Statistics of Extremes, traffic jams and natural disasters
(a) 20,000 data of daily water level of the river Danube, measured at Nagymaros from 1st of January 1901 [7]. (b) Seasonality of the average daily water for one year [see Eq. (1)]. (c) Probability density distribution \( P(\Delta h) \) of the water level fluctuations \( h \) [see Eq. (2)]. Note that the vertical scale is logarithmic. (d) Power spectrum of the detrended time series obtained by the standard FFT method. Dotted lines show two scaling regimes, at low frequencies \( (f<0.05 \text{ day}^{-1}) \) the characteristic exponent is 1:20:1, at large frequencies \( (f>0.1 \text{ day}^{-1}) \) the exponent value is 3:3 0:1.

Quelle: Imre M. Janosia; Jason A.C. Gallas: Growth of companies and water-level fluctuations of the river Danube; Physica A 271 (1999) 448-457
Rescaled probability density distribution $P_0 = p_2(h_0)P(r|\ln h_0)$ as a function of the rescaled logarithmic rate of change $r_0 = p_2[r - r(h_0)]=(h_0)$ for the data shown in Fig. 2a. The data approximately collapse upon the universal curve Eq. (5) (thin solid line).

Quelle: Imre M. Janosia; Jason A.C. Gallas: Growth of companies and water-level fluctuations of the river Danube; Physica A 271 (1999) 448-457
First Measurements to fundamental diagrams by Greenshields (1934)
Speed Density Relation $V(\rho)$
The first Fundamental Diagram
q - v Diagram from 5 minutes interval

[A9 München – Holledau, Zeitraum 27.07.-09.08.2000, i.l. 20160 measurement values]
Typical bottleneck situations

Source: R. Kühne, R. Mahnke, J. Hinkel: Modelling the Effects of Corridor Control Systems on Road Capacity; ISHC Yokohama Juli 2006
Speed vs. Flow on I-10 westbound in 5 minute intervals from 4am to 6pm

Results from Performance Monitoring System on Californias Highways (data from 26000 sensors)

- 600 recurrent bottlenecks = 50% of weekday peak delays,
- 28% additionally peak-period congestion delay is caused by collisions,
- 10% of it accounting for 90% of all collision-induced delay

Total vehicle-hours of travel (left) and sources of congestion (right) during peak periods
Demonstration of two 5-h-periods on two cross sections of the A9 München-Holledau
Definition of Traffic Breakdown

1) speed drop: $\Delta v > 15 \text{ km/h}$
2) speed after drop: $v_2 < 75 \text{ km/h}$
3) minimum traffic flow: $q_1 > 1000 \text{ veh/h}$

Result: breakdown y/n at $q_1$
temporal ratio of different traffic flow classes (%) 
zeitlicher Anteil der Verkehrsstärke-Klassen [%]

traffic flow classes [veh/h]
absolut number of traffic breakdowns

traffic flow classes [veh/h]

mit SBA

with traffic control

without traffic control

Vortrag_Januar_2006; Prof. Kuhne
probability of traffic breakdown

traffic flow class [veh/h]

- with traffic control
- without traffic control
Approximation for the breakdown probability function for a two lane section

decomposition into free flow and congested traffic

number of vehicles within the cluster: $n$
minimum cluster size: $n_{esc}$

balance

$$\dot{P}(n,t) = +q \ P(n-1,t) - q \ P(n,t)$$
$$+ \frac{1}{\tau} \ P(n+1,t) - \frac{1}{\tau} \ P(n,t)$$
Taylor expansion

\[ P(n\pm 1, t) \approx P(n, t) \pm \partial_n P(n, t) + \frac{1}{2} \partial_n^2 P(n, t) \]

into balance equation gives

Fokker Planck equation

\[ P(n, t) = -(q - \frac{1}{\tau}) \partial_n P(n, t) + \frac{1}{2} (\frac{1}{\tau} + \frac{1}{\tau}) \partial_n^2 P(n, t) \]

dim. less variables (~suppressed)

\[ x = \frac{n}{n_{esc}} \quad \tilde{t} = \frac{1}{\tau n_{esc}} \quad \beta = (\tau q - 1) n_{esc} \]

\[ \dot{P}(x, t) = (\partial_x \Phi + \partial_x^2) P(x, t) \]

with potential

\[ \Phi = -2\beta x \quad 0 < x < 1 \]
traffic state dynamics

\[ \dot{x} = -\Phi' + \Gamma(t) \quad \Gamma(t) = \text{fluctuating force} \]

\[ <\Gamma(t)> = 0 \quad <\Gamma(t)\Gamma(t')> = 2\delta(t-t') \]
First passage time

