Comparison of Orientation Angle Estimation Methodes over Coherent Scatterers

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Abstract

In this paper, a study concerning the comparison of several different procedures for the estimation of scatterers orientation angle for the coherent backscattering case is performed. Routines were proposed for the estimation of the orientation angle of distributed as well as of deterministic (coherent) scatterers. On deterministic scatterers it is expected that all routines converge to the same value, and we verify this statement. The procedures that can be applied to the general case of non-symmetrical scatterers were found out to belong to two main classes that are fundamentally different. We indicate the cases for which the classes of routines diverge due to their fundamental difference. The Coherent Scatterers (CSs) technique is used to support the analysis and confirm the existence of general non-symmetric point-like scatterers in SAR images of urban environments. For that, experimental data using the DLR's ESAR system at L-band over the city of Dresden, in Germany, is used.

1 Introduction

In radar remote sensing, many polarimetric approaches for the estimation of scatterers orientation angle, in the plane orthogonal to the radar Line of Sight (LOS), have been proposed for the coherent (deterministic scatterer) and non-coherent (distributed scatterer) cases. Among the non-coherent ones are the circular polarization algorithm of Lee [1], [2] the polarization signature approach [3], and the beta angle from Cloude/Pottier coherence matrix eingendecomposition [4]. For the coherent case, Cameron decomposition [5], the cross-polarization minimization procedure [6], and the scatterer optimum polarization states [7], [8], represent the main procedures. The sphere-dihedral-helix decomposition of Krogager contains the orientation of the dihedral component, however, the dihedral component orientation does not correspond directly to the general scatterer orientation. In the case of a deterministic scatterer, it is intuitive to expect that all procedures, for both coherent and non-coherent cases, converge toward the same value. In this paper we verify that this is not always the case and show the reasons for their divergences, theoretically, through simulation, and experimentally (using the Coherent Scatterers - CSs - method on ESAR data).

The validity interval of some of the procedures is on $(-\pi/4, \pi/4\,]$ and of others on $(-\pi/8, \pi/8\,]$. In order to have a general comparison, the analysis in this paper is restricted to the cases of real arguments for the arctangent function in the expressions of the orientation angle. This restricts the analysis of orientation angles to the interval $(-\pi/8, \pi/8\,]$.

2 Orientation Angle Estimation

Hereafter the reciprocal backscattering case in which the scatterers may be fully represented by symmetrical scattering matrices ($S_{HV} = S_{VH}$) is assumed.

The beta orientation angle was shown to be constrained to the subset of symmetrical scatterers (scatterers whose scattering matrices can be condiagonalized by unitary rotation transformation [7]) [9]. This drawback happens for distributed as well as for coherent scattering as shown in [9] and hence the procedure will not be considered here. However, the modification of the beta angle proposed in [10] belongs, as will be shown, to a class of procedures that can retrieve the orientation angle of more generalized scatterers. The polarization signature method requires a maximization procedure and as it belongs, by definition, to a class of routines that will be treated in the following, it will also not be specifically addressed.

We remain with a set of procedures that may be applied to general non-symmetrical scatterers and we divide them in two classes: the procedures that minimize the cross-polarization channel of the scattering matrix (in a linear basis) by a unitary rotation operator; and the procedures that make the cross-polarization channel completely null (condiagonalize the scattering matrix) by a general unitary consimilarity (or congruence) transformation (orthonormal change of basis).

The first class of routines will be referred here as the Rotation Transformation Based Routines - RTBR, while the second class as the general Consimilarity (or Congruence) Transformation Based Routines - CTBR.

As we shall see, although in many cases the orientation angle estimated by both classes of routines is the same, this is not the case for general asymmetric scatterers.

2.1 CTBR and RTBR Estimation

The scattering matrix S for the backscattering reciprocal case, in the linear Horizontal-Vertical basis, is symmetric

$$S = \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix} = \begin{pmatrix} a+b & c \\ c & a-b \end{pmatrix} \quad (1)$$

where a, b and c are the Pauli components.
Symmetrical scattering matrices, and only this class of matrices, can be condiagonalized by a general consimilarity transformation that corresponds to an orthonormal change of basis for both transmission and reception [11]

$$S \rightarrow U^T(\psi,\tau) S U(\psi,\tau) = S_d = diag\,[\lambda_1, \lambda_2] \quad (2)$$

where S_d is diagonal, λ_1 and λ_2 are coneigenvalues of S,

$$\begin{aligned} U(\psi,\tau) = & \; R(\psi)\, T(\tau) \\ = & \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \cos\tau & j\sin\tau \\ j\sin\tau & \cos\tau \end{pmatrix} \end{aligned} \quad (3)$$

