

# Forest and the Random Volume over Ground

## - nature and effect of 3 possible error types

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### Abstract

The potential of Polarimetric SAR Interferometry (Pol-InSAR) lies in the 3-dimensional resolution of different scattering processes. This proves especially useful for the interpretation of volume scatterers – such as forests - and makes physical parameter extraction possible. Due to the complex structure of forests, the interpretation of Pol-InSAR data is not straight forward, but requires the consideration of a scattering model. The Random Volume over Ground-model has frequently been employed for the forest height extraction from L-band Pol-InSAR data, and the results encourage considerations about spaceborne configurations. In this sense, it is certainly useful to benefit from an analysis of possible error types and error sources. This article addresses different error types and discusses their impact on the model-based height estimation.

## 1 Introduction

Pol.InSAR data inversion at L-band can be used to invert forest heights [1,2]. The basic principle beyond this inversion is an interpretation of the volume-related decorrelation of the interferometric coherence, which depends on the vertical distribution of the scattering processes. The phase of the interferometric coherence lies between the ground and the forest canopy. High extinction in dense forest environment drags the phase center towards the canopy top. For low extinctions, the location of the phase center depends strongly on the ground contribution.

The Random Volume over Ground-model (RVoG) interprets the interferometric coherence in dependence of the scatterer profile in height, and allows to invert forest height with respect to extinction and ground influence. The mathematical description of the RVoG for a constant extinction along the volume is given by Eqs. 1/ 2 [1,2].

Volume-only coherence  $\gamma_V$  ( $m=0$ ):

$$\gamma_V = \frac{\int \exp(2 \cdot \sigma \cdot z' / \cos \theta) \cdot \exp(-i \cdot kz \cdot z') dz'}{\int \exp(2 \cdot \sigma \cdot z' / \cos \theta) dz'} \quad (1)$$

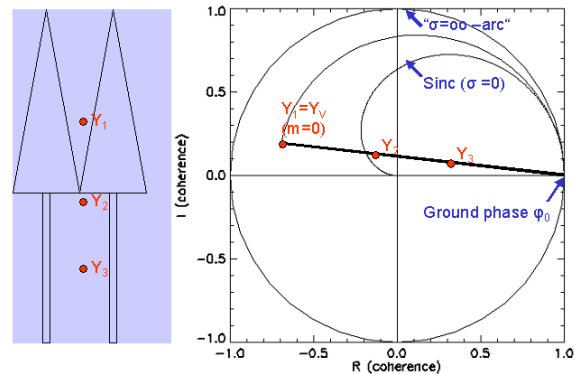
(Complex) coherence  $\gamma$  with  $m>0$ :

$$\gamma = \frac{m + \gamma_V}{m + 1} \cdot \exp(i\phi_0) \quad (2)$$

with the system parameters: look angle  $\theta$ , vertical wavenumber  $kz = 4\pi/\lambda \cdot \Delta\theta/\sin(\theta)$ , and the forest parameters: extinction ' $\sigma$ ' [Np/m], height dimension  $z$  [m], and ground contribution  $m$ . ' $m$ ' is calculated as the ratio of the distance  $\gamma_V - \gamma$  to the distance  $\gamma - \phi_0$ :

$$m = \frac{\gamma_V - \gamma}{\gamma - \exp(i\phi_0)} \quad (3)$$

Assuming the interferometric volume response as polarization independent, and the ground response as polarization dependent, the coherences at different polarizations differ as a function of the ground contribution. For a constant extinction along the volume depth, the polarimetric coherences form a line in the unit circle (Fig. 1). Characteristic features of the model are the ground phase  $\phi_0$  (in Fig. 1 at  $0^\circ$ ), the volume-only coherence  $\gamma_V$  where  $m=0$ , the sinc-solution where ground-to-volume ratio  $m=0$  and extinction  $\sigma=0$ , and " $\sigma=\infty$  -arc" at  $|\gamma_V|=1$  (on the unit circle).  $\gamma_V$  is assumed to be the (complex) coherence that is most distant from the ground phase  $\phi_0$  in the unit circle (Fig. 1). The inversion space for  $\gamma_V$  in the unit circle is then given by the range of heights and extinctions, and is a function of  $kz$  (Fig. 2b,c).

