Performance Prediction and Verification for Bistatic SAR Synchronization Link

Marwan Younis, Robert Metzig, Gerhard Krieger
German Aerospace Center (DLR), Microwaves and Radar Institute

Rainer Klein
EADS Astrium GmbH, Microwave Engineering & Technologies

Abstract

The paper describes the configuration of a synchronization link sharing hardware components with the SAR instrument. Such a system is intended for the TanDEM-X mission. The resulting restrictions on the phase synchronization scheme, timing, and accuracy are investigated. The statistical properties of the phase compensation signal are used to derive a figure-of-merit for synchronization performance. The focus is on the influence of the synchronization link RF hardware on the quality of the derived compensation signal. A system model taking the various influence factors into account is described. The system model is parameterized using statistical analysis from measurement data.

1 Introduction

Bistatic SAR systems have a high potential for scientific, commercial and security applications. One of the benefits is the possibility to generate highly accurate digital elevation models using bistatic interferometry. Examples for proposed bi- and multi-static satellite missions with interferometric capabilities are TanDEM-X and Cartwheel. Both are based on radar instruments placed on different spacecrafts, which gives rise to several technical challenges for the system realization.

A factor which may severely degrade the performance of a bistatic SAR is the phase instability of the two oscillators involved. Investigations have shown [1], that, unless highly stable oscillators are used, the oscillators phase noise has to be compensated by establishing a synchronization link to directly exchange signals providing information on the oscillator phase noise. The method is based on recording the received demodulated phases, which are then used to derive a compensation signal to correct the SAR data. The paper describes the synchronization as intended for implementation in TanDEM-X [2].

2 Synchronization Scheme

For TanDEM-X synchronization each satellite repeatedly transmits a pulsed synchronization signal, however, there is a time delay between the transmit instances of the TerraSAR-X and TanDEM-X satellite, thus the synchronization is also alternate. A parameterized timing diagram is shown in **Figure 1**. At time t satellite 1 transmits the synchronization signal of duration T_{p}, which is received τ_{12} seconds later by satellite 2. Similarly, after an internal system delay of τ_{sys}, satellite 2 transmits its synchronization signal at $t+\tau_{\mathrm{sys}}$, which is received by satellite 1 with a delay corresponding to the signal travel time τ_{21}. This procedure is repeated at the synchronization rate f_{syn}. The synchronization lasts over the data take time T_{data}.

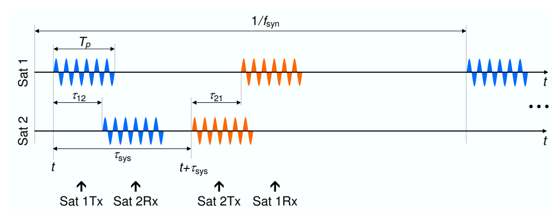


Figure 1: Timing diagram for the exchange of synchronization pulses.

3 Synchronization System Model

The contribution of the oscillator phase noise to the SAR signal phase error can be compensated by establishing a method of phase referencing. In the following a mathematical model is established, which is later used as a basis for deriving quantitative estimates for the performance of the synchronization link.

3.1 Oscillator Signal Phase

Satellite i transmits a synchronization signal, which is received by satellite j, where $i, j \in \{1, 2\}$. The frequency of oscillator i at start of data take t_0 is $f_i = f_0 + \Delta f_i$, with the nominal frequency f_0, and a constant frequency offset Δf_i. The phase $\varphi_i(t)$ at time t is the integration over frequency:

$$\varphi_i(t) = 2\pi \int_{t_0}^{t} f_i(t)dt + \varphi_{\mathrm{ini}\,i} + n_{\varphi i}(t) \qquad (1)$$

with the initial —time independent— phase $\varphi_{\mathrm{ini}\,i}$, and the oscillators phase noise[1] $n_{\varphi i}(t)$ [3].
At the receive instance $t + \tau_{ij}$ the phase $\varphi_j(t + \tau_{ij})$ of oscillator j is:

$$\varphi_j(t+\tau_{ij}) = 2\pi \int_{t_0}^{t+\tau_{ij}} f_j(t)dt + \varphi_{\mathrm{ini}\,j} + n_{\varphi j}(t+\tau_{ij}) \qquad (2)$$

The demodulated phase $\varphi_{ji}(t)$ available at satellite j for a signal transmitted by satellite i is the difference between (2) and (1) after including the system and path contributions. The data take start time t_0 can be set to zero without restricting generality. The phase differences $\varphi_{ji}(t)$ and $\varphi_{ij}(t)$ are used to obtain the compensation phase.

