Meteorol. Zeitschrift, N.F. 6, 275–280 (Dezember 1997)


# Potential vorticity: A short history of its definitions and uses

ALAN J. THORPE, Reading and HANS VOLKERT, Oberpfaffenhofen

**Summary.** There has, in recent years, been something of a renaissance in the use of potential vorticity to interpret atmospheric dynamics. This owes a lot to the review paper of HOSKINS, MCINTYRE and ROBERTSON (1985). However, the definition and use of potential vorticity dates back to about 1940. In this paper the different roots of the conception of the dynamical quantity now called potential vorticity are traced during the period 1940 to 1955. Some background information about the principal scientists involved is provided as is a brief account of the use of the early ideas during the following two decades.

**Potentielle Vorticity: Ein geschichtlicher Abriß zu Definitionen und Anwendungen**

**Zusammenfassung.** Die vergangenen Jahre sahen eine Art Renaissance des Gebrauchs der potentiellen Vorticity zur Interpretation der Atmosphärendynamik. Diese basiert zu einem Gutteil auf dem Übersichtsartikel von HOSKINS, MCINTYRE und ROBERTSON (1985). Doch Definition und Anwendung von potentieller Vorticity stammen aus der Zeit um 1940. In diesem Aufsatz werden die verschiedenen Wurzeln für den Entwurf einer dynamischen Größe, die heute potentielle Vorticity genannt wird, während des Zeitraums 1940 bis 1955 nachgezeichnet. Dabei wird auch Hintergrundinformation geliefert über die wegbereitenden Wissenschaftler sowie eine kurze Erwähnung der Anwendungen der frühen Ideen während der folgenden beiden Jahrzehnte.

## 1. Historical perspective

The aim of this paper is to provide an analysis of the various strands which went into the early development of the definition and use in meteorology of the quantity now called potential vorticity. We focus especially on the role of the scientists who were the four key protagonists: two of them working in the USA, Carl-Gustaf Rossby and Jule Charney, and two in Germany, Hans Ertel and Ernst Kleinschmidt. A point which emerges is the (perhaps inappropriate) transfer of the terminology introduced by Rossby to the quantity defined by Ertel (and its various approximations).

### 1.1. Rossby's potential vorticity

The first use of the term "potential vorticity" in the literature is by Carl-Gustaf ROSSBY[1] in a paper in 1940 which appears in a supplement to Volume 66 of the Quarterly Journal of the Royal Meteorological Society. Rossby's approach used the shallow water equations and it is worth quoting his definition: "The potential vorticity represents the vorticity the air column would have if it were brought isentropically to a standard latitude and stretched or shrunk vertically to a standard depth or weight." The analogy to potential temperature defined in the 19th century by VON BEZOLD (1889) is clear.

The potential vorticity defined by Rossby, $P_R$, in his 1940 paper is as follows:

$$P_R = (\xi_\theta + f)\frac{\Delta p_0}{\Delta p} - f_0 \qquad (1)$$

where

$\xi_\theta$ is the relative vorticity about an axis perpendicular to an isentropic surface

$\Delta p$ is the pressure depth of a column of air whose top and bottom have a chosen potential temperature difference $\Delta\theta_0$

$\Delta p_0$ is a chosen reference $\Delta p$ for the same column

$f_0$ is the Coriolis parameter of a chosen reference latitude

$f$ is the Coriolis parameter.

It should be noted that $P_R$ is a relative "vorticity" and, of course, has dimensions of $s^{-1}$. The $P_R$ is conserved for dry adiabatic frictionless flow and as the column moves changing its $\Delta p$, then its isentropic relative vorticity, $\xi_\theta$, changes to maintain the constancy of $P_R$. It is straightforward to define an absolute version of Rossby's potential vorticity by forming $P_R + f_0$. Thus $P_R$ is the relative vorticity the air column would have if it were to move to the standard latitude and occupy the standard pressure depth.

Having noted the constancy of $P_R$ for adiabatic frictionless flow, equation (1) can be used by re-arranging it to give:

---

[1] Carl-Gustaf Rossby: born 1898 in Stockholm, Sweden, died 1957 in Stockholm, Sweden.
Main affiliations: Univ. Bergen, Norway with V. and J. Bjerknes (1919–1920); Swedish Meteorological and Hydrological Institute, Stockholm (SMHI, Swedish Met. Service; 1922–1925); Massachusetts Institute of Technology, Cambridge (MIT; 1928–1941); Univ. of Chicago (1941–1947); Univ. of Stockholm, Sweden (1947–1957).
For more information see e.g. BOLIN (1959), LEWIS (1996).

