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The Influence of Radiation on the Diffusional Growth of Ice Crystals

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Abstract

The influence of radiation on the diffusional growth/evaporation process of ice crystals is investigated. A theoretical analysis of the heat balance equation shows that only crystals having sizes of more than a few tens of microns are affected and that radiation can become dominant for particles that exceed a few 100 of microns in size. Furthermore the emissivity of spherical ice crystals is determined as a function of their temperature and size. Radiation fields for various situations are computed and their effect on the growth of ice crystals is discussed. It turns out that the determination of the radiation field is inevitable in studies of the microphysical evolution of ice clouds when they contain large crystals.

Zusammenfassung

Der Einfluß von Strahlung auf das Diffusionswachstum von Eiskristallen

Der Einfluß von Strahlung auf das diffusive Wachstum (bzw. das Verdampfen) von Eiskristallen wird untersucht. Die theoretische Analyse des Wärmehaushalts zeigt, daß nur Kristalle, die größer als einige 10 Mikrometer sind, von der Strahlung beeinflußt werden. Die Strahlung ist allerdings dominierend für Kristalle, die einige 100 Mikrometer Größe übertreffen. Außerdem wird die Emissivität sphärischer Eisteilchen in Abhängigkeit von ihrer Temperatur und Größe bestimmt. Das Strahlungsfeld wird für verschiedene Situationen berechnet und dessen Auswirkung auf das Kristallwachstum diskutiert. Es zeigt sich, daß man nicht umhinkommt, das Strahlungsfeld zu berechnen, wenn man die mikrophysikalische Entwicklung einer Eiswolke studieren will, die größere Kristalle enthält.

1 Introduction

In the life cycle of a cloud radiation acts in a twofold manner: (i) Hydrometeors absorb and emit radiation while they are exchanging energy with the cloud air. The air experiences differential heating and cooling which leads to temperature gradients in the cloud. This can excite convective motions. (ii) The hydrometeors themselves are heated and cooled by radiation. This affects their diffusional growth or evaporation rates. There is a vast literature on topic (i), and one can find profiles of radiative heating or cooling in clouds (i.e. profiles of the radiation flux divergence) in many publications (e.g. Stephens, 1978; Trautmann and Zdunkowski, 1986; Fu and Liou, 1992). However, the literature on topic (ii) is scarce.

The first study in this direction seems to be the paper of Hall and Pruppacher (1976) who considered the influence of radiation on the falling distances of precipitating ice particles: radiative cooling of ice crystals prevents them from quick evaporation, thus they can fall longer distances. Stephens (1983) performed a similar study and employed a refined radiation calculation. In the same year Wendling studied the influence of the 8–12 μm radiation on growing ice crystals and compared the effects for different crystal habits. In a recent paper, Knollenberg et al. (1993) calculated the radiative energy gains and losses for ice crystals in tropical cumulonimbus anvils. They found indications for the possibility that radiation plays a dominant role in cloud microphysics and stressed the necessity of research on this topic.

Ramaswamy and Detwiler (1986) and Zhang et al. (1989) have taken into account the radiative effects on crystal growth in their thermodynamical model of cirrus cloud evolution. These authors conclude that the influence of the radiation is non-negligible. However, their computation of the radiative energy gain and loss of individual crystals is rather crude: they simply compute the radiative flux divergences and distribute them among the individual ice crystals, applying an ad hoc weighting that is not physically established (cf. Eq. (11) of Ramaswamy and Detwiler, and Eqs. (2.5–2.7) of Zhang et al.).

In the present paper I investigate the effect of radiation on the heat balance of individual ice crystals and the consequences for their diffusional growth or evaporation. The intention is to achieve a better understanding of the physics of this effect. The subsequent consequences for ice cloud evolution are not considered here. First the heat budget equation is analysed. In doing so the complicated radiative transfer computations are circumvented by formulation of the radiative energy gains and losses as grey body fluxes. Already this analysis will yield the most important finding, namely that radiation only affects the growth (evaporation) of large crystals having sizes of more than a few tens of microns. This statement is valid in all circumstances, independent of the thermodynamic state of the environmental air (temperature, pressure, relative humidity) and independent of sedimentation velocity (ventilation) and kinetic effects. For crystals a few 100 micron in size radiation dominates the diffusional growth and can pose upper size limits even in highly supersaturated air. From the heat budget equation I will derive an explicit formulation for the diffusional growth (evaporation) of ice crystals (or other hydrometeors) that are subject to a radiation field.

Calculations of the grey body fluxes for realistic cases will show that unfortunately it is almost impossible to predict whether in a given situation the crystals would be heated or cooled. The reason for this is the dependence of the crystal's energy gain on many parameters: the state of the atmosphere from ground to top (cloudiness, profiles of temperature, pressure, water vapour partial density etc.), the position of the sun, and the size and location of the crystal itself. This means that the determination of the radiation field is inevitable in studies of the microphysical evolution of clouds, in particular when large crystals are present.

The heat budget of ice crystals is analysed in Section 2. Their emissivity and the radiation field are calculated for various situations in Section 3. I discuss the results in Section 4 and present the conclusions in Section 5.

2 The Heat Balance of an Ice Crystal

The heat balance for a growing or evaporating ice crystal can be expressed in the following way (cf. any textbook on cloud physics):

$$mc_{ice}\frac{dT_s}{dt} = L\frac{dm}{dt} - \frac{dE}{dt}. \tag{1}$$

Here, m is the crystal's mass, c_{ice} is the specific heat of ice, so mc_{ice} is the heat capacity of the crystal, L is the latent heat of sublimation, dm/dt is the growth or evaporation rate and dE/dt is the rate of energy exchange of the crystal with its environment. dT_s/dt is the rate of temperature change resulting from the energy released to or transported away from the crystal.

The crystal exchanges energy with its environment by conduction and radiation:

$$\frac{dE}{dt} = \frac{dE_{con}}{dt} + \frac{dE_{rad}}{dt}. \tag{2}$$

In Section 4 it will be shown that ventilation factors and kinetic correction terms have no effect on the present analysis. They will therefore be neglected. Also spherical particles will be assumed in order to simplify the subsequent equations and with regard to the radiative transfer computations in the next section. This is probably not too serious a restriction of generality since Stephens (1983) demonstrated that cylindrical and spherical ice particles absorb similar amounts of radiative energy per unit surface area and that infrared absorption is not much affected by the orientation of an ice crystal (Stephens, 1980). Then the following simple relations are obtained (e.g. Rogers, 1979, chapter 6):

$$\begin{aligned} dm/dt &= 4\pi r D(\rho_\infty - \rho_s) \\ dE_{con}/dt &= 4\pi r K(T_s - T_\infty). \end{aligned} \tag{3}$$

Here, r is the particle radius, D and K are the diffusivities of vapour and heat in moist air, T_s and ρ_s are the temperature of the crystal itself and the H_2O vapour partial density immediately at the crystal's surface, T_∞ and ρ_∞ are the temperature and the H_2O vapour partial density in the environment. The above pair of differential equations is based on the premises of stationary growth (i.e. the growth process does not change the ambient vapour field) and the single particle approximation. The assumption of stationarity produces only negligible errors (Rogers, 1979, chapter 6); the case of a crystal population consisting of many particles that compete for the available water vapour can be treated by inclusion of a third prognostic equation for $\rho_\infty(t)$. This is done anyway in a dynamical cloud model, but for the present purpose it is not necessary.

