot e^{-2\pi i\,(f_1\,t_1 + f_2\,t_2)} \;. \tag{4}$$

As an implication of phase deramping, the data may now be focused using Fourier transform (spectral analysis) techniques. Figure 3 demonstrates that for data focusing, the applied transform should match the corresponding image model.

C. Point Scatterer Response

The PSR used in this study is derived from the standard SAR case (see Appendix) as

$$\psi_r(t_1, t_2) = \theta_{t_1,r}\{\; rect_r(t_1, t_2) \cdot \\ e^{-2\pi i\,(f_1 t_1 + k_1(r)\,t_1^2 \,+\, f_2 t_2 + k_2 t_2^2)}\;\}\,. \tag{5}$$

Compared to (4), the PSR is range dependent, possesses quatratic phase terms in both directions and is deformed according to range migration; $\theta_{t_1,r}$ is a range migration operator that applies deformation to ψ. The amount of range migration depends on fly-by time t_1 and minimum range distance r. All difficulties that make SAR processing a challenge are summarized in (5): First of all, the shape of ψ depends on range r. The equation moreover reveals that:

- The support of ψ_r in range *migrates* towards far range according to the sensor position in azimuth.
- The support of ψ_r in azimuth *increases* with range according to the antenna opening angle.
- The azimuth frequency modulation of ψ_r *decreases* with range according to less distance variations in far range.

Range Migration (RM) is the most disturbing effect in SAR processing. It prevents the transform of being separable, i.e. the two-dimensional problem cannot be separated into two one-dimensional problems. But it has been shown that range migration can be corrected efficiently by applying a chirp scaling operation in the range-Doppler domain [6]. It equalizes the different degrees of migration from near to far range to that of only one reference migration, usually at mid range. After that, range migration is totally removed by shifts in range but varying with the Doppler frequency in azimuth frequency direction.

The correction of range migration is usually embedded efficiently in a number of consecutive one-dimensional Fast Fourier Transforms. Let us assume in the following that RM has been corrected already. If FFTs can be replaced by wavelet techniques then also RM correction should work efficiently.

D. Processing Separability

After range migration correction, the two-dimensional PSR function is separable according to

$$\begin{aligned}\theta^{-1}_{t_1,r}\,\{\psi_r(t_1,t_2)\} &= rect_1(t_1)_r \cdot e^{-2\pi i\,(f_1 t_1 + k_1(r)\,t_1^2)} \;\cdot \\ &\qquad rect_2(t_2)\cdot e^{-2\pi i\,(f_2 t_2 + k_2 t_2^2)} \\ &= \psi_1(t_1)_r \,\cdot\, \psi_2(t_2)\end{aligned}$$

where subscript r is written to remind that the function or operation still depends on r but in contrast to time variables t_1 and t_2, parameter r is a fix entity. According to the separability, SAR processing now reduces to

- one-dimensional convolutions in range using $\psi_2(t_2)$ and
- one-dimensional convolutions in azimuth using $\psi_1(t_1)_r$.

Convolutions in range are usually done first. They can be done even *before* RM correction due to the circumstance that range shifts (in RM correction) and convolution in range are exchangeable operations.

E. Simplified Processing Model

It has been pointed out so far that SAR processing reduces to one-dimensional convolutions if Range Migration is already corrected or can be neglected for some reason. In the following, we use the term 'SAR processing' or 'SAR transform' in the sense that an one-dimensional convolution

$$\mathcal{C}_\psi s\,(b) := (s * \psi)(b) = \int_{-\infty}^{+\infty} s(t)\,\psi(t-b)\,dt \tag{6}$$

is to perform using the reference chirp

$$\psi(t) = rect(t)\cdot e^{-2\pi i\,(f\,t + k\,t^2)} \tag{7}$$

as convolution kernel.

F. Matrix Interpretation and Non-Orthogonality

Convolution is often written in matrix form

$$y = M_\psi\, x \tag{8}$$

where $M_\psi \in \mathbb{R}^N$ is a band matrix and $x, y \in \mathbb{R}^N$ are discrete one-dimensional signals representing image and raw data, respectively. The columns of M_ψ form the raw data function basis. The first column is the discrete chirp signal (7). It generates all other columns via translation. Matrix M_ψ includes all translated chips ψ such that raw data y are superpositions of chirp translations weighted by radar reflectivity x.