0941-2948/97/0006-0275 $ 02.70

$$\xi_\theta = \frac{\Delta p}{\Delta p_0}(P_R + f_0) - f \qquad (2)$$

Hence knowing an air-parcel's $P_R$ and its latitude and pressure depth we can use equation (2) to find its isentropic relative vorticity.

Rossby notes, in that paper, that his collaborator Victor Starr suggested to him that this quantity might be used for the identification of air-masses. In his 1940 paper Rossby discussed at length the planetary flow patterns after making approximations such as time averaging over one week and the superposition of several equal density layers in the vertical. His (shallow water) potential vorticity was suggested as an important tracer, but of a somewhat auxiliary nature. His motivation clearly stemmed from a multi-year research initiative at MIT.

Closely following on from Rossby's paper the new ideas were pursued by STARR and NEIBURGER (1940), SPAR (1943) and PLATZMAN (1949) by applying them to atmospheric cases. One of the motivations of this work was in air-mass identification for which it was felt there should be three conserved quantities. In other words an air-parcel could be uniquely identified only if three conserved quantities were known. These three were potential temperature, specific humidity, and potential vorticity. For example if an isentropic chart is displayed then an air-parcel must remain somewhere on that surface. To determine its location on the surface one can use specific humidity (as long as no condensation or evaporation takes place) but this only provides a location anyway along a humidity isoline. To fix the parcel's position along that isoline the potential vorticity was used. The usefulness of this method seems to rely on how orthogonal such isolines are to one another; for example, if the potential vorticity isolines were to be nearly parallel to the humidity isolines then it would remain difficult to pinpoint a given air-parcel.

## 1.2. Ertel's potential vorticity

Independently, Hans ERTEL[2] published a paper in 1942 in the Meteorologische Zeitschrift dealing with a general vortex theorem („Wirbelsatz") in relation with assumed hydrodynamical invariants („hydrodynamische Invariante") which are individually conserved for each fluid element. He was a partly self taught academic who obtained his PhD in 1932 three years after he was exceptionally allowed to enter university without a final secondary school exam (Abitur). On little more than four two-column pages he deduced over 40 equation steps the differential version of a very general vortex theorem from the hydrodynamical equations. As a special case he mentioned adiabatic motion and stated in a formula that the scalar product of the absolute vorticity vector and the potential temperature gradient multiplied by the specific volume does not change with time, but he did not assign a name to this complicated quantity. He went on to derive the integral version of the vortex theorem. Ertel's purely theoretical approach was quite different to Rossby's, not least because it made no connection to observational data.

The adiabatic vorticity invariant introduced by Ertel 1942, $P_E$, is defined as:

$$P_E = \frac{1}{\rho}\underline{\zeta} \cdot \nabla\theta \qquad (3)$$

where $\rho$ is the air density
$\underline{\zeta}$ is the absolute vorticity vector

The dimensions of $P_E$ are $\mathrm{Km^2s^{-1}kg^{-1}}$. It should be noted that $P_E$ can be written in terms of isentropic relative vorticity in the following alternative way if the hydrostatic approximation can be made:

$$P_E = -g(\xi_\theta + f)\frac{\partial\theta}{\partial p} \qquad (4)$$

Interestingly there is a very general reference to Rossby (ERTEL 1942, p. 280): "... the adiabatic case of the vortex theorem may turn out to be a useful tool in 'isentropic analysis' (C. G. Rossby)", but no citation of Rossby's work[3].

## 1.3. Charney's potential vorticity

In a separate line of reasoning in 1948 Jule Charney[4], published a paper which essentially defined the quasi-geostrophic theory. In that paper he derived a conservation principle for what he called the absolute potential vorticity

[2] Hans Ertel: born 1904 in Berlin, Germany; died 1971 in Berlin, Germany.
Main affiliations: Institut für Meteorologie, University of Berlin (1934–1941), Zentralanstalt für Meteorologie und Geodynamik, Wien (1942); Institut für Meteorologie und Geophysik, Univ. Innsbruck (1943–1945); Institut für Meteorologie und Geophysik, (Humboldt)Univ. (East-)Berlin (1947–1962) and Deutsche Akademie der Wissenschaften (East-)Berlin (1949–1969).
For more information see MAUERSBERGER (1971), SCHRÖDER (1994).