The radiational energy exchange dE_{rad}/dt consists of an absorption and an emission part. The emission part can be written down easily as the luminosity of a spherical grey body:

$$dE_{rad,\,em}/dt = 4\pi r^2 \varepsilon \sigma T_s^4, \tag{4}$$

where σ is the Stefan-Boltzmann constant. The emissivity ε depends – as will be seen lateron – on the crystal size and weakly on its temperature. Anticipating the results of Section 3.1 I will use $\varepsilon = 0.14$ (0.90, 1.07, 0.93) for $r = 1$ (10, 100, 1000) μm. For the theoretical treatment of the crystal's heat balance it is useful to devise a similar expression for the absorbed energy, as

$$dE_{rad,\,abs}/dt = 4\pi r^2 \alpha \sigma T_\infty^4. \tag{5}$$

This expression is interpretable as follows: The crystal is irradiated by the environment which radiates as a grey body of emissivity α. Obviously, the whole radiative transfer problem is contained in the determination of α which generally depends on the state of the atmosphere, the solar zenith angle, and the location and size of the ice crystal.

However, for a first estimate of the radiational effects on ice crystal growth it suffices to assume reasonable values of α, namely $\alpha < \varepsilon$ for a cooling situation (e.g. $\alpha/\varepsilon = 0.5$), and $\alpha > \varepsilon$ for a heating case (e.g. $\alpha/\varepsilon = 1.5$). This means that one can find out some principles about the influence of radiation on crystal growth without actually solving the radiative transfer problem.

The heat balance of the growing or evaporating crystal is expressed as $dT_s/dt = 0$. Thus, in equilibrium

$$4\pi r L D(\rho_\infty - \rho_s) - 4\pi r K(T_s - T_\infty) - 4\pi r^2 \sigma(\varepsilon T_s^4 - \alpha T_\infty^4) = 0. \tag{6}$$

For the analysis this equation is reformulated in the following way:

$$L D \rho_\infty + K T_\infty + r\sigma\alpha T_\infty^4 = L D \rho_s + K T_s + r\sigma\varepsilon T_s^4. \tag{7}$$

In this equation, the left-hand side is a constant for given external parameters ($p_\infty, \rho_\infty, T_\infty, \alpha$) and a certain size r of the crystal under consideration. It will be written "φ_0". Also the "constants" D, L and K depend on the external parameters. The number of free parameters can be reduced by choosing a certain atmosphere (i.e. a relation between p_∞ and T_∞) and a certain supersaturation with respect to ice, e.g. a water saturated situation: $\rho_\infty = \rho_{sat,\,liq}(T_\infty)$.

The right-hand side of the equation is a function of the ice crystal's temperature T_s. It will be denoted "$\varphi(T_s)$". The vapour density at the crystal's surface corresponds to saturation with respect to ice: $\rho_s = \rho_{sat,\,ice}(T_s)$. Equilibrium is thus equivalent to

$$\varphi(T_s) = \varphi_0 \quad \text{or} \quad \varphi(T_s) - \varphi_0 = 0. \tag{8}$$

A free parameter that occurs both in $\varphi(T_s)$ and in φ_0 is the crystal radius r. The zeroes of $\varphi(T_s) - \varphi_0$ have been determined for $1\,\mu m \le r \le 1000\,\mu m$ and for $r = 0$. Obviously, the case $r = 0$ corresponds to a no radiation case, and the other cases can be easily compared with this one. The comparison then demonstrates the effect of the radiation. Although spherical ice crystals as large as 1 mm do not exist in clouds they serve here as a model for large crystals in other habits like plates and columns that reach maximum dimensions (diameter, length) of a few mm (e.g. Heymsfield and Platt, 1984). This simplification may be allowed when the surface area of the considered spherical crystal is equal to that of the natural nonspherical particle since then both absorb similar amounts of radiative energy (Stephens, 1983).

For the choices $\alpha/\varepsilon = 1.5$ (i.e. radiative heating) and $\alpha/\varepsilon = 0.5$ (cooling) the zeroes of $\varphi(T_s) - \varphi_0$ have been determined for four atmospheric conditions that occur in a midlatitude winter standard atmosphere (McClatchey et al., 1972). These are listed in Table 1.

The results are presented in the following way: Let T_s^* be the zero of $\varphi(T_s) - \varphi_0$. For the case $r = 0$ (no radiation) the difference $\Delta T_0 = T_s^* - T_\infty$ is determined. For the other cases ($r \neq 0$, with radiation) the additional difference $\Delta T_{rad} = T_s^*(r) - T_s^*(0)$ is calculated. ΔT_0 is positive for a growing ice crystal (assumed here) and negative for an evaporating one. ΔT_{rad} is positive for a heating situation and negative for a cooling one. Finally, it is possible to express ΔT_{rad} in terms of the effective change of the relative humidity with respect to ice saturation, ΔrH, as was defined by Hall and Pruppacher (1976) and also used by Stephens (1983):

$$\Delta rH = -\frac{1}{\rho_{sat,\,ice}(T_\infty)} \left(\frac{\partial \rho_{sat,\,ice}}{\partial T}\right)_{T = T_s^*(0)} \Delta T_{rad}. \tag{9}$$

Table 1 Atmospheric conditions used for the analysis of the heat balance of an ice crystal.

Case	T_∞/K	p_∞/hPa	α/ε
a	219	256	1.5
b	219	256	0.5
c	238	400	1.5
d	238	400	0.5
e	262	790	1.5
f	262	790	0.5
g	272	1018	1.5
h	272	1018	0.5

The results of the calculation are presented in Table 2 and in Figures 1 and 2.

From the balance Eq. (6) it is immediately clear that the influence of radiation on crystal growth or evaporation is the stronger the larger the crystal is, since both radiation terms are proportional to r^2 whereas all the other terms are proportional to r. This realization is reflected in the results compiled in Table 2. The radiation influence is – independent of the thermodynamic situation (T_∞, p_∞) – vanishingly small for r = 1 μm particles, almost negligible for 10 μm, weak for 100 μm, and strong for particles with r = 1 mm. Figures 1 and 2 show that the radiative effects expressed as ΔT_{rad} or ΔrH are

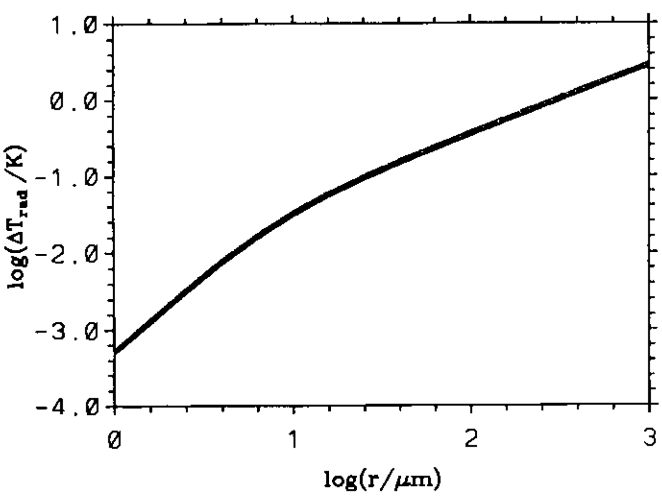

Figure 1 Radiation induced warming of a growing ice crystal as a function of the crystal radius. The four curves refer to the four warming cases ($\alpha/\varepsilon = 1.5$) of Table 1: a (solid line), c (dotted), e (short dashed), and g (long dashed).

