Advanced Bistatic and Multistatic SAR Concepts and Applications

Gerhard Krieger

Microwaves and Radar Institute, DLR
# Future SAR Systems: Motivation

## Application Areas for SAR Data Products

### Agriculture
- Precision Farming Suite
- Crop Ripeness
- Crop Inventory
- Yield Prediction Cereals

### Forestry
- Strategic Forestry Inventory
- Reconnaissance Inventory
- Inventory Update

### Cartography
- Topo Map
- Regional Planning Map
- Environmental Planning Map
- Infrastructure Planning Map

### Marine
- Ship Detection Service
- Oil Spill Monitoring
- Sea Ice Monitoring

### Risk / Disaster
- Flood Damage Assessment
- Fire Damage Assessment
- Storm Damage Assessment

### Security
- Reconnaissance Imagery (VHR-SAR)

### Geology
- Geology Structure Map
- Geology Image Map
- Geology Elevation Map
- Oil Seep Detection

### Transportation
- Dynamic Traffic Monitoring
- Maps of Roads, Channels, ...

## Requirements
- **wide area coverage and short revisit times**
- **high radiometric and geometric resolution**
- **new data products:**
  - high precision DEMs
  - 3-D volume images (biomass, soil, ice, ...)
  - dynamic maps (ocean currents, traffic, ...)
  - ...
- **high reliability and cost efficiency**
Future SAR Systems: Paradigm Shift to Satellite Clusters

- Smaller, simpler satellites → reduced cost & time
- Modular design → upgradable, improved reliability
- Spatially distributed → improved revisit time / coverage / adaptability
- Separated, sparse apertures → improved performance and resolution

Large, multi-functional satellites

Virtual satellite - web of cooperating satellites

Paradigm Shift
Bistatic and Multistatic SAR Systems

Definitions:
• Radar systems with a spatial separation between transmitter and receiver are called bistatic.
• Systems with multiple receivers are called multistatic.

**Fully active system**
(TechSAT21, Radarsat 2/3, TanDEM-X)

+ more observables & baselines
+ phase synchronisation in ping/pong mode
+ high redundancy & great flexibility

**Semi active system**
(BISSAT, Cartwheel, Pendulum)

+ reduced size, weight and costs
+ increased sensitivity (no Tx/Rx switches)
+ receiver camouflage, robustness to jamming
Examples for Suggested Missions

**TanDEM-X (Germany)**

**Radarsat 2/3 (Canada)**

**TechSat 21 (USA)**

**BISSAT (Italy)**

**TerraSAR-L Cartwheel (ESA)**

**Voice, Habitat (Earth Explorer Proposals)**
Potentials of Bistatic and Multistatic SAR Systems

- **Bistatic Imaging**
  - Potential: Resolution Enhancement
  - Applications: Cross-Track Interferometry, Along-Track Interferometry
  - Features: Interference Suppression, Silent Operation, Increased Radiom Sensitivity, Double Differential InSAR

- **Moving Target Indication**
  - Potential: Frequent Monitoring
  - Applications: Wide Swath Imaging
  - Features: Interference Suppression, Silent Operation, Increased Radiom Sensitivity, Double Differential InSAR

- **Resolution Enhancement**
  - Potential: SAR Tomography
  - Applications: Interference Suppression, Silent Operation, Increased Radiom Sensitivity, Double Differential InSAR
Geostationary Illuminator / LEO Receivers

Basic Idea:
• constant illumination by geostationary transmitter
• signal reception by multiple low-cost receivers

Illuminator:
• geostationary orbit
• high Tx power (CW)
• large antenna area
• optional: steerable antenna

Receivers:
• passive (receive only)
• low power, small antennas
• low-cost micro-satellites
• low earth orbit

Advantages:
• substantially improved revisit times without cost explosion
• multiple missions may share one illuminator
Bistatic SAR Systems: Basic Definitions

**Bistatic Triangle**: $r = r_{Tx} + r_{Rx}$

$→$ for a given range $r$ the target is located on an **ellipsoid** with semi major axis $a = \frac{r_{Tx} + r_{Rx}}{2}$

$r_{Tx} + r_{Rx} = \text{const.}$ $Tgt$
Iso-Range Contours

Tangent Plane:

\[ r(x, y) = r_{Tx}(x, y) + r_{Rx}(x, y) = \text{const.} \]

(Intersection of Ellipsoid with Tangent Plane)
Slant Range Resolution: (after Pulse Compression)

\[ \Delta r_s = \frac{c_0}{B_r} \]

\( B_r \): bandwidth of range chirp
\( c_0 \): velocity of light

Ground Range Resolution:

\[ \Delta r_{grd} \leq \frac{l}{\| \text{grad}[r(x,y)] \|} \cdot \frac{c_0}{B_r} \]

\( B_r = 300 \text{MHz} \)
Ground Range Resolution

**Special Case:**

\[ r(x, y) = \sqrt{(x - x_{Tx})^2 + z_{Tx}^2} + \sqrt{(x - x_{Rx})^2 + z_{Rx}^2} \]

\[ r_{Tx}(x, y) \quad r_{Rx}(x, y) \]

\[ \frac{\partial}{\partial x} [r(x, y)] = \frac{x - x_{Tx}}{\sqrt{(x - x_{Tx})^2 + z_{Tx}^2}} + \frac{x - x_{Rx}}{\sqrt{(x - x_{Rx})^2 + z_{Rx}^2}} \sin \theta_{Tx} - \sin \theta_{Rx} \]

\[ \Delta r_{grd} = \frac{c_0}{B_r |\sin \theta_{Tx} - \sin \theta_{Rx}|} \]
Bistatic Iso-Doppler Contours

Doppler Frequencies:

\[ f_{Dop} = -\frac{1}{\lambda} \left[ \frac{\partial}{\partial t} \left( r_{Tx} + r_{Rx} \right) \right] \approx \]

\[ \approx -\frac{1}{\lambda} \left[ \frac{\vec{v}_{Tx} \cdot \vec{r}_{Tx}}{\|\vec{r}_{Tx}\|} + \frac{\vec{v}_{Rx} \cdot \vec{r}_{Rx}}{\|\vec{r}_{Rx}\|} \right] \]

\[ \Delta f_{Dop} = \frac{1}{\Delta t_{int}} \]

\[ \Delta a_{z_{grad}} = \frac{1}{\Delta t_{int} \cdot \|\text{grad}[f_{Dop}(x,y)]\|} \]

\[ f_{Dop} = \text{const.} \]
Resolution Cell: Area and Diameter

Range Resolution Vector:
\[ \tilde{a} = \frac{\text{grad}(r(x,y))}{\|\text{grad}(r(x,y))\|^2} \cdot \frac{c_0}{B_r} \]