As matrix M_ψ is generated by ψ, its inverse $M_\psi^{-1} = M_\phi$ is generated by the convolution inverse $\phi = \mathcal{F}^{-1}\{1/\mathcal{F}\{\psi(t)\}\}$ of ψ determined via the Fourier transform $\mathcal{F}$. Then, the operation

$$x = M_\psi^{-1} y \tag{9}$$

corresponds to image formation. It reverses raw data formation (8). Obviously, M_ψ is invertible and $M_\psi^{-1} M_\psi = I$ is the identity matrix. For orthogonal transforms, it is required that $M_\psi^{\mathsf{T}} M_\psi = I$. This applies to the Fourier transform for example. But for convolutions in general it does not apply as the supports of two basis functions may overlap and, hence, their scalar product can hardly be zero. SAR processing as transform is non-orthogonal because the scalar product between two basis functions $< \psi(t), \psi(t - \cdot) >$ is non-zero if their supports overlap.

IV. Wavelet Transform

In this section, we compare the convolution (6) with wavelet transforms (10). Wavelet transforms allow to analyse raw signals with translated and scaled versions of only one wavelet called the basic wavelet.

A. Admissibility Condition

The wavelet transform of signal $s \in L^2(\mathbb{R})$ with respect to some wavelet ψ is a function

$$\mathcal{W}_\psi s(a,b) := |a|^{-\frac{1}{2}} \int_{-\infty}^{+\infty} s(t)\, \psi(\frac{t-b}{a})\, dt \tag{10}$$

of scaling parameters $a \in \mathbb{R} \setminus \{0\}$ and translation parameters $b \in \mathbb{R}$. Here, any square-integrable function $\psi \in L^2(\mathbb{R})$ fulfilling the admissibility condition

$$0 < c_\psi := \int_{\mathbb{R}} \frac{|\widehat{\psi}(\omega)|^2}{|\omega|}\, d\omega < \infty \tag{11}$$

where $\widehat{\psi}$ is the Fourier transform of ψ, is called a *wavelet* and every wavelet defines a wavelet transform.

Let us now rewrite this formula in terms of convolution

$$\mathcal{W}_\psi s(a,b) := (s * \psi_a)(b) \tag{12}$$

where $\psi_a = |a|^{-\frac{1}{2}}\, \psi(\cdot/a)$ is a scaled version of ψ. Then it can be seen that the Wavelet transform performs a convolution on each scale a and this can be employed for SAR processing. It only requires the calculation of one scale, say $a = 1$. At this point, the question arises how Discrete Wavelet Transforms can be faster than FFTs despite their additional parameter b compared to (1) which only depends on a.

The answer is that the calculation of all parameters $a \in \mathbb{R} \setminus \{0\}$ is highly redundant and does not lead to an efficient algorithm. Rather, parameter a is restricted to a discrete subset and also $(s * \psi_a)(b)$ is not calculated directly. The transform rather 'updates' the already calculated convolution from the lower scale. Starting at the trivial case (e.g. focused) it iterates through increasing scales a and finally reaches the finest scale (e.g. defocused) after $log_2(m)$ steps if m is the convolution filter length. Inverse convolution (image formation) is achieved using the inverse convolution kernel (*Inverse Filter*) or the complex conjugate kernel (*Matched Filter*) instead.

TABLE I
Integral Transform Kernel Functions

Fourier Transform	Wavelet Transform	SAR Transform
$e^{-2\pi i f t}$	$rect(t)\, e^{-2\pi i f t}$	$rect(t)\, e^{-2\pi i (ft + kt^2)}$
$\psi(\frac{t}{a})$	$\psi(\frac{t-b}{a})$	$\psi(t-b)$
dilations	dilations & translations	translations
orthogonal	orthogonal or non-orth.	non-orthogonal

B. Chirp As Basic Wavelet

A chirp signal $\psi(t) = w(t)\, e^{-2\pi i\, (f_0 + f(t)) t}$ is described using some window function $w(t)$ for restricting time $t \in \mathbb{R}$, some constant frequency component f_0 and a time varying frequency component $f(t) = k\, t$ with modulation rate k.

It is easy to see that chirps are well located in both, time and frequency domain but, moreover, every square-integrable zero-mean function ψ on compact support (let window $w(t)$ be zero outside some time interval) fulfills the admissibility condition [2] and is therefore a *wavelet*. It means that each chirp defines an Integral Wavelet Transforms (IWT).

