Weights Estimation of Phased Arrays moving in a Test Field

Thomas Molkenthin, Jens Fischer, David Hounam, Marco Schwerdt
German Aerospace Center (DLR), Microwaves and Radar Institute, D-82234 Wessling, Germany

1 Abstract

Due to the high performance and degree of flexibility of microwave instruments using phased array antennas, a comprehensive characterisation of the antenna is essential and a major challenge [1]. New concepts are needed to keep the costs and effort acceptable. A new characterization technique is proposed, the so-called Weight-Estimation Method (WEM), which permits the exitation coefficients, or weights of the array elements to be estimated from a limited number of far-field measurements. With these weights, i.e the gains and phase settings applied to the elements of the phased array, the complete antenna pattern in the range of +/- 90 degree about the boresight can be derived. The concept is applicable for any phased array system sensing electromagnetic signals. The paper describes the measurement concept, the estimation method and presents simulated results for measurements in a compact range and with the antenna in motion. The method was developed to characterize phased array antennas of synthetic aperture radars (SAR) and promises to simplify the on-board calibration circuitry of future instruments.

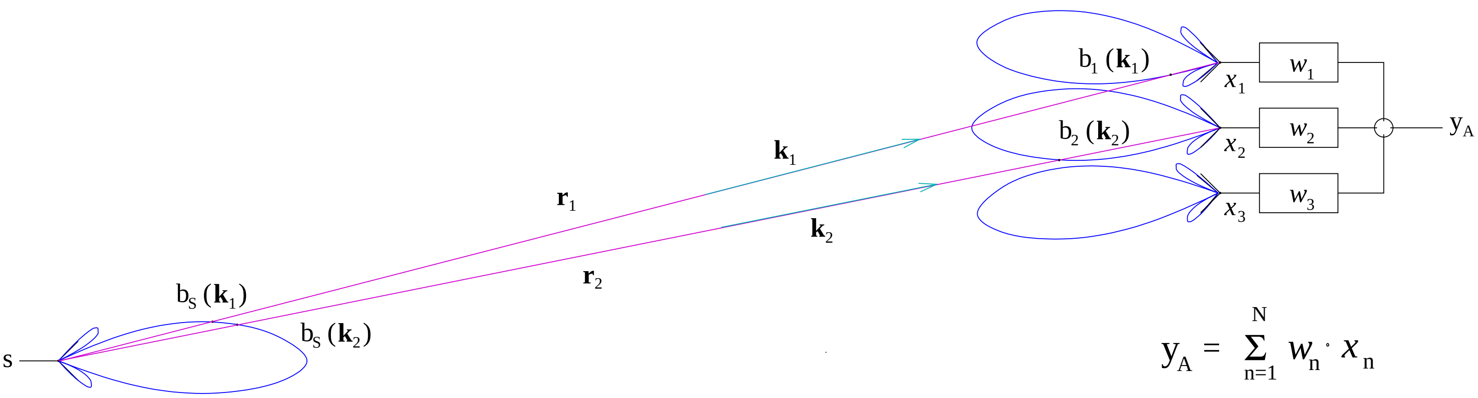


Figure 1: Diagram of the system used to describe the Weight Estimation Method with a sounding antenna S (bottom left) and the array to be characterized A (top right)

2 Introduction

Phased arrays are antenna systems composed of radiating array elements and a network that distributes the signals and adjusts the element gain and phase. For an active phased array, each antenna element is composed of a radiator and a transmit/receive module (TRM) which contains the gain and phase adjustment. Active phased arrays often allow corrections to be made to the weights by commanding different gain and phase settings. Before being put into service, the complete phased array has to be characterized and its initial state defined. Because the components can alter with time, the antenna may need to be characterized regularly in the final configuration. If the weights actually applied by the TRMs are different from the commanded ones, the shape of the phased array antenna pattern will differ from the desired one. Thus, it is important to know the actual weights in order to induce corrective actions. Knowledge of the weights enables the complete pattern to be determined.

Figure 1 shows the WEM concept with the unknown array at top right and the sounding antenna bottom left. If the other components keep their initial properties, the weight w_n of every array element can be estimated provided the complex signals x_i at the radiators are known and the antenna input/output y is available as a complex number. If it is possible to identify N (N = number of array elements) linearly independant sets of signals $\mathbf{x}_j$, the system of linear equations can be inverted and the element weights estimated.

