

# An Integrated Model for Aeroelastic Simulation of large flexible Aircraft using MSC.Nastran

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## **Abstract**

Traditionally, in the field of aircraft flight loads, dynamic response and aeroelastics different modelling approaches are employed depending on the type of analysis. Manoeuvre simulation concentrates on the effects of large amplitude rigid body motion whereas gust load and aeroelastic analysis require accounting for small amplitude rigid body and elastic motion. It is desirable to arrive at an integral model formulation that unifies the various modelling principles.

An integral full model of the large flexible aircraft is developed using MSC.Nastran SOL144, SOL145 and SOL100. Traditionally the aeroelastic model is based on a finite element model with unsteady aerodynamics. To develop an integral formulation where steady state aerodynamics is considered the static aeroelasticity capabilities of MSC.Nastran are used.

# 1 Introduction

The integral model is build using the MSC.Nastran DMAP language to combine the quasi-steady and unsteady solution. Additionally thrust forces are included in the formulation. The originally calculated rigid body modes are replaced by decoupled rigid body modes in order to be consistent to the flight mechanics rigid body motion.

The paper begins with the mathematical description of aeroelasticity and flight mechanics. Terms that are needed for an integral formulation are identified comparing aeroelastic and flight mechanical equations of motion.

Then the realization of the integral model equation with various MSC.Nastran solutions (MSC.Nastran SOL144, SOL145 and SOL100) and DMAP alters is described.

Further a practical case is studied to show the capabilities of the integral formulation.

## 2 Mathematical Formulation

In this section the equations of motion in the various disciplines will be described. The equations in aeroelasticity are given first, followed by the nonlinear and linearized equations of flight mechanics. Then the integral model formulation is derived comparing the aeroelastics and flight mechanics equations.

### 2.1 Aeroelasticity

The field of aeroelasticity deals with the interaction of aerodynamic and elastic forces at flexible structures. Aeroelastic phenomena arise when structural deformations induce additional aerodynamic forces [1]. The interaction of aerodynamic, elastic and inertial forces is known as dynamic aeroelasticity whereas static aeroelasticity only involves aerodynamic and elastic forces.

#### Dynamic Aeroelasticity

The equation of motion in dynamic aeroelasticity can be written according to [3] as:

$$[-\omega^2 \mathbf{M}_{hh} + i\omega \mathbf{B}_{hh} + \mathbf{K}_{hh} - q_\infty \mathbf{Q}_{hh}(Ma, k)] \mathbf{u}_h(\omega) = \mathbf{P}_h(\omega) \quad (2.1)$$

in which  $\mathbf{M}_{hh}$ ,  $\mathbf{B}_{hh}$  and  $\mathbf{K}_{hh}$  denote the model mass, damping and stiffness matrix and  $\mathbf{P}_h$  the modal external load vector that acts on the system. The modal displacement vector  $\mathbf{u}_h$  is a function of the circular frequency  $\omega$ . The aerodynamic force matrix  $\mathbf{Q}_{hh}(Ma, k)$  depends on Mach number  $Ma$  and the reduced frequency  $k$  and is obtained from the selected aerodynamic theory. The factor  $q_\infty$  is the dynamic pressure.

## Static Aeroelasticity

Static aeroelasticity occurs when the aircraft is in steady trimmed flight. Structural load distribution and lift distribution is determined by solving the basic equation for static aeroelasticity [3]:

$$\mathbf{M}_{aa}\ddot{\mathbf{u}}_a + [\mathbf{K}_{aa} - q_\infty \mathbf{Q}_{aa}]\mathbf{u}_a = \mathbf{P}_a + q_\infty \mathbf{Q}_{ax}\mathbf{u}_x \quad (2.2)$$

The index  $a$  denotes the structural degrees of freedom set and  $x$  denotes the aerodynamic extra points (aerodynamic control surface deflection and overall rigid body motion).

## 2.2 Flight Mechanics

The nonlinear equation of motion for flight mechanics will be given using the Nastran Basic coordinate frame. For comparison with the aeroelastic equations which describe small variations from the equilibrium condition, the flight mechanic equations will be uncoupled and linearized.

