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Mode Transition Behavior in Hybrid Dynamic Systems

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Mode Transition Behavior in Hybrid Dynamic Systems

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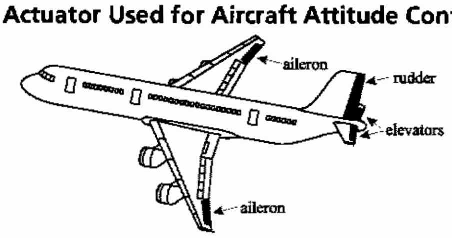
Introduction

Mode Transitions in Hybrid Models of Physical Systems

- **hybrid because**
  - **continuous, differential equations**
  - **discrete, finite state machine**
- **overview of phenomena involved**

Illustrated by Hydraulic Actuator Used for Aircraft Attitude Control Surfaces

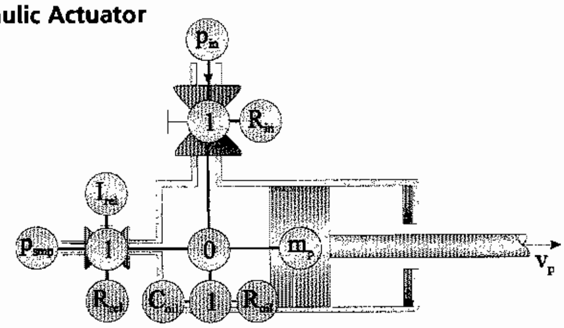


Modeling of Physical Systems

Ideal Picture Model (Schematic)

Identify Behavioral Phenomena

For Example, A Hydraulic Actuator



Equation Generation

Compile Constituent Equations

- R_{in} — $f_{in}R_{in} = p_{Rin}$
- R_{oil} — $f_R R_{oil} = p_{Roil}$
- C_{oil} — $C_{oil}\dot{p}_C = f_R$
- m_p — $m_p\dot{v}_p = A_p p_{cyl}$
- R_{rel} — $f_{rel}R_{rel} = p_{rel}$
- I_{rel} — $I_{rel}\dot{f}_{rel} = p_{rel}$
- *0*, **cylinder chamber** — $v_p = f_{in} - f_{rel}$
- *1*, **relief flow pipe** — $p_{rel} = p_{smp} - f_{rel}R_{rel} + p_{cyl}$
- *1*, **intake pipe** — $p_{Rin} = p_{in} - p_{cyl}$
- *1*, **oil compression** — $p_{Roil} = p_{oil} - p_C$

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Equation Processing

Before Simulation

- the number of equations is reduced
  - substitution/elimination
- equations are sorted
  - each equation computes one variable
- equations are solved
  - high index problems may require differentiation of certain equations

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Hybrid Behavior

Introduce Valves

- make highly nonlinear behavior piecewise linear
  - intake valve $if \;\; v_{in} \;\; then \;\; p_{Rin} = p_{in} - p_{cyl} \;\; else \;\; f_{in} = 0$
  - relief valve $if \;\; v_{rel} \;\; then \;\; p_{rel} = p_{smp} - f_{rel} R_{rel} + p_{cyl} \;\; else \;\; f_{rel} = 0$

Switching Between Modes of Continuous Behavior

- intake valve, v_{in}, external switch (control law)
- relief valve, v_{rel}, autonomous switch triggered by physical quantities

$$v_{rel} = p_{cyl} > p_{th}$$

- different sets of equations

Computational Causality

When Switching Equations

- computational causality may change
  - re-ordering
  - re-solving

Example

- when the intake valve closes, equations change
  - from $p \quad = p \quad - p_{cyl}$
  - to $f_{in} = 0$
- therefore, in this equation
  - p_{Rin} becomes unknown
  - f_{in} becomes known

Implicit Modeling

Deal With Causal Changes Numerically

Valve Behavior

- residue on f_{in} $\quad 0 = \mathit{if} \ v_{in} \ \mathit{then} \ -p_{Rin} + p_{in} - p_{cyl} \ \mathit{else} \ f_{in}$
- residue on f_{rel} $\quad 0 = \mathit{if} \ v_{rel} \ \mathit{then} \ -p_{rel} + p_{smp} - f_{rel} R_{rel} + p_{cyl} \ \mathit{else} \ f_{rel}$