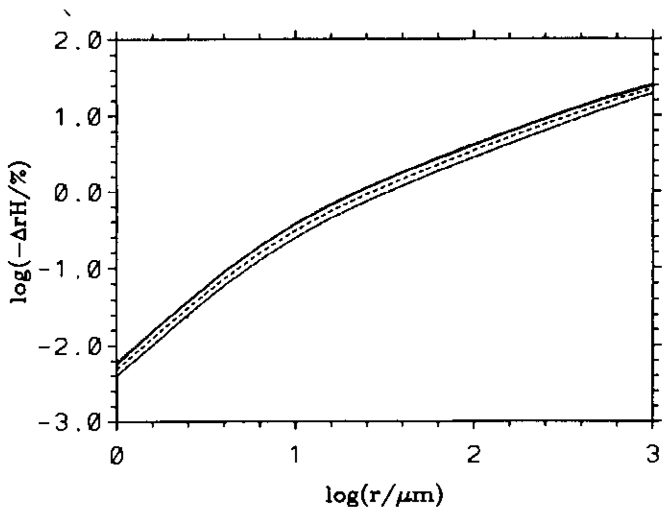

Figure 2 Radiation induced increase of the relative humidity (with respect to ice) for a growing ice crystal as a function of the crystal radius. The four curves refer to the four cooling cases ($\alpha/\varepsilon = 0.5$) of Table 1: b (solid line), d (dotted), f (short dashed), and h (long dashed).

Table 2 Radiation induced changes of the temperature, ΔT_{rad}, and relative humidity, ΔrH (in percent), of a growing ice crystal. The change of temperature that is caused by latent heat release alone is ΔT_0.

Case	ΔT_0	ΔT_{rad} [K]				ΔrH [%]			
	[K]	1 μm	10 μm	100 μm	1 mm	1 μm	10 μm	100 μm	1 mm
a	.12	.0005	.03	.33	2.7	- 0.01	- 0.36	- 4.31	- 45.7
b	.12	-.0005	-.03	-.34	- 2.8	0.01	0.37	4.10	25.0
c	.41	.0006	.03	.39	3.1	- 0.01	- 0.37	- 4.39	- 45.0
d	.41	-.0006	-.03	-.40	- 3.3	0.01	0.37	4.17	25.9
e	.45	.0006	.03	.38	3.0	0.00	- 0.29	- 3.44	- 33.3
f	.45	-.0006	-.03	-.40	- 3.3	0.01	0.30	3.40	22.9
g	.06	.0005	.03	.35	2.8	0.00	- 0.24	- 2.88	- 27.2
h	.06	-.0005	-.03	-.36	- 3.1	0.00	0.25	2.76	19.7

proportional to r^m where the exponent m varies between about 2 for the smallest and 1 for the largest crystals considered. In the Appendix it is demonstrated that this dependence arises from the dependence of the emissivity ε on the crystal radius. The radii of ice spheres considered by Hall and Pruppacher (1976) and Stephens (1983) range from 40 µm up to 160 µm. The relative humidity changes ΔrH found by these authors are generally a few percent which is in accordance with the present results. Also their results show that the influence of radiation is the stronger the larger the ice particle is. I have repeated the above calculations also with 1 % and 10 % supersaturation with respect to ice in the environment. This altered the amount of latent heat released to the ice particle; hence the equilibrium temperatures T_s^* of the crystals now differ from those of the situation considered above. Accordingly, the values of ΔT_0 differ from those given in Table 2. Although the crystal temperature and therefore also the emission term ($\sim \varepsilon\sigma(T_s^*)^4$) are changed now, the additional effects of radiation, which correspond to ΔT_{rad} and ΔrH, hardly differ from those in Table 2. This is easily explained: Even if T_s^* changed by $\delta T_s^* = 1$ K because of variations in the environmental conditions, the relative change $\delta T_s^*/T_s^*$ would be less than half a percent. This corresponds to a change of $1.005^4 = 1.02$, i.e. 2 percent at most in the emission term. Thus, the effect of radiation on the growth or evaporation of ice crystals is almost not modified by relative humidity variations in the environment.

Finally, I derive an explicit differential equation for dm/dt or dr/dt as an analytical approximation for the crystal growth. Since $|\Delta T| = |T_s^* - T_\infty| << T_\infty$, one may write

$$(T_s^*)^4 \approx T_\infty^4 + 4T_\infty^3 \Delta T \tag{10}$$

and neglect higher powers of ΔT. This can be inserted in the balance equation in the form dm/dt = (1/L) dE/dt and yields:

$$dm/dt = (4\pi r/L)\,[G_r(T_\infty)\,\Delta T + r\sigma(\varepsilon - \alpha)\,T_\infty^4] \tag{11}$$

with

$$G_r(T_\infty) = K + 4\varepsilon(r)\,r\sigma T_\infty^3. \tag{12}$$

Following Srivastava and Coen (1992) one approximates the saturation vapour density difference as a linear function of the temperature difference ΔT:

$$\rho_s \approx \rho_{sat,ice}(T_\infty) + \left(\frac{\partial \rho_{sat,ice}}{\partial T}\right)_{T=T_\infty} \Delta T. \tag{13}$$

Inserting this and the former approximation into the balance Eq. (6) and resolving for ΔT yields

$$\Delta T = \frac{LD\,s\,\rho_{sat,ice}(T_\infty) - r\sigma(\varepsilon-\alpha)T_\infty^4}{G_r(T_\infty) + LD\left(\frac{\partial \rho_{sat,ice}}{\partial T}\right)_{T=T_\infty}}, \tag{14}$$

s being the ambient supersaturation with respect to ice. The expression for ΔT is now inserted in Eq. (11). This yields after some simple algebra

$$dm/dt = \frac{4\pi r D\,\rho_{sat,ice}(T_\infty)s + 4\pi r^2\sigma(\varepsilon-\alpha)T_\infty^4\Gamma_r(T_\infty)/L}{1 + \Gamma_r(T_\infty)} \tag{15}$$

with

$$\Gamma_r(T_\infty) = LD\left(\frac{\partial \rho_{sat,ice}}{\partial T}\right)_{T=T_\infty} / G_r(T_\infty). \tag{16}$$

For spherical ice crystals, dr/dt = (dm/dt) $(4\pi r^2 \rho_{ice})^{-1}$. Thus

$$dr/dt = \frac{D\,\rho_{sat,ice}(T_\infty)s/r + \sigma(\varepsilon-\alpha)T_\infty^4\Gamma_r(T_\infty)/L}{\rho_{ice}(1 + \Gamma_r(T_\infty))} \tag{17}$$

is the desired analytical approximation for the diffusional growth under the influence of radiation.

