Robustness Assessment of Flight Controllers for a Civil Aircraft using $\mu-$Analysis.

G.H.N. Looye[1,2], S. Bennani[2], A. Varga[1], D. Moormann[1], G. Grübel[1]

[1]Deutsches Zentrum für Luft- und Raumfahrt (DLR)
Institut für Robotik und Systemdynamik
DLR-Oberpfaffenhofen
D-82234 Wessling, Germany
Gertjan.Looye@dlr.de

[2]Faculty of Aerospace Engineering
Group for Control & Simulation
Delft University of Technology
Kluyverweg 1, 2629 HS Delft
The Netherlands
S.Bennani@lr.tudelft.nl

Contents

- Introduction
- Review of LFTs and μ
- Research Civil Aircraft Model (RCAM)
- LFT-modeling of RCAM
- μ-Analysis results
- Conclusions

Background:

- GARTEUR FM(AG08): Project on robust flight control.
  - definition of two design benchmark for
    - civil aircraft (3D trajectory tracking, stab. augmentation)
    - military aircraft (Fly-by-wire, stab. augmentation);
  - application of modern and classical techniques by member organizations

    ... resulting in:
    - 12 designs (10 methods) for Research Civil Aircraft Model (RCAM),
    - 6 designs (5 methods) for High Incidence Research Model (HIRM);
  - evaluation and comparison of design techniques.
- For RCAM stability analysis by DLR-OP and TUD using:
  - worst-case parameter optimization,
  - ... and μ-analysis.

- We are basically interested in:
  1. is the controlled system stable over operating envelope ? (allowed parameter ranges)
  2. if so, what can we say about a stability margin?
  3. ... preferably in terms of uncertain physical parameters

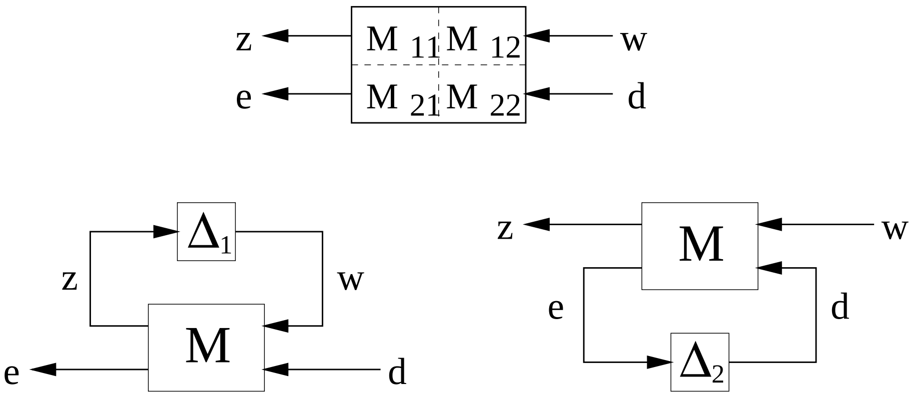
- We need a suitable manner to represent our uncertain parameters in the system model:

  Linear Fractional Transformations (LFTs);

- ... and a *robustness indicator* in terms of uncertain parameters:

  structured singular value μ.

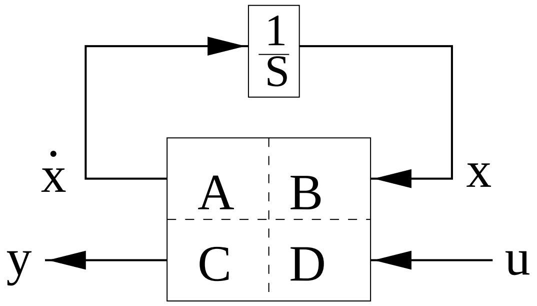
Linear Fractional Transformations (LFTs):



$$e = F_u(M, \Delta_1)d \qquad\qquad z = F_l(M, \Delta_2)w$$

$$\begin{aligned} F_u &= M_{22} + M_{21}(I - \Delta_1 M_{11})^{-1}\Delta_1 M_{12} \\ F_l &= M_{11} + M_{12}(I - \Delta_2 M_{22})^{-1}\Delta_2 M_{21} \end{aligned}$$

