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Simplification of the Discrete Angle Radiative Transfer Method for Clouds with Translational Symmetry

by K. M. GIERENS

DLR, Institut für Physik der Atmosphäre, Oberpfaffenhofen, 82230 Weßling, Germany

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Abstract

A simplification of the Discrete Angle Radiative Transfer method is presented for the case of clouds with translational symmetry. For translational symmetry, it is not necessary to explicitly calculate the intensities for the direction parallel to the symmetry axis. Instead, they can be taken into account implicitly by adding a correction term to the scattering probabilities. This correction term is derived.

Zusammenfassung

Vereinfachung des Discrete Angle Radiative Transfer Verfahrens für Wolken mit Translationssymmetrie

Es wird eine Vereinfachung des Discrete Angle Radiative Transfer Verfahrens für den Fall von Wolken mit Translationssymmetrie angegeben. Bei dieser Art von Symmetrie müssen die Intensitäten in Richtung der Symmetrieachse nicht explizit berechnet werden; sie können stattdessen implizit über ein Korrekturglied für die Streuwahrscheinlichkeiten berücksichtigt werden. Dieses Korrekturglied wird hergeleitet.

1 Introduction

In a recent paper (Gierens, 1993, paper I) the application of the Discrete Angle Radiative Transfer Method (DART, Lovejoy et al., 1990) to finite three dimensional clouds has been described. The application was mainly designed for implementation into a 3-d hydrodynamics code with cubic grid cells. Therefore, the 6-stream version of DART was chosen, i.e. the DA(3,6) version (the label (n, m) means that m streams, in a n-dimensional space are considered, see Lovejoy et al., 1990).

Occasionally, one has to deal with clouds that possess translational symmetry, i.e. their physical properties vary only in two dimensions (say, x and z), and they are constant in the third one (say y). Examples of such clouds are contrails and wave clouds. For such clouds, it is desirable to save computing time and memory by using a 2-d radiative transfer model instead of a 3-d one, e.g. the DA(2,4) version.

In clouds with translational symmetry there is no net energy transport parallel to the symmetry axis (since the net fluxes are zero). But this does not imply that there is no radiation flowing into this direction. It does only imply that intensities[1] for $+y$ and $-y$ directions are equal. There are applications for which one would like to know the intensity of this radiation, e.g. if one wants to compute the actinic flux onto cloud particles. However, in a pure 2-d radiative transfer model there is no possibility to compute the intensity in the y-direction. In order to overcome this problem one has to go back to the 3-d model.

In the present paper it will be shown how a 2-d model for clouds with translational symmetry can be derived from the DA(3,6) model, so that the

[1] The meaning of "intensity" is somewhat diffuse in the context of DART because it has properties of both radiance and flux density (see paper I, Sect. 2.1). Therefore the notion "radiance" has been avoided.

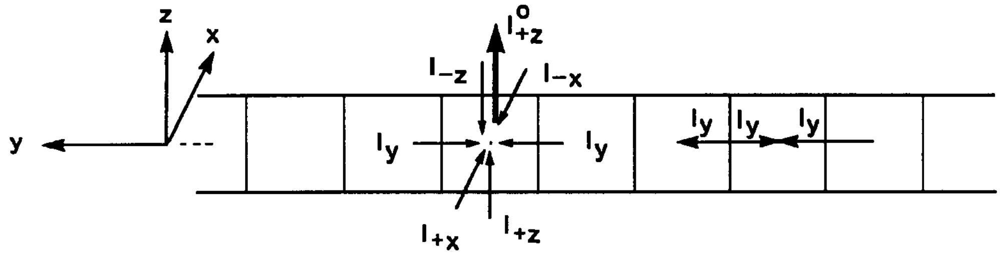

Figure 1 A stack of an infinite number of identical cubic cells representing a column of a cloud with translational symmetry. The intensity of radiation flowing parallel to the symmetry axis is I_y. The symmetry of the situation requires that $I_{+y} = I_{-y}$ and that $\partial I/\partial y \equiv 0$ for any of the intensities.

information on the radiation flowing along the cloud axis is retained and can be used for subsequent calculations. This goal is achieved by an analytical computation of the intensity for the y-direction, which yields a correction term for the cell scattering probabilities.