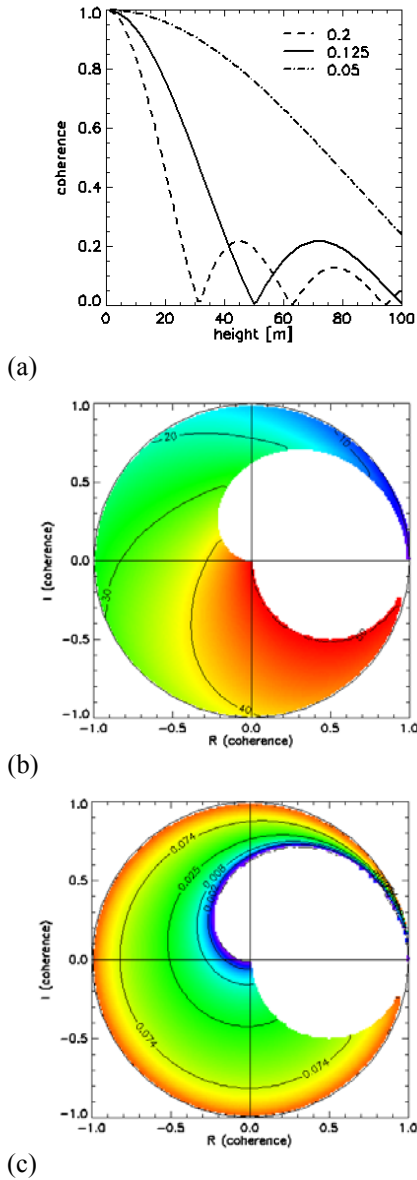


**Figure 1** In a forest, the interferometric phases of different polarizations  $Y_1$ - $Y_3$  are located at different heights. In the unit circle, the Random Volume over Ground-model predicts polarimetric coherences to form a line.

Due to the potential of this algorithm to determine forest height, it is important to address the nature and effect of possible errors and error sources.

Three error types are considered in the following: (1) a residual ground contribution  $m$  in the assumed  $\gamma_V$ , (2) an offset  $\Delta\phi$  from the true ground phase  $\phi_0$ , and (3) an additional decorrelation source in  $\gamma_V$ .

The presented error analysis is focused on the height estimation error. Section 2 models a certain interferometric system configuration for the analysis. In section 3, the error is quantified in dependence of error magnitude, height and extinction. Section 4 discusses the impact of the error in a larger frame of possible error sources.



**Figure 2** (a) Choosing of an appropriate  $kz$  for the interferometric system configuration (0.2, 0.125, 0.050), (b) height inversion space [m], (c) extinction inversion space [Np] in the unit circle.

## 2 Methods

The error analysis was conducted for a certain PolInSAR system configuration. Effectively, a  $kz$  for achieving maximum sensitivity for a desired range of forest heights has been chosen. A good proxy can be obtained by setting  $\sigma=0$  and  $m=0$  in the RVoG (sinc-solution). For forest heights up to 50 m, a  $kz=0.125$  corresponds to a coherence of 0, providing a coherence range from 0 to 1 (Fig. 2a, solid line). A larger  $kz$  introduces ambiguities in the height-coherence relation (Fig. 2a dashed line); for a smaller  $kz$ , the coherence-height relation loses sensitivity due to a decreasing coherence-range (dash-dotted line). The resulting inversion space in the unit circle for different heights and extinctions is shown in Fig. 2b/c. Heights up to 50 m and extinctions up to 0.223 Np/m ( $\sim 2$ dB/m) are considered. Typical extinction values for forests in L-band are considerably lower with 0.01-0.04 Np/m ( $\sim 0.1$ -0.3 dB/m) [3].

## 3 Results

In Fig. 3-5, the results for the three mentioned error types are presented in a certain figure scheme: (a) sketch of error type in the unit circle, (b/ c/ d) relative error for 10/ 20/ 30m height as a function of extinction. The three heights represent different regions of the inversion space in the unit circle. In this sense, they are equivalent to other  $kz$ -height pairs corresponding to different configurations.