[3] We know that Ertel and Rossby met. In 1937 Ertel spent a two month research visit at the meteorological division of MIT headed by Rossby (MAUERSBERGER 1971; see also PHILLIPS 1990, p. 203). The foreword of his treatise in German on dynamical meteorology (ERTEL 1938) is signed: "at present Cambridge, Mass., December 1937, Massachusetts Institute of Technology, Meteorological division – H. Ertel". In the 246 often detailed footnotes Rossby's earlier papers are frequently referred to. In 1948, after the war, Rossby visited Ertel in Berlin (SCHRÖDER 1994) and there is a joint paper (ERTEL and ROSSBY 1949), which apparently was entirely written by Ertel, but did neither introduce nor even contain the concept of potential vorticity (the contrary has been erroneously stated by CRESSMAN, 1996, p. 25).

[4] Jule G. Charney: born 1917 in San Francisco, USA; died 1981 in Boston, USA.
Main affiliations: Institute for Advanced Study, Princeton (1948–1956); Massachusetts Institute of Technology, Cambridge (MIT; 1956–1981).
For more information see LINDZEN et al. (1990).

and made a rather profound summary of the new theory: "... the motion of large-scale disturbances is governed by the laws of conservation of potential temperature and absolute potential vorticity, and by the conditions that the horizontal velocity be quasi-geostrophic and pressure quasi-hydrostatic." The quantity, whose conservation Charney derived, is now called the quasi-geostrophic potential vorticity and it has a relationship, albeit one that is not intuitively obvious, with the quantities discussed by Rossby and Ertel.

CHARNEY (1948) uses the term "absolute potential vorticity" for a quantity, $P_Q$, which arises from quasi-geostrophic theory:

$$P_Q = f + \xi + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma^2}\frac{\partial \psi}{\partial p}\right) \tag{5}$$

where

$\xi = \nabla^2\psi$ is the geostrophic relative vorticity on a pressure surface

$\sigma^2$ is the static stability of a horizontally homogeneous background atmosphere

$\psi$ is the geostrophic stream function

The $P_Q$ is conserved by the geostrophic flow and has units of $s^{-1}$. The relationship between $P_Q$ and the other potential vorticities is not a direct one.

It is intriguing to note that a year later in CHARNEY (1949) there is a full derivation of Ertel's theorem, without reference to ERTEL (1942). There, Charney, says that '... we obtain essentially Rossby's equation for the conservation of "potential vorticity" (see ROSSBY 1940)'. This probably sowed the seeds of the transfer of the name potential vorticity to Ertel's quantity.

### 1.4. Kleinschmidt's contribution

In a seemingly independent and more profound way Ernst Kleinschmidt[5] freely used Ertel's "adiabatic vorticity invariant" to discuss the dynamics of cyclones in a three-part paper in Meteorologische Rundschau in 1950 and 1951 (THORPE 1993). This link with Ertel's quantity was made eight years after Ertel's paper and involved a case study of cyclogenesis which had occurred in March 1943. Apparently Kleinschmidt was unaware of Rossby's work or at least did not make to the connection between Ertel's and Rossby's theorems. Kleinschmidt had referred to Ertel already in his theoretically oriented PhD-thesis of 1941 and had obtained thorough practical experience with data as a forecaster during the war. His pioneering work developed the ideas of conservation, non-conservation, and invertibility of $P_E$. He did not use a specific term for the new variable, but only declared "to make use of a hydrodynamical state variable, which will continue to become an important one". Once he called it "Ertel's quantity Z" and stayed with this letter for the remainder of this treatise.

The series of papers Kleinschmidt wrote in the early 1950's are staggeringly original. An indication of the originality and imagination involved in Kleinschmidt's use of potential vorticity is given in a figure he included in his 1950 paper; here reprodueed as Figure 1. The draughtsmanship seems to owe something to Salvador Dali! Rarely since then has such imagination been put into a meteorological diagram. It attempts to do what even the latest computer graphics seem to have difficulty with namely to provide a perspective three-dimensional depiction of a potential vorticity anomaly (referred to as the "Höhenkörper" by Kleinschmidt).