3.2 Compensation Phase

The compensation phase $\varphi_{\mathrm{c}}(t)$ is difference:

$$\varphi_{\mathrm{c}}(t) = \frac{1}{2}\big(\varphi_{21}(t) - \varphi_{12}(t)\big) \qquad (3)$$

where $\varphi_{21}(t)$ and $\varphi_{12}(t)$ are the demodulated phases of the synchronization signals recorded by satellite 2 and 1, respectively.
The advantage of using the phase difference —the compensation phase is actually the difference of the difference, since the phases $\varphi_{12}(t)$ and $\varphi_{21}(t)$ already represent a phase difference— is that the antenna, link path, and all common Tx/Rx system phase variations will cancel out as long as their contribution do not change within the time a pair of synchronization signals are exchanged. The compensation phase can then be used to correct the time varying oscillator phase noise errors and the frequency offset of the SAR signal.

3.3 Approach and Definition of Terms

The basic idea behind phase synchronization is to determine the phase noise and frequency offset differences in (2) and (1). Thus, the phase noise, described through its spectral density is the useful, i.e. "wanted", quantity. The phase spectrum $S_\varphi(f)$, at oscillator frequency f_{osc}, is described analytically by a linear superposition of five different frequency components according to [3]. At nominal up-converted RF frequency f_0 the valid phase noise spectral density is given by $\gamma^2 S_\varphi(f)$, where γ is the ratio of RF to oscillator frequency $\gamma = f_0/f_{\mathrm{osc}}$.
A general signal flow block diagram is shown in **Figure 2** for a priori phase correction, i.e. before SAR focusing. Several factors will influence the phase of the synchronization link as shown in the lower signal path in the figure. Here the transfer function $H_{\mathrm{LP}}(f)$ describes the effect of alternate pulse synchronization. The *receiver* noise $\varphi_{SNR}(t)$ determined by the signal-to-noise ratio SNR is of special interest; its influence on the signal *phase* is described by the receiver phase noise spectral density function $S_{\varphi_{SNR}}(f)$. The hardware system error is represented $\varphi_{\mathrm{sys}}(t)$. Further, for pulsed synchronization the phase is sampled, which requires a later interpolation of the compensation phase. We may choose to filter the compensation phase with an —arbitrary— transfer function $H_{\mathrm{syn}}(f)$. Finally the *compensated* SAR phase (SAR phase after subtracting the compensation phase) is filtered through the azimuth compression. This filter is described through the transfer function $H_{\mathrm{az}}(f)$ and is dependent on the azimuth processing [1].

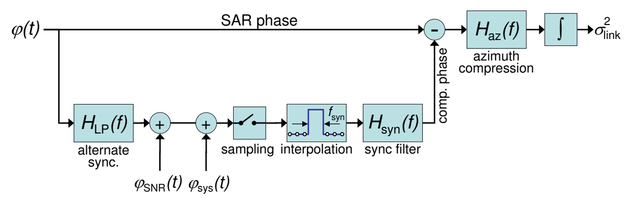


Figure 2: Synchronization and SAR signal flow diagram.

The performance of the synchronization link is determined by the quality of the phase noise reconstruction. The figure-of-merit is the phase variance σ^2_{link} after subtracting the oscillator phase noise and performing the azimuth compression (see **Figure 2**). The link phase error, i.e. the residual contribution to the synchronized SAR, is represented by the standard deviation (STD) σ_{link}.
Note that according to the above model, the SAR transfer function $H_{\mathrm{az}}(f)$ is applied on the signal's phase instead of the signal itself. It can be shown that this is valid —from the synchronization point of view, where the SAR focusing is understood as an averaging process— for small values of the residual phase error σ_{link}.