2 Derivation of the Correction Term

Consider a column of cubic grid cells that is infinitely long in the y-direction. From the assumption of translational symmetry in this direction, it follows that all the cells in the column are equivalent. Furthermore, the $+y$ and $-y$ directions are equivalent. Now, let us consider a single cell out of the column (Figure 1) and let us compute an arbitrary intensity emerging from this cell. The intensity I^0_{+z} is given by the following combination of the intensities incident on the considered cell (cf. Eq. (2) of paper I):

$$I^0_{+z} = T I_{+z} + R I_{-z} + S (I_{+x} + I_{-x} + 2 I_y) . \quad (1)$$

The intensities $I_{\pm x}$ and $I_{\pm z}$ are supposed to be computed by the 2-d radiative transfer model, i.e. for the moment these are considered known. The task is, to determine I_y as a function of the known intensities and the transmission, reflection and scattering probabilities T, R and S.

From Figure 1 and the symmetry of the problem it can be seen that

$$I_y = T I_y + R I_y + S (I_{+x} + I_{-x} + I_{+z} + I_{-z}) . \quad (2)$$

Thus,

$$I_y = \frac{S}{1 - T - R} \cdot (I_{+x} + I_{-x} + I_{+z} + I_{-z}) . \quad (3)$$

Now, Eq. (1) reads

$$I^0_{+z} = \left(T + \frac{2S^2}{1-T-R} \right) I_{+z} + \left(R + \frac{2S^2}{1-T-R} \right) I_{-z} + \left(S + \frac{2S^2}{1-T-R} \right) (I_{+x} + I_{-x}) . \quad (4)$$

This is the formulation of a DA(2,4) system (Lovejoy et al., 1990), because only four streams in two dimensions ($\pm x$ and $\pm z$) are retained in this equation. Obviously, one can derive the DA(2,4) system from the DA(3,6) system for translational symmetry by adding simply a correction term $2S^2/(1 - T - R)$ to the probabilities under consideration.

3 Discussion

The correction term $2S^2/(1 - T - R)$ implicitly accounts for all the possible photon paths along the cloud symmetry axis. This becomes evident when it is expanded:

$$\frac{2S^2}{1-T-R} = 2S^2 \sum_{n=0}^{\infty} (T+R)^n = S^2 \cdot 2 (1 + T + R + T^2 + 2TR + R^2 + T^3 + 3T^2R + 3TR^2 + R^3 + \cdots) . \quad (5)$$

Written in this form, the physical content of the correction factor becomes clear: Each photon path along the symmetry axis is a random walk that consists of a random series of transmissions and reflections. The corresponding probabilities are summed up by the term in brackets. For every such walk there is an adjoint path, viz. its reflected image. This is accounted for by the factor 2. Any random walk along the symmetry axis is initiated and terminated by a scattering event. The corresponding probability is S^2. Obviously, the correction term takes into regard the infinite number of possible photon paths along the symmetry axis at a time. Other techniques that account for the radiation flowing along the symmetry axis, like cyclic boundary conditions or mirrors ($R = 1$, $T = S = 0$) adjacent to a single $x - z$ plane, possess severe problems concerning their convergence behaviour. They converge very slowly because they explicitly simulate the random walk of the photons step by step and iteration by iteration. If absorption is weak, the random walk can go rather far. Then many iterations are required.

The new method, like the DA(3,6) method (paper I) guarantees convergence for mathematical reasons, because in the scattering operator there are now the row sums Σ,

$$\Sigma = T + R + 2S + 4 \cdot \frac{2S^2}{1 - T - R} \begin{cases} = 1 & \text{for } A = 0 \\ < 1 & \text{for } A > 0 \end{cases}$$

where $A = 1 - T - R - 4S$ is the absorption probability within a cell.

4 Summary

Clouds with translational symmetry (e.g. contrails, wave clouds) can be treated easily with the DA(3,6) version of the Discrete Angle Radiative Transfer Method, when the following steps are performed:

a) Compute the transmission, reflection and scattering probabilities T, R and S;

b) Compute the corresponding corrected probabilities

$$T' = T + K, \quad R' = R + K, \quad S' = S + K,$$

with $K = 2S^2/(1 - T - R)$;

c) Compute the 2-d intensity field ($I_{\pm x,z}$) using T', R', and S';

d) Finally the y-intensity I_y can be computed by means of Eq. (3).

References

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