Kleinschmidt's use of potential vorticity really set the groundwork for the way it is used today. He used the conservation property to monitor air-parcel trajectories. However he also notes that in cloudy regions potential vorticity is not conserved due to latent heat release and this source/sink of potential vorticity may have important consequences. In addition he suggested that if the distribution of potential vorticity were known then it could be "inverted" to find the (balanced) winds and temperatures consistent with that distribution. HOSKINS, MCINTYRE and ROBERTSON (1985) much later refer to this as the invertibility principle.

## 2. Inter-relationships between potential vorticities

The practice of applying the term potential vorticity for the quantity introduced by Ertel has been followed since the early 1950's leading to the uncomfortable use of the name "potential vorticity" for a quantity which does not have dimensions of vorticity. A simple scaling could be used to obtain the "correct" units but this was not pursued by HOSKINS et al. (1985) who instead introduced a "potential vorticity unit" as an abbreviation (1 PVU = $10^{-6}m^2s^{-1}Kkg^{-1}$).

It is possible to state precisely the relationship between the various definitions of potential vorticity. Comparing equations (1) and (4) we find that for motions for which the hydrostatic approximation is applicable:

$$P_R = P_E \frac{\Delta p_0}{g\Delta\theta_0} - f_0 \tag{6}$$

(Note that the pressure depths $\Delta p$ and $\Delta p_0$ are defined as positive quantities.)

The Rossby potential vorticity, in contrast to the Ertel quantity, requires values of the constants $\Delta p_0$, $\Delta\theta_0$ and $f_0$ to be provided. Values given by ROSSBY 1940 are: $f_0 = 10^{-4}s^{-1}$, $\Delta p_0 = 100$ mb and $\Delta\theta_0 = 4$ K which gives the numerical value of the factor which appears in equation (6) to be:

$$\frac{\Delta p_0}{g\Delta\theta_0} = 250\ \mathrm{kgm^{-2}K^{-1}} \tag{7}$$

[5] Ernst Kleinschmidt: born 1912 in Friedrichshafen, Germany; died 1971 in Göttingen, Germany.
Main affiliations: Reichswetterdienst, partly attached to the air force (1938–1946); Kaiser-Wilhelm- (from 1948: Max-Planck-) Institut für Strömungsforschung, Göttingen (1946–1971).
For more information see THORPE (1993).