3 The WEM concept

The concept characterizes a phased array of interest A with the help of a single calibrated sounding antenna S whose properties are well known. At any time t, each antenna has a position, velocity and attitude in space. With the WEM method, a series of discrete far-field measurements is performed over an angular range sufficient to estimate the individual array weights. As will be seen, these measurements include ones taken in sidelobe regions. In the following the necessary conditions for the WEM estimation are derived. The concept can be used when the phased array is receiving or transmitting.

Hence, the signals x_n at the radiators of A must be estimated from the measured data and the knowledge of the antenna patterns of the phased array elements $b_n(\mathbf{k})$, as well as the sounding antenna $b_S(\mathbf{k})$. The antenna patterns of the array elements must include mutual coupling effects. When a collection of N 'linear independant' measurements is available, the weights of A can be estimated. Applying the antenna pattern equation of a phased array with the weights estimated, the antenna pattern of the phased array can be determined:

$$b_A(\mathbf{k}) = \sum_{n=1}^{N} w_n \cdot b_n(\mathbf{k}) \cdot \exp\left[-j\,\mathbf{k}^T \cdot \mathbf{r}_{nC}\right] \tag{1}$$

We will first consider the case where the antenna of interest A is receiving a plane wave radiated by the sounding antenna S whose properties are known. Provided that the far-field conditions are fulfilled and the propagation path is homogeneous, the transmitted wave can be regarded as a monochromatic, purely polarized plane wave. The antenna S is fed with a signal s radiating a field with polarisation $\rho_S \in \mathbb{R}^3$ and modulated according to the antenna pattern $b_S(\mathbf{k})\in \mathbb{C}$. The wavenumber $\mathbf{k} \in \mathbb{R}^\mathbf{3}$ is given in a common coordinate system. In the far field of S, the signal at the position in space $\mathbf{r}_C$ ($\mathbf{r}_C$ is the reference coordinate of the center of the antennas), is given by

$$s_C = \left[\frac{\exp\left(-j\frac{2\pi}{\lambda}r_{CS}\right)}{4\pi r_{CS}} \cdot b_S(\mathbf{k}_{CS})\right] \cdot s$$

where $r_{CS} = \|\mathbf{r}_C - \mathbf{r}_S\|$, $\mathbf{r}_S$ being the position of the antenna S. $\mathbf{k}_{CS}$ is the wavenumber of length $\frac{2\pi}{\lambda}$ and direction $\mathbf{r}_C - \mathbf{r}_S$. The signal s_C has to be considered as the one received by an isotropic antenna with the same polarisation as S. If a receiving antenna R with pattern $b_R(\mathbf{k})$ and polarisation ρ_R is placed at $\mathbf{r}_C$, the received signal x will be

$$x = b_R(\mathbf{k}_{RS}) \cdot p_{RS} \cdot f_{CS} \cdot b_S(\mathbf{k}_{RS}) \cdot s$$

where the scalar f_{CS} is defined as

$$f_{CS}(r_{CS}, \lambda) = \frac{\exp\left(-j\frac{2\pi}{\lambda}r_{CS}\right)}{4\pi r_{CS}}$$

and the scalar p_{RS} takes into account the polarisation mismatch losses.

$$p_{RS}(\rho_R, \rho_S, \mathbf{z}_S) = \sqrt{\|\rho_R\|^2 - \left(\rho_R^T \cdot \frac{\rho_S \times \mathbf{z}_S}{\|\rho_S \times \mathbf{z}_S\|}\right)}$$

p_{RS} is the length of the projection of ρ_R on the plane spanned by ρ_S and $\mathbf{z}_S$, $\mathbf{z}_S$ being the normalized boresight vector of antenna S. $\times$ is the cross product. In the next step, antenna R is replaced by a phased array A. The signals x_n at the array radiators are defined by

$$x_n = b_n(\mathbf{k}_n) \cdot p_n \cdot \frac{\exp\left(-j\frac{2\pi}{\lambda}r_{nS}\right)}{4\pi r_{nS}} \cdot b_S(\mathbf{k}_n) \cdot s$$

$\mathbf{k}_i$ is the wavenumber emanating from the source S and pointing to $\mathbf{r}_i$, the position of the ith element.