For the derivation of the linearized flight and uncoupled mechanics equations of motion for longitudinal and lateral motion the following assumptions are made:

- earth is an inertial reference frame
- flight in undisturbed air
- symmetric aircraft with constant mass
- quasi-steady aerodynamics

The principle of linear momentum with its corresponding external forces is given by [4]:

$$m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix}_b = \mathbf{S}_{ba} \begin{bmatrix} -W \\ -Q \\ -A \end{bmatrix}_a + \mathbf{S}_{bg} \begin{bmatrix} 0 \\ 0 \\ G \end{bmatrix}_g + \mathbf{S}_{bt} \begin{bmatrix} F_B \\ 0 \\ 0 \end{bmatrix}_t + \mathbf{S}_{ba} \begin{bmatrix} -R_D \\ 0 \\ 0 \end{bmatrix}_a \quad (2.3)$$

where

- $u, v, w$  = velocity components in basic coordinate frame
- $W, Q, A$  = drag, aerodynamic side force and lift
- $G$  = aircraft weight
- $F_B$  = thrust
- $R_D$  = ram drag
- $\mathbf{S}$  = coordinate system transformation matrix

The principle of angular momentum with its corresponding external moments is given by [4]:

$$\begin{aligned}
& \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{bmatrix}_f \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}_f + \begin{bmatrix} p \\ q \\ r \end{bmatrix}_f \times \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{bmatrix}_f \begin{bmatrix} p \\ q \\ r \end{bmatrix}_f = \\
& \mathbf{S}_{fa} \begin{bmatrix} L \\ M \\ N \end{bmatrix}_a + \begin{bmatrix} -x_{T_{res}} \\ 0 \\ z_{T_{res}} \end{bmatrix}_f \times \mathbf{S}_{ft} \begin{bmatrix} F_B \\ 0 \\ 0 \end{bmatrix}_t + \begin{bmatrix} -x_{T_{res}} \\ 0 \\ z_{T_{res}} \end{bmatrix}_f \times \mathbf{S}_{fa} \begin{bmatrix} -R_D \\ 0 \\ 0 \end{bmatrix}_a \quad (2.4)
\end{aligned}$$

where

- $p, q, r$  = roll, pitch and yaw rate
- $L, M, N$  = aerodynamic moments
- $I$  = inertial tensor of the aircraft
- $x_{T_{res}}$  = x-location of resulting thrust
- $z_{T_{res}}$  = z-location of resulting thrust

The coordinate systems [5] in Eq. (2.3) and Eq. (2.4) are as follows denoted:

- $a$  = aerodynamic coordinate system
- $b$  = Nastran basic coordinate system
- $f$  = body (aircraft) fixed coordinate system
- $g$  = geodetic coordinate system
- $t$  = propulsion fixed coordinate system

Equation Eq. (2.3) is already given in the Nastran basic coordinate system whereas Eq. (2.4) has to be transformed from the body fixed to the Nastran basic coordinate frame.

Now, assuming that sideslip  $\beta = 0$  and roll angle  $\Phi = 0$ , the longitudinal and lateral motion can be uncoupled. The formulations Eq. (2.3) and Eq. (2.4) will then be linearized with respect to the desired working point. Executing the linearization leads to the following equations of motion.

## Longitudinal Motion

The linearized equation of motion for longitudinal motion in Nastran basic coordinate frame becomes [4]:

$$\begin{aligned} m\Delta\ddot{x}_b &= -A_0 \cos \gamma_0 \Delta\gamma - \Delta A \sin \gamma_0 + W_0 \sin \gamma_0 \Delta\gamma - \Delta W \cos \gamma_0 \\ &\quad - F_{B0} \sin(\sigma_{T_{res}} + \theta_0) \Delta\theta + \Delta F_{B0} \cos(\sigma_{T_{res}} + \theta_0) \\ &\quad + R_{D0} \sin \gamma_0 \Delta\gamma - \Delta R_{D0} \cos \gamma_0, \end{aligned} \quad (2.5a)$$