Implicit Numerical Solver (e.g., DASSL)

- designed to handle this formulation

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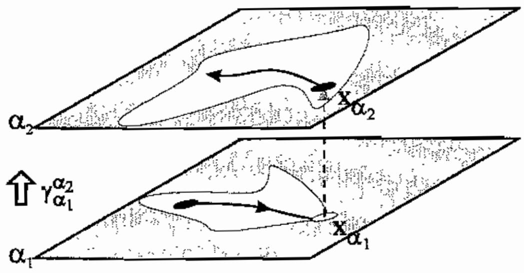
Hybrid Dynamic Behavior

Geometric View

- modes of continuous, smooth, behavior
- patches of admissible state variable values



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Specification Parts

Hybrid Behavior Specification

- a function, f, that defines continuous, smooth, behavior for each mode

  $f_{\alpha_i}: E_{\alpha_i}\dot{x} + A_{\alpha_i}x + B_{\alpha_i}u = 0$

- an inequality, γ, that defines admissible state variable values

  $\gamma_{\alpha_i}^{\alpha_{i+1}}: C_{\alpha_i}x + D_{\alpha_i}u \geq 0$

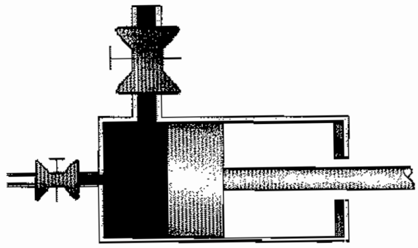
Dynamics

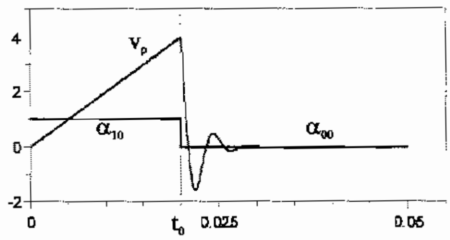
Behavior Characteristics

- C^0, i.e., no jumps in state variables
- steep gradients

Example

- when the intake valve closes, piston velocity quickly reduces to 0





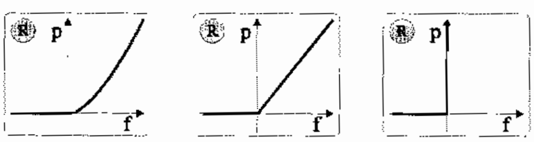
The Next Step

Remove Steep Gradients

- e.g., singular perturbation

Algebraic Constraints Between State Variables

- high index systems
- subspace with admissible (continuous) dynamic behavior
- discontinuities (jumps) in state behavior



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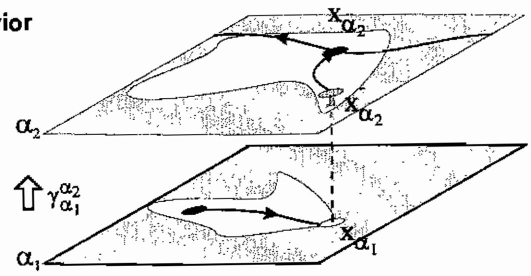
Hybrid Dynamic Behavior - Refined

Geometric View

- modes of continuous, smooth, behavior
- patches of admissible state variable values
- manifold of dynamic behavior



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Specification Parts

Hybrid Behavior Specification

- a function, f, that implicitly defines for each mode
  - continuous, smooth, behavior
  - state variable value jumps

  $$f_{\alpha_i}: E_{\alpha_i}\dot{x} + A_{\alpha_i}x + B_{\alpha_i}u = 0$$
- an inequality, γ, that defines admissible generalized state variable values

  $$\gamma_{\alpha_i}^{\alpha_{i+1}}: C_{\alpha_i}x + D_{\alpha_i}u \geq 0$$
- for explicit reinitialization (semantics of x^-)

  $$f_{\alpha_i}: E_{\alpha_i}\dot{x} + A_{\alpha_i}x + B_{\alpha_i}^{u}u + B_{\alpha_i}^{x}x^{-} = 0$$