Using the Hadamard Product $\odot$ which is defined as the entry wise multiplication of the entries of two matrices $\mathbf{A}_{m\times n}$ and $\mathbf{B}_{m\times n}$

$$\mathbf{A} \odot \mathbf{B} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} := \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

the vector $\mathbf{x}$ of all signals x_n is

$$\mathbf{x} = \mathbf{b}_A \odot \mathbf{p} \odot \mathbf{f} \odot [\mathbf{b}_S \cdot s]$$

where the vector $\mathbf{f}$ is the field-kernel with the components

$$f_n = \frac{\exp\left(-j\frac{2\pi}{\lambda}r_{nS}\right)}{4\pi r_{nS}}$$

$\mathbf{b}_A(\mathbf{k}_n)$ and $\mathbf{b}_S(\mathbf{k}_n)$ representing the antenna responses and $\mathbf{p}$ the polarisation match vector with:

$$p_n = \sqrt{\|\rho_n\|^2 - \left(\rho_n^T \cdot \frac{\rho_S \times \mathbf{z}_S}{\|\rho_S \times \mathbf{z}_S\|}\right)} \tag{2}$$

In the normal case that the extents of the array are very small compared to the distance between the center $\mathbf{r}_C$ of the receiving array A and the sounding antenna S, the waves impinging on the array can be abstracted as plane waves. In case of narrow-band signals, each element receives the same signal, but retarded by a phase delay ϕ_n which depends on $\mathbf{k}$ and the element position $\mathbf{r}_{nC} = \mathbf{r}_n - \mathbf{r}_C$ relative to the reference point $\mathbf{r}_C$ on the array center. The element phase delays can be put into an array manifold vector $\mathbf{v}_A$ [2] , where each element is defined by

$$v_{An} = exp\left(-j\,\mathbf{k}_n^T \cdot \mathbf{r}_{nC}\right)$$

Finally, the vector of the array element contributions is given by:

$$\mathbf{x} = \mathbf{b}_A \odot \mathbf{p} \odot \mathbf{v}_A \odot (\mathbf{b}_S \cdot f_{CS} \cdot s)$$

where the field-kernel has been reformulated

$$\mathbf{f} = f_{CS} \cdot \mathbf{v}_A$$

Athmospheric losses are neglected here, but could be taken into account in $\mathbf{p}$.

The signal sensed by the phased array is the weighted sum of the signals x_i sensed by the individual elements:

$$y_A = \mathbf{w}_A^T \cdot \mathbf{x} \tag{3}$$

The actual weights $\mathbf{w}_A$ can be estimated if it is possible to identify N measurements y_j where $\mathbf{x}_j$ are linearly independant.

$$\mathbf{y}_A^T = \mathbf{w}_A^T \cdot \mathbf{X}\ ;\ \mathbf{X} = \left[\mathbf{x}_1 \vdots \mathbf{x}_2 \vdots \ldots \vdots \mathbf{x}_N\right] \tag{4}$$

In this case, we have a system of linear equations which can be inverted:

$$\text{if } \det(\mathbf{X}) \neq 0 \text{ then } \mathbf{w}_A^T = \mathbf{y}^T \cdot \mathbf{X}^{-1} \tag{5}$$

where $\mathbf{X}^{-1}$ is the inverse of

$$\mathbf{X} = \left[\mathbf{B}_A \odot \mathbf{P} \odot \mathbf{V}_A \odot \mathbf{B}_S \odot \left(\mathbf{1}_{n\times 1} \cdot \mathbf{s}^T\right)\right] \cdot f_{CS}$$

If the array of interest is emitting signals, the above constellation can be reversed. In this case, the signals at the radiators x_i must be estimated using the signals y_S measured at the sounding antenna output. Since the reciprocity principle applies,

$$\left[\mathbf{B}_S \odot \mathbf{P} \odot \mathbf{B}_A \odot \mathbf{V}_A \odot \mathbf{X}\right]^T \cdot \mathbf{1}_{n\times 1} = \frac{\mathbf{y}_S}{f_{CS}}$$

In other terms,

$$\sum_i \left(b_{Snj} \cdot p_{nj} \cdot b_{Anj} \cdot v_{Anj}\right) \cdot x_{nj} = \frac{y_j}{f_{CS}}$$

To obtain the matrix $\mathbf{X}$, a linear equation system must be solved. This done, the weights can be estimated.