$$\begin{aligned} m\Delta\ddot{z}_b &= -A_0 \sin \gamma_0 \Delta\gamma + \Delta A \cos \gamma_0 - W_0 \cos \gamma_0 \Delta\gamma - \Delta W \sin \gamma_0 \\ &\quad + F_{B0} \cos(\sigma_{T_{res}} + \theta_0) \Delta\theta + \Delta F_{B0} \sin(\sigma_{T_{res}} + \theta_0) \\ &\quad - R_{D0} \cos \gamma_0 \Delta\gamma - \Delta R_{D0} \sin \gamma_0 \end{aligned} \quad (2.5b)$$

$$\begin{aligned} I_{yy}\Delta\ddot{\theta}_b &= -\Delta M - R_{D0} z_{T_{res}} \sin \alpha_0 (\Delta\theta - \Delta\gamma) \\ &\quad + \Delta R_D z_{T_{res}} \cos \alpha_0 - \Delta F_B z_{T_{res}} \cos \sigma_{T_{res}} + \Delta F_B x_{T_{res}} \sin \sigma_{T_{res}} \\ &\quad + R_{D0} x_{T_{res}} \cos \alpha_0 (\Delta\theta - \Delta\gamma) + \Delta R_D x_{T_{res}} \sin \alpha_0, \end{aligned} \quad (2.5c)$$

$$\Delta\dot{\theta} = -\Delta q_b \quad , \quad \Delta\dot{\theta} = -\Delta\dot{\theta}_b$$

where 0 is the steady value at the working point and  $\Delta$  is a small variation from the steady value.

The variations of lift  $\Delta A$ , drag  $\Delta W$  and moment  $\Delta M$  are functions of the following variables:

$$\Delta A = \Delta A(V, h, Ma, \alpha, \dot{\alpha}, q, \delta_{el}, \delta_{ia}, \delta_{oa}) \quad (2.6a)$$

$$\Delta W = \Delta W(V, h, Ma, \alpha, \dot{\alpha}, q, \delta_{el}, \delta_{ia}, \delta_{oa}) \quad (2.6b)$$

$$\Delta M = \Delta M(V, h, Ma, \alpha, \dot{\alpha}, q, \delta_{el}, \delta_{ia}, \delta_{oa}) \quad (2.6c)$$

## Lateral motion

The linearized equation of motion for lateral motion in Nastran basic coordinate frame [4] becomes:

$$m\Delta\ddot{y}_b = \Delta Q + W_0 \Delta\beta - A_0 \Delta\phi, \quad (2.7a)$$

$$I_{xx}\Delta\ddot{\phi}_b + I_{xz}\Delta\ddot{\psi}_b = \Delta L - M_0 \Delta\beta, \quad (2.7b)$$

$$I_{zz}\Delta\ddot{\psi}_b + I_{xz}\Delta\ddot{\phi}_b = -\Delta N, \quad (2.7c)$$

$$\begin{aligned} \Delta\dot{\phi} &= \Delta p_b \quad , \quad \Delta\dot{\phi} = \Delta\dot{\phi}_b, \\ \Delta\dot{\psi} &= -\Delta r_b \quad , \quad \Delta\dot{\psi} = -\Delta\dot{\psi}_b \end{aligned}$$

The variations of lateral force  $\Delta Q$  and moments  $\Delta L$ ,  $\Delta N$  are functions of the following variables:

$$\Delta Q = \Delta Q(\beta, \dot{\beta}, q, \delta_{ru}, \delta_{ia}, \delta_{oa}) \quad (2.8a)$$

$$\Delta L = \Delta L(\beta, \dot{\beta}, q, \delta_{ru}, \delta_{ia}, \delta_{oa}) \quad (2.8b)$$

$$\Delta N = \Delta N(\beta, \dot{\beta}, q, \delta_{ru}, \delta_{ia}, \delta_{oa}) \quad (2.8c)$$

## 2.3 Integral Model Approach

### Deficiencies of the aeroelastic model with respect to longitudinal motion

Comparing Eq. (2.5a), Eq. (2.5b), Eq. (2.5c) with 2.1 the following terms can be identified which are not included in the common aeroelastic formulation: derivatives of propulsion, derivatives of drag, derivatives of Mach number and steady aerodynamic forces at small variation from the working point in  $h, V, \gamma$ .