Handling of Systems With High Index

DASSL Handles Index 2 Systems

- implicit formulation for continuous behavior

Requires Consistent Initial Conditions When Mode Changes Occur

- compute from implicit formulation to make jump space (projection) explicit
- for example, sequences of subspace iteration
  - space of dynamic behavior: $V^{n-1} = A^{-1} E V^n,\ V^0 = R^n$
  - jump space: $T^{n+1} = E^{-1} A T^n,\ T^0 = \{0\}$
- or, decomposition in (pseudo) Kronecker Normal Form

Projections

Linear Time Invariant Index 2 System

- derive pseudo Kronecker Normal Form (numerically stable)

$$\begin{bmatrix} E_{11} & 0 & 0 \\ 0 & 0 & E_{22,12} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_f \\ \dot{x}_{i,1} \\ \dot{x}_{i,2} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12,1} & A_{12,2} \\ 0 & A_{22,11} & A_{22,12} \\ 0 & 0 & A_{22,22} \end{bmatrix}\begin{bmatrix} x_f \\ x_{i,1} \\ x_{i,2} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_{2,1} \\ B_{2,2} \end{bmatrix} u = 0$$

- after integration (no impulsive input behavior), consistent values are

$$x_f = x_f^- - E_{11}^{-1} A_{12,1} A_{22,11}^{-1} E_{22,12} (x_{i,2} - x_{i,2}^-)$$

$$x_{i,1} = A_{22,11}^{-1}(-B_{2,1} u + E_{22,12} \dot{x}_{i,2}) - A_{22,12} x_{i,2}$$

$$x_{i,2} = -A_{22,22}^{-1} B_{2,2} u$$

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The Hydraulic Actuator

Generalized State Jumps for Each Mode

Mode	Projection
α_{00}	$f_{rel} = 0$ $v_p = 0$
α_{01}	$v_p = (m_p v_p^- - I_{rel} f_{rel}^-)/(m_{rel} + m_p)$ $f_{rel} = (m_p v_p^- - I_{rel} f_{rel}^-)/(m_{rel} + m_p)$
α_{10}	$v_p = v_p^-$ $f_{rel} = 0$
α_{11}	$v_p = v_p^-$ $f_{rel} = f_{rel}^-$

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A Scenario

Intake Valve Is Open

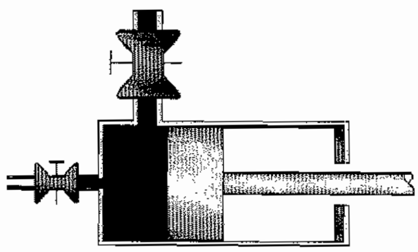
- piston starts to move

Intake Valve Closes

- piston inertia causes pressure build-up
- pressure reaches critical value

Relief Valve Opens

- cylinder pressure decreases



⇒ Interaction Between Mode Transition Behavior

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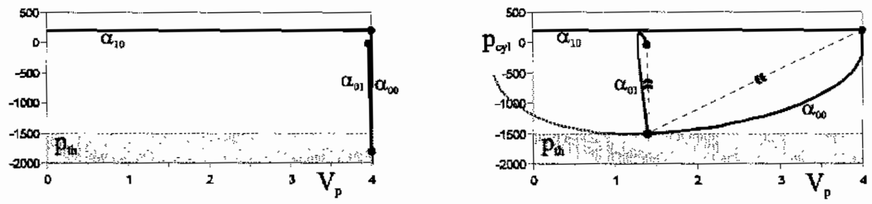
Phase Space of Cylinder Scenario