3 The Radiation Terms: Determination of α and ε

In the previous section it was possible to analyse the heat balance of an ice crystal without actually solving the radiative transfer problem by introduction of two dimensionless parameters, α and ε, so that one could write the absorbed and emitted powers in the form of grey body fluxes. In this Section I compute the emissivity ε as a function of particle size and temperature, and the absorptivity α, which depends on the radiation field and the size of the ice crystal. In order to have realistic radiation fields, these are determined for a variety of situations that occur in standard atmospheres (taken from McClatchey et al., 1972). Unlike the case of dynamical cloud models, for the present investigation it is not required to take into account the influence of the ice crystal back on the incident radiation field. For the calculation of α the radiation field is simply regarded to be given.

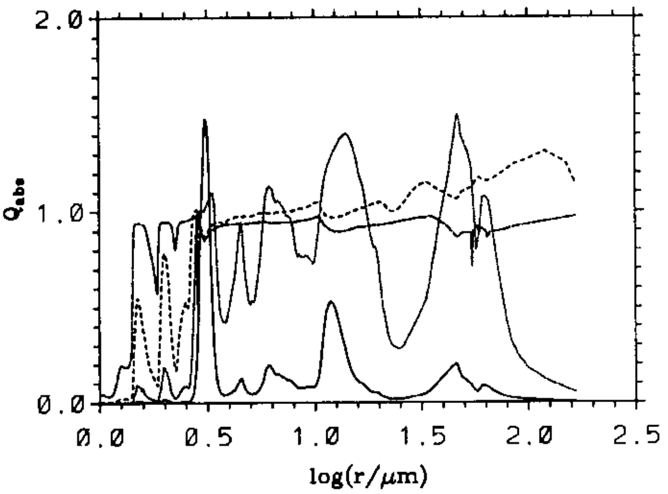

Figure 3 Absorption efficiency of spherical ice crystals as a function of wavelength. The crystals have radii of 1 μm (solid line), 10 μm (dotted), 100 μm (short dashed), and 1 mm (long dashed).

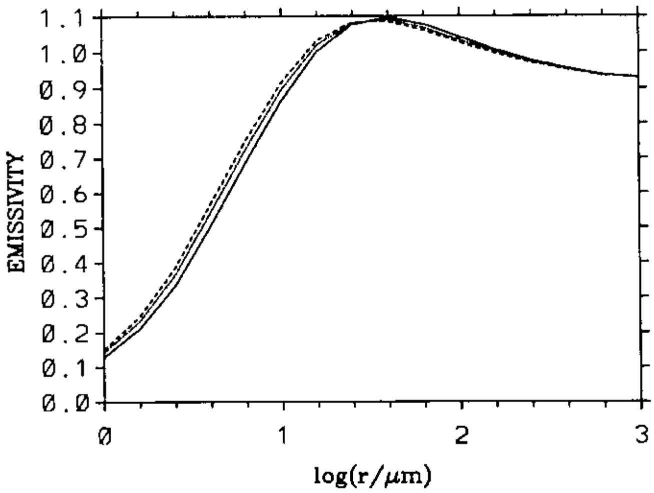

Figure 4 Variation of the emissivity of spherical ice crystals with their radius. Temperatues of − 60 °C (solid), − 30 °C (dotted), and 0 °C (dashed) have been assumed.

3.1 The Emissivity of Small Ice Particles

According to Kirchhoff's law, the monochromatic emissivity of a body having temperature T is given by its absorptivity times the Planck function $B_\lambda(T)$. The absorptivity of spherical particles, $Q_{abs}(\lambda, r)$, is the absorption efficiency factor from the Mie-scattering theory. The dependence of Q_{abs} on wavelength λ and particle radius r is shown in Figure 3 where Q_{abs} is plotted for $1\ \mu m \leq \lambda \leq 167\ \mu m$ and for radii $r \in \{1, 10, 100, 1000\ \mu m\}$ (cf. Knollenberg et al., 1993). For the calculation of Q_{abs} the refraction indices for ice tabulated by Warren (1986) have been used.

The emissivity ε is the Planck-weighted mean of $Q_{abs}(\lambda, r)$:

$$\varepsilon(T, r) = \frac{\int_0^\infty Q_{abs}(\lambda, r)\, B_\lambda(T)\, d\lambda}{\int_0^\infty B_\lambda(T)\, d\lambda}, \qquad (18)$$

or

$$\varepsilon(T, r) = [(\sigma/\pi)T^4]^{-1} \int_0^\infty Q_{abs}(\lambda, r)\, B_\lambda(T)\, d\lambda. \qquad (19)$$

These integrals have been evaluated for radii from 1 μm up to 1 mm and for temperatures in the range $-60\ °C \leq T \leq 0\ °C$. It turns out that the dependence of ε on T is very weak so that $\varepsilon(T, r)$ is almost constant within the investigated temperature range. The reason for this is that the maximum of the Planck function varies only little between − 60 °C ($\lambda_{max} = 23.9\ \mu m$) and 0 °C ($\lambda_{max} = 18.7\ \mu m$). However, the dependence of ε on the particle size is strong, in particular for the very small particles, i.e. $r \leq 20\ \mu m$ (see Figure 4). This is obviously a consequence of the strong dependence of Q_{abs} on the particle size. The absorption efficiency is generally low for $r = 1\ \mu m$, except for the strong peak at $\lambda = 3\ \mu m$. This peak however is too far away from the maxima of the Planck functions used. Thus its contribution to the emissivity is vanishingly small.

For $r \leq 20\ \mu m$ the absorption efficiency increases monotonically with r in the important wavelength region around $\lambda = 20\ \mu m$.Hence, ε increases monotonically from values of ≈ 0.15 at $r = 1\ \mu m$ to $\varepsilon \approx 1$ at $r = 20\ \mu m$. For larger particles ($r \geq 20\ \mu m$), $\varepsilon \approx 1 \pm 0.1$ and varies only weakly with size, since also Q_{abs} varies little with r at λ_{max}. For intermediate sized crystals (20–100 μm) ε exceeds unity. These particles are blacker than a black body! This seemingly paradox is explained by the fact that $Q_{abs} > 1$ for these sizes in a considerable portion of the spectrum. In this situation, the crystal is able to absorb photons that pass it in a distance of a few wavelengths (interprete photons as wave packets that have an extension of about λ). In thermodynamic equilibrium the ice particle must reemit these photons in order to keep a constant temperature.

3.2 Absorption of Solar Radiation

The solar irradiation on the ice crystal consists of a direct unscattered contribution and an indirect scattered one. We can see it with our eyes, that in the visual spectrum the direct contribution is much stronger than the indirect one. In the NIR (near infrared, $\lambda = 1 - 4\ \mu m$) the unscattered part dominates even more, since the Rayleigh scattering, which produces the scattered component, varies with λ^{-4}. An ice crystal, however, can only absorb NIR radiation (and longer wavelengths). It is therefore justified to neglect the scattered component of the solar radiation and to regard only the direct contribution.