Therefore the following terms have to be included in the integral model for longitudinal motion [4]:

Additional terms for comparison of Eq. (2.1) with		
Eq. (2.5a)	Eq. (2.5b)	Eq. (2.5c)
$-A_0\Delta\gamma_0$	$\Delta A(V, Ma, h)$	$-\Delta M(V, Ma, h)$
$-\Delta W(V, h, Ma, \alpha, \dot{\alpha}, q, \delta_{el}, \delta_{ia}, \delta_{oa})$	$-W_0\Delta\gamma$	$-R_{D0}z_{T_{res}}\sin\alpha_0(\Delta\theta-\Delta\gamma)$
$-F_{B0}\sin(\sigma_{T_{res}}+\alpha_0)\Delta\theta$	$F_{B0}\cos(\sigma_{T_{res}}+\alpha_0)\Delta\theta$	$\Delta R_{D0}(h, Ma, n_1)z_{T_{res}}\cos\alpha_0$
$\Delta F_B(h, Ma, n_1)\cos(\sigma_{T_{res}}+\alpha_0)$	$\Delta F_B(h, Ma, n_1)\sin(\sigma_{T_{res}}+\alpha_0)$	$-\Delta F_B(h, Ma, n_1)z_{T_{res}}\cos\sigma_{T_{res}}$
$\Delta R_{D0}(h, Ma, n_1)$	$R_{D0}\Delta\gamma$	$\Delta F_B(h, Ma, n_1)x_{T_{res}}\sin\sigma_{T_{res}}$
–	–	$R_{D0}x_{T_{res}}\cos\alpha_0(\Delta\theta-\Delta\gamma)$
–	–	$\Delta R_{D0}(h, Ma, n_1)x_{T_{res}}\sin\alpha_0$

Table 2.1: Additional terms for longitudinal motion

### Deficiencies of the aeroelastic model with respect to lateral motion

Comparing Eq. (2.7a), Eq. (2.7b), Eq. (2.7c) with 2.1 it can be seen that steady aerodynamic forces at small variation from the working point in  $\beta, \Psi, \Phi$  are not included in the aeroelastic formulation [4].

The propulsion forces and the dependence on the working point can be neglected in the lateral flight mechanics equations of motion. Therefore these terms will not be included in the integral model formulation.

The following terms have to be added to the aeroelastic model for lateral motion:

Additional terms for comparison of Eq. (2.1) with		
Eq. (2.7a)	Eq. (2.7b)	Eq. (2.7c)
$W_0\Delta\beta$	$-M_0\Delta\beta$	–
$W_0\Delta\psi$	–	–
$-A_0\Delta\phi$	–	–

Table 2.2: Additional terms for lateral motion

## Modelling of the Additional Terms

The induced drag is modelled and obtained from lift and angle of attack by

$$W \approx \sum_p A_j \alpha_j \quad (2.9)$$

The variation of drag with Mach number is obtained from total aircraft derivatives.

Unsteady propulsion and ram drag resulting from engine motion are derived for longitudinal motion and transformed to model coordinates.

## 3 Integral Model Process

The integral model process is realized using MSC.Nastran. The Solution sequence to come to the integral formulation consists of three solutions (Fig. 3.1):