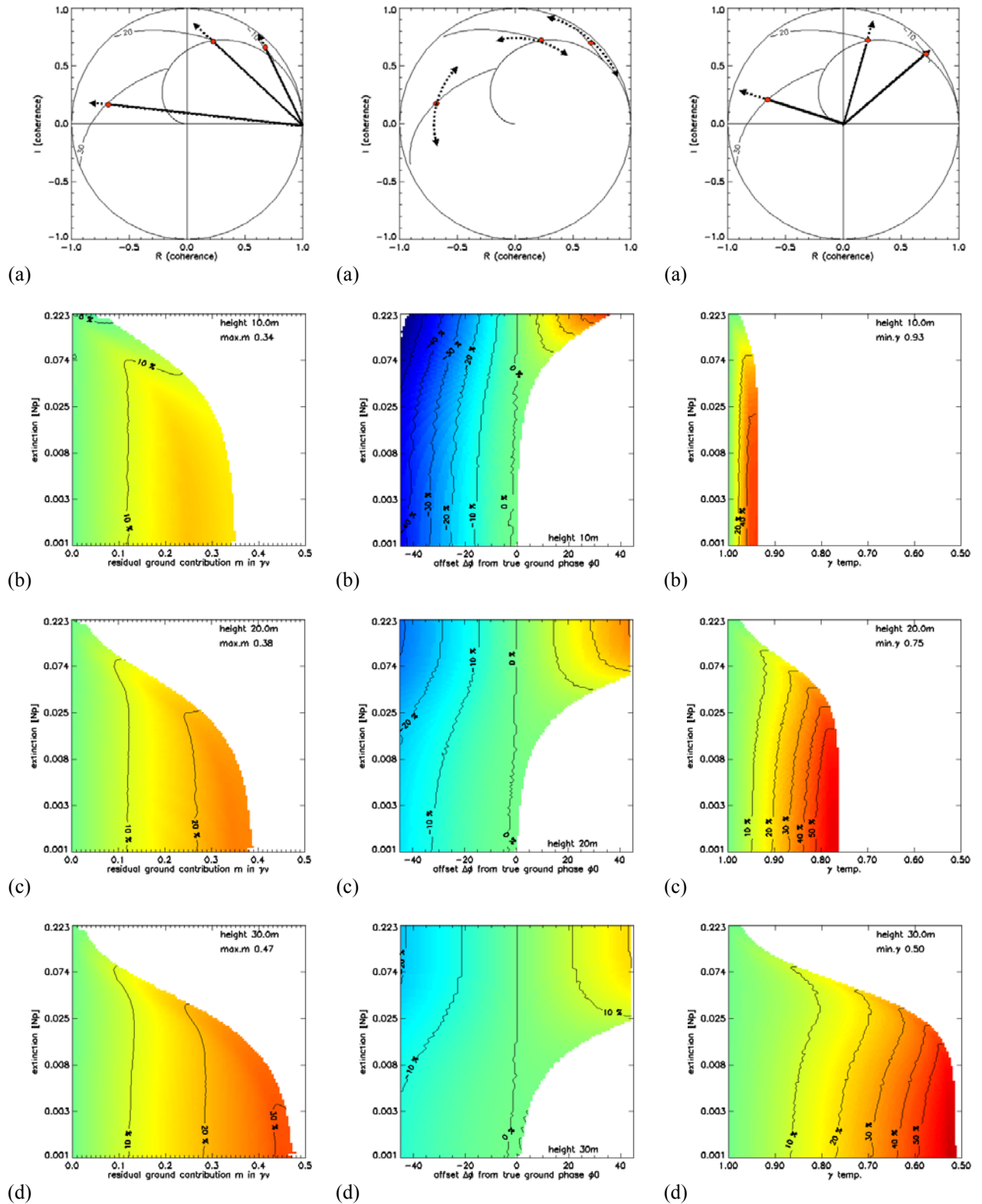
### 1. Residual ground contribution $m$ in assumed $\gamma_V$ :

If the assumed volume-only coherence  $\gamma_{V(m)}$  contains a residual  $m$ , then the true volume-only coherence  $\gamma_V$  can be calculated (according to Eq. 2, setting  $\phi_0=0^\circ$ ):

$$\gamma_V \cdot (1 + m) - m = \gamma_{V(m)} \quad (4)$$

As Fig. 3a shows, for a residual  $m$ , the true volume-only coherence  $\gamma_V$  is located on a line extending from the ground phase  $\phi_0$  (Fig. 3a). Since the  $m$  of a coherence  $\gamma$  is defined as the ratio of  $(\gamma_V - \gamma) / (\gamma - \exp(i\phi_0))$ , the maximum value that a residual  $m$  of  $\gamma_V$  can assume, is given by the difference between the sinc-curve and the unit circle. At maximum  $m$  can therefore approach 1, but typically  $m$  is smaller by at least one order of magnitude.

From Fig. 3a, it is obvious that any residual  $m$  leads to a height overestimation. The maximum values  $m$  can assume are dependent on the location of  $\gamma_V$  in the inversion space. For the modeled scenario, the maximum  $m$ -values for 10/ 20/ 30m height are  $m=0.34/ 0.138/ 0.47$ . As can be observed from Fig. 3b-d, the relative height error appears to be widely independent of extinction and has a constant behaviour over different heights. Quantitatively, an overestimation of 10/ 20/ 30% corresponds to a residual  $m=0.12/ 0.28/ 0.44$ .



**Figure 3** Height error due to residual ground contribution  $m$  in  $\gamma_V$  in dependence of extinction. (a) error type sketched in the unit circle, (b/ c/ d) relative height error for 10/ 20/ 30m height.

**Figure 4** Height error due to offset  $\Delta\phi$  from true ground phase  $\phi_0$  in dependence of extinction. (a) error type sketched in the unit circle, (b/ c/ d) relative height error for 10/ 20/ 30m height.

**Figure 5** Height error due to an additional decorrelation source  $\gamma_T$  in  $\gamma_V$  in dependence of extinction. (a) error type sketched in the unit circle, (b/ c/ d) relative height error for 10/ 20/ 30m height.