However, the design of a dedicated Discrete Wavelet Transform (DWT) for SAR processing requires much more than a suitable wavelet because calculating all parameters $a \in \mathbb{R} \setminus \{0\}$, $b \in \mathbb{R}$ is not possible. A minimum subset of the discrete parameters a and b must be found such that signal $s(t)$ is reconstructible from the calculated wavelet coefficients in (12).

V. Transform Comparison

In this section, we compare Fourier (1), Wavelet (10) and SAR transform (6) with each other.

A. Basis Change

In a Hilbert space, some signal $s(t)$ can be decomposed into components of basis functions ψ_k

$$s(t) = \sum_k < s, \tilde{\psi}_k > \psi_k(t) \tag{13}$$

by calculating all scalar products

$$< s, \tilde{\psi}_k > = \int_{-\infty}^{+\infty} s(t)\, \overline{\tilde{\psi}_k(t)}\, dt \tag{14}$$

where $\overline{\psi_k}$ is the complex conjugate of ψ_k and $\tilde{\psi}$ is the dual of ψ, i.e. $< \tilde{\psi}_l, \psi_k > = \delta_{k,l}$. The function basis is orthonormal if $\tilde{\psi} \equiv \psi$. The calculation of scalar products corresponds to changing the underlying function basis for a function $s(t)$. Signal $s(t)$ can be reconstructed using the scalar products with (13). Fourier, Wavelet and SAR transform definitions follow formula (14) but use

- different kernels $\psi(t)$ and
- different basis generation methods (dilation, translation).

Moreover, (14) is convolution in case of Wavelet and SAR transform. Table I lists the two integral transform kernels for

Fourier and SAR transform and an example kernel for the Wavelet transform in the first row. They are also schematically depicted in Figure 4. The second row explains the way a function basis is generated from the kernel. Note that the Fourier transform only uses dilation a, the Wavelet transform uses both, dilation a and translation b, and the SAR transform only uses translation b. The Fourier transform does not fulfill the admissibility condition (11). Hence, its kernel function is strictly no wavelet but follows the general concept of generating a basis from only one function.

Furthermore, note that (2) is the two-dimensional version of (13) and that calculating radar reflectivity $\sigma(nT)$ in (2) corresponds to calculating scalar products

$$\sigma(nT) = < raw(t), \psi^{-\mathbf{1}}(t - nT) > = \psi^{-\mathbf{1}}(t) * raw(t)$$

and this corresponds to inverse convolution performed in SAR processing.

B. Fast Transforms

We have seen that Fourier, Wavelet and SAR transform are closely related and for Fourier and Wavelet transforms fast algorithms are known. The idea is now to copy the technique used in FFTs and in DWTs to find a direct sliding transition from SAR raw to image data and in that way to speed-up SAR processing.

Fast algorithms base on the *Divide and Conquer* concept. It splits data into two data sets, even and odd samples mostly, and applies the transform to both, separately. This leads to algorithms in the order of $O(n\,log\,n)$. In the DWT algorithm, the descent is only in the left-most data branch which is even faster and results in approximately $2\,O(n)$ operations. However, for SAR processing it will not be sufficient to descent into one branch. A fast convolution algorithm will lie in the order of $O(n\,log\,n)$ operations if full resolution images are required.

The *Lifting Scheme* is a fully time-domain wavelet technique that allows fast convolutions without any Fourier theory. It splits the data set into two, e.g. even and odd pixels, then predicts the odd pixels from the even pixels and calculates the differences. If the predict method matches the signal model then the difference is zero and the data set has been compressed to only half its spatial extent. Now we save the even pixels and the differences (which are expected zero). Note that this is a fully invertible operation. The procedure can be repeated until only one even pixel is left. If the prediction was right, then the data have been compressed to only one pixel and zeros else. This not only realizes an excellent data compression technique because many zeros have been generated, it is also an excellent Impulse Compression technique which can be used for SAR image formation. Figure 4 shows that the Fourier transform bases on the same idea but on a circle.

VI. Conclusion

In this paper, we modeled SAR raw and image data and described SAR processing as linear non-orthogonal transform.