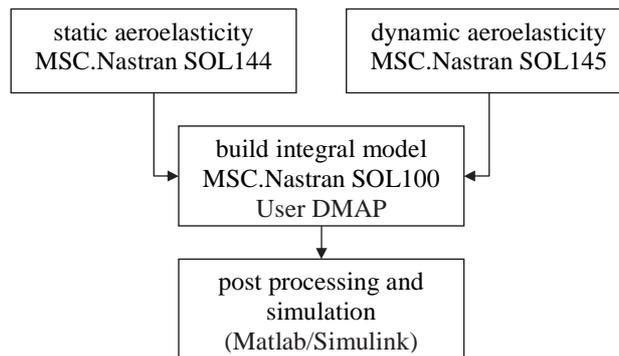


Figure 3.1: Integral Model Process

- First a static aeroelastic solution (SOL144) is used to calculate the trim condition. A DMAP alter stores the downwash and pressures for further steps.
- In the next step a dynamic aeroelastic analysis (SOL145) is performed. Additional terms for integral modelling are calculated using a DMAP alter. Also relevant data is stored for the next Nastran run.
- The third step (SOL100) collects data from the two previous Nastran runs. Additional aerodynamic forces are calculated and added to unsteady aerodynamic forces via a DMAP alter. A detailed description of each step is given in the following sections.

### 3.1 Static Solution 144

The elastic trim analysis is performed using SOL144. The trim condition is specified using the TRIM card. The unrestrained analysis is used. For aerodynamic theory Doublet Lattice is selected and improved with aerodynamic corrections.

MSC.Nastran provides three ways of experimental aerodynamic corrections. Correction factors can be input using the matrix  $\mathbf{W}_{kk}$ , experimental pressures can be input using the vector  $\mathbf{FA2J}$  and adjustments to the downwash to account for, e.g., the effects of camber and twist, can be input using the vector  $\mathbf{W2GJ}$  [3]. All correction matrices are included using DMI entries.

In the integral model formulation the matrix  $\mathbf{W}_{kk}$  is used to correct the lift slope of the fuselage panels:

$$\mathbf{P}_k = q\mathbf{W}_{kk}\mathbf{S}_{kj}\mathbf{A}_{jj}^{-1}\mathbf{w}_j \quad (3.10)$$

The initial downwash from camber and the effect from twist is included using the  $\mathbf{W2GJ}$  matrix,  $\mathbf{w}_j^g$ :

$$\mathbf{w}_j = \mathbf{D}_{jk}\mathbf{u}_k + \mathbf{D}_{jx}\mathbf{u}_x + \mathbf{w}_j^g \quad (3.11)$$

The DMAP Alter writes the computed data, e.g. pressure and downwash, (via OUTPUT2) for subsequent input (via INPUTT2) into the MSC.Nastran SOL100 run.

#### 3.1.1 Generation of W2GJ

To achieve a realistic trim solution, the lift distribution at  $0^\circ$  angle of attack has to be accounted for. Therefore the initial downwash vector  $\mathbf{W2GJ}$  has to be generated. This vector depends on the geometric properties of the lifting surfaces such as the twist and the mean camber line of the airfoil at each spanwise station. The elements of  $\mathbf{W2GJ}$  represent the initial angles of attack of each individual aerodynamic box element.

The twist is given along the span of a lifting surface and is interpolated at the spanwise midpoints of each aerodynamic box. Boxes in chordwise direction have the same value.

The angle of attack due to camber is explained by means of the well known airfoil family of the NACA-four-digit series. E.g. the NACA2415 airfoil has a maximum thickness  $t$  at 15% with a maximum camber  $m$  of 2% located at 40% ( $p = 0.4$ ) from the airfoil leading edge. The mean camber line is then given by:

$$y_c = \frac{m}{p^2}(2px - x^2) \quad \text{for } 0 \leq x < p \quad (3.12a)$$

$$y_c = \frac{m}{(1-p)^2}((1-2p) + 2px - x^2) \quad \text{for } p \leq x \leq 1 \quad (3.12b)$$

Differentiating the camberline equation with respect to  $x$  yields the equation for the slope. The angle of attack due to camber is the slope evaluated at

the collocation points, i.e. at the 3/4-chord location of each individual box. Special care has to be taken, to ensure a consistency of chordwise location and the numbering of the boxes.

The slopes due to camber and twist are added and the boxes have to be sorted according to their ID:

$$\alpha_j = \alpha_j^{twist} + \alpha_j^{camber} \quad (3.13)$$

These local angles of attack constitute the elements for the DMI input of the **W2GJ** vector.