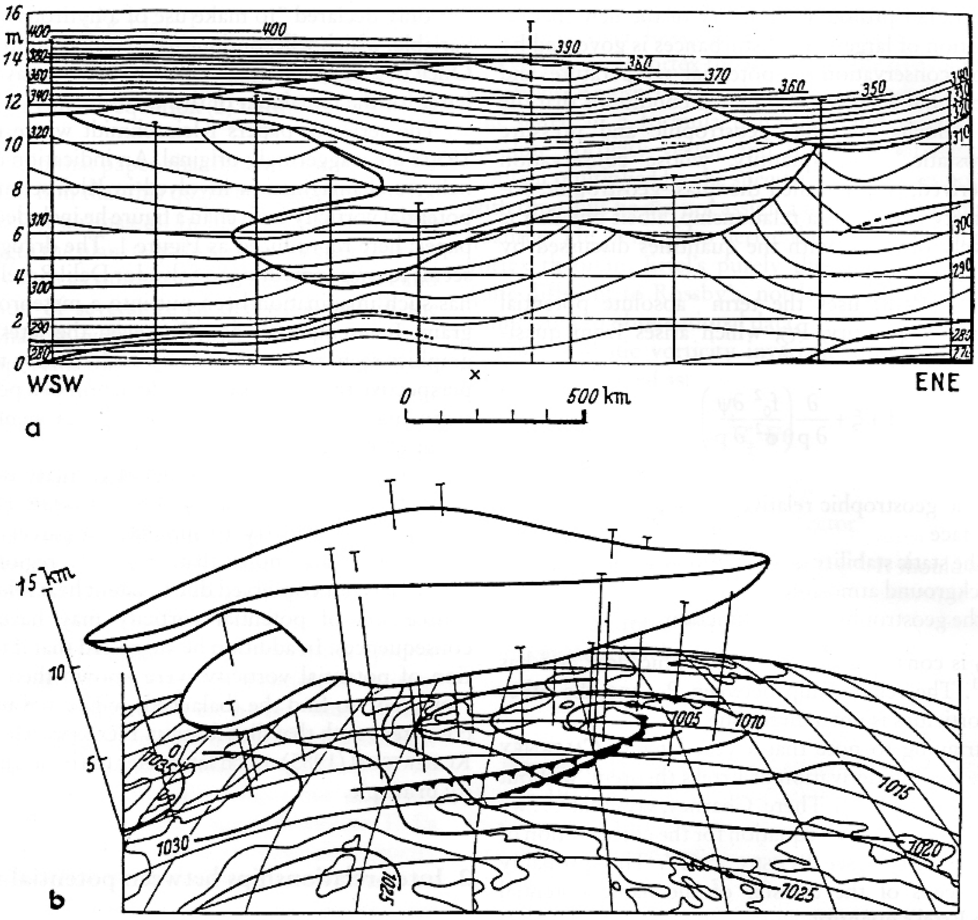


Fig. 1. (a) A vertical cross-section through the cyclone of 6 March 1943 at 05 Z showing isentropes. Note the outlined body of increased static stability between the troposphere and stratosphere. This elevated body (Höhenkörper) is associated, by Kleinschmidt, with high PV of a value intermediate between that of the lower troposphere and the stratosphere. This figure appeared as figure 2 of Kleinschmidt (1950). — (b) A three-dimensional depiction of the Höhenkörper itself above the surface synoptic chart. This appeared as figure 4 of Kleinschmidt (1950).

Abb. 1. (a) Vertikalschnitt durch die Zyklone vom 6. März 1943 um 05 Z. Dargestellt sind Linien gleicher potentieller Temperatur. Beachtenswert ist der eingerahmte Bereich mit erhöhter statischer Stabilität zwischen Tropo- und Stratosphäre. Dieser ‚Höhenkörper' wird von Kleinschmidt in Zusammenhang gebracht mit erhöhter potentieller Vorticity, d. h. mit Werten zwischen denen in der unteren Troposphäre und denen in der Stratosphäre. Die Abbildung erschien als Abb. 2 in Kleinschmidt (1950). — (b) Dreidimensionales Abbild des Höhenkörpers über der synoptischen Bodenkarte; erschienen als Abb. 4 in Kleinschmidt (1950).

Note that if $P_E = 1$ PVU then $P_R = 1.5 \times 10^{-4} s^{-1}$. It remains an historical oddity that of these different forms it became the fashion to use $P_E$ but to give it the name coined by Rossby for $P_R$. This is a confusing practice as the units of $P_E$ are not those of vorticity. A possible resolution of this difficulty is to define a scaled version of $P_E$ suggested by equation (7):

$$P_s = 250\, P_E \tag{8}$$

where the number 250 has the dimensions given in equation (7) and then $P_s$ has the dimensions of $s^{-1}$ if $P_E$ is in SI units. Then the quantity $P_s$ is the absolute potential vorticity formed by adding $f_0$ to $P_R$. So, for example, a value of $P_s$ = $1\times10^{-4} s^{-1}$ (equal to the typical mid-latitude planetary vorticity) corresponds to a value of $P_E = 4\times10^{-7} Km^2s^{-1}kg^{-1}$ = 0.4 PVU.

The relationship of $P_Q$ to $P_E$, or $P_R$, is less direct. It can be shown that the derivatives of $P_E$ on an isentropic surface bear a simple relationship to the derivatives of $P_Q$ on a pressure surface. However if we divide the atmosphere into a horizontally homogeneous reference (denoted by an overbar) and a perturbation, as is done in quasi-geostrophic theory, then on the large-scale we can approximate $P_E$ by:

$$P_E \approx -g\left[f_0 \frac{\partial \theta'}{\partial p} + (f+\xi)\frac{\partial \bar{\theta}}{\partial p}\right] \tag{9}$$

Comparing equations (5) and (9), and noting that

$$\theta' = -\theta_0 f \rho \frac{\partial \psi}{\partial p},$$

we can see that there is a relationship between $P_Q$ and $P_E / \left(-g\frac{\partial \bar{\theta}}{\partial p}\right)$. This relationship is complicated by the presence, in the quasi-geostrophic form, of the static stability parameter inside the vertical derivative. This feature does not occur in the approximated form of $P_E$ in equation (9).

## 3. Epilogue

In the preceding sections we have mentioned the contributions of the different protagonists during the early development of the potential vorticity concept in America and Europe. The interplay of at least two contrasts which appear to be unique to this powerful but complicated quantity has been highlighted: a far-sighted approximation (Rossby) versus a very abstract deductive approach (Ertel) and new terminology in two important scientific languages, English and German with the latter one losing its status as a relevant communication medium during World War II.