The direct solar NIR radiation is black body radiation with $T = 5783$ K (Goody and Yung, 1989, A.9.3) that is attenuated along the path from the top of the atmosphere to the location of the ice crystal. Let the (wavelength dependent) transmission along this path be $T_\lambda(\zeta_\odot, z)$ with $\zeta_\odot$ the zenith angle of the sun, and z the altitude of the ice crystal in the atmosphere. Then, the intensity that hits the particle is $T_\lambda B_\lambda(5783\,K)$. The intensity is assumed to be independent of direction within the small solar solid angle $\omega_\odot$ (= 6.13×10^{-5} sr). Outside of $\omega_\odot$ the intensity is zero. The particle absorbs the solar radiation with an absorption cross section of $\pi r^2 Q_{abs}(\lambda, r)$. With these prerequisites the gain of solar radiation can now be formulated as:

$$dE_{rad,\,sol}/dt = \pi r^2 \omega_\odot \int_{1\mu}^{4\mu} T_\lambda(\zeta_\odot, z)\, Q_{abs}(\lambda, r)\, B_\lambda(5783\,K)\, d\lambda. \quad (20)$$

The solar NIR is attenuated in 4H_2O bands, designated as Ψ (1.25–1.54 μm), Ω (1.70–2.08 μm), X (2.27–2.99 μm), and 3.2 μ (2.99–3.57 μm). The mean absorption A_λ (or mean transmission $1 - A_\lambda$) in these bands can be computed with empirical formulae that have been derived by Howard et al. (1955, 1956). These formulae and their corresponding parameters are given in Kondratyev's book (1969,eq. 3.51 and Table 3.5). Using these formulae, the mean transmission for various standard atmospheres (McClatchey et al., 1972), sun zenith angles $\zeta_\odot$ and ice crystal locations z have been computed. The results are compiled in Table 3. Attenuation in the four NIR CO_2 bands (see Kondratyev, 1969, Table 3.14) is negligible.

Now, with the transmission probabilities at hand, the NIR energy gain $dE_{rad,\,sol}/dt$ can be computed. The corresponding value of α, α_{sol}, is definied as

$$\alpha_{sol} = (4\pi r^2 \sigma T_\infty^4)^{-1} dE_{rad,\,sol}/dt. \quad (21)$$

The results are summarized in Table 4. It turns out that as a function of the particle radius $\alpha_{sol}(r) \propto r^x$ where $x \approx 1/3$. This is valid in the whole range from 1 μm to 1 mm.

3.3 Absorption of Terrestrial Radiation

Consider an ice crystal of arbitrary shape and a fixed direction in space that is denoted by a pair of angles (ϑ, φ). Let the particle's cross section in a plane perpendicular to (ϑ, φ) be $\Sigma(\vartheta, \varphi)$. The external radiation field is specified in the (ϑ, φ) coordinate system and the ice crystal is assumed to be small enough, so that $I_\lambda(\vartheta, \varphi)$ would be independent of position at the location of the ice particle if the particle were absent. The energy gain of the ice particle in the wavelength region $\lambda \geq 4$ μm is then given by:

$$dE_{rad,\,ter}/dt = \int_{4\mu}^{\infty} d\lambda \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta\, \Sigma(\vartheta, \varphi)\, Q_{abs}(\lambda, r)\, I_\lambda(\vartheta, \varphi). \quad (22)$$

It is assumed that the radiation field is independent of the azimuth angle φ and that the crystal is a sphere, so that $\Sigma(\vartheta, \varphi) = \pi r^2$. Then:

$$dE_{rad,\,ter}/dt = (2\pi)(\pi r^2) \int_{4\mu}^{\infty} d\lambda \int_0^{\pi} \sin\vartheta\, d\vartheta\, Q_{abs}(\lambda, r)\, I_\lambda(\vartheta). \quad (23)$$

Table 3 Mean transmission in the NIR H_2O bands for various standard atmospheres (McClatchey et al., 1972), sun zenith angles $\zeta_\odot$ and ice crystal locations z. The temperatures T_∞ at these locations are given, too.

No.	Atmosphere	$\zeta_\odot$	z/km	T_∞/K	Mean transmission			
					Ψ	Ω	X	3.2 μ
1	midlat. winter	60 °C	10	219	.90	.87	.72	.93
2	midlat. summer	30 °C	10	235	.90	.87	.72	.93
3	US standard	45 °C	10	223	.90	.88	.73	.93
4	tropic	0 °C	15	204	.965	.956	.900	.976

Table 4 Solar energy gain (Watt) and values of α_{sol} for various standard atmospheres. Sun zenith angles $\zeta_{\odot}$, ice crystal locations z and temperatures T_{∞} are as in Table 3.

No.	$dE_{rad,sol}/dt$				α_{sol}			
	1 μm	10 μm	100 μm	1 mm	1 μm	10 μm	100 μm	1 mm
1	4.02×10^{-11}	8.49×10^{-9}	2.29×10^{-6}	4.80×10^{-4}	0.02	0.05	0.14	0.29
2	4.02×10^{-11}	8.49×10^{-9}	2.29×10^{-6}	4.80×10^{-4}	0.02	0.04	0.11	0.22
3	4.03×10^{-11}	8.53×10^{-9}	2.30×10^{-6}	4.80×10^{-4}	0.02	0.05	0.14	0.27
4	4.40×10^{-11}	9.38×10^{-9}	2.50×10^{-6}	4.80×10^{-4}	0.04	0.08	0.20	0.42

Table 5 Terrestrial energy gain (Watt) and values of α_{sol} for various standard atmospheres. Sun zenith angles $\zeta_{\odot}$, ice crystal locations z and temperatures T_{∞} are as in Table 3.

No.	$dE_{rad,ter}/dt$				α_{ter}			
	1 μm	10 μm	100 μm	1 mm	1 μm	10 μm	100 μm	1 mm
1	2.54×10^{-10}	1.56×10^{-7}	1.79×10^{-5}	1.61×10^{-3}	0.16	0.95	1.09	0.98
2	3.32×10^{-10}	1.98×10^{-7}	2.20×10^{-5}	1.99×10^{-3}	0.15	0.91	1.01	0.92
3	1.36×10^{-10}	9.71×10^{-8}	1.13×10^{-5}	1.00×10^{-3}	0.08	0.55	0.64	0.57
4	2.38×10^{-10}	1.41×10^{-7}	1.60×10^{-5}	1.45×10^{-3}	0.19	1.14	1.30	1.17

Table 6 Values of α and α/ε for the four standard atmospheric situations of Table 3.

No.	α				α/ε			
	1 μm	10 μm	100 μm	1 mm	1 μm	10 μm	100 μm	1 mm
1	0.18	1.00	1.23	1.27	1.20	1.12	1.18	1.37
2	0.17	0.95	1.12	1.14	1.13	1.07	1.08	1.23
3	0.10	0.60	0.78	0.84	0.67	0.67	0.75	0.90
4	0.23	1.22	1.50	1.59	1.53	1.37	1.44	1.71

If the ice crystal were perfectly black ($Q_{abs} \equiv 1$) and inside a hohlraum ($I_\lambda(\vartheta, \varphi) = B_\lambda(T)$), the total energy gain ($0 \le \lambda \le \infty$) would be $4\pi r^2 \sigma T^4$.[1]

The radiation field I_λ has been computed for various standard atmospheres (McClatchey et al., 1972) using a Matrix-Operator method (after Plass et al., 1973). The code computes intensities for a set of zenit angles $\vartheta_i = \arccos \mu_i$, so that one can write:

$$dE_{rad,\,ter}/dt = $$

$$= (2\pi)(\pi r^2) \int_{4\mu}^{\infty} d\lambda\, Q_{abs}(\lambda, r) \sum_i I_\lambda(\mu_i)\, w_i, \quad (24)$$

where w_i are certain weighting factors. The calculated energy gains and the corresponding values of α_{ter} are presented in Table 5. The size dependence of $\alpha_{ter}(r)$ follows closely that of $\varepsilon(r)$ so that $\alpha_{ter}(r)/\varepsilon(r) \approx$ const. This can indeed be shown mathematically and follows ultimately from the insensitivity of $\varepsilon(r)$ with respect to the temperature in ranges relevant for the atmosphere.