and $R(\psi)$ and $T(\tau)$ are a real rotation orthogonal matrix and a complex symmetric unitary coninvolutory matrix, respectively, with orientation angle $-\pi/2 \leq \psi \leq \pi/2$ and helicity angle $-\pi/4 \leq \tau \leq \pi/4$. The polarization state of the basis that condiagonalize the S matrix is called the maximum co-pol power polarization state. It was Kennaugh that first showed that this polarization state exist and is *the same for both transmission and reception* in the case of backscattering reciprocity.
CTBR are based on Eq. 2. The estimated orientation angle is the angle associated to the polarization state that maximizes the co-polarization (corresponding to a cross-polarization null) power under a general orthonormal change of basis (general consimilarity transformation).
On the other hand, the orientation angle estimated from RTBR is the angle associated to the polarization state that maximizes the co-polarization (or minimize the cross-polarization) powers under a *(restricted) orthonormal change of linear basis* (unitary rotation transformation). In this case, the scattering matrix will not be condiagonalized for general non-symmetric scatterers and consequently the cross- and co-polarization powers are not the maximum and minimum powers, respectively, that could be obtained back from the scatterer if a general consimilarity transformation would be applied. However, as we shall see, the orientation angle corresponding to these states may be, although not always, the same as the angle for CTBR.
Scattering matrices corresponding to the class of symmetrical scatterers in the Huynen sense [7] can be condiagonalized by pure rotations having $\tau = 0$. In this case, the entire range of the power spectra can be obtained by transformations of linear basis (pure rotations) and both CTBR and RTBR are equivalent. In fact, for the class of symmetrical scatterers, all routines mentioned in the introduction are able to retrieve the correct orientation angle converging to the same value.
The scattering matrix S can be condiagonalized by different procedures, all belonging to the CTBR class: The Kennaugh/Graves optimal polarization state derived from the scattering power matrix [12] was shown to be equivalent to the "three stage procedure" and to the polarization transformation ratio formalism, both proposed by Boerner [8]. The polarization signature method proposed by Schuler [3] is in principle the same, as it searches for the polarization states that maximize the co-pol power. Recently, the optimum states and the set of Huynen invariants have been also analytically derived in [13].
In the Appendix, the Cameron orientation of scatterer maximum symmetry axis, the circular polarization algorithm of Lee, the minimum cross-polarization channel, and the modified β procedures, are shown to be equivalent for the backscatterer reciprocal coherent case and when restricted to the orientation angle interval $(-\pi/8, \pi/8\,]$. All these procedures belong to the RTBR class. Some of the proofs were also given in [6].

2.2 Differences Between CTBR and RTBR

In Eq. 2 the S_d diagonal matrix containing the coneigenvalues λ_1 and λ_2 can be expressed as the sum of two Pauli components: one corresponding to a sphere and the other to a dihedral scatterer and their relative phase:

$$S_d = \cos\alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + e^{j\phi_\alpha} \sin\alpha \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (4)$$

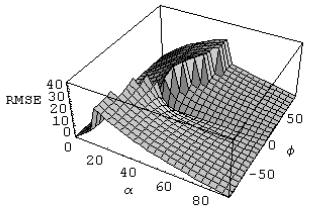
with $0 \leq \alpha \leq \pi/2$ and $-\pi/2 \leq \phi_\alpha \leq \pi/2$.
This representation is used by Touzi [9] together with Eq. 3 in the Target Scattering Vector Model (TSVM). It corresponds to the projection of the Kennaugh/Huynen coneigenvalues onto the Pauli basis, in order to extend the Cloude/Pottier parametrization of the scattering matrix. Similar to the coneigenvalues of S, the parameters α_s and ϕ_α are related to the intrinsic scattering mechanism represented by S.
It can be shown that for the general case of non-symmetrical targets ($\tau \neq 0$) *that have* $\phi_\alpha \neq 0$ (i.e., the coneigenvalues $\lambda_{1,2}$ are complex), the two classes of procedures lead to different values for the orientation angle.
In order to demonstrate this behaviour, Fig. 1 shows the Root Mean Square Error - RMSE of the orientation angle estimated by RTBR as a function of the S matrix parameters of Eq. 2. The input orientation angle was always the same $\psi = 10°$. Fig. 1(a) shows ψ RMSE as a function of α_s and ϕ_α for $\tau = 15°$. For $\tau = 0°$ the ψ RMSE is zero.
Note that when $\tau \neq 0$, RTBR is equal to CTBR only when $\phi_\alpha = 0$ or $\alpha_s = 90°$ (dihedral) and when $\tau = 0$ both routines give the same result independently of ϕ_α and α_s (symmetric scatterer case).