Following the initial suggestion by Rossby that potential vorticity be used as an air-parcel tracer there were two distinct pioneering ways in which it has been used in dynamical meteorology. The first is that implied by the quasi-geostrophic potential vorticity, and theory, developed by CHARNEY (1948). This had proved to be extremely productive in the interpretation and theorizing about the synoptic scale aspects of extra-tropical weather systems. It forms the cornerstone of large-scale meteorology.

The second application to dynamical meteorology stems from the Ertel theorem and Kleinschmidt's use of it in synoptic analysis and via the invertibility concept. Both Ertel and Kleinschmidt worked in small groups or even in isolation, so their concept did not flourish in German meteorological research. Apparently it was in America where Rossby's imaginative term and Ertel's general quantity of a physical dimension different from vorticity first became coupled by CHARNEY (1949) and then continuously applied by Reed, Danielsen and Shapiro among others. Most of these applications used the approximation of isentropic potential vorticity as a marker for stratospheric air without giving an indication about the origin of the variable.

In contrast CHARNEY and PHILLIPS (1953) were apparently aware of all the earlier studies and put the concept of potential vorticity in relation to the then very recent technique of numerical modelling. They refer to the isentropic form of Ertel's quantity as "potential vorticity" and show how it plays a central role in an envisaged quasi-geostrophic, semi-Lagrangian numerical model with potential temperature as the vertical coordinate[6]. REED (1955) used the potential vorticity to help define the existence of upper fronts and tropopause folds in extratropical cyclones by noting that the stratosphere has a distinctly larger value of potential vorticity than the troposphere. BLECK (1973) provided an exception to the dearth of citations to the earlier papers as his section "Early work on the potential vorticity theorem" referred to the contributions by Rossby, Ertel and Kleinschmidt.

The silent transition of terminology from Rossby's to Ertel's quantity was probably aided by the lack of a concise German term for Ertel's quantity. In his German textbook RAETHJEN (1953) clearly distinguished between Rossby's "potential vorticity" (p. 206) and Ertel's "adiabatic vortex invariant" (p. 207). KLEINSCHMIDT (1955) called Ertel's quantity "potential vorticity" in the English abstract, but strangely "potential vortex value" (potentieller Wirbelwert) in the German main text. In their detailed handbook contribution ELIASSEN and KLEINSCHMIDT (1957) introduced Ertel's theorem in full generality under the heading of "potential vorticity". Its conservation is stated for adiabatic and frictionless motion with a short credit to Rossby for its derivation "in a somewhat simplified form".

Over the 50 years since its inception potential vorticity has been used in a huge number of meteorological applications: invertibility, instability criteria, tracers, electrostatic analogy, attribution, diabatic and frictional generation, and the theory of balanced equations. Today, potential vorticity is in ubiquitous use in dynamical meteorology, even jargon as "PV thinking" appears to be en vogue. But the celebrated quantity remains to be complicated, "dimensionally awkward" (BLECK 1973) and its relevance is difficult to appreciate for newcomers to the field as well as to non-theoreticians. We hope that a better knowledge of the hidden path of the concept's early development and reception is also conductive to a better understanding.

## References

Bezold, W. von, 1889: Zur Thermodynamik der Atmosphäre. — Meteorol. Z. **6**, 287–294.

Bleck, R., 1973: Numerical forecasting experiments based on the conservation of potential vorticity on isentropic surfaces. — J. Appl. Meteorol. **12**, 737–752.

Bolin, B. (ed.), 1959: The Atmosphere and Sea in motion. — Scientific contributions to the Rossby Memorial Volume, Rockefeller Institute Press, New York, 509 pp.

Charney, J. G., 1948: On the scale of atmospheric motions. — Geofys. Publ. **17**, no. 2, 17 pp.

— 1949: On a physical basis for numerical prediction of large-scale motions in the atmosphere. — J. Meteorol. **6**, 371–385.

[6] In CHARNEY and PHILLIPS (1953) short credits are given to ROSSBY (1940) for the "discovery of the law of conservation of potential vorticity" (footnote 5, p. 72); to KLEINSCHMIDT (1950) for "deducing flow patterns corresponding to simple prescribed distributions of potential vorticity" (footnote 9, p. 95); and to ERTEL (1942) for the "general form of the potential vorticity theorem" (in brackets at the end of p. 97). But the subtleties of the different definitions are not mentioned. More details may be found in the quoted technical report by SHUMAN (1951) of which we could not obtain a copy.