## 2. Offset $\Delta\phi$ in true ground phase $\phi_0$ :

For a height inversion of a given volume-only coherence, an error in the ground phase is equivalent to a rotation of the assumed volume-only coherence  $\gamma_{V(\Delta\phi)}$  by  $\Delta\phi$  (setting  $\phi_0=0^\circ$ , Fig. 4a):

$$\gamma_V = \gamma_{V(\Delta\phi)} \cdot \exp(i\Delta\phi) \quad (5)$$

As will be discussed in Section 4, the error in the ground phase estimation can be large and may make it necessary to consider a different coherence as the volume-only coherence  $\gamma_V$ . In Fig. 4b-d,  $\gamma_V$  was assumed not to change, and a ground phase error of  $\Delta\phi = \pm 45^\circ$  is plotted. In general, a negative phase deviation means the heights are underestimated, while a positive phase deviation leads to overestimation. The effect of  $\Delta\phi$  is largest for the 10m height, where  $45^\circ$  lead to  $\sim 40\%$  relative height error for an extinction of  $\sigma = 0.001$  Np/m, and  $>60\%$  height error for  $\sigma = 0.223$  Np/m. Notably, the *absolute* height error stays to some degree similar between the heights for a given extinction. Regarding the extinction dependence it is worth noting, that for small shifts in  $\phi_0$ , the error for an extinction of  $\sigma = 0$  (sinc-solution) is double the error for  $\sigma = \infty$  (on the unit circle). This corresponds to the phase / height ratio between these boundaries.

## 3. Decorrelation $\gamma_T$ in $\gamma_V$

A decorrelation in the volume-only coherence over a non-decorrelated ground was described in [1] as a temporal decorrelation effect and will be denoted  $\gamma_T$  subsequently. The error extends radial from the center of the unit circle, and the true volume-coherence  $\gamma_V$  can be calculated from the assumed one  $\gamma_{V(decor.)}$  by:

$$\gamma_V = \frac{1}{|\gamma_T|} \cdot \gamma_{V(decor.)} \quad (6)$$

Temporal decorrelation always leads to a height overestimation. The maximum decorrelation (or minimum  $\gamma_T$ ) is given by the difference between the sinc-curve and the " $\sigma=\infty$ -arc" at  $|\gamma_V|=1$  (on the unit circle). In the modeled scenario, the minimum possible values for 10/ 20/ 30m height are for  $\gamma_T = 0.93/ 0.75/ 0.50$ . This maximum decorrelation is equivalent to a height overestimation of 100%, and the height error scales relative to the maximum decorrelation. Therefore, the same decorrelation has a stronger effect on the 10m height than on the 20m and 30m height (Fig. 5b-d). The height error is comparably independent of the extinction. The relative height error is more sensitive when decorrelation increases.

## 4 Discussion

Three error types in the height inversion of Pol-InSAR data with the Random Volume over Ground-model were considered. In summary, residual m-errors, decorrelations, and positive ground phase deviations result in a height overestimation, while only negative ground phase deviations may cause underestimation. The errors in the height estimation are almost equal for the considered range of extinctions.

In order to estimate how critical each error type for a given system can be, the importance and magnitudes must be discussed:

- (1) The presence of a residual ground contribution in  $\gamma_V$  is an inversion limitation. The resulting relative height error is very similar for different heights, and should not exceed 20%. It occurs more likely for low heights and/or extinctions, and can be minimized by increasing exploiting the polarimetric space (e.g. Pauli and optimum coherences).
- (2) Errors in the ground phase estimation result from uncertainties in the fitted line. They occur statistically when the "visible line" [1] is small due to similar ground contributions in the polarizations, or systematically when the forest structure deviates from the random scatterer distribution [1]. The absolute height error for  $\sigma < 0.025$  Np/m lies around 3-5m for  $-45^\circ$  phase deviation for the considered heights. Large errors in the ground phase occur if the decision between the two possible ground phase solutions was not correct. Then, it is also necessary to consider a different coherence as the volume-only coherence.
- (3) Temporal decorrelation in  $\gamma_V$  leads to an overestimation up to 100% especially for low heights. It is the most critical error considered, but can be avoided to some degree: (1) temporal decorrelation can be minimized with single pass interferometry or short revisit times; (2) the sensitivity of low heights to decorrelation can be avoided with a second baseline with a higher  $kz$ -value. Other decorrelation sources like range-, SNR-, and co-registration- decorrelation also affect  $\gamma_V$ , and may additionally lead to an offset in the determined ground phase  $\phi_0$  [4].

The next steps in the error analysis are (1) an analytical error quantification in dependence of system and forest parameters, and (2) the quantification of error types in dependence of the error source. It is expected that the error analysis contributes to system design aspects, parameter accuracy estimations, and error minimization in the data interpretation.

## References

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