Figure 3.2 depicts a comparison of the local liftcoefficients with and without accounting for the initial downwash given by the **W2GJ** vector.

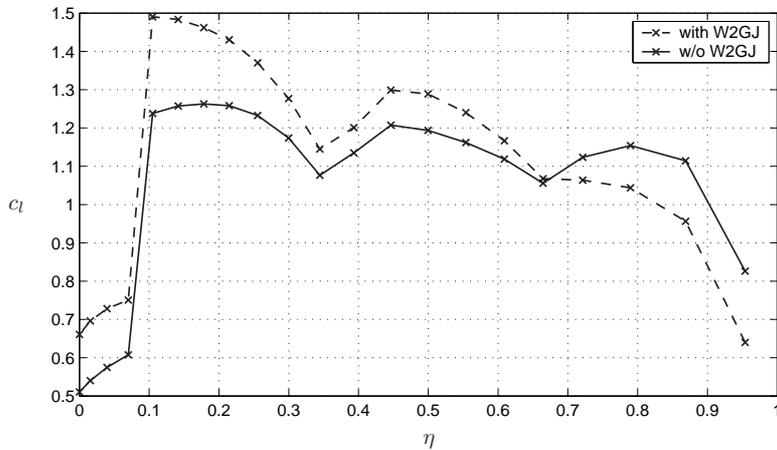


Figure 3.2: local lift coefficients  $c_l$  of the wing with and without W2GJ

### 3.2 Dynamic Solution 145

In the next step a dynamic aeroelastic analysis (SOL145) is performed. The solution SOL145 is not used for flutter analysis but as basic solution structure for the integral model procedure.

The vibration modes are obtained from the normal mode analysis in the MODERS subDMAP. The rigid body and elastic mode shapes are orthonormalized with respect to the mass matrix. The integral model approach requires rigid body mode shapes in analogy to flight mechanics i.e. unit translations and rotations in the direction of the basic coordinate frame and its origin in the center of gravity. The original rigid body mode shapes are therefore replaced with the rigid body modes shapes in analogy to flight mechanics. The new rigid body mode shapes are generated using the VECPLOT DMAP module. The new mode shapes are then used to generalize the mass stiffness matrices.

The doublet lattice theory is selected for unsteady aerodynamic theory. Since there are no unsteady aerodynamic forces at the fuselage caused by

roll the matrix  $\mathbf{D}_{kk}$  is used to identify the fuselage panels. In the DMAP alter the aerodynamic forces resulting from fuselage panels due to roll mode (roll mode column in QKH) are eliminated:

```
MATGEN ,/CPVEC/4/1/NCQKH/1/1/NRQKH/4/1/1 $
MATPRN CPVEC// $
PARTN QKH,CPVEC,DKK/Q1,Q2,Q3,Q4/1 $
MERGE Q1,Q2,Q3,,CPVEC,DKK/QKH/1 $
EQUIV QKH,QKH/ALWAYS $
MPYAD GPKH,QKH,/QHH/1 $
```

Thrust forces are also included in the integral model. Therefore DMIG entries are used to specify the thrust parameters in the Nastran input deck. The equations for thrust forces are implemented in the DMAP alter, e.g.  $x$ -force of inner engine:

```
MESSAGE // 'ENGINE-ALTER' /
$ X-FORCE INNER ENGINE
SRE13=(DFGDH*COS(SIGIE+ALPHA)*COS(VUIE)-DRDDH)
-(DFGDMA*COS(SIGIE+ALPHA)*COS(VUIE)-DRDDMA)*DMA/AV*DAVDH)$
SRE14= FG*SIN(ALPHA+SIGIE)*COS(VUIE) $
SRE15=+FG*SIN(VUIE)*COS(ALPHA+SIGIE) $
SIM11=DFGDMA*COS(SIGIE+ALPHA)*COS(VUIE)*DMA-DRDDMA*DMA$
```

Gyroscopic effects from engines are particularly important on large flexible aircraft. The gyroscopic engine forces depend on the power setting and the time derivative of engine pylon motion. The gyroscopic engine forces are modelled using the B2PP functionality. Then the structural damping matrix includes the gyroscopic forces.