Charney, J. G., N. A. Phillips, 1953: Numerical integration of the quasi-geostrophic equations for barotropic and simple baroclinic flows. — J. Meteorol. 10, 71–99.

Cressman, G. P., 1996: The origin and rise of numerical weather prediction. — In: J. R. Fleming (ed.), Historical Essays on Meteorology 1919–1995. — Amer. Meteorol. Soc., Boston, 21–39.

Eliassen, A., E. Kleinschmidt, 1957: Dynamic Meteorology. — In: S. Flügge (ed.), Handbuch der Physik **48**, 1–154.

Ertel, H., 1938: Methoden und Probleme der dynamischen Meteorologie. — Ergebnisse der Mathematik und ihrer Grenzgebiete, 5. Band, Lieferung 3, Springer, Berlin, 153–278.

— 1942: Ein neuer hydrodynamischer Wirbelsatz. — Meteorol. Z. **59**, 277–281.

Ertel, H., C.-G. Rossby, 1949: A new conservation-theorem of hydrodynamics. — Pure Appl. Geophys. **14**, 189–193.

Hoskins, B. J., M. E. McIntyre, A. W. Robertson, 1985: On the use and interpretation of isentropic potential vorticity maps. — Q. J. R. Meteorol. Soc. **111**, 877–946.

Kleinschmidt, E., 1950: Über Aufbau und Entstehung von Zyklonen. — (I. Teil) Meteorol. Rdsch. **3**, 1–6; (II. Teil), Meteorol. Rdsch. **3**, 54–61.

— 1951: Über Aufbau und Entstehung von Zyklonen. — (III. Teil) Meteorol. Rdsch. **4**, 89–96.

— 1955: Die Entstehung einer Höhenzyklone über Nordamerika. — Tellus **7**, 96–108.

Lewis, J. M., 1996. C. G. Rossby: Geostrophic adjustment as an outgrowth of modelling the Gulf Stream. — Bull. Amer. Meteorol. Soc. **77**, 2711–2728.

Lindzen, R. S., E. N. Lorenz, G. W. Platzman (eds.), 1990: The atmosphere — a challenge: The science of Jule Gregory Charney. — Amer. Meteorol. Soc., Boston, xvii + 321 pp.

Mauersberger, P., 1971: Nachruf Prof. Dr. Hans Ertel. — Z. f. Meteorol. **22**, 314–317.

Phillips, N. A., 1990: The emergence of quasi-geostrophic theory. — In: R. S. Lindzen et al. (eds.), The atmosphere — a challenge: The science of Jule Gregory Charney. — Amer. Meteorol. Soc., Boston, 177–206.

Platzman, G. W., 1949: The motion of barotropic disturbances in the upper troposphere. — Tellus 1(3), 53–64.

Raethjen, P., 1953: Dynamik der Zyklonen. — Akad. Verlagsgesellschaft, Leipzig, xii + 384 pp.

Reed, R. J., 1955: A study of a characteristic type of upper-level frontogenesis. — J. Meteorol. **12**, 226–237.

Rossby, C.-G., 1940: Planetary flow patterns in the atmosphere. — Q. J. R. Meteorol. Soc. **66** suppl., 68–87.

Schröder, W., 1994: Meteorologist's profile – Hans Ertel. — Weather **49**, 284–288.

Shuman, F. G., 1951: Solutions of a differential equation of pressure tendency. — Techn. Rept. No. 9, Research on atmospheric pressure change, Dept. Meteorol., Mass. Inst. Tech., 135 pp.

Spar, J., 1943: The correlation between specific humidity and potential vorticity in the atmosphere. — Bull. Amer. Meteorol. Soc. **24**, 196–200.

Starr, V., M. Neiburger, 1940: Potential vorticity as a conservative property. — J. Marine Res. **3**, 202–210.

Thorpe, A. J., 1993: An appreciation of the meteorological research of Ernst Kleinschmidt. — Meteorol. Z., N.F. **2**, 3–12.



Prof. Alan J. Thorpe
Department of Meteorology
University of Reading
PO Box 243, Earley Gate
Reading, RG6 6BB, U.K.

Dr. Hans Volkert
Institut für Physik der
Atmosphäre
DLR Oberpfaffenhofen
D-82230 Weßling


Received 17 June 1997, in revised form: 26 August 1997