4 Optimal Locations for Measurements

We consider the unknown antenna being measured at defined points in space. We have seen that a good estimate of the weights depends on the degree of linear independance of the columns of $\mathbf{X}$. Equation (5) can be transposed, yielding $\mathbf{X}^T\mathbf{w} = \mathbf{y}$. Thus, $\mathbf{y}$ is a linear combination of the columns of $\mathbf{X}^T$which span the signal space under consideration. The normalized columns of $\mathbf{X^T}$ are called basis vectors and form the matrix χ. The optimum would be when the basis vectors are orthogonal to one another, so that $\det(\chi) = 1$.

The matrix X is composed of the signals at the radiators at the different measurement directions.

$$x_{nj} = b_n(\mathbf{k}_{nj}) \cdot p_{nj} \cdot v_R(\mathbf{k}_{nj}) \cdot f_{CS} \cdot b_S(\mathbf{k}_{nj}) \cdot s_n$$

The wavenumbers $\mathbf{k}_{nj}$ at one measurement point j become parallel to one another when $r_{CS} \gg \max(r_{nC})$, i.e. the items of one row of χ are driven by the same wavenumber $\mathbf{k}_j$. The challenge of WEM is to find a set of N wavenumbers $\mathbf{k}_j$ (base wavenumbers) among the wavenumbers possible for the measurements, yielding the largest possible determinant of χ.

$$maximize\ over\ \mathbf{k}_1, ...\mathbf{k}_N\ :\ \det(\chi)$$

The coordinates of the wavenumber $\mathbf{k}$ which fully characterizes a plane wave are shown in Figure 2.

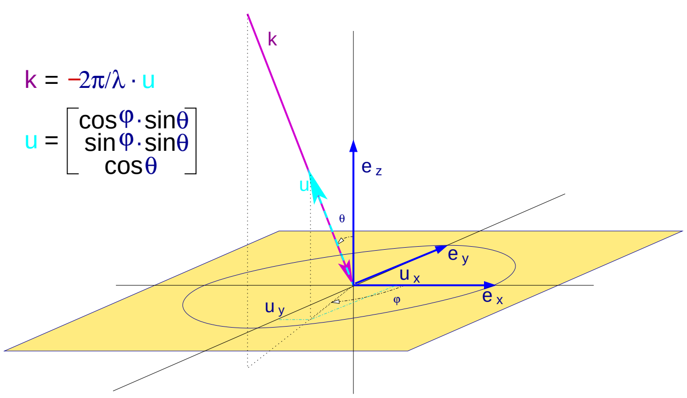


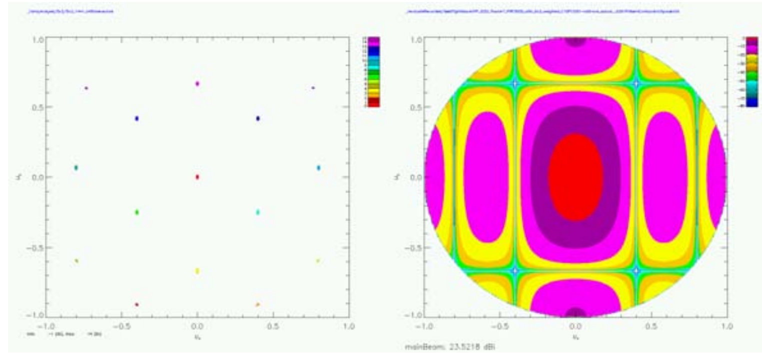
Figure 2: The coordinates of a wavenumber $\mathbf{k}$

The projection of the vector $\mathbf{u} = -\frac{2\pi}{\lambda}\mathbf{k}$ on the x and y achses of the local coordinate system are $\mathbf{u}_x$ and $\mathbf{u}_y$. They are used to define the plane wave direction on a 2-D plot. The ellipse with the axes $\mathbf{e}_x$ and $\mathbf{e}_y$ defines the visible region.

To demonstrate WEM, a very simple phased array is used. This array will be referenced as “5x3”. It is a regularly spaced rectangular array with 5 by 3 isotropic elements. The horizontal and vertical element spacing is $\lambda/2$.