4 Discussion

Table 6 collects the values of $\alpha = \alpha_{sol} + \alpha_{ter}$ and of α/ε for the four selected standard atmosphere situations of Table 3. The corresponding values of ΔT_{rad} and ΔrH are presented in Table 7. α/ε depends on the crystal size. Whether and how strong radiation cools or heats an ice crystal depends not only on the state of the atmosphere and the crystal's position therein, but also on its size.

Except for the US standard atmosphere in which a cirrus layer has been inserted between 9 and 10 km altitude, in all other investigated standard situations

[1] Stephens (1983) has erroneously introduced an additional factor of $\cos\vartheta$ in his Eq. (6), which is not justified by his basic Eq. (3). Accordingly, he would get for a black sphere inside a hohlraum the energy gain $2\pi r^2 \sigma T^4$, which is wrong by a factor of two.

Table 7 Values of ΔT_{rad} and ΔrH for the four standard atmospheric situations of Table 3.

No.	ΔT_{rad}/K				ΔrH/%			
	1 μm	10 μm	100 μm	1 mm	1 μm	10 μm	100 μm	1 mm
1	2×10^{-4}	7×10^{-3}	.12	1.98	0.00	-0.09	-1.54	-30.9
2	1×10^{-4}	4×10^{-3}	.05	1.33	0.00	-0.04	-0.59	-17.1
3	-3×10^{-4}	-.02	-.17	-.56	0.00	0.24	2.12	6.50
4	4×10^{-4}	.02	.24	3.20	-0.01	-0.25	-3.58	-69.5

the ice crystals are heated by the radiation which would reduce their growth rate via the accompanying lowering of the effective relative humidity. The heating is particularly strong in the tropical atmosphere with $\alpha/\varepsilon \approx 1.5$. The reduction of the effective relative humidity for the 1 mm particle reaches 70 % in this case. If the environmental supersaturation (with respect to ice) were less than 70 %, large particles could not grow any longer. This means that radiative heating poses an upper size limit for ice crystal growth.

In the two midlatitude atmospheres the radiative heating of ice crystals is more moderate; the reduction of relative humidity for 1 mm particles is about 30 % in the winter and 17 % in the summer atmosphere. Interestingly, the radiative effects on the crystal are larger in winter than in summer, although the crystal absorbs ≈ 25 % more energy in the summer. The reason for this are the different temperatures in the two cases (219 and 235 K), with the accompanying increase of the emission rate in summer by 32.5 %.

Only the US standard atmosphere yields a cooling situation. In this case the growing crystals were located at the top of a cirrus layer inserted in this atmosphere. This layer is assumed to be 1 km thick. Its optical thickness in the infrared spectral region is 0.71 which means that the IR irradiation of ice crystals at the top of the layer at $z = 10$ km is diminished by 50 %. Accordingly, the relative humidity for 1 mm particles is enhanced by about 7 %. ΔrH increases with the size of the particle, so this situation is favourable for the growth of large particles.

Next, the variation of the radiative effects with altitude z in the atmosphere is discussed. For this, the α values for $z = 5$ km in the midlatitude winter atmosphere have been computed. The results are $\alpha = 0.12$ (0.76, 0.90, 0.88) for $r = 1$ (10, 100, 1000 μm). In this case the radiation cools the ice particles when they are located deeper in the atmosphere. Although the IR energy gains are higher by about 30 %, also the ice crystals temperature is higher so that the increase of emission dominates the increase of absorption. For the US standard atmosphere at the same altitude, $\alpha = 0.16$ (0.95, 1.10, 1.06), so that just the opposite behaviour is found: the crystals are heated deeper in the atmosphere. Here, the energy gain in 5 km altitude is three times as large as in 10 km (presumably because of the cirrus layer), whereas the temperature variation is similar to that in the midlatitude winter atmosphere. Thus, here the increase of absorption dominates the increase of emission.

Regarding the results in Table 6, one finds that in all heating situations (cases 1, 2, and 4) α/ε is lower for particles having intermediate sizes (10 and 100 μm) than for the very small (1 μm) and very big (1 mm) particles. This means that situations are possible where small and big particles are heated (α/ε slightly exceeding 1) whereas those with intermediate size are cooled (α/ε slightly less than 1). Such a situation has been investigated by Knollenberg et al. (1993). They gave the following explanation for the differential cooling/heating behaviour: Very small crystals effectively absorb the solar NIR radiation in the 3 μm band, but they hardly emit at longer wavelengths. Thus they are heated. Larger particles also effectively emit and absorb in the 15 μm and 60–80 μm bands. However, these authors had a case where the infrared radiation field was attenuated by a thick cumulonimbus cloud. Accordingly, the emission of IR radiation exceeded the absorption and the crystals were cooled. Finally, the very large particles additionally absorb the solar NIR in the 1.5 μm and 2 μm bands, which sufficed to heat them. Obviously the absorption of solar NIR radiation is the major source of the differential cooling/heating. At night, when $\alpha_{sol} = 0$, α/ε is almost constant. The mid and far infrared radiation field together with the crystal temperature control whether there is cooling or heating. In the case considered by Knollenberg et al. the IR field is weak, therefore those crystals would have been cooled at night. In the present cloud free cases 1 and 4 the IR radiation is stronger, so crystals would be slightly heated. Case 2 yields at night a situation with $\alpha/\varepsilon = 1.00$ (1.02, 0.97, 0.99) for the four considered crystal sizes. The corresponding temperature changes are $\Delta T_{rad} = 0$ (+ 0.001, − 0.03, − 0.11) K, i.e. radiative equilibrium for the very smallest particles, very weak heating for small particles (10 μm) and slight cooling for the larger ones.

Furthermore the effect of the neglected kinetic correction terms and ventilation factors on the results has been investigated. These factors enter the heat balance equation as altered diffusivities for vapour and heat:

$$\begin{aligned} D &\rightarrow DF_\beta f_m \\ K &\rightarrow KF_\alpha f_Q . \end{aligned} \tag{25}$$

(I follow the notation of Hall and Pruppacher, 1976). It was shown that it depends primarily on the size of an ice crystal whether radiation influences the heat balance or not. Therefore the dependence of the correction factors on crystal size has to be investigated. The ventilation factors depend on the crystal size via their dependence on the Reynolds number. Both, f_m and f_Q vary from about 1 to 3.4 when r is increased by a factor of 1000 from 1 μm to 1 mm. Hence their variation is negligible. The kinetic correction terms depend directly on the particle radius:

$$\begin{aligned} F_\beta &= r/(r + l_m^*) \\ F_\alpha &= r/(r + l_Q^*) . \end{aligned} \tag{26}$$