Doppler Resolution Vector:
\[ \tilde{b} = \frac{\text{grad}(f_{Dop}(x,y))}{\|\text{grad}(f_{Dop}(x,y))\|^2} \cdot \frac{1}{t_{int}(x,y)} \]

\[ A_{\text{res}} = \frac{\|\tilde{a} \cdot \tilde{b}\|}{\sin(\varphi)} = \frac{\|\tilde{a}\|^2 \cdot \|\tilde{b}\|^2}{\|\tilde{a} \times \tilde{b}\|} \]

\[ \varnothing_{\text{res}} = \frac{\sqrt{\|\tilde{a}\|^2 + \|\tilde{b}\|^2 + 2 \cdot \|\tilde{a}\| \cdot \|\tilde{b}\| \cdot \cos(\varphi)}}{\|\sin(\varphi)\|} \]
Bistatic Radar Equation (1)

Antenna Gain: \[ G_{Tx} = \frac{4\pi A_{Tx}}{\lambda^2} \]

Power Density of Isotropic Radiator:
\[ S_{\text{sphere}}(r) = \frac{P_{Tx}}{4\pi r^2} \]

Power Density at Resolution Cell:
\[ S_{\text{rescell}} = S_{\text{sphere}}(r_{Tx}) \cdot G_{Tx} = \frac{P_{Tx} G_{Tx}}{4\pi r_{Tx}^2} \]
Bistatic Scattering Coefficient

Bistatic scattering coefficient depends on:

- Incident angle $\theta_{\text{Tx}}$
- Scattering angle $\theta_{\text{Rx}}$
- Out-of-plane angle $\phi$
- Frequency
- Polarization
- Surface / Object
Bistatic Scattering Coefficient: Example

Data from Domville (1967)
- X-Band scattering for rural land
- vertical polarization (VV)
- only in-plane scattering
- see also Willis, 1991

Forward scattering may increase the radar cross section by 10 dB and more.
Bistatic Radar Equation (2)

Power at Receiver:

\[ P_{Rx} = \frac{P_{Tx}G_{Tx}}{4\pi r_{Tx}^2} \cdot A_{res}\sigma_B^0 \cdot \frac{A_{Rx}}{4\pi r_{Rx}^2} \]

Radiated Power from Resolution Cell:

\[ P = \frac{P_{Tx}G_{Tx}}{4\pi r_{Tx}^2} \cdot A_{res}\sigma_B^0 \]
Signal to Noise Ratio

**Single Pulse SNR:**

\[
SNR_1 = \frac{P_{Tx} G_{Tx}}{4\pi r_{Tx}^2} \cdot \frac{A_{res} \sigma_B^0}{4\pi r_{Rx}^2} \cdot \frac{A_{Rx}}{k T_s B_n FL}
\]

**Independent Samples:**

\[
n_{rg} = B_r \cdot \tau_P \approx B_n \cdot \tau_P = B_n \cdot \frac{P_{avg}}{P_{Tx} \cdot PRF}
\]

\[
n_{az} = t_{int} \cdot PRF
\]

**SNR after Coherent Integration:**

\[
SNR_n = n_{rg} \cdot n_{az} \cdot SNR_1 = \frac{P_{avg} G_{Tx}}{4\pi r_{Tx}^2} \cdot \frac{A_{res} \sigma_B^0}{4\pi r_{Rx}^2} \cdot \frac{A_{Rx}}{k T_s FL} \cdot \frac{t_{int}}{t_{int}}
\]

\[\text{usually } A_{res} \sim \frac{1}{t_{int}} \rightarrow \text{SNR independent of azimuth resolution}\]
Noise Equivalent Sigma Zero (NESZ)

- The Noise Equivalent Sigma Zero (NESZ) measures the sensitivity of a given SAR.
- The NESZ corresponds to the bistatic scattering coefficient for which the SNR is equal to one.

\[
SNR = \frac{P_{\text{avg}} G_{\text{Tx}} A_{\text{Rx}} \cdot A_{\text{res}} \cdot t_{\text{int}}}{(4\pi)^2 r_{\text{Tx}}^2 r_{\text{Rx}}^2 kT_s FL} \cdot \sigma_B^0 = 1 \quad (0 \text{ dB})
\]

\[
\text{NESZ} = \sigma_B^0 (SNR = 1) = \frac{(4\pi)^2 r_{\text{Tx}}^2 r_{\text{Rx}}^2 kT_s FL}{P_{\text{avg}} G_{\text{Tx}} A_{\text{Rx}} \cdot A_{\text{res}} \cdot t_{\text{int}}}
\]

- Lower NESZ values are better.
- For spaceborne SAR, typical values of the NESZ are in the order of -20 dB.
- The SNR is given by: \( SNR = \sigma_B^0 - \text{NESZ} \quad [\text{dB}] \)
NESZ Example: Geostationary Illuminator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>3.1 cm</td>
</tr>
<tr>
<td>Max. Bandwidth</td>
<td>300 MHz</td>
</tr>
<tr>
<td>Average Transmit Power</td>
<td>1000 W</td>
</tr>
<tr>
<td>Antenna Size Tx</td>
<td>100 m²</td>
</tr>
<tr>
<td>Antenna Size Rx</td>
<td>6 m²</td>
</tr>
<tr>
<td>Noise Figure + Losses</td>
<td>5 dB</td>
</tr>
<tr>
<td>Receiver Altitude</td>
<td>400 km</td>
</tr>
<tr>
<td>Ground Resolution</td>
<td>3 m</td>
</tr>
<tr>
<td>Max. Res. Cell Diameter</td>
<td>6 m</td>
</tr>
</tbody>
</table>

Ground-looking SAR enables synergy with other instruments (e.g. optical sensors, altimeters, ....)
Digital Beamforming in Passive Receivers

Digital beamforming on receive makes effective use of the total signal energy in the large illuminated footprint:

⇒ mapping of a wide swath or multiple spots
  (in spite of extended antennas in elevation)

⇒ very long synthetic apertures
  (also with long receiver apertures)
  – high azimuth resolution
  – more independent looks
  – improved sensitivity