4.1 Measurements in anechoic chambers

An orthogonal set of basis vectors can be obtained when the whole visible region can be exploited. This is possible, for instance, in an anechoic compact range where the antenna can be oriented at will in azimuth and elevation under far-field conditions. Figure 3 shows the coordinates of the direction cosines of a set of plane waves providing a set of orthogonal basis vectors for the "5x3" array.

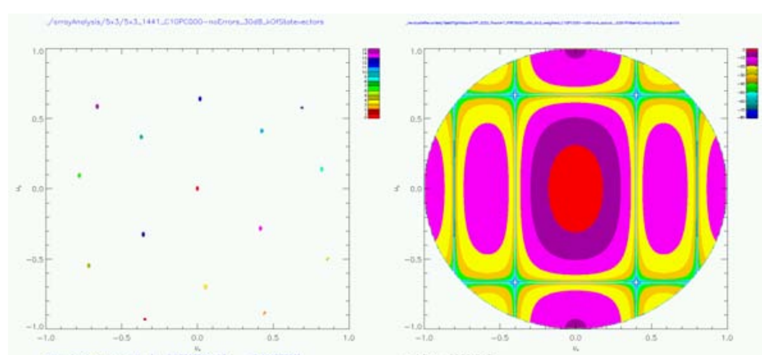


(a) Dir. cosines of the base wavenumbers (b) Simulated antenna pattern

Figure 3: Set of wavenumbers providing a set of orthogonal basis vectors for the “5x3” array.

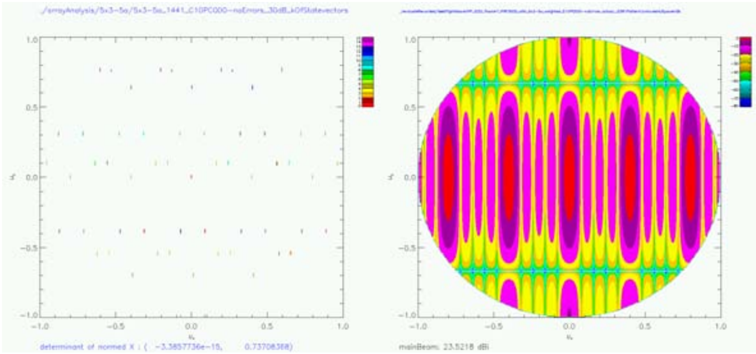
The pattern of the wavenumber direction cosines depend on the array geometry. Note, the array response also depends on the weights applied, refer to (3). In order to achieve a good estimate, all N measurements have to be above the noise level. Only incoherent thermal noise need be considered. Starting with an a priori knowledge of the weights commanded, an adequate selection of measurement points can be made. Probably, the columns of χ will no longer be orthogonal, but χ will be sufficiently conditioned to obtain satisfactory estimates. We will now compare the basis vector sets of two array configurations "5x3" and “5x3-5a”. The latter is similar to "5x3", but the element spacing in azimuth is 5 times the wavelength. The plane waves for the basis vectors for "5x3" are depicted in the first row, and for “5x3-5a” in the second one.

For the selection of the basis vectors, only measurements with array outputs in the range 0 dB to -30 dB of the maximum response are considered to eliminate noisy contributions. As seen in Figure 5, one basis vector can result from several wavenumbers due to the grating lobes. Depending on the weights applied, a dedicated set of basis vectors is needed to optimize the estimation.



(a) Dir. cosines of the base wavenumbers (b) Simulated antenna pattern

Figure 4: Simulation of the “5x3” array for the unweighted boresight beam. It can be seen that some base wavenumber fall within a null in the pattern and need to be rejected.



(a) Dir. cosines of the base wavenumbers (b) Simulated antenna pattern

Figure 5: Simulation of the "5x3-5a" array for the unweighted boresight beam. It can be seen that several wavenumbers can trigger one basis vector.

4.2 Measurements of array antennas on spaceborne and airborne platforms

The situation is different when the measurement points can only be on the tracks of a satellite or aircraft carrying the phased array. With the knowledge of the scheduled ephemeris data of the satellite and the commanded antenna weights, a satisfactory selection is possible, but the risk to get an ill conditioned χ is high. Therefore, several measurements should be taken in addition to the scheduled measurement points. We thus obtain an over determined system. The pseudo-inverse χ^+ can be calculated with singular value decomposition, and a principal component analysis is recommended for noise discrimination. A least squares type estimation will be obtained.