Probability of state \( n \) anywhere between 0 and \( n_{\text{esc}} \) at time \( t \) when started with \( n=0 \) at time \( t=0 \):

\[
W(t) = \int_{0}^{n_{\text{esc}}} P(n,t|0,0) \, dn
\]

Drop of probability of state \( n \) anywhere between 0 and \( n_{\text{esc}} \):

\[- \frac{dW(t)}{dt} = - \int_{0}^{n_{\text{esc}}} \dot{P}(n,t|0,0) \, dn\]

\( \equiv \) probability that state exceeds \( n=n_{\text{esc}} \):

\[\mathcal{P}(t) = - \int_{0}^{n_{\text{esc}}} \dot{P}(n,t|0,0) \, dn\]

Inserting Fokker-Planck equation:

\[\dot{P}(n,t|0,0) + \partial_{n} j(n,t|0,0) = 0\]

\[\rightarrow \mathcal{P}(t) = - \int_{0}^{n_{\text{esc}}} \dot{P}(n,t|0,0) \, dn = j(n,t|0,0) \bigg|_{n=0}^{n=n_{\text{esc}}} = j(n_{\text{esc}},t|0,0)\]
separation $P(x,t) = e^{\frac{\Phi(x)}{2}} e^{-\lambda t} \varphi(x)$ gives $(-\partial^2_x + \beta^2)\varphi_v(x) = \lambda_v \varphi_v(x)$ with $(-\beta + \partial_x)\varphi(0) = 0$

ground state $\varphi_0 = \begin{cases} N_0 \sin k_0 (x-1) \\ N_0 \sinh \kappa_0 (x-1) \end{cases}$ $\lambda_0 = \begin{cases} k_0^2 + \beta^2 \\ -\kappa_0^2 + \beta^2 \end{cases}$

excited states $\varphi_v = N_v \sin k_v (x-1)$ $\lambda_v = k_v^2 + \beta^2$ $N_v^2 = \frac{2}{1+\beta/\lambda_v}$ $\nu = 1, 2, ...$

eigenvalues

$k_0 \cot k_0 = -\beta$ $\beta > -1$

$k_0 \coth \kappa_0 = -\beta$ $\beta < -1$

$k_v \cot k_v = -\beta$ $\nu = 1, 2, ...$
time development of probability distribution

\[ P(x,t|0,0) = \delta(x) \]

\[ P(x,t|0,0) = e^{-\frac{\Phi(x)}{2} + \frac{\Phi(0)}{2}} \sum_\nu \phi_\nu(x) \phi_\nu(0) e^{-\lambda_\nu t} \]

\[
\begin{cases}
  e^{-\frac{x^2}{4} + \beta x - \beta^2 t} + e^{-\frac{(x-2)^2}{4} + \beta x - \beta^2 t}, & \beta > -1 \\
  e^\beta x N_\nu^2 \sinh \kappa_\nu (x-1) \sinh(-\kappa_\nu) e^{\lambda_0 t}, & \beta < -1
\end{cases}
\]
First passage time distribution (left: density; right: cumulative distribution) as probabilistic interpretation of observing a traffic breakdown during a time interval \((t, t+dt)\) or during the observation time \(0 \ldots T\)

Source: R. Kühne, R. Mahnke, J. Hinkel: Modelling the Effects of Corridor Control Systems on Road Capacity; ISHC Yokohama Juli 2006
cumulative breakdown probability

\[ W(T) = 1 - \int_{0}^{1} dx \, P(x, T|0,0) = \frac{1}{2} \left( 1 - \text{erf} \left( \frac{1}{2\sqrt{T}} - \beta \sqrt{T} \right) \right) + \frac{e^{2\beta}}{2} \left( 1 - \text{erf} \left( \frac{1}{2\sqrt{T}} + \beta \sqrt{T} \right) \right) \]
Beispiel einer Streckenbeeinflussungsanlage

[Quelle: Engl, H. und F. Lämmel, Highway Deutschland, 1996]
Comparison of two q –v Diagrams from 5 minutes intervals

[A9 München – Holledau, Zeitraum 27.07.-09.08.2000, d.h. 20160 Messwerte]
Cumulative breakdown probability distribution as function of the traffic volume $q$ as control parameters for different critical cluster sizes $m$ modelling the influence of traffic control measures on the breakdown probability.

Source: R. Kühne, R. Mahnke, J. Hinkel: Modelling the Effects of Corridor Control Systems on Road Capacity; ISHC Yokohama Juli 2006