⇒ interference suppression

⇒ ambiguity reduction

⇒ multiple phase center MTI
Digital Beamforming on Receive

Transmitter

Receiver

Multiple beams with adaptable antenna patterns

Mixing

Analog Digital Conversion

Digital Signal Processing

Digital Beam Forming

Focusing and Higher-Level Processing

SAR Processing
Parasitic SAR with Communication Satellite

**Basic Idea:**
- illumination by a transmitter of opportunity
- sufficient SNR is provided by very long coherent integration time and moderate resolution

**Illuminator:**
- e.g. digital communication satellite
- geostationary orbit

**Receivers:**
- passive, low-cost mini- or micro-satellites
- e.g. geosynchronous orbit for long integration time (Prati et al., 1998)

**Advantages:**
- free transmitter
- receiving part can be designed using commercial DAB- or TV-SAT components
Performance of a Parasitic SAR with Communication Satellite and Geosynchronous Receiver

Power Budget Example (cf. Prati, Rocca, Giancola, Monti Guarnieri, 1998):

<table>
<thead>
<tr>
<th>Geostationary Illuminator &amp; Geosynchronous Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power Density on Ground</strong></td>
</tr>
<tr>
<td>$P_{\text{ground}} = \frac{P_{\text{Tx}} G_{\text{Tx}}}{4\pi f_{\text{Tx}}^2 B_{\text{Tx}}}$</td>
</tr>
<tr>
<td>Effective Irradiated Power ($P_{\text{Tx}} G_{\text{Tx}}$)</td>
</tr>
<tr>
<td>Transmit Range</td>
</tr>
<tr>
<td>Transmit Bandwidth</td>
</tr>
<tr>
<td><strong>Power at Receiver Satellite</strong></td>
</tr>
<tr>
<td>$P_{\text{rec}} = \frac{P_{\text{ground}} B_{\text{Rx}}}{4\pi f_{\text{Rx}}^2} \cdot A_{\text{Rx}} \cdot A_{\text{res}} \sigma_0$</td>
</tr>
<tr>
<td>Receiver Bandwidth</td>
</tr>
<tr>
<td>Receive Range</td>
</tr>
<tr>
<td>Receiver Antenna Area</td>
</tr>
<tr>
<td>Ground Resolution</td>
</tr>
<tr>
<td>Sigma</td>
</tr>
<tr>
<td><strong>SNR</strong></td>
</tr>
<tr>
<td>$SNR = \frac{P_{\text{rec}}}{kT_{\text{FB}}} = \frac{P_{\text{rec}}}{kT_{\text{F}}} \cdot t_{\text{int}}$</td>
</tr>
<tr>
<td>Receiver Noise Figure + Losses</td>
</tr>
<tr>
<td>Receiver Temperature</td>
</tr>
<tr>
<td>Integration Time</td>
</tr>
<tr>
<td><strong>SNR</strong></td>
</tr>
<tr>
<td>$SNR = \frac{P_{\text{rec}}}{kT_{\text{F}}} = \frac{P_{\text{rec}}}{kT_{\text{F}}} \cdot t_{\text{int}}$</td>
</tr>
<tr>
<td>(NESZ: -25.1 dB)</td>
</tr>
</tbody>
</table>
Combination of multiple receiver signals enables:

- Cross-track interferometry for cost efficient acquisition of high quality global DEMs
- Along-track interferometry (e.g. oceanography) & moving target indication
- Increased geometric resolution by super-resolution techniques in azimuth and range
- Retrieval of vegetation and volume parameters by polarimetric interferometry
- Real 3-D imaging of semitransparent volume scatterers by SAR tomography
- Ambiguity reduction and high resolution wide swath SAR imaging
- Improved classification (e.g. joint evaluation of multiple mono- and bistatic RCS)
- …
Basic Principle of Cross-Track Interferometry

\[ \Delta r \sim \Delta h \]

single-pass:

repeat-pass:

first pass

second pass

(after days ... months)
Single-Pass Cross-Track Interferometry with Multiple Satellites:

→ **no temporal decorrelation** (as opposed to repeat-pass interferometry)
→ **no atmospheric distortions** (as opposed to repeat-pass interferometry)
→ **large interferometric baselines** (as opposed to e.g. SRTM)
Relative Movement in Satellite Clusters

- Relative satellite movement is described in a **rotating reference frame**

- **Linearization** of the equations of motions in a circular Kepler orbit leads to Clohessy-Wiltshire (or Hill’s) Equations:

\[
\begin{align*}
\ddot{x} - 2n \dot{y} - 3n^2 x &= 0 \\
\ddot{y} + 2n \dot{x} &= 0 \\
\ddot{z} + n^2 z &= 0
\end{align*}
\]

with \( n = \sqrt{\frac{GM_\oplus}{a_{sat}^3}} \)

- Solution to Clohessy-Wiltshire Equations:

\[
\begin{align*}
x_i(t) &= A_i \sin \left( \frac{2\pi}{T_0} t + \alpha_i \right) \\
y_i(t) &= 2A_i \cos \left( \frac{2\pi}{T_0} t + \alpha_i \right) + \Delta y_i \\
z_i(t) &= B_i \sin \left( \frac{2\pi}{T_0} t + \beta_i \right)
\end{align*}
\]

with \( T_0 = 2\pi \sqrt{\frac{a_{sat}^3}{GM_\oplus}} \)

- Vertical cross-track baseline
- Along-track displacement
- Horizontal cross-track baseline
2-SAT Pendulum (Hartl 1989, Zebker, 1992)

- horizontal cross-track separation at equator by different ascending nodes
- requires along-track displacement to avoid satellite collision at orbit crossing
- insufficient baselines for polar regions

Mathematical expression:

\[ z(t) \approx A \cdot \cos\left(\frac{2\pi}{T_0} t\right) \]
HELIX satellite formation enables safe operation

- horizontal cross-track separation at equator by different ascending nodes
- vertical (radial) separation at poles by orbits with different eccentricity vectors (periodic motion of libration has to be compensated by regular manoeuvres)
Interferometric Cartwheel (Massonnet, 1998)

- all satellites share the same orbital plane
- arguments of apogee differ by $120^\circ$ for a Cartwheel with 3 satellites
- provides a stable vertical baseline for all orbit positions
- relative movement of the receiver satellites can be approximated by an ellipse
Interferometric Performance Analysis

- scattering model
- instrument parameters
- post-spacing
- orbits
- Radar Equation
- Ambiguity Analysis
- Volume Scattering
- Quantization Analysis
- Processing & Coreg.
- Instrument & Sync.
- Coherence Estimation
- Number of Looks
- Phase Error Derivation
- Height Error Derivation
- Baseline Analysis
- height errors
Coherence Estimation

\[ \gamma_{tot} = \gamma_{SNR} \cdot \gamma_{quant} \cdot \gamma_{amb} \cdot \gamma_{geo} \cdot \gamma_{az} \cdot \gamma_{vol} \cdot \gamma_{temp} \cdot \gamma_{proc} \]