Direct generalization of:



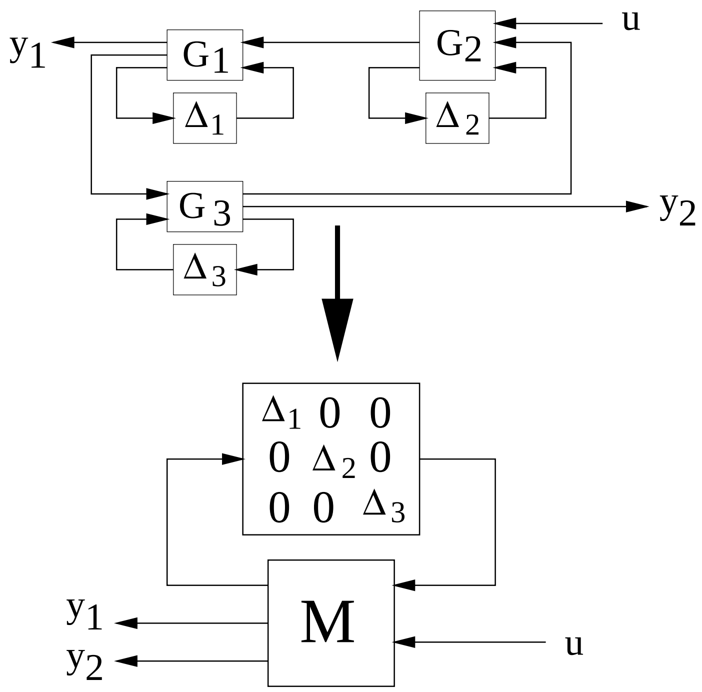
$$y = D + C(I - \frac{1}{s}A)^{-1}\frac{1}{s}B$$

What makes LFT's so useful?

Typical algebraic operations like:

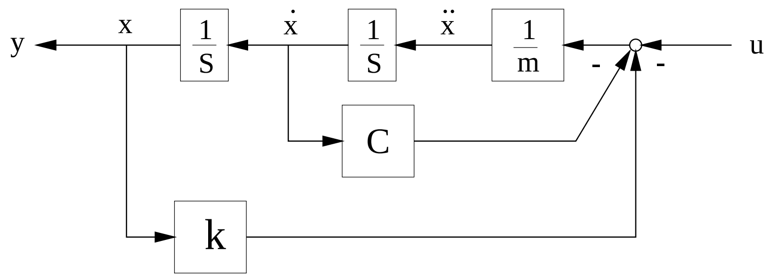
- cascade connections
- parallel connections
- feedback connections (inverse!)
- frequency response

... preserve the LFT structure.



- Interconnections of LFT's are again LFT's
- Unstructured uncertainty at component level
  ...becomes structured uncertainty at system level

The mass-spring-damper example:



- Model uncertainty in k. c and $\frac{1}{m}$:

$$k = \bar{k}(1 + 0.4\delta_k) \qquad c = \bar{c}(1 + 0.3\delta_c) \qquad \frac{1}{m} = \frac{1}{\bar{m}(1+0.5\delta_m)}$$

$$-1 \leq \delta_m,\ \delta_c,\ \delta_k \leq 1$$

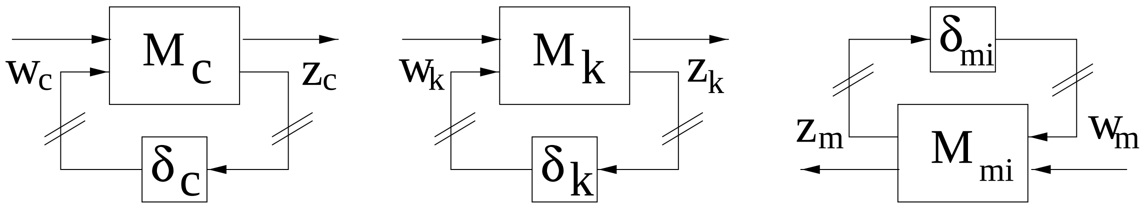
- These expressions are LFT's:

$$F_l = M_{11} + M_{12}\ (I - \Delta\ M_{22})^{-1}\ \Delta\ M_{21}$$

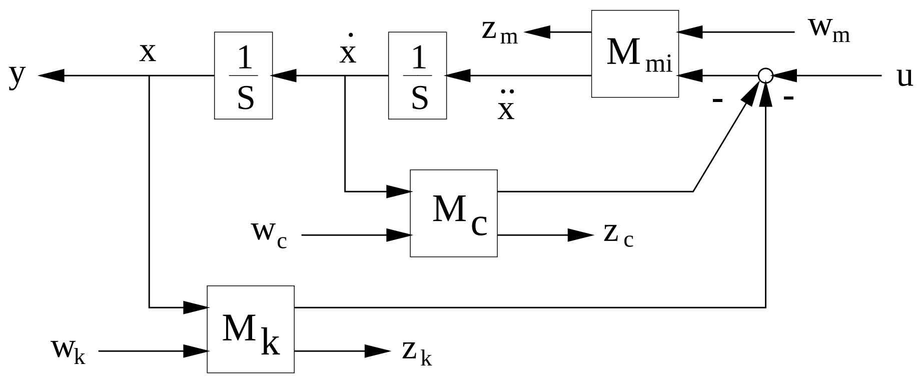
e.g: $c = \bar{c} + 0.3\bar{c}\ (1 - \delta_c\ 0)^{-1}\ \delta_c\ 1$

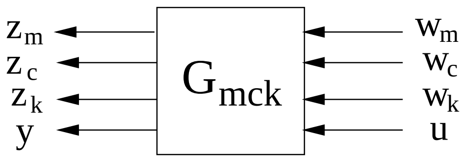
- M for each parameter now looks like:

$$M_c = \begin{pmatrix} \bar{c} & 0.3\bar{c} \\ 1 & 0 \end{pmatrix} \quad M_k = \begin{pmatrix} \bar{k} & 0.4\bar{k} \\ 1 & 0 \end{pmatrix} \quad M_{mi} = \begin{pmatrix} -0.5 & \frac{1}{\bar{m}} \\ -0.5 & \frac{1}{\bar{m}} \end{pmatrix}$$



* The block-diagram, δ's omitted:



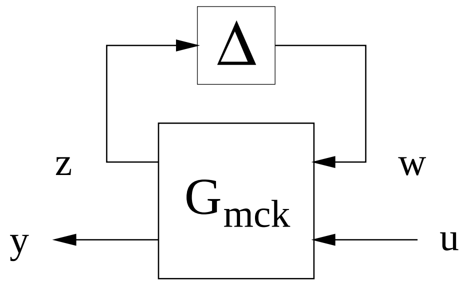
... OR: 

G_{mck} known

* Now define:

$$\Delta = \begin{pmatrix} \delta_m & 0 & 0 \\ 0 & \delta_c & 0 \\ 0 & 0 & \delta_k \end{pmatrix}$$

* We have a new LFT:

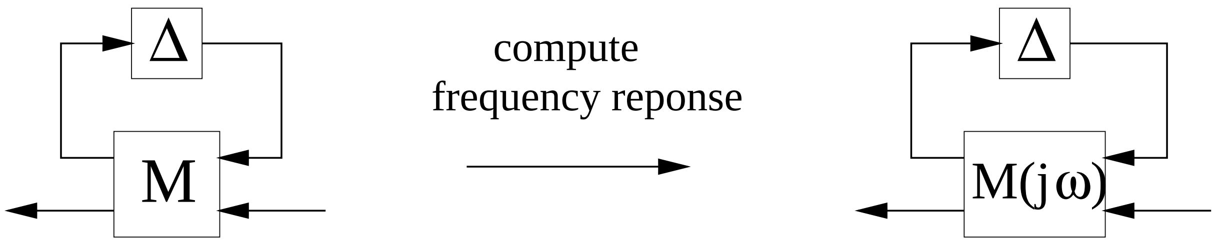


$\bar{\sigma}(\Delta) \leq 1$ structured Δ

G_{mck} known

Unstructured uncertainty at component level

...becomes structured uncertainty at system level



... Δ is from some set $\boldsymbol{\Delta}$ with specific diagonal structure, $\bar{\sigma}(\Delta) \leq 1$