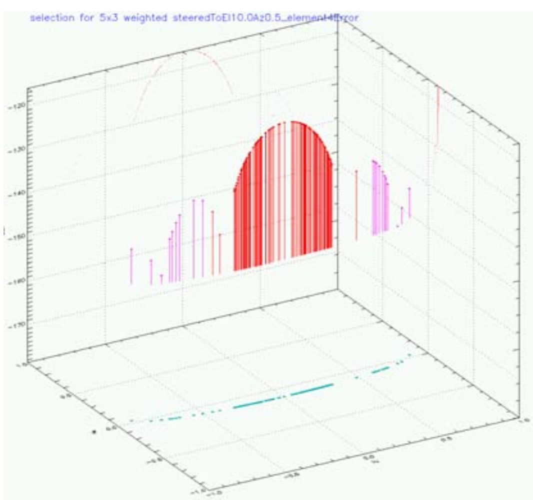


Figure 6: Measurement for the chosen set of basis vectors for the "5x3" array. The projection of the measurement points on the bottom plane show the associated base wavenumbers.

The weights of the '5x3' array and the '5x3-5a' array were adjusted to steer the beam to $10°$ elevation and $0.5°$ azimuth and were estimated using WEM in a simulation. One element has the weight 0 in amplitude to simulate a total element failure. The phased array flies on a sun synchroneous orbit. The measurements are taken on the pass where the center of the array footprint comes closest to the position of antenna S.

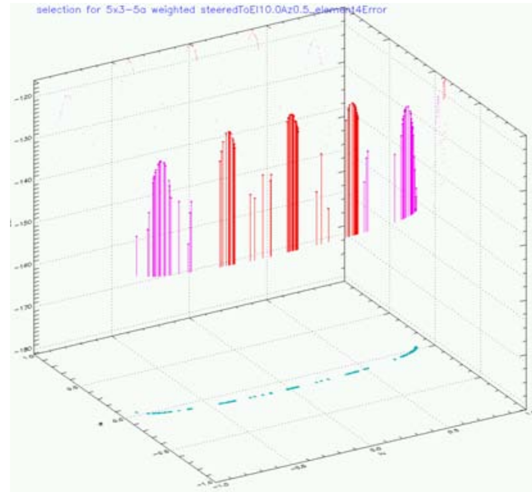


Figure 7: Measurement for the chosen set of basis vectors for the "5x3-5a" array.

The results show that the estimated weights have no systematic bias and the variance of the estimated weight amplitudes and phases are less than -45 dB and $0.5°$, respectively. In addition to the incoherent thermal noise of the phased array components, coherent terrestral noise received by the radiators must be considered. A better signal to noise ratio can be obtained with matched filtering. But the remaining disturbances degrade of course the estimation results. Additionally, the Doppler effect has to be taken into account.

For airborne array antennas, the distances are not so large with the consequence that the signals are higher, leading to a better signal to noise ratio. But the positioning of the phased array for the measurements isn't so precise.

5 Conclusion

The simulations show that the WEM method allows accurate characterisation of phased arrays in their operational configuration with a limited number of measurements. The measurement can take place with the sounding antenna outside the main beam of the phased array, although for signal-to-noise reasons the main should be used if possible. It is only important that the sounding antenna is within the pattern of the array's radiating elements. Hence, a single ground station housing the sounding antenna and its equipment could be used to fully characterise array antennas on moving platforms. Using a high-gain sounding antenna tracking the phased array, the field produced in the vincinity of the phased array can be very strong, so that measurements can be made even in low sidelobe regions.

If a full characterisation of a phased array can be performed externally in this way, the circuitry for characterising the antenna onboard can be greatly simplified or even dispensed with altogether.

References

[1] Schwerdt, M.; Hounam, D.; Alvarez-Peres, J.-L.; Molkenthin, T.: *The Calibration Concept of TerraSAR-X: A Multiple Mode, High Resolution SAR*, Canadian Journal of Remote Sensing, 2005

[2] Harry L. Van Trees: *Optimum Array Processing* Wiley-Interscience, 2002