4 Synchronization Performance

In the following the link phase error is given for the synchronization scheme in section 2 including the error contributions resulting from the synchronization link itself as described in the model of section 3.3. The results are given for an X-band radar with $f_0 = 9.65\,\mathrm{GHz}$ using an oscillator similar in performance to the TanDEM-X oscillator. Ionospheric effects are negligible for this frequency range and satellite separations in the order of a few 100 meters.
The compensation phase according to (3) is computed at discrete time instances $t_k = k/f_{\mathrm{syn}}$ for

[1] It is irrelevant whether f_i is the oscillator frequency or the up-converted RF frequency as long as the representation of the oscillator phase noise is consistent. Consequently the term *oscillator* phase noise identifies the phase noise originating from the oscillator without specifying the frequency.

$k = 0, 1, \ldots \lfloor T_{\text{data}} \cdot f_{\text{syn}} \rfloor$ with $\lfloor T_{data} \cdot f_{\text{syn}} \rfloor$ the total number of synchronization pulses during data take. Using (1) and (2) the compensation phase is:

$$\begin{aligned} \varphi_{\text{c}}(t_k) = \; & \frac{1}{2}\big(\varphi_{SNR1}(t_k + \tau + \tau_{\text{sys}}) - \varphi_{SNR2}(t_k + \tau)\big) \\ & + \pi(\Delta f_2 - \Delta f_1) \cdot (\tau + \tau_{\text{sys}} + 2t_k) - \pi f_D \tau_{\text{sys}} \\ & + \Delta\varphi_{sys1}(t_k, \tau_{\text{sys}} + \tau) - \Delta\varphi_{sys2}(t_k, \tau_{\text{sys}}) \\ & + n_{\varphi 2}(t_k + \tau) + n_{\varphi 2}(t_k + \tau_{\text{sys}}) \\ & - n_{\varphi 1}(t_k + \tau + \tau_{\text{sys}}) - n_{\varphi 1}(t_k) \end{aligned} \quad (4)$$

the first line in the above expression is the phase variation due to receiver noise φ_{SNR}. The second line contains the frequency offset and the Doppler terms. Here, the frequency offset term $2\pi(\Delta f_2 - \Delta f_1)t_k$ results in a linear phase ramp, which can be extracted to correct the frequency offset error of the SAR signal; consequently this term is neglected for the further analysis. The third line represents the phase introduced by the transmit and receive hardware system of the two satellites. The last two lines in (4) represent the wanted phase noise compensation terms.

Doppler Phase

The Doppler phenomenon, with the Doppler frequency $f_D = f_0 v_{sat}/c_0$ due to the relative velocity v_{sat} between the two satellites, manifests itself for alternate synchronization pulses, because of the unequal signal travel times $\tau_{12} \neq \tau_{21}$ due to the changing satellite separation between the transmit instances t_k and $t_k + \tau_{\text{sys}}$. However, the Doppler phase contribution is constant for constant v_{sat}. Only a relative satellite acceleration, i.e. a time dependent $v_{sat}(t)$ will cause a phase error. For severe intersatellite acceleration a Doppler phase compensation is necessary which requires the satellite separation to be known.

Receiver Noise

The receiver noise, consisting of thermal noise and the noise collected by the antenna, will introduce both amplitude and phase fluctuation to the synchronization signal. Here, the *phase* variations described by their spectral density function are of interest. For band limited white Gaussian noise the spectral density function $S_{\varphi_{SNR}}(f)$ is related to the SNR through [4]:

$$S_{\varphi_{SNR}}(f) = \frac{1}{2B_{\text{w}} \cdot SNR} \quad (5)$$

with the receiver (noise) bandwidth B_{w}.
The signal-to-noise ratio is improved through the azimuth compression where the compensated SAR signal is averaged over a period T_{a}, which is equivalent to a low-pass filter $H_{\text{az}}(f)$. The receiver noise variance then is:

$$\frac{1}{2}\sigma^2_{SNR} = \frac{1}{4f_{\text{syn}} \cdot SNR} \int_{-f_{\text{syn}}/2}^{f_{\text{syn}}/2} |H_{\text{syn}}(f)H_{\text{az}}(f)|^2 df \quad (6)$$

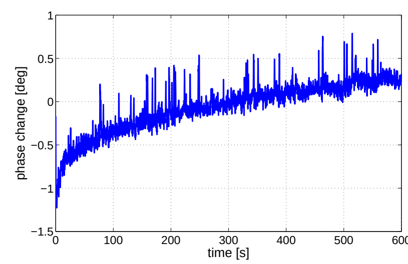
where uncorrelated noise and equal SNR values are assumed for both receivers.