We are interested in smallest Δ for which M goes unstable...

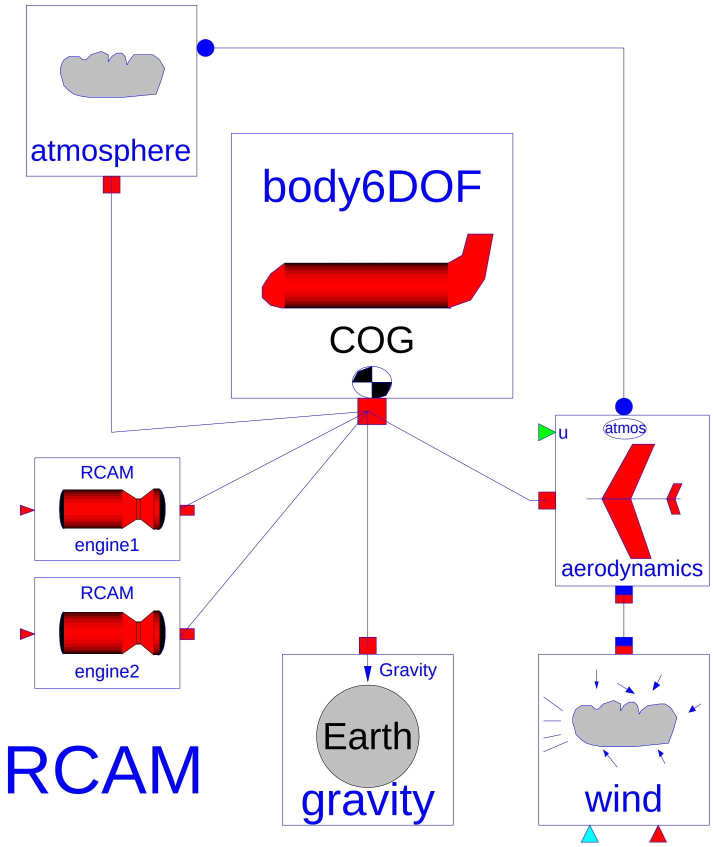
- or, the *smallest* Δ for which a first pole travels over the imaginary axis,
- at which very moment and frequency the LFT is singular,
- approach: walk along imaginary axis, and 'compute':

$$\mu_{\boldsymbol{\Delta}}(M_{11}(j\omega)) := \frac{1}{\min_{\Delta \in \boldsymbol{\Delta}} \{\bar{\sigma}(\Delta) : \mathsf{det}(I - M_{11}(j\omega)\Delta) = 0\}}$$

System is *robustly stable* $\iff \mu_{\boldsymbol{\Delta}}(M_{11}(j\omega)) < 1 \quad \forall \omega$

Plenty of tools available for approximating μ:

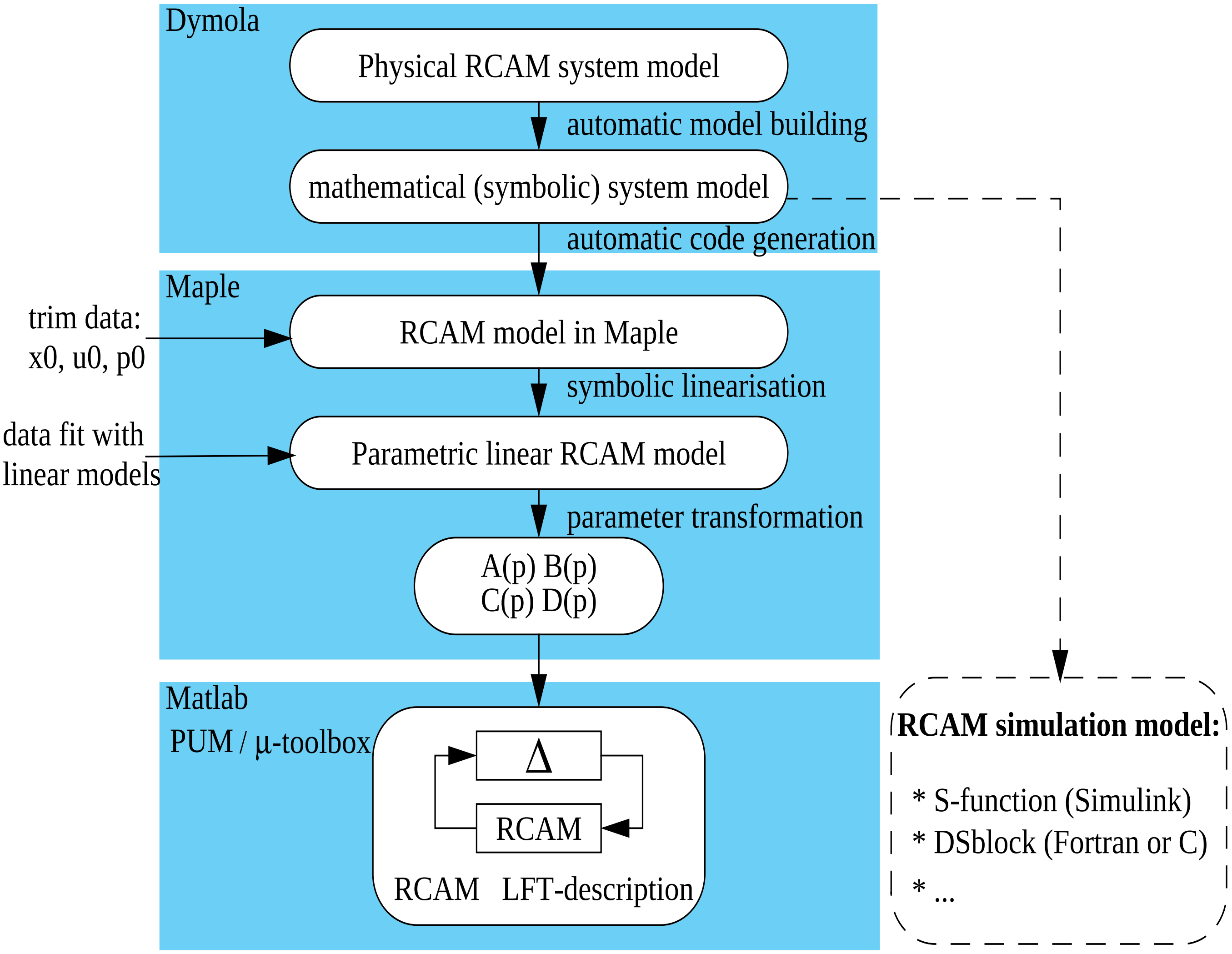
- D-G scaling (upper bound),
- power algorithms (lower bound),
- branch-and-bound schemes,
- frequency sweep (lower bound) [CERT-ONÉRA],
- ... LMI techniques.