Gust forces are calculated in analogy to SOL146 by including the GUST module in the DMAP alter:

```
GUST CASES,DLT,FRL,DIT,QhJL,,ACPT,CSTMA,/
PHF1/
S,N,NOGUST/BOV/MACH/Qx $
```

All necessary tables and matrices from SOL145 are written for subsequent use in the SOL100 run.

### 3.3 Combination using SOL100

A solution SOL100 (USERDMAP) run is used to combine the results from the previous SOL144 and SOL145 runs.

Another aerodynamic correction is performed with the  $\mathbf{W}_{kk}$ -functionality. The matrix  $\mathbf{W}_{kk}$  correct the lift slope of each Doublet lattice panel.

An important step in the integral model procedure is the modelling of the additional terms in analogy to flight mechanics. This required the approximation of drag by induced drag. The distribution of induced drag is

computed in the SOL100 DMAP alter from the lift and angle of attack at each panel.

Further more pressure and moment coefficients are computed. Then the additional forces are modelled in analogy to flight mechanics and added to the unsteady aerodynamic forces:

$$\mathbf{Q}_{hh}(Ma, k) = \mathbf{Q}_{\Delta A}(Ma, k) + \mathbf{Q}_{TKB}(Ma, k) + \mathbf{Q}_{AK}(Ma, k) \quad (3.14)$$

where  $\Delta A$  denotes the unsteady aerodynamic forces,  $TKB$  the trust forces resulting from engine motion and  $AK$  the additional aerodynamic forces.

All system matrices are written to a file for post processing and simulation in matlab.

## 4 Analysis and Simulation

### 4.1 State Space Model

The unsteady aerodynamic forces  $\mathbf{Q}_{hh}$  are approximated using the Karpel approximation [2]:

$$\mathbf{Q}_{hh}(k) = \mathbf{A}_0 + \mathbf{A}_1jk - \mathbf{A}_2k^2 + \mathbf{D}(jk\mathbf{I} - \mathbf{R})^{-1}\mathbf{E}jk \quad (4.15)$$

The equation of motion for the integral model can than be written in state space form

$$\begin{bmatrix} \dot{u}_h \\ \ddot{u}_h \\ \dot{x}_L \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} \\ \mathbf{0} & E & \frac{v}{c_{ref}}R \end{bmatrix} \begin{bmatrix} u_h \\ \dot{u}_h \\ x_L \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \delta \\ F_{ex} \end{bmatrix} \quad (4.16)$$

with

$$\begin{aligned} \mathbf{A}_{21} &= -M^*(K_{hh} - \frac{\rho}{2}v^2A_0) & \mathbf{B}_{21} &= M^*\Phi_{boore}^TK_{Au} \\ \mathbf{A}_{22} &= -M^*(B_{hh} - \frac{\rho}{2}c_{ref}vA_1) & \mathbf{B}_{22} &= M^*\Phi_{exfo}^T \\ \mathbf{A}_{23} &= M^*\frac{\rho}{2}v^2D & M^* &= (M_{hh} - \frac{\rho}{2}c_{ref}^2A_2)^{-1} \end{aligned} \quad (4.17)$$

and used for simulation.

### 4.2 Simulation Results

The response of an elevator step is shown in figure 4.3 and the response of an aileron stairs input is shown in figure 4.4.