Similar behaviour happens when the RMSE of the RTBR estimated ψ is shown as a function of α_s and τ for $\phi_\alpha = 50°$ (Fig. 1(b)). For $\phi_\alpha = 0°$ the RMSE is always zero. RTBR lead to the same result as CTBR for $\phi_s \neq 0$ only when $\tau = 0$ or $\alpha_s = 90°$. When $\phi_\alpha = 0$, both procedures return the same value independently from α_s and τ.

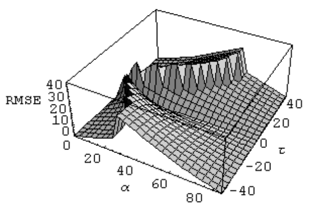
Fig. 1(c) shows the RTBR ψ RMSE as a function of ϕ_α and τ for $\alpha_s = 50°$ while Fig. 1(d) for $\alpha_s = 90°$. Note that when $\alpha_s \neq 90°$ RTBR and CTBR are equal only when $\phi_\alpha = 0$ or $\tau = 0$. Finally, when $\alpha_s = 90°$, both routines are equivalent independently of ϕ_α and τ.

CTBR lead always to the correct input orientation since the input parameters (ψ, τ, α_s, ϕ_α) were defined by using the CTBR structure (Eq. 2).

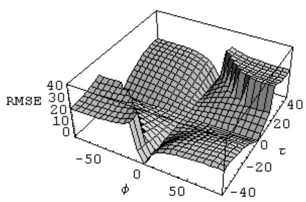
When RTBR is not equal to CTBR, it does not mean that RTBR lead to wrong results. *Their concept of scatterer orientation angle is different* , as addressed in section 2.1. For the estimation of scatterers orientation under asymmetric scattering conditions in the *general form* ($\tau \neq 0$ and $\phi_\alpha \neq 0$), the *definition of scatterer is mandatory*. In these cases, the orientation angle is not easily decoupled from the scatterer as it seems to be for symmetric scatterers.



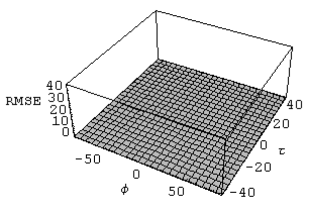
(a) $\tau = 15°$



(b) $\phi_\alpha = 50°$



(c) $\alpha_s = 50°$



(d) $\alpha_s = 0°$

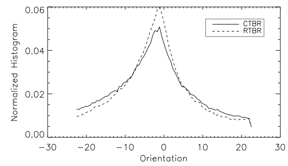
Figure 1: RTBR orientation angle Root Mean Square Error (RMSE) for different scattering matrix parameters.

3 Coherent Scatterers

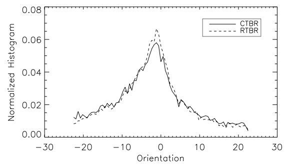
The coherent scatterers (CSs) method [14] to detect scatterers with a deterministic (point-like) scattering behaviour was used to complement the theoretical results addressed before. The fact that CSs have a coherent behaviour, being completely described by their scattering matrices, justify their application on the above study.

ESAR full polarimetric data at L-band was acquired over the city of Dresden in Germany. Fig. 2(a) shows the histograms of the orientation angles estimated with the CTBR and RTBR, of the CSs detected in the scene. Its possible to see that the histograms are similar however not equal. Fig. 2(b) shows the histograms of the estimated orientation angles for the CSs with $-5° \leq \phi_\alpha \leq 5°$ while Fig. 2(c) shows the orientation of the CSs with $-5° \leq \tau \leq 5°$. Note that in both cases, as expected, the histograms are highly matched indicating the agreement between both classes of routines for such cases.

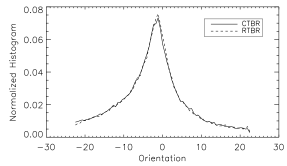
Fig. 2(d) and 2(e) show the estimated orientation angles of the CSs with $|\phi_\alpha| \geq 70°$ and $|\tau| \geq 20°$, respectively, for both CTBR and RTBR. It can be seen in these cases that the orientation angles obtained from the two procedures diverge, as expected.