The following parameters may vary:

parameter	nominal	minimum	maximum
X_{cg}	$0.23\bar{c}$	$0.15\bar{c}$	$0.31\bar{c}$
Z_{cg}	$0.0\bar{c}$	$0.0\bar{c}$	$0.21\bar{c}$
mass	120 000 kg	100 000 kg	150 000 kg
delay	0.05 s	0.05 s	0.10 s

The design speed is 80 m/s



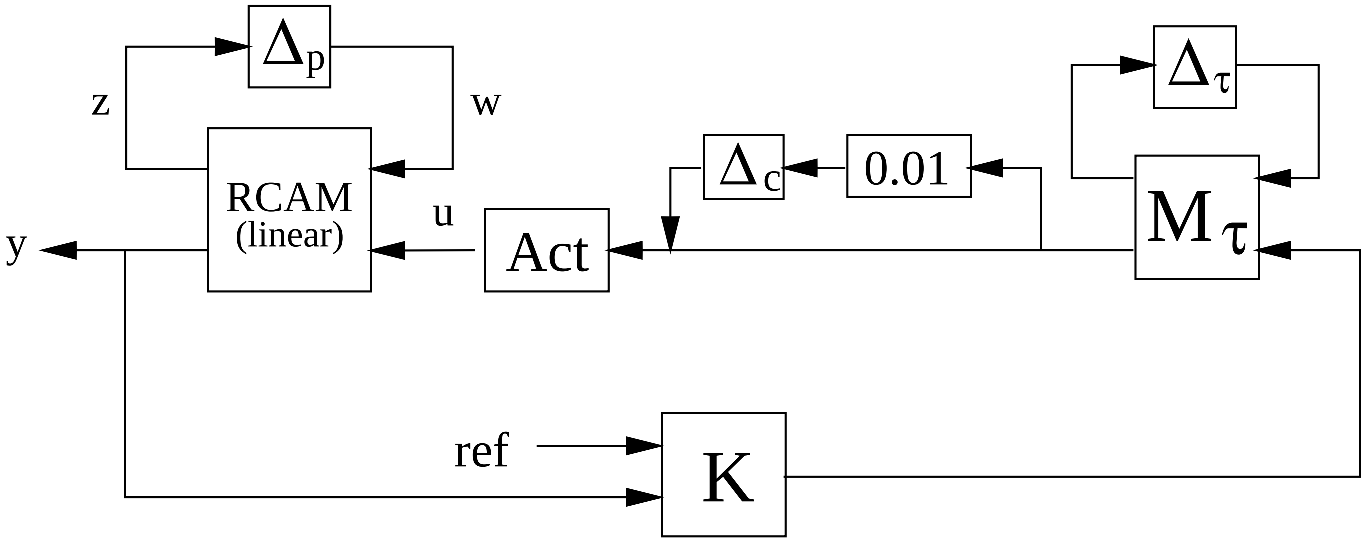
Example: $A(7,7) = X_u = -\frac{qS}{m}\frac{2}{V_0}(C_{D_0} - \alpha_0 C_{L_0})$

We end up with:

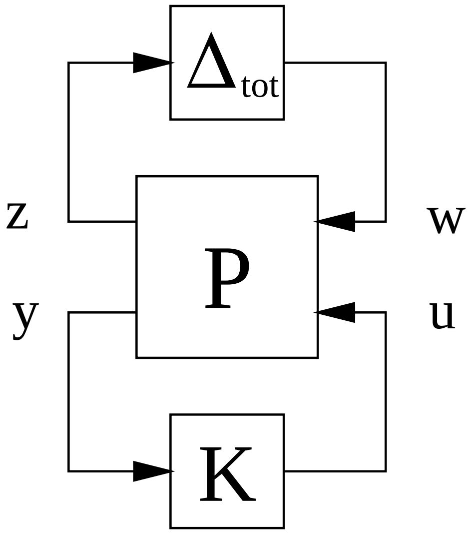
- linear nominal model, 12 states, 5 contr., 15 meas.;
- Δ-block structure:

$$\Delta_p = \begin{pmatrix} \delta_m I_{17} & 0 & 0 \\ 0 & \delta_{xcg} I_{15} & 0 \\ 0 & 0 & \delta_{zcg} I_3 \end{pmatrix}$$

We complete the system:

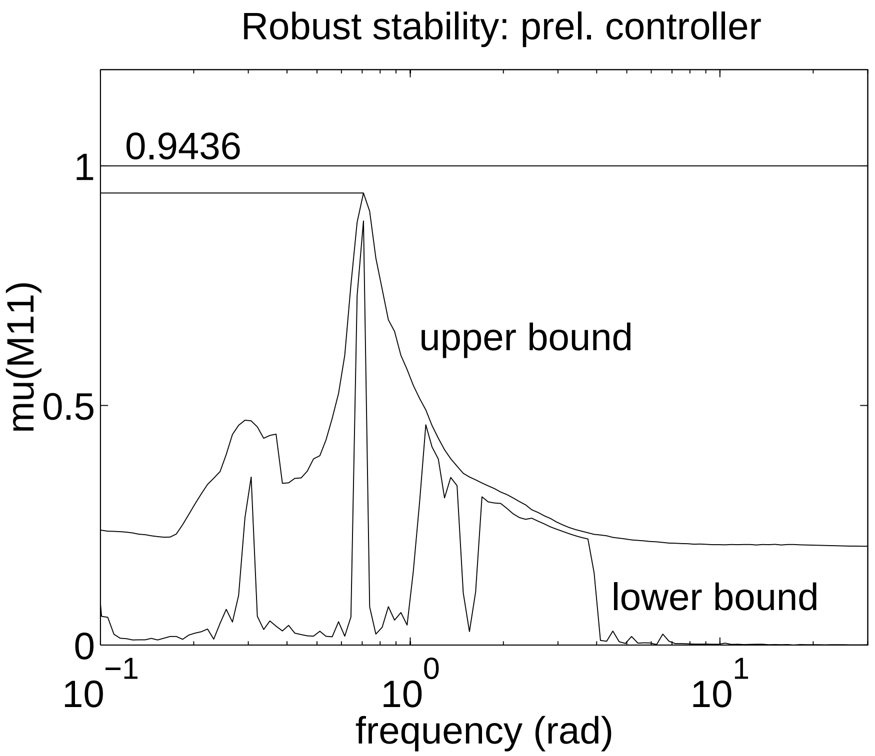


Or, in LFT form:



$$\Delta_{tot} = \begin{pmatrix} \Delta_p & 0 & 0 \\ 0 & \Delta_\tau & 0 \\ 0 & 0 & \Delta_c \end{pmatrix}$$

For K we can plug-in any of the controllers, after linearizing the Simulink structure...



* Figure out worst-case parameters:

parameter	$\delta_{..}$	value
mass	δ_m	1.1092
X_{cg}	δ_{xcg}	1.1092
Z_{cg}	δ_{zcg}	1.1092
delay	δ_τ	1.1092
compl. pert.	$\delta_{c1..5}$	0

... $\bar{\sigma}(\Delta) = 1.1092$, and $1/\bar{\sigma}(\Delta) \approx 0.90$

* East-bound pole:

0.95 Δ_{crit}	Δ_{crit}	1.05 Δ_{crit}
-0.0137 $\pm$ 0.6954i	0.0 $\pm$ 0.6899i	$+$0.0131 $\pm$ 0.6844i