- **System Noise** (radar equation)
  \[ \gamma_{amb} \approx \frac{1}{1 + RASR \cdot 1 + AASR} \]

- **Ambiguities**

- **Volume Decorrelation**

- **Processing & Coregistration Errors**
  \[ \gamma_{proc} = 0.97 \]

- **Quantization Noise** (4 bit)
  \[ \gamma_{quant} = 0.99 \]

- **Baseline Decorrelation**
  \[ \gamma_{geo} = 1 \] (range filtering)

- **Doppler Decorrelation**
  \[ \gamma_{az} = 1 \] (azimuth filtering)

- **Temporal Decorrelation**
  \[ \gamma_{temp} = 1 \]
Computation of Coherence Loss from Limited SNR

Computation of Noise Equivalent Sigma Zero:

\[ \text{NESZ} = \frac{4^4 \pi^3 r^3 v \sin(\theta_{\text{inc}}) k TB_{rg} FL}{P_{Tx} G_{Tx} G_{Rx} \lambda^3 c_0 \tau_p PRF} \]

Derivation of Single Channel SNR:

\[ \text{SNR}[dB] = \sigma_0[dB] - \text{NESZ}[dB] \]

Derivation of Coherence:

\[ \gamma_{\text{SNR}} = \frac{1}{\sqrt{1 + \text{SNR}_1^{-1}} \cdot \sqrt{1 + \text{SNR}_2^{-1}}} \]

- \( r \): slant range
- \( v \): satellite velocity
- \( \theta_{\text{inc}} \): incident angle
- \( k \): Boltzmann constant
- \( T \): system temperature (290K)
- \( B_{rg} \): chirp bandwidth
- \( F \): system noise figure
- \( L \): losses
- \( P_{Tx} \): peak transmit power
- \( G_{Tx} \): gain of transmit antenna
- \( G_{Rx} \): gain of receive antenna
- \( \lambda \): wavelength
- \( c_0 \): velocity of light
- \( \tau_p \): pulse duration
- \( \text{PRF} \): pulse repetition frequency
- \( \sigma_0 \): backscatter coefficient
Noise Equivalent Sigma Zero (Example)

NESZ and Sigma Nought (for 50% & 90%, soil & rock, VV)

TanDEM-X
(bistatic strip-map)

$\sigma^0 = 50 \%$

$\sigma^0 = 90 \%$

Incident Angle [deg]

Ground Range [km]

[dB]
Total Coherence (Example)

TanDEM-X
(bistatic strip-map)

\[ \sigma^0 = 50\% \]

\[ \sigma^0 = 90\% \]

Total Coherence (soil & rock, W)

Ground Range [km]

Incident Angle [deg]
Derivation of Interferometric Phase Errors

Phase Error PDF:

\[
p_{\phi}(\phi) = \frac{\Gamma\left(n + \frac{1}{2}\right)(1 - \gamma^2)^n \gamma \cos \phi}{2\sqrt{\pi} \Gamma(n)(1 - \gamma^2 \cos^2 \phi)^{n+\frac{1}{2}}} + \frac{(1 - n^2)^n}{2\pi} \text{ \text{F\left(n,1; \frac{1}{2}; \gamma^2 \cos^2 \phi\right)}}
\]

(cf. Lee et al., 1994)

Standard Deviation:

\[
\sigma_\phi = \sqrt{\int_{-\pi}^{\pi} \phi^2 p_\phi(\phi) \cdot d\phi}
\]

90 Percentile:

\[
\Delta \phi^{90\%} : \int_{-\Delta \phi^{90\%}}^{\Delta \phi^{90\%}} p_\phi(\phi) \cdot d\phi = 0.9
\]
Interferometric Phase Errors

Standard Deviation

90 Percentile
Independent Looks

**Ground Range Resolution**
(with range filtering)

\[
\Delta r_g = \frac{c_0}{2B_{rg} \sin(\theta_{inc} - \alpha)} \cdot \frac{B_{\perp, crit}}{B_{\perp, crit} - B_{\perp}}
\]

with critical baseline

\[
B_{\perp, crit} = \frac{2B_{rg} \lambda r \tan(\alpha)}{c_0}
\]

and

- \( c_0 \): velocity of light
- \( B_{rg} \): chirp bandwidth
- \( \lambda \): wavelength
- \( r \): slant range
- \( \theta_{inc} \): incident angle
- \( \alpha \): local slope
- \( B_{\perp} \): effective baseline

**Azimuth Resolution**
(with azimuth filtering)

\[
\Delta a_z = \frac{v_{grd}}{B_{proc} - \Delta f} \quad \text{with} \quad \Delta f = |f_{Dop,1} - f_{Dop,2}|
\]

with Doppler Centroid approximation

\[
f_{Dop,i} = \frac{1}{\lambda} \left[ \frac{v_{Tx} \cdot p_{Tx} + v_{Rx,i} \cdot p_{Rx,i}}{||p_{Tx}||} \right]
\]

and

- \( v_{grd} \): satellite velocity on ground
- \( B_{proc} \): processed bandwidth (B = 2266 Hz)
- \( \Delta f \): relative shift of Doppler Centroids
- \( \lambda \): wavelength
- \( v_{(Tx,Rx)} \): velocity vector (transmitter/receiver)
- \( p_{(Tx,Rx)} \): vector from satellite to scene center

\[
N_{\text{looks}} = \frac{\Delta x \cdot \Delta y}{\Delta r_g \cdot \Delta a_z}
\]

with independent post spacing \( \Delta x \) and \( \Delta y \)
Multilook Interferometric Phase Errors (Example)

TanDEM-X (bistatic strip-map)

$\sigma^0 = 90\%$ & $\Delta \varphi = 90\%$

$\sigma^0 = 50\%$ & $\sigma_\varphi$ (stdv.)
Derivation of Relative Height Errors

Standard deviation of height errors:

$$\sigma_h = \frac{\lambda r \sin(\theta_i)}{2\pi B_\perp} \cdot \sigma_\varphi \quad \text{(flat terrain)}$$

HRTI-3 requires point-to-point height errors at 90% confidence levels:

$$\Delta h_{\text{HRTI}} \approx \frac{\lambda r \sin(\theta_i)}{2\pi B_\perp} \cdot \varphi_{90\%} \cdot \sqrt{2}$$
Relative Height Errors (Example)