Hardware System Error

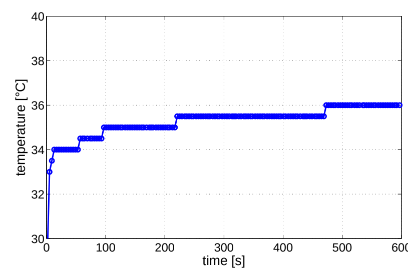
The phase of the hardware system, dominated by active and passive radar RF components, will change within the duration of data take. Concerning the performance of the synchronization link, all the contributions from components common to the Tx and Rx path will cancel out, due to two way operation [5]. The remaining phase drift contributions are given in (4) by

$$\begin{aligned} & \Delta\varphi_{sys1}(t_k, \tau_{\text{sys}} + \tau) - \Delta\varphi_{sys2}(t_k, \tau_{\text{sys}}) = \quad (7) \\ & + \frac{1}{2}\big(\varphi_{\text{Rx1}}(t_k) - \varphi_{\text{Tx1}}(t_k)\big) - \frac{1}{2}\big(\varphi_{\text{Rx2}}(t_k) - \varphi_{\text{Tx2}}(t_k)\big) \end{aligned}$$

with the subscripts $_{\text{Rx}}$, $_{\text{Tx}}$ indicating the phase of the transmit or receive path, respectively. In the above expression, the change in hardware phase within the periods τ and τ_{sys} is ignored.



(a) phase variation $\Delta\varphi_{sys1}(t_k)$



(b) temperature variation

Figure 3: A measured phase variation of a single synchronization path and corresponding temperature variation of the phase critical RF components for a $600\,\text{s}$ data take.

In **Figure 3** an exemplary measured phase variation over time is shown for one synchronization path (one system) for a $f_{\text{syn}} = 10\,\text{Hz}$ and a $100\,\text{MHz}$ chirp [6]. Clearly there is a correlation between temperature and phase variation, which suggests using the recorded temperature data to remove the systematic phase variation. However, even without correction the maximum phase change (for the given measurement) within the duration of a data take is $|\Delta\varphi_{sys1}(t_k)| = +\frac{1}{2}|\varphi_{\text{Rx1}}(t_k) - \varphi_{\text{Tx1}}(t_k)| = 0.39\,°$.

Filter Effect on Sampled Phase Noise

From (4) it is recognized that each oscillator's phase noise is sampled at different time instances. This is equivalent to a low-pass comb filter having the transfer function:

$$H_{\mathrm{LP}}(f) = \exp(-j\pi f \tau_{\mathrm{sys}}) \cdot \cos(\pi f \tau_{\mathrm{sys}}) \tag{8}$$

Thus here the compensation phase will contain a low-pass filtered oscillator noise, where the amount of attenuation depends on the system delay τ_{sys}. This results in a filter mismatch error σ_{filt} given by [7]:

$$\sigma_{\mathrm{filt}}^2 = 2\gamma^2 \int_0^{f_{\mathrm{syn}}/2} S_\varphi(f)|H_{\mathrm{az}}(f)|^2 \cdot |H_{\mathrm{LP}}(f)H_{\mathrm{syn}}(f)-1|^2 df \tag{9}$$

Interpolation Error

The interpolation error is because frequency components outside the range $-\frac{1}{2}f_{\mathrm{syn}} < f < +\frac{1}{2}f_{\mathrm{syn}}$ are lost due to the sampling and hence can not be reconstructed. The interpolation variance is [1]

$$\sigma_{\mathrm{int}}^2 = 2\gamma^2 \int_{f_{\mathrm{syn}}/2}^{\infty} S_\varphi(f)|H_{\mathrm{az}}(f)|^2 df, \tag{10}$$