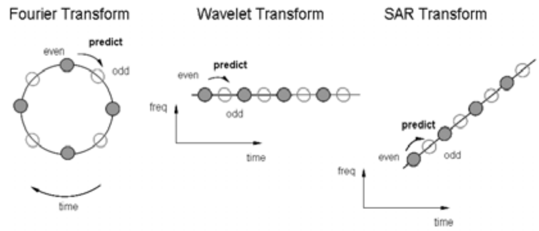

Fig. 4. Lifting Scheme. Visualization of one upscale/downscale step. The difference between odd values and predicted odd values is expected zero. If this is true then odd values are redundant and hence the waveform is compressed to only half its previous spatial extent. Iterating several times results in pulse compression on the one hand and waveform generation on the other hand.

SAR Imaging corresponds to forward transforming radar reflectivity to SAR raw data and SAR Processing refocuses the imaged radar reflectivity. SAR Processing as transform has been compared to both Fourier and Wavelet transform due to common properties. All three transforms calculate scalar products which corresponds to changing the underlying function basis in a signal. We derived the Point Scatterer Response function for SAR processing and compared it to the other transform kernels. All three transforms use the concepts of scaling and/or translation for generating basis functions from the transform kernel.

Finally, we studied the Lifting Scheme known in wavelet theory. It has the potential of allowing fast sliding transitions from SAR raw data to SAR images without using Fourier techniques. This is subject to further investigations towards a fast time-domain convolution algorithm for SAR processing.

VII. Appendix

In azimuth, we approximate the varying distance R between SAR sensor and a scatterer on ground located at time $t_1 = 0$ at a range of r by

$$R(t_1, r) = \sqrt{r^2 + (vt_1)^2} \approx r + \frac{(vt_1)^2}{2r}$$

such that the echo time delay is $2R(t_1, r)/c$. Subtracting the constant part $2r/c$ of the delay variation, the scatterer migrates by the amount of $(vt_1)^2/(c\,r)$ seconds in the recorded range position, i.e. a shift in range direction of

$$\theta_{t_1,r}\{\ \psi(\cdot,\cdot)\ \} = \psi(\ \cdot,\ \cdot - \frac{(vt_1)^2}{c\,r}\ )$$

must be corrected in SAR processing when focusing the Point Scatterer Response ψ located at coordinates t_1 and r. Therewith, the point scatterer response is written as

$$\psi_r(t_1, t_2) = \theta_{t_1,r}\{\ rect_r(t_1, t_2) \cdot e^{-2\pi i\,(f_1 t_1 + k_1(r)\,t_1^2\ +\ f_2 t_2 + k_2 t_2^2)}\ \}\,.$$

with azimuth frequency modulation (depending on r)

$$f_1(t_1, r) = f_1 + k_1(r)\,t_1$$

and range frequency modulation

$$f_2(t_2) = f_2 + k_2\,t_2.$$

The azimuth modulation rate $k_1(r) = 2v^2/\lambda r$ is a function of r but it also depends on wavelength λ and platform velocity v, f_1 is the Doppler centroid frequency. In range we assume a down chirp with modulation rate $k_2 = -(1/2) \cdot (\triangle f/\triangle t)$ where $\triangle f$ is the chirp bandwidth and $\triangle t$ the chirp duration. Factor $1/2$ is an integration constant arising when integrating frequency $f(t) = k\,t$ to phase $\Phi(t) = k\,t^2/2$. The chirp center frequency f_2, usually zero, is given here for symmetry reasons. The rectangular function

$$rect_r(t) = \begin{cases} 1 \quad \text{if} \quad -T_a(r)/2 \ \leq t_1 \leq T_a(r)/2 \quad \text{and} \\ \qquad\qquad -T_c/2 \leq t_2 \leq T_c/2 \\ 0 \quad \text{sonst} \end{cases}$$

describes the visibility of an individual scatterer on ground as it is actually 'seen' by the SAR sensor. Originally rectangular shaped, it is itself subject to range migration and, hence, follows the deformation line of a range migration trajectory at range r. Let

$$T_a(r) = \frac{r}{v}(\tan(\frac{\theta}{2} - \Psi) + \tan(\frac{\theta}{2} + \Psi))$$

be the scatterer azimuth illumination time as a function of r, also depending on the sensor velocity v, antenna opening angle θ and squint angle Ψ, T_c is the chirp duration. Antenna weighting in azimuth and pulse weighting in range are approximated in $rect$ being constant one.

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