Relative Height Accuracy (90% Point-to-Point and Stdv)

TanDEM-X:
Bistatic
Strip map
B = 500 m
$\Delta x = 12$ m

90% point-to-point errors

single point standard deviation
Relative Height Errors (Example)

TanDEM-X:
Bistatic
Strip map
B = 1000 m
Δx = 12 m
Relative Height Errors (Example)

TanDEM-X enables large baselines which allow for ultra high resolution DEMs with height accuracies in the sub-meter range, but …

\[ \Delta \varphi = 2\pi \]

(height of ambiguity)

Compromise on Accuracy for Global DEM

- use reduced baselines
- additional acquisitions for difficult terrain

→ acquisition scenario for global DEM

Multi-Baseline Data Acquisitions

large baseline for high sensitivity

small baseline for resolving ambiguities
Multi-Baseline Data Acquisitions (Example)

Trinodal Pendulum

→ multiple baselines with fixed baseline ratio in one pass

Performance Example (TerraSAR-L)

Short Baseline (~1km)  
(height of ambiguity: 100 m)

Large Baseline (~10km)  
(height of ambiguity: 10 m)

~ 0.5 m  
(@ 12m x 12m)

Large Baseline → Excellent Height Accuracy  
Small Baseline → Easy Phase Unwrapping
Polarimetric SAR Interferometry

Inversion of Coherent Scattering Model
(e.g. Random Volume + Ground)

\[ h_V, \sigma, \phi_0, m_{1-3} \]

(from K. Papathanassiou & S. Cloude, 2001)
PollInSAR L-Band Performance Example

**Monostatic Repeat Pass System**

\( \gamma_{\text{temp}} = 0.5 \)

- Poor Inversion Accuracy

**Multistatic Single Pass System**

\( \gamma_{\text{temp}} = 1.0 \), but smaller Rx antennas

- Good Inversion Accuracy

\[ \text{pdf}(\mu_{\text{min}}) \]

\[ \text{pdf}(\mu_{\text{max}}) \]

Separation of phase center pdfs

- Polarisations
PolInSAR L-Band Performance Example

<table>
<thead>
<tr>
<th></th>
<th>Small Baseline (500 m)</th>
<th>Large Baseline (2 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H = 10 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="#" alt="Diagram" /></td>
<td><img src="#" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>poor phase center separation</td>
<td>good performance</td>
</tr>
<tr>
<td>H = 40 m</td>
<td><img src="#" alt="Diagram" /></td>
<td><img src="#" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
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<td></td>
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</tbody>
</table>

*Multibaseline data acquisitions optimize performance*

*Further information about vertical structure by joint multibaseline data evaluation:*

→ fusion of PolInSAR techniques with tomography
**Tomography with Micro-Satellite Array**

**Basic Idea:** Cluster of receiver satellites to form an additional aperture in elevation

- allows real three-dimensional imaging, i.e. a geometric resolution in height direction
- avoids the intrinsic height ambiguity in conventional SAR imaging
- accurate modeling and retrieval of vegetation parameters
- not affected by layover or foreshortening effects
- cross-track distance between the satellites defines the height ambiguity value for tomographic processing
- total tomographic baseline defines the height resolution

*Tomographic results* (from A. Reigber & A. Moreira, 1999)

*Airborne Polarimetric SAR Tomography*

Upper Image: Polarimetric color composite (L-band) of a tomographic slice in the height/azimuth-direction

Lower Image: Schematic view of the imaged area

(from A. Reigber & A. Moreira, 1999)
Tomography with Semi-Active Micro-Satellite Array

Fundamental relations:
- height resolution:
  \[ \Delta h \approx \frac{\lambda \cdot r_0 \cdot \sin \theta_{\text{inc}}}{L \cdot \cos \theta_{\text{look}}} \]
- required sampling:
  \[ d < \frac{\lambda \cdot r_0 \cdot \sin \theta_{\text{inc}}}{h_v \cdot \cos \theta_{\text{look}}} \]

Example:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.23 m</td>
</tr>
<tr>
<td>( r_0 )</td>
<td>700 km</td>
</tr>
<tr>
<td>( \theta_{\text{inc}} )</td>
<td>30°</td>
</tr>
<tr>
<td>( \theta_{\text{look}} )</td>
<td>30°</td>
</tr>
<tr>
<td>( L )</td>
<td>20 km</td>
</tr>
<tr>
<td>( d )</td>
<td>2 km</td>
</tr>
</tbody>
</table>

\[ \Delta h = 3 \text{ m} \]
\[ h_v < 30 \text{ m} \]
Multistatic SAR Imaging

- improved detection, segmentation, and classification in SAR images
- separation of different scattering mechanisms (e.g. coherent from non-coherent components)
- radargrammetry and multi-shadow evaluations
- speckle reduction without resolution degradation
- multibaseline coherence analyses
- acquisition of bi- and multistatic Doppler spectra (e.g. multiple ocean wave spectra)
- downward looking receivers (fusion with other sensors)

Extended Observation Space

- Scattering angles $\theta_{Rx}$
- Out-of-plane angles $\phi$
- Doppler frequencies
- Polarimetry (e.g. $\sigma_{HV} \neq \sigma_{VH}$)
Multistatic Scattering Coefficients: Example

Bistatic Airborne Radar Experiment: (February 2003, Nimes, France)

Color composite of three bistatic images:

E-SAR (DLR)  Ramses (ONERA)
Along-Track Interferometry

highly accurate measurements of the radial displacement between two radar observations separated by a short time lag.
SAR Imaging with Four Phase Centres

**short baseline**
(\(\Delta t \approx 0.2\) ms)

**long baseline**
(\(\Delta t \approx 10-200\) ms)

sensitive to fast movements

sensitive to slow movements

SAR imaging with four phase centres enables highly accurate velocity estimates for slow and fast object movements
Applications of Along-Track Interferometry

Ocean Currents

Coastal Surveillance

Ice Drift & Ice Flow

Traffic Monitoring
Multi-Baseline ATI and GMTI

$S_1(x,t)$, $S_2(x,t)$, $S_3(x,t)$, $S_N(x,t)$

Clutter

$\cos \varphi$

$f_D$

fast, slow

steep notch enables good clutter suppression

EUSAR 2006 Tutorial – Slide 57

Microwaves and Radar Institute
gerhard.krieger@dlr.de
Ambiguity Reduction and Wide Swath Imaging