Aliasing Error

This error results from the cyclic repetition of the spectrum at integer multiples of the sampling frequency f_{syn}; thus, frequency components outside the range $-\frac{1}{2}f_{\mathrm{syn}} < f < +\frac{1}{2}f_{\mathrm{syn}}$ will be folded into the original spectrum causing the aliasing error with the variance [1, 7]

$$\sigma_{\mathrm{alias}}^2 = 2\gamma^2 \sum_{i=1}^{\infty} \int_{-f_{\mathrm{syn}}/2}^{f_{\mathrm{syn}}/2} S_\varphi(f + i \cdot f_{\mathrm{syn}}) \cdot |H_{\mathrm{LP}}(f + i \cdot f_{\mathrm{syn}})H_{\mathrm{syn}}(f)H_{\mathrm{az}}(f)|^2 df \tag{11}$$

4.1 Total Error

The total phase variance is the sum given by:

$$\sigma_{\mathrm{link}}^2 = \frac{1}{2}\sigma_{SNR}^2 + \sigma_{\mathrm{sys}}^2 + \sigma_{\mathrm{filt}}^2 + \sigma_{\mathrm{int}}^2 + \sigma_{\mathrm{alias}}^2 \tag{12}$$

Figure 4 shows the interpolation, aliasing and receiver noise contributions to the total phase error versus the synchronization rate calculated for the TanDEM-X satellite. Here τ_{sys} is adapted to the synchronization rate so as to maximize the low-pass filter effect $\tau_{\mathrm{sys}} = 1/2f_{\mathrm{syn}}$ (in this case the transmit instances of satellite 1 are interleaved midway between those of satellite 2) in which case the aliasing error is minimized.

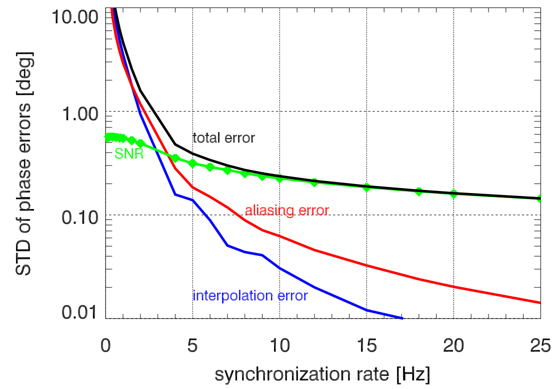


Figure 4: Contributions to the link error versus the synchronization rate f_{syn} for $SNR = 35\,\mathrm{dB}$, $T_{\mathrm{a}} = 0.5\,\mathrm{s}$, $\tau_{\mathrm{sys}} = 1/2f_{\mathrm{syn}}$, and $H_{\mathrm{LP}}(f)H_{\mathrm{syn}}(f) = 1$.

Acknowledgment

We would like to thank Martin Stangl, Uwe Schönfeldt, and Harald Braubach from EADS Astrium GmbH for their valuable support when performing the characterization measurements for TanDEM-X.

References

[1] G. Krieger and M. Younis, "Impact of oscillator noise in bistatic and multistatic SAR," *IEEE Geoscience and Remote Sensing Letters*, accepted for publication, 2006.

[2] H. Braubach and M. Volker, "Method for drift compensation with radar measurement with the aid of reference radar signals," U.S. Patent US 2005/0 083 225 A1, Apr. 21, 2005.

[3] J. Rutman, "Characterization of phase and frequency instabilities in precision frequency sources: Fifteen years of progress," *Proceedings of the IEEE*, vol. 66, no. 9, pp. 1048–1074, Sept. 1978.

[4] F. G. Stremler, *Introduction to Communication Systems*, 2nd ed. Addison-Wesley, 1982.

[5] M. Younis, G. Krieger, R. Metzig, and M. Werner, "TanDEM-X bistatic synchronization system performance," German Aerospace Center, Microwaves and Radar Institute, Tech. Note TDX-TN-DLR-1102, Sept. 2005.

[6] R. Metzig, M. Younis, M. Zink, and M. Werner, "TanDEM-X requirements on TerraSAR-X insturment on-ground tests," German Aerospace Center, Microwaves and Radar Institute, Tech. Note TX-SEC-TN-4206, Oct. 2005.

[7] M. Younis, R. Metzig, and G. Krieger, "Performance prediction of a phase synchronization link for bistatic SAR," *IEEE Geoscience and Remote Sensing Letters*, accepted for publication, 2006.