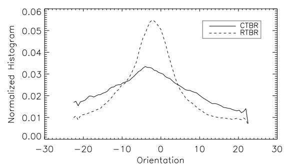
(a) All detected CSs



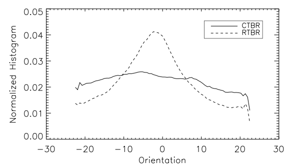
(b) CSs with $|\phi_\alpha| \leq 5°$



(c) CSs with $|\tau| \leq 5°$



(d) CSs with $|\phi_\alpha| \geq 70°$



(e) CSs with $|\tau| \geq 20°$

Figure 2: CSs orientation angle histograms estimated by CTBR and RTBR.

4 Conclusions

It was shown that orientation angle estimation procedures for the general case of non-symmetrical scatterers can be divided in two classes: RTBR and CTBR. The two classes are fundamentally different, however in many cases they converge to the same value. For the cases where they diverge, the orientation angle can not be viewed as being easily decoupled from the underlying scatterer, as it was for the case of symmetrical scattering. It was also demonstrated through the CSs technique that general asymmetric point-like scatterers, for whose RTBR and CTBR diverge, are present in urban areas.

5 Appendix

The third Pauli component after the application of the rotation operator on the S matrix become

$$c' = b \sin 2\psi + c \cos 2\psi \tag{5}$$

with power $|c'|^2$

$$|c'|^2 = (b_R \sin 2\psi + c_R \cos 2\psi)^2 + (b_I \sin 2\psi + c_I \cos 2\psi)^2$$

$$|c'|^2 = |b|^2 + (|c|^2 - |b|^2)\cos^2 2\psi + (b_R c_R + b_I c_I)\sin 4\psi \tag{6}$$

The minimum power $|c'|^2_{min}$ of $|c'|^2$ that can be obtained by pure rotations of S is given when its first derivative

$$\frac{\partial}{\partial\psi}|c'|^2 = 2(|b|^2 - |c|^2)\sin 4\psi + 4\,(b_R c_R + b_I c_I)\cos 4\psi \tag{7}$$

is zero, i.e.

$$\tan 4\psi = -\frac{2\,(b_R c_R + b_I c_I)}{|b|^2 - |c|^2} = -\frac{bc^* + b^* c}{|b|^2 - |c|^2} \tag{8}$$

and its second derivative is positive:

$$\begin{aligned} \frac{\partial^2}{\partial\psi^2}|c'|^2 = \quad & 8(|b|^2 - |c|^2)\cos 4\psi \\ & -16\,(b_R c_R + b_I c_I)\sin 4\psi > 0 \end{aligned} \tag{9}$$

Eqs. 8 and 9 are in agreement with Cameron's diagonalization angle in the interval $(-\pi/8, \pi/8\,]$ [5].
Lee's expression of the circular polarization algorithm within the interval $(-\pi/8, \pi/8\,]$ is given by [1], [2]:

$$-4\theta = \arctan\left[\frac{-4Re\{\langle (S_{HH} - S_{VV})\, S^*_{HV}\rangle\}}{-\langle |S_{HH} - S_{VV}|^2\rangle + 4\langle |S_{HV}|^2\rangle}\right] \tag{10}$$

where, for the coherent case become:

$$\tan 4\theta = \frac{-2Re\{(S_{HH} - S_{VV})\,(2S_{HV})^*\}}{|S_{HH} - S_{VV}|^2 - |2S_{HV}|^2} = \frac{-2Re\{bc^*\}}{|b|^2 - |c|^2} \tag{11}$$

and clearly Eq. 11 is equal to Eq. 8.
Concerning the modified β angle [10], Cloude/Pottier Pauli scattering vector parametrization [4] is given by

$$k_p = \frac{1}{\sqrt{2}}\left[e^{j\phi_1}\cos\alpha,\ e^{j\phi_2}\sin\alpha\cos\beta,\ e^{j\phi_3}\sin\alpha\sin\beta\right]. \tag{12}$$

For a deterministic scatterer, only one eingenvalue of the coherence matrix is non-zero and β is directly given by Eq. 12. Using Eqs. 11 and 12 we have

$$\tan 4\theta = \frac{-2\sin^2\alpha\sin\beta\cos\beta\cos(\phi_2 - \phi_3)}{\sin^2\alpha\left(\cos^2\beta - \sin^2\beta\right)} \tag{13}$$

and

$$\tan 4\theta = -\tan 2\beta\cos(\phi_2 - \phi_3) \tag{14}$$

Eq. 14 is similar to the modified β in [10]. When $\phi_2 = \phi_3$ the scatterer is symmetric and $\beta = 2\theta = 2\psi$.

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