- single transmitter illuminates **wide image swath**
- multiple receivers record scattered signal
- N receivers allow **reduction of PRF** by a factor of 1/N **without** raising **azimuth ambiguities**:
  - increase of swath width by factor N at **full azimuth resolution** (as opposed to ScanSAR)
  - variability in optimum receiver displacement:
    \[ x_i - x_l \approx \frac{2 \cdot v}{PRF} \left( \frac{i-1}{N} + k_i \right) \quad i \in \{1,2,...,N-1\}, k_i \in \mathbb{Z} \]
  - reconstruction possible for other displacements
  - performance can be optimized by PRF adaptation
  - requires stable oscillators or RF synchronization and accurate estimation of relative displacement
- major application: **high resolution** imaging of a **wide image swath** with **small antennas** (e.g. distributed L-Band SAR with multiple microsatellites)
Multi-Channel Model for Sparse Array

Multiple Aperture Array:

System Model:

PRF

Bistatic Azimuth Impulse Response:

\[ h_i(t; \Delta x_i) = A_{Tx}(vt) \cdot A_{Rx,i}(vt - \Delta x_i) \cdot \exp\left[-j \frac{2\pi}{\lambda} \left(\sqrt{r_0^2 + (vt)^2} + \sqrt{r_0^2 + (vt - \Delta x_i)^2}\right)\right] \]
Coherent Reconstruction

\[
A(f) = \begin{bmatrix}
H_1(f) & H_2(f) & H_3(f) \\
H_1(f + B/3) & H_2(f + B/3) & H_3(f + B/3) \\
H_1(f + 2B/3) & H_2(f + 2B/3) & H_3(f + 2B/3)
\end{bmatrix}
\]

\[
A^{-1}(f) = \begin{bmatrix}
P_{11}(f) & P_{12}(f + B/3) & P_{13}(f + 2B/3) \\
P_{21}(f) & P_{22}(f + B/3) & P_{23}(f + 2B/3) \\
P_{31}(f) & P_{32}(f + B/3) & P_{33}(f + 2B/3)
\end{bmatrix}
\]

**Model with Quadratic Phase Approximation**

<table>
<thead>
<tr>
<th>Monostatic SAR Response ($\Delta x=0$)</th>
<th>Joint Antenna Footprints</th>
<th>Phase Shift</th>
<th>Time Delay</th>
<th>PRF</th>
<th>Reconstruction</th>
<th>Monostatic SAR Processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(t)$</td>
<td>$h_m(t, r_0)$</td>
<td>$A_1(f)$</td>
<td>$\varphi = -\frac{\pi \Delta x_1^2}{2 \lambda r_0}$</td>
<td>$\Delta t = \frac{\Delta x_1}{2v}$</td>
<td>$P_i(f)$</td>
<td>$h_m^*(t, r_0)$</td>
</tr>
<tr>
<td>$h_m(t, r_0) = -j\frac{2\pi}{\lambda} \left[ 2r_0 + \frac{(vt)^2}{r_0} \right]$</td>
<td></td>
<td>$A_2(f)$</td>
<td>$\varphi = -\frac{\pi \Delta x_2^2}{2 \lambda r_0}$</td>
<td>$\Delta t = \frac{\Delta x_2}{2v}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_3(f)$</td>
<td>$\varphi = -\frac{\pi \Delta x_3^2}{2 \lambda r_0}$</td>
<td>$\Delta t = \frac{\Delta x_3}{2v}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Bistatic Azimuth Impulse Response:**

$$h_i(t; \Delta x_i) \cong A_{Tx}(vt) \cdot A_{Rx,i}(vt - \Delta x_i) \cdot \exp \left[ -j \frac{4\pi}{\lambda} r_0 \left( 1 + \frac{\Delta x_i^2}{8r_0^2} \right) \right] \cdot \exp \left[ -j \frac{2\pi}{\lambda} \left( \frac{vt - \Delta x_i}{2} \right)^2 \right]$$
Sparse Array Reconstruction: Raw Data

Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRF</td>
<td>1167 Hz</td>
</tr>
<tr>
<td>Antenna Length (Tx)</td>
<td>4.8 m</td>
</tr>
<tr>
<td>Antenna Length (Rx)</td>
<td>4.8 m</td>
</tr>
<tr>
<td>Displacement (Rx1)</td>
<td>300.0 m</td>
</tr>
<tr>
<td>Displacement (Rx2)</td>
<td>614.0 m</td>
</tr>
<tr>
<td>Displacement (Rx3)</td>
<td>934.1 m</td>
</tr>
<tr>
<td>Processed Bandwidth</td>
<td>2600 Hz</td>
</tr>
<tr>
<td>Wavelength (X-Band)</td>
<td>3.1 cm</td>
</tr>
<tr>
<td>Antenna Pointing</td>
<td>±0.0°</td>
</tr>
</tbody>
</table>

PRF = $\frac{B}{3}$

$H_1(f) \rightarrow H_2(f) \rightarrow H_3(f)$

Ambiguities

$P_1(f) \rightarrow P_2(f) \rightarrow P_3(f)$

No ambiguities

Signal Reconstruction (Suppressed Ambiguities)
Sparse Array Reconstruction: Focused Data

$$PRF = \frac{B}{3}$$

AMBIGUITIES

Simulation Parameters

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<td>±0.0°</td>
</tr>
</tbody>
</table>

SIGNAL RECONSTRUCTION (SUPPRESESSED AMBIGUITIES)
Sparse Array Reconstruction: Nonuniform Distance + Noise

Simulation Parameters

- **PRF**: 1167 Hz
- **Antenna Length (Tx)**: 4.8 m
- **Antenna Length (Rx)**: 4.8 m
- **Displacement (Rx1)**: 300.0 m ± 1.7 m
- **Displacement (Rx2)**: 617.0 m ± 0.4 m
- **Displacement (Rx3)**: 934.1 m ± 1.3 m
- **Processed Bandwidth**: 2600 Hz
- **Wavelength (X-Band)**: 3.1 cm
- **Antenna Pointing**: 0.01°
- **SNR**: -5 dB

**Raw Data** (Point Target)

**Processed Signal** (Matched Filter)

**Sparse Array Processing**

**Signal Reconstruction & SAR Processing**

 ambiguities

 ambiguity ~ -2 dB !

 ambiguity < -25 dB !
Superresolution with Multistatic Satellite Arrays

Increased geometric resolution of SAR images by:

- along-track displacement of receiving satellites:
  - different Doppler centroids
  - super-resolution in azimuth by coherent combination of shifted Doppler spectra

- across-track displacement of receiving satellites:
  - different incident angles
  - super-resolution in range by coherent combination of images with different ground range spectra
Challenges in Bistatic and Multistatic SAR Imaging

**Orbit Control & Relative Position Sensing**

**Bi- and Multistatic SAR Processing**

**Bistatic Calibration & Antenna Co-Pointing**

**Time and Phase Synchronisation**
System Model for Bistatic Radar

\[ s(t) = \exp\{jm[-2\pi f_T \tau + 2\pi (f_T - f_R) t + \phi_T(t - \tau) - \phi_R(t)]\} \]

- **Azimuth Modulation**
- **Frequency Offset** \( \Delta f \)
- **Phase Noise** \( \phi(t) \)
Degradation of Azimuth Impulse Response

**Constant Frequency Offset**

- **Frequency Offset:** $\Delta f$
- **Phase Error:** $\varphi(t)$
- **Impulse Response:** Linear Displacement

**Linear Frequency Drift**

- **Phase Error:** $\varphi(t)$
- **Impulse Response:** Mainlobe Dispersion

**Higher-Order Phase Error**

- **Phase Error:** $\varphi(t)$
- **Impulse Response:** Increase of Sidelobes
Model of Oscillator Phase Errors

- $\varphi(t)$ is modelled by a stochastic process
Model of Oscillator Phase Errors

• \( \phi(t) \) is modelled by a stochastic process with acf

\[
R_{\phi}(\tau) = \langle \phi(t) \cdot \phi(t+\tau) \rangle
\]
Model of Oscillator Phase Errors

- \( \varphi(t) \) is modelled by a stochastic process with acf

\[
R_{\varphi}(\tau) = \langle \varphi(t) \cdot \varphi(t+\tau) \rangle
\]

- The phase spectrum

\[
S_\varphi(f) \leftrightarrow R_{\varphi}(\tau)
\]

describes phase fluctuations per Hertz bandwidth at Fourier frequency \( f \)
• $\varphi(t)$ is modelled by a stochastic process with acf

\[ R_\varphi(\tau) = \langle \varphi(t) \cdot \varphi(t+\tau) \rangle \]

• The phase spectrum

\[ S_\varphi(f) \leftrightarrow R_\varphi(\tau) \]

describes phase fluctuations per Hertz bandwidth at Fourier frequency $f$

• $S_\varphi(f)$ is often approximated by:

\[ S_\varphi(f) = a \cdot f^{-4} + b \cdot f^{-3} + c \cdot f^{-2} + d \cdot f^{-1} + e \cdot f^0 \]

- random walk frequency noise
- frequency flicker noise
- white frequency noise
- flicker phase noise
- white phase noise

$S(f)$ [dB rad2/Hz]

- random walk frequency noise $\sim f^{-4}$
- flicker frequency noise $\sim f^{-3}$
- white frequency noise $\sim f^{-2}$
- flicker phase noise $\sim f^{-1}$
- white phase noise $\sim f^0$

- $f_0 = 10$ MHz
- $\sigma_a(\tau=1s) \approx 1 \cdot 10^{-11}$
- $\sigma_a(\tau=10s) \approx 2 \cdot 10^{-11}$
- $\sigma_a(\tau=100s) \approx 6 \cdot 10^{-11}$
- $L(f=1Hz) = -90$ dB/c
- $L(f=10Hz) = -120$ dB/c
Typical Realisations from $R_\varphi(\tau)$ \ $\bullet$ \ $S_\varphi(f)$

Realisations with $\varphi(t=0) = 0^\circ$

Constant phase ramp removed
Bistatic SAR Response: Simulation Example

- *azimuth displacement*
- *mainlobe dispersion*
- *increased sidelobes*

- $\lambda = 0.24 \text{ m}$
- $T_{az} = 4 \text{ sec}$
- $r_0 = 800 \text{ km}$
- $v = 7 \text{ km/s}$
Bistatic SAR Response: Simulation Example

- **azimuth displacement**
- **phase errors** (interferometry)
- **increased sidelobes**

- \( \lambda = 0.24 \text{ m} \)
- \( T_{az} = 4 \text{ sec} \)
- \( r_0 = 800 \text{ km} \)
- \( v = 7 \text{ km/s} \)
Integrated Side-Lobe Ratio (ISLR)

**Definition:**

\[
\text{ISLR} = \frac{\text{Signal Energy in Mainlobe}}{\text{Integrated Signal Energy in Sidelobes}}
\]

**Derivation from Phase Spectrum \( S_\phi(f) \):**

\[
\text{ISLR} \approx 2 \left( \frac{f_0}{f_{osc}} \right)^2 \cdot \int_{\frac{1}{T_a}}^{\infty} S_\phi(f) \cdot df
\]

(for \( \sigma_{\phi, HF} \ll 1 \text{ rad} \))
Mainlobe Dispersion

- mainlobe dispersion is mainly due to quadratic phase errors
- typical requirement: $\varphi_Q < 45^\circ$ (~ 3 % dispersion)
- quadratic phase errors may be derived from second derivative of phase

**Quadratic Phase Errors:**

$$
\sigma_Q^2 = 2 \cdot \left( \frac{f_0}{f_{osc}} \right)^2 \cdot \frac{\pi T_a}{4} \cdot T_a \cdot \int_0^1 f^4 \cdot S_\phi(f) \cdot df
$$

**Mainlobe Dispersion:**

$$
\frac{\Delta az_{bistat}}{\Delta az_{ideal}} \approx \sqrt{1 + \left( \frac{\sigma_Q}{\pi} \right)^2} \quad \text{for} \quad \varphi_Q < \pi
$$
Azimuth Displacement

- small frequency offset $\rightarrow$ large azimuth displacement
- possible solutions:
  - use of ground control points
  - simultaneous mono- and bistatic data acquisition
  - relative phase referencing between Tx and Rx

\[ \Delta x = \frac{c_0 r_0}{2 v_0} \cdot \frac{\Delta f}{f_{osc}} \]

Residual Azimuth Displacement:

\[ \sigma^2_{\Delta x_{res}} = \left( \frac{c_0 r_0}{v_0} \right)^2 \cdot \int_0^\infty f^2 S_{\phi}(f) \left[ \frac{\sin(\pi T_a f)}{\pi T_a f} \right]^2 \left[ 1 - \frac{\sin(2\pi ft)}{2 \sin(\pi ft)} \right]^2 df \]