The dependence is proportionality for $r << l_{(m, Q)}$. The critical lengths can be written in the form:

$$\begin{aligned} l_m^* &= 1.42 \times 10^{-5}\,\mathrm{m} \left(\frac{T}{T_0}\right)^{1.44} \left(\frac{p_0}{p}\right) f_m \\ l_Q^* &= 1.86 \times 10^{-6}\,\mathrm{m} \left(\frac{T}{T_0}\right)^{1/2} \left(\frac{p_0}{p}\right) f_Q , \end{aligned} \tag{27}$$

with $p_0 = 101\,325$ Pa and $T_0 = 273.15$ K. (For the derivation of these expressions I have used Eqs. (6), (9) and (13) of Hall and Pruppacher, with $\beta = 0.1$, $\alpha = 1$ and $K = 2 \times 10^{-2} \mathrm{Jm^{-1}s^{-1}K^{-1}}$). For typical values of T and p in cirrus layers, l_m^* and l_Q^* range from 10 to 100 μm. Thus effects on the results must be expected for small particles. Therefore, the calculations for the situations of Table 3 have been repeated and the new values of ΔT_{rad} and ΔrH were compared with those of Table 7. As expected, the results for 1 mm particles are hardly altered, those for r = 100 μm a little, and those for the smaller particles are strongly affected. However, the differences of the results are large only in a relative sense; in an absolute sense they are completely negligible. The values of ΔrH for the 1 micron particles are now (in percent): − 0.02 (− 0.01, 0.03, − 0.08) in the four regarded cases. For the 10 micron crystals we found $\Delta rH = -0.15$ (− 0.08, 0.40, − 0.57). Obviously, the neglection of the correction factors was justified and the conclusion that the growth of small particles is hardly affected by radiation remains valid.

Except for a location deep within a cloud (Stephens' (1983) blackbody depth) an ice particle experiences a radiation field that is markedly anisotropic. The crystal's upper hemisphere is irradiated by radiation from the cold sky, whereas the lower hemisphere sees the warmer layers of the atmosphere and the ground. Thus it is imaginable that there are temperature gradients within ice crystals that affect the diffusional growth. It will be shown now that this effect is small and may be neglected: in the extremest cases (i.e. no radiation from the sky at all) the temperature difference between the crystal's both hemispheres reaches 10 percent of ΔT_{rad}, the average temperature shift of the crystal as a whole that is caused solely by radiation.

The argumentation is as follows. The relaxation time τ of a spherical ice crystal of radius r is $\tau = r^2 \rho_{ice} c_{ice}/k$, where at − 50 °C $\rho_{ice} = 924$ kg/m^3, $c_{ice} = 1738$ J/kg/K, and the heat conductivity of ice k = 2.780 Watt/m/K (Ražnjević, 1976). For r = 1 mm, $\tau \approx 0.6$ s. This is the time needed to establish the temperature gradient. It is much shorter for smaller particles, since $\tau \sim r^2$. Suppose now that the radiation from the sky is suddenly switched off. Furthermore neglect for the moment the latent heat release and the heat conduction with the air. Then the upper half of the crystal cools down by emission with a rate given by:

$$dT/dt = -(3\varepsilon\sigma T^4)/(r \rho_{ice} c_{ice}) . \tag{28}$$

Suppose the lower half of the crystal to keep its temperature constant. Then after the relaxation time the temperature difference between the two hemispheres is:

$$\Delta T = |dT/dt|\,\tau = (3\varepsilon\sigma/k)\, rT^4 . \tag{29}$$

For T = − 50 °C and r = 1 mm, $\Delta T = 0.14$ K. In all realistic cases the actual temperature variation inside an ice crystal is smaller. This generally is less than 10 percent of all ΔT_{rad} values computed for the present paper. This is also true for smaller crystals because $\Delta T \sim r$.

Constructing a simple model for the heat conduction between two hemispheres of an ice crystal, it is possible to set up a coupled pair of heat balance equations for the two half crystals, having temperatures T_1 and T_2. The numerical solution of the equation system for an extreme case as above (i.e. no radiation from the sky) generally yielded $|T_1 - T_2| < 0.1\,\Delta T_{rad}$. The numerical results thus corroborate the statement found from the above simple argumentation, namely that temperature gradients within ice crystals are too small for a significant modification of the diffusional growth process. A considerable modification can only be achieved by a reduction of the heat conductivity k by at least 1 order of magnitude. Such small values of k may perhaps pertain to ice crystals that contain air bubbles or to conglomerates of many small crystallites. Such particles, however, have radiative properties that cannot be computed with Mie calculations. They are therefore beyond the scope of the present investigation but may be practically important.

There are at least two application grounds for the method developed in the present paper, i.e. the formulation of the direct radiative effects on the ice crystals (or other hydrometeors) via grey body fluxes. The first one may think of is the investigation of the temporal evolution of particle size (or mass) spectra for a particle population subject to a radiation field with the aid of the explicit growth Eqs. (15) and (17) of Section 2. Secondly, the method may be implemented in cloud models like those of Ramaswamy and Detwiler (1986) or Zhang et al. (1989). An improvement of these models could be reached as soon as the radiation field and hence $\alpha(r)$ is determined. Then the radiative influence on the individual crystals can be formulated directly instead of indirectly via the flux divergence and a weighting factor that is not physically established (Eq. (11) of Ramaswamy and Detwiler, and Eqs. (2.5)–(2.7) of Zhang et al.). Also the explicit growth equations of Section 2 could be incorporated in these models. In such a cloud model the effect of the temporal evolution of the crystal size spectrum on the radiation field and on $\alpha(r)$ must be taken into account by updating the radiation field at certain timesteps and a subsequent new determination of the crystals absorptivities.

5 Conclusions

In this paper I have studied the influence of radiation on the growth and evaporation of ice crystals in the atmosphere. The investigation commenced with an analysis of the heat balance equation for an ice crystal. The formulation of radiative energy gain and loss as grey body fluxes allowed to perform the analysis without actually solving the radiative transfer problem. The results of this section were: (i) The influence of radiation on ice crystals grows with their size; it is negligible for crystal radii less than a few tens of microns, but it is huge for mm sized particles whose surface relative humidity can be altered by more than 50 %. (ii) The effect of radiation on crystal growth (or evaporation) is not modified by the water vapour content in the surrounding air. Later it was found furthermore that effects of temperature gradients on the diffusional growth of ice crystals may be neglected.

Next, radiative transfer computations were performed in order to determine realistic grey body fluxes for ice crystals in various standard atmosphere situations. The emissivity of spherical ice particles was calculated as a function of their temperature and size and the T-dependence was found to be weak between $-60\,°C$ and $0\,°C$. However, ε depends strongly on crystal size, and its values for typical ice crystal sizes in the atmosphere cover the whole range from $\varepsilon \approx 0$ up to $\varepsilon \gtrsim 1$. The absorption parameter α has been determined for the direct solar near infrared and the terrestrial radiation. The ratio ε/α in the calculations varied between about 0.5 and 1.5 at night and 0.7 and 1.7 at daytime. The solar contribution to α ranges from 10 to 25 % in a clear atmosphere and can reach 35 % in a cloudy one. It must have reached 50 % in the cumulonimbus anvil observed by Knollenberg et al. (1993). For a given situation, $\alpha_{ter}(r)/\varepsilon(r) \approx$ const, whereas $\alpha_{sol}(r) \propto r^x$ with $x \approx 1/3$.