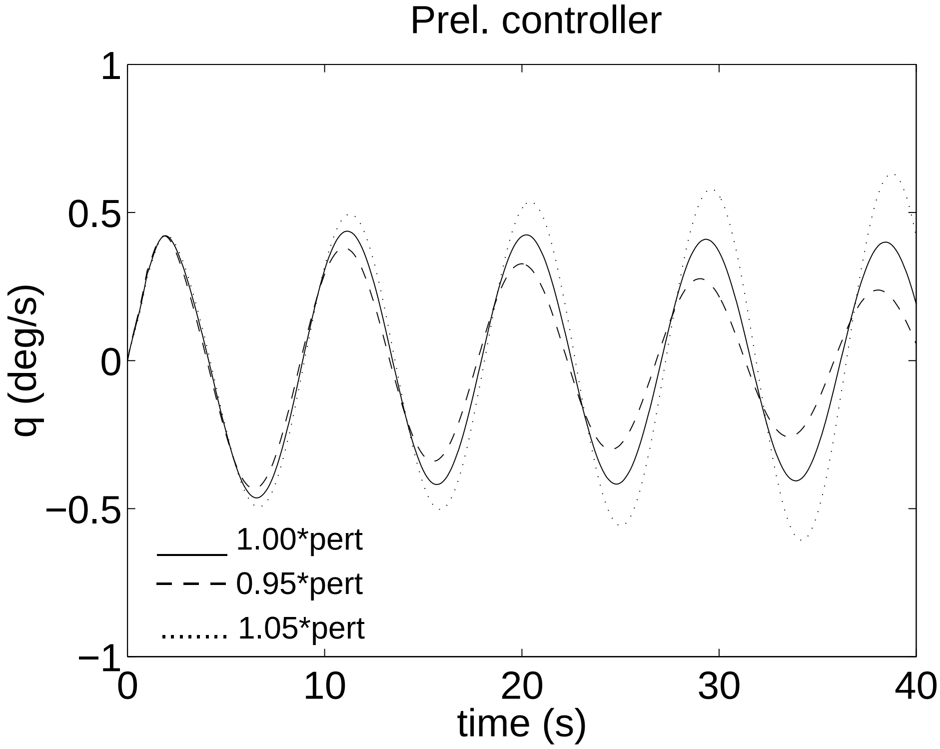
* The critical parameter values are:

$$\begin{array}{lclclcl} m & = & 125\,000 & + & 25\,000 * 1.092 & = & 152\,300\,\text{kg} \\ X_{cg} & = & 0.23\bar{c} & + & 0.08\bar{c} * 1.092 & = & 0.317\bar{c} \\ Z_{cg} & = & 0.105\bar{c} & + & 0.105\bar{c} * 1.092 & = & 0.220\bar{c} \\ \tau & = & 0.075 & + & 0.025 * 1.092 & = & 0.102\,\text{s} \end{array}$$

... indeed outside the specified parameter ranges

Perform *nonlinear simulation* with *original model:*

... actually, using simulation code generated from very same DYMOLA model



No.	method	$\mu(M_{11})$ (upper bnd)	peak freq. (rad/s)	worst-case parameters δ_m	δ_{xcg}	δ_{zcg}	δ_τ	mode (lon/lat)	remarks
MO-16	multi-model multi-objective	0.35	6.0	-	+	+	+	lon	
MM-12	modal multi-model-synth.	0.36	8.0	-	-	-	+	lon	
EA-22	eigenstructure assignment	0.39	0.5	+	+	+	+	lon	
FL-15	fuzzy logic	0.44	5.5	-	-	-	+	lon	linearized
MS-11	μ-synthesis	0.49	2.9	-	-	+	+	lat	diff. LFT!!!
			0.6	+	-	-	+	lon	
CC-13	classical control	0.51	0.8	+	+	+	+	lon	
LY-14(2)	Lyapunov	0.57	0.8	+	+	+	+	lon	
MF-25	Model following	0.65	0.6	+	+	+	+	lon	
EA-18	eigenstructure assignment	0.83	7.0	-	+	+	+	lon	
HI-prel	H_∞ loopshaping	0.94	0.7	+	+	+	+	lon	example
MS-19	μ-synthesis	1.36	15.1	-	-	+	+	lon	diff. LFT!!!
HI-21	H_∞ mixed sensitivity	1.53	1.3	+	+	+	+	lon	

μ-Analysis is a very promising tool for (post-design) robustness analysis

- Linear Fractional Transformations:
  - a powerful framework for representing uncertainty;
    ... but, (especially for aircraft) not trivial to obtain;
  - automated LFT generation is great relief
    ... but, better methods for reduction Δ need to be implemented.
- μ-computation:
  - used frequency gridding: tricky,
  - worst-case parameters, and verified in nonlin. simulations,
  - upperbound conservativeness 1–9 %,

Outlook:

- new algorithms for automated LFT generation from parametric nonlinear models
- apply improved optimization algorithms for μ-computations
- beyond μ, LTI...

reports on this work publicly available via `http://www.nlr.nl`