- X-Band $(T_a=1s, r_0=700km)$
- L-Band $(T_a=4s, r_0=700km)$
Pulse Synchronisation

*LO frequency offset will lead to slightly different PRFs:*

→ range walk

→ *shift of the receiving window*
Pulse Synchronisation

**Solutions to synchronization:**

- Pulse synchronization with link to common reference (e.g. GPS)
- Automatic detection of swath signal (e.g. by power analysis of received signal)
- Direct data link between transmitter and receiver
- Continuous recording

Example for synchronization with GPS:

- GPS 1 (Tx) 1 pps → STALO 1 (Rx) → PRF counter 1 → PRF 1 (Tx)
- GPS 2 (Rx) 1 pps → STALO 2 (Rx) → PRF counter 2 → PRF 2 (Rx)
- reset PRF counters (e.g. each second)
Solutions for Phase Referencing

Continuous USO Synchronisation

Mutual Exchange of Radar Pulses

cf. M. Eineder, IGARSS 2003

Ground Control Points + 'Hyper-Stable' Oscillators

Alternating Transmit Mode

\[ \sigma_a < 10^{-12} \]

\[ \tau = 1..100\sigma \]
Derivation of Required Sync Frequency

Relative Phase

\[ \varphi(t) \]

\[ t \]

\[ \Phi(f) \]

\[ f \]
Derivation of Required Sync Frequency

Relative Phase

Sampled Reference Signal

\[ \varphi(t) \]

\[ \Phi(f) \]

\[ \frac{1}{T_c} \quad \frac{1}{T_c} \]
Derivation of Required Sync Frequency

Relative Phase

\[ \varphi(t) \]

Sampled and Interpolated Reference Signal

\[ T_c \]

\[ \frac{1}{T_c} \]

\[ \frac{1}{T_c} \]

\[ \Phi(f) \]

Derivation of Required Sync Frequency

Relative Phase

\[ \Phi(t) \]

Sampled and Interpolated Reference Signal

Residual Error

\[ \Phi(f) \sim \frac{1}{T_c} \sim \frac{1}{T_c} \]

\[ \Phi(f) \]

\[ \frac{1}{T_c} \]

\[ \frac{1}{T_c} \]
Derivation of Required Sync Frequency

Relative Phase

Sampled and Interpolated Reference Signal

Residual Error

\[ \sigma^2_\varphi = 2 \cdot \left( \frac{f_0}{f_{osc}} \right)^2 \cdot \left[ \int_{1/(2T_c)}^{\infty} S_\varphi(f) \cdot \left( \frac{\sin(\pi T_a f)}{\pi T_a f} \right)^2 \cdot df \right] + \sum_{i=1}^{\infty} \int_{-1/(2T_c)}^{1/(2T_c)} S_\varphi(f + \frac{i}{T_c}) \cdot \left( \frac{\sin(\pi T_a f)}{\pi T_a f} \right)^2 \cdot df \]
Predicted Phase Error

**Normal USO**

Interferometric Phase Error

\[ \sigma^2 = 2 \cdot \left( \frac{f_0}{f_{osc}} \right)^2 \cdot \left[ \int_{1/(2T_c)}^{\infty} S_\phi(f) \cdot \left( \frac{\sin(\pi T_a f)}{\pi T_a f} \right)^2 \cdot df \right] + \sum_{i=1}^{\infty} \int_{-1/(2T_c)}^{1/(2T_c)} S_\phi(f + \frac{i}{T_c}) \cdot \left( \frac{\sin(\pi T_a f)}{\pi T_a f} \right)^2 \cdot df \]

\( f_0 = 10 \text{ MHz} \)
\( \sigma_a(t=1s) \approx 1 \times 10^{-11} \)
\( \sigma_a(t=10s) \approx 2 \times 10^{-11} \)
\( \sigma_a(t=100s) \approx 6 \times 10^{-11} \)

**Hyper-Stable Oscillator**

Interferometric Phase Error

\( f_0 = 10 \text{ MHz} \)
\( \sigma_a(t=1s) \approx 5 \times 10^{-14} \)
\( \sigma_a(t=10s) \approx 1 \times 10^{-13} \)
\( \sigma_a(t=100s) \approx 4 \times 10^{-13} \)

*X-Band (T_a = 1s)*

*L-Band (T_a = 4s)*
Predicted Phase Error

No Phase Referencing  \( T_C = 10 \text{ sec} \)  \( T_C = 1 \text{ sec} \)

Interferometric Phase

\[
\text{Interferometric Phase} \quad \text{interferometric phase} = 17.78 \text{deg} \\
\text{Interferometric Phase} \quad \text{interferometric phase} = 0.33 \text{deg}
\]
Close Formation Flight: Collision Avoidance

**Autonomous Control**
- onboard computation of relative orbit maneuvers to ensure safe distance
- requires real-time relative position estimation and communication link between the satellites
- has to take into account fuel constraints (DART !)

**Safe Orbit Design**
- avoid orbit crossings, e.g. by slight eccentricity vector offset (HELIX)
- arbitrary satellite shifts along their orbits
- very short along-track baselines for a given latitude range
Baseline Control and Baseline Estimation

- **All satellites are exposed to almost identical orbit perturbations**
  - precise baseline control with low fuel consumption (e.g. to ensure fixed baseline ratios)
  - very small differential acceleration (< 100 nm/s² → baseline error of less than 1 mm for 1000 km swath)

- **Precise baseline estimation by**
  - double-difference GPS/Galileo carrier-phase measurements
  - accurate orbit propagation model
  - several studies predict a 3-D accuracy in the order of 1-2 mm
References: General

References: Frequent Monitoring, Parasitic Radar

References: Bistatic Scattering

References: SAR Interferometry

General Reviews about Interferometry:

Interferometry with Multistatic Satellite Configurations:
### References: Multi-Baseline Interferometry & Tomography

References: Polarimetric SAR Interferometry

General Papers about Polarimetric SAR Interferometry:


Polarimetric SAR Interferometry with Multistatic Satellite Array:


References: Along-Track Interferometry and GMTI

- Romeiser R et al., Study on concepts for radar interferometry from satellites for ocean (and land) applications, http://www.ifm.uni-hamburg.de/~wwwrs/project_koriolis.htm.
- Carande R, Dual baseline and frequency along-track interferometry, Proc. IGARSS 1992, Houston, Texas, USA.
References: High Resolution and Wide Swath SAR Imaging


References: Bistatic SAR Processing

References: Bistatic SAR Experiments & Synchronisation

Thank you for your attention