Unfortunately, it seems to be impossible to predict the degree of heating or cooling for a given ice particle in a given situation without a solution of the radiative transfer problem. Whereas it is easy to compute the radiative energy loss of a crystal, the estimation of α is difficult since it depends on the state of the atmosphere, the crystals position therein and its size. E.g. in a clear midlatitude winter atmosphere ice crystals are cooled at $z = 5$ km but heated 5 km above. In the US standard atmosphere with a cirrus layer between 9 and 10 km altitude it is just opposite. Whereas because of the blocking of the radiation from the ground, cooling is probable when there are clouds underneath the considered ice layer, very small and very large crystals are heated in an anvil above a tropical cumulonimbus (Knollenberg et al., 1993), because they efficiently absorb solar radiation.

Radiation can pose an upper size limit for diffusional growth of ice crystals. Large crystals can be heated by radiation by a few degrees so that the relative humidity at the surface is reduced by some

10 percent. If the supersaturation in the environmental air is not higher than the radiation induced reduction, diffusional growth ceases. On the other hand, it is possible that crystals grow in a subsaturated environment by radiative cooling. However, since only relatively big particles react on the radiation, they would have to be furnished from a remote location. The radiative cooling of smaller particles is too weak that they could grow to larger sizes in subsaturated air.

Appendix
The Variation of ΔT_{rad} and ΔrH with the Crystal Radius

Figure 1 showed that $\Delta T_{rad} \sim r^m$ where $1 \lesssim m \lesssim 2$. It will be demonstrated now that this behaviour is explained from the variation of the emissivity ε on r. Let, as before $T_s^*(r)$ denote the zero of $\varphi(T_s) - \varphi_0$ (cf. Section 2), and $\rho_s(r) = \rho_{sat,ice}[T_s^*(r)]$. Then from the balance Eq. (7) one gets the following identiy:

$$LD[\rho_\infty - \rho_s(r)] - K[T_s^*(r) - T_\infty] - r\sigma[\varepsilon T_s^*(r)^4 - \alpha T_\infty^4] = 0 \tag{30}$$

and in particular for r = 0 (i.e. no radiation):

$$LD[\rho_\infty - \rho_s(0)] - K[T_s^*(0) - T_\infty] = 0. \tag{31}$$

Subtracting (30) from (31) yields:

$$LD[\rho_s(r) - \rho_s(0)] - K[T_s^*(r) - T_s^*(0)] + r\sigma[\varepsilon T_s^*(r)^4 - \alpha T_\infty^4] = 0, \tag{32}$$

where the desired difference $\Delta T_{rad} = T_s^*(r) - T_s^*(0)$ is contained in the second bracket. In the calculation that yielded ΔT_{rad} as a function of r, the ratio α/ε was kept constant, so that one may write $(\alpha/\varepsilon)T_\infty^4 = \Theta^4 =$ const. Resolving for ΔT_{rad} one arrives with:

$$\Delta T_{rad} = r\varepsilon(r)\,\sigma[\Theta^4 - T_s^*(r)^4]K^{-1} - LD[\rho_s(r) - \rho_s(0)]K^{-1}. \tag{33}$$

The resolution for ΔT_{rad} is not yet complete since the right-hand side contains functions of $T_s^*(r)$. However, since $\Delta T_{rad} << T_s^*$ it is allowed to perform the following expansions:

$$\rho_s(r) \approx \rho_s(0) + \left(\frac{\partial \rho_{sat,ice}}{\partial T}\right)_{T = T_s^*(0)} \Delta T_{rad} \tag{34}$$

and using the identity $T_s^*(r) = T_s^*(0) + \Delta T_{rad}$:

$$T_s^*(r)^4 \approx T_s^*(0)^4 + 4T_s^*(0)^3 \Delta T_{rad}$$

where higher powers of ΔT_{rad} have been neglected. With these expansions the resolution for ΔT_{rad} is complete:

$$\Delta T_{rad} = r\varepsilon(r)\frac{\sigma[\Theta^4 - T_s^*(0)^4]}{K + 4r\varepsilon(r)\sigma T_s^*(0)^3 + LD\left(\frac{\partial \rho_{sat,ice}}{\partial T}\right)_{T = T_s^*(0)}} = r\varepsilon(r)\frac{C_1}{C_2 + r\varepsilon(r)C_3} \tag{36}$$

with "constants" $C_{1,2,3}$. For the atmospheric parameters, say, of case a) in Section 2, $C_2 = 0.02$ W/m/K and $C_3 = 2.83$ W/m^2/K so that even for the largest crystals $C_2 >> r\varepsilon(r)C_3$. Thus

$$\Delta T_{rad} = r\varepsilon(r)(C_1/C_2) \tag{37}$$

is a very good approximation. It follows that ΔT_{rad} varies approximately according to $\Delta T_{rad} \sim r^{n+1}$ when $\varepsilon \sim r^n$. This can be seen by inspecting Figure 5 where the slopes d log ΔT_{rad}/d log r for case a of Section 2 and d log ε/d log r for −60 °C have been plotted together as functions of r. The change of the effective relative humidity ΔrH displays the same dependence on the crystal radius as ΔT_{rad} since (cf. Eq. (9)) it is proportional to the latter.

Note however, that in a given atmospheric situation α/ε is a function of particle radius (see Section 3):

$$\alpha(r)/\varepsilon(r) = \alpha_{ter}(r)/\varepsilon(r) + \alpha_{sol}(r)/\varepsilon(r) \approx c + c'r^x/\varepsilon(r), \tag{38}$$

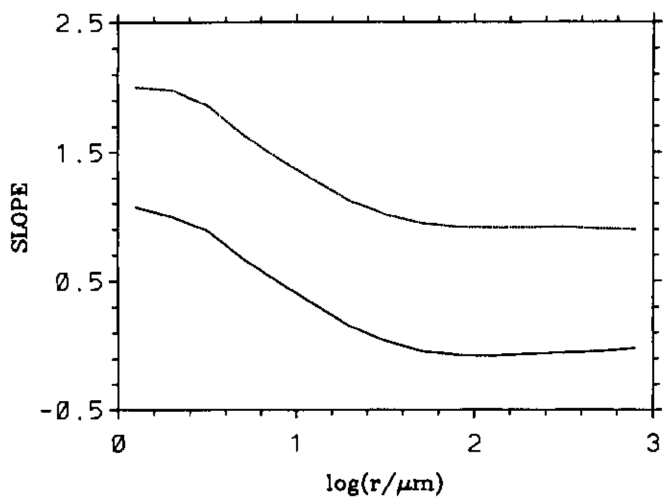

Figure 5 d log ΔT_{rad}/d log r for case a of sect. 2 (dotted) and d log ε/d log r for −60 °C (solid) as functions of the crystals radius r. The curves are nearly parallel with a distance of 1. This shows that the radiation induced warming (or cooling) of a growing ice crystal is almost proportional to the crystal radius and the emissivity ε: $\Delta T_{rad} \sim r\varepsilon(r)$.

with $x \approx 1/3$. This yields

$$\Delta T_{rad} \approx r\varepsilon(r)\,(C_4/C_2) + r^{1+x}(C_5/C_2) \qquad (39)$$

with $C_4 = \sigma[cT_\infty^4 - T_s^*(0)^4]$ and $C_5 = c'\sigma T_\infty^4$.

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