Projection Is Aborted

- immediately
- after partial completion

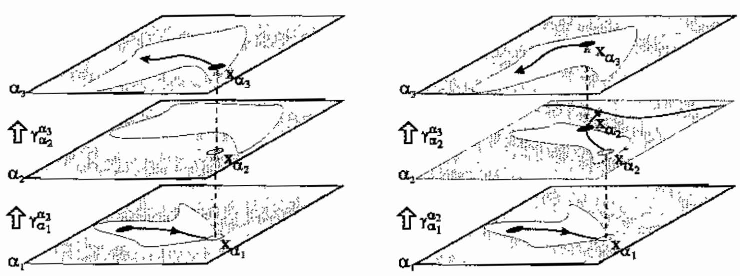


(a) (b)

Sequences of Mode Changes

(a) State Outside of a Patch in the New Mode

(b) During Projection State Values are Reached Outside of a Patch in the New Mode



(a) (b)

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Impulses

High Index Systems May Contain Impulsive Behavior

- **in case of the hydraulic cylinder, $p > p_{th}$, would always hold if not $v_p = v_p^-$**
- **unknown where the patch is abandoned**

In-Depth Analysis of Switching Conditions

- **solve for required $x(t)$**
- **compute earliest $t = t_s$ at which $\gamma(x(t), u(t), t) \geq 0$**
- **substitute t_s to compute $x(t_s)$**

Complex Switching Structure

Additional Difficulty When Interacting Fast Transients (e.g., collision)

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Detailed Analysis of the Projection

Cylinder Example (Imaginary Eigenvalues, $\lambda = \lambda_r + i\,\lambda_i$)

- **from detailed model**
  - **solve for p**

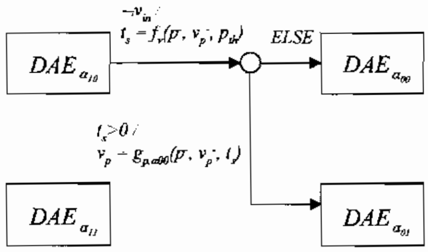
$$p(t) = e^{\lambda_r t}(p^- \cos(\lambda_i t) - \frac{1}{\lambda_i}(\frac{1}{C_1} v_p^- + \lambda_r p^-)\sin(\lambda_i t))$$

  - **substitute t at which $p(t) > p_{th}$**

$$v_p = e^{\lambda_r t_s}(v_p^- \cos(\lambda_i t) - (\frac{R_2}{I_1} v_p^- - \frac{p_1}{I_1} + \lambda_r v_p^-)\frac{\sin(\lambda_i t_s)}{\lambda_i})$$

Complex Switching Structure

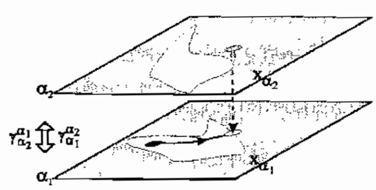
Explicit Re-Initialization

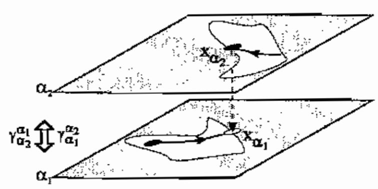


Chattering

What If the New Mode Switches Back

- immediately ⇒ inconsistent model, no solution
- after infinitesimal period of time ⇒ chattering behavior, solve with
  - equivalent control
  - equivalent dynamics





Equivalent Dynamics

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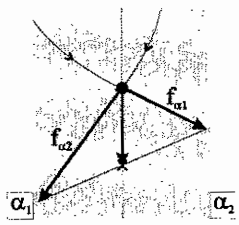
- fast component
  - remove
- slow component
  - weighted mean of instantaneous vector fields (Filippov Construction)
- sliding behavior



Ontology

Phase Space Transition Behavior Classification

- mythical (state invariant)
- pinnacle (state projection aborted)
- continuous
  - interior (continuous behavior)
  - boundary (further transition after infinitesimal time advance)
  - sliding (repeated transitions after each infinitesimal time advance)

Combinations of Behavior Classes

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Conclusions

Mode Transition Behavior

- **Rich**
- **Complex**

Requires

- **special algorithms/computations**
- **model verification analyses**

How to Efficiently Generate Behavior (e.g., for Real-time Applications)?

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