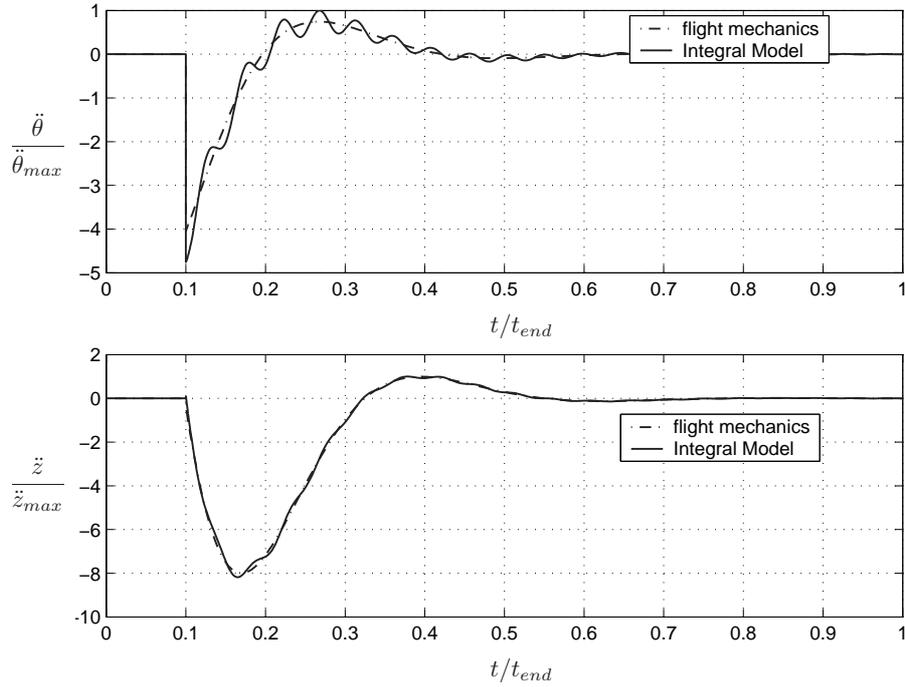


Figure 4.3: Accelerations at the center of gravity resulting from an elevator step input.

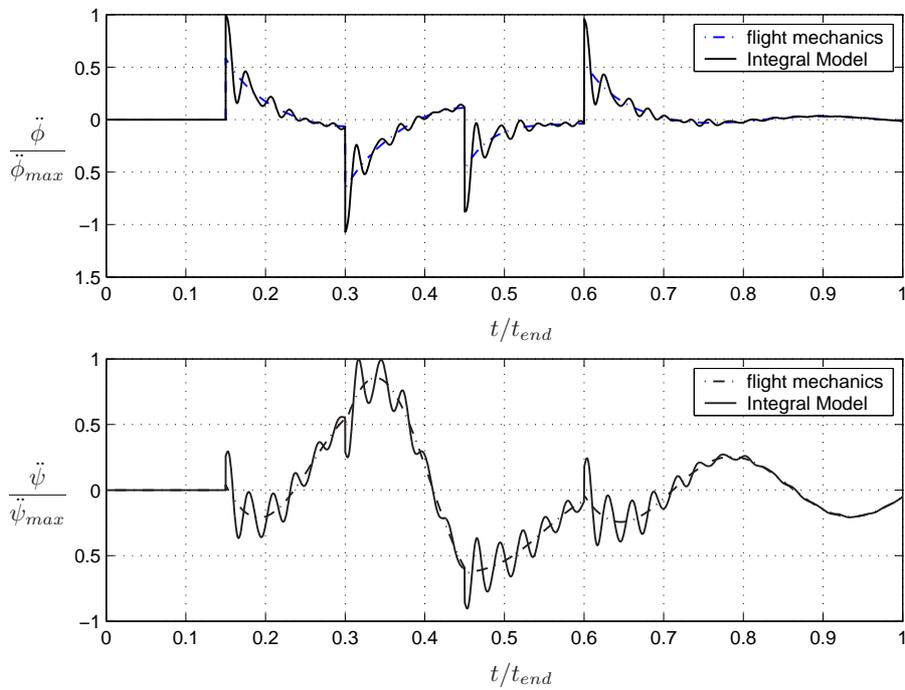


Figure 4.4: Accelerations at the center of gravity resulting from an aileron stair input.

## 5 Conclusions

A procedure for developing an integral model for flight mechanics and aeroelastic simulation using MSC.NASTRAN has been presented. The procedure uses the static aeroelastic solution (SOL144), dynamic aeroelastic solution (SOL145) and the USERDMAP (SOL100). Each solution is combined with a DMAP alter to realize the integral model equations.

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