Transport Asymmetry in Skewed Convective Circulations

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ABSTRACT

Two simple kinematical models are used to describe the asymmetry in bottom-up and top-down diffusion in the convective boundary layer. Both models resolve the mean circulation with narrow updrafts and wider downdrafts but differ in the treatment of horizontal branches of the circulation. The models indicate that the transport asymmetry arises from the nonstationarity of the concentration field, the skewness of the flow field, and the inhomogeneous manner by which the species is introduced into the updrafts and downdrafts.

1. Introduction

Large-eddy simulations (Wyngaard and Brost 1984; Moeng and Wyngaard 1984) and several analytical studies (Sawford and Guest 1987; Wyngaard 1987; Weil 1990; Wyngaard and Weil 1991) have revealed a "transport asymmetry" in the convective boundary layer (CBL): A species introduced through an area source at the top of the CBL with zero flux through the bottom (i.e., one undergoing "top-down" diffusion) has a smaller mean diffusivity than one introduced at the bottom with zero flux at the top ("bottom-up" diffusion). As a consequence, the vertical concentration difference across the mixed layer of the CBL is smaller for bottom-up than for top-down diffusion. The local diffusivities may be ill conditioned, but the mean diffusivity is about 0.2 $w_* H$ for top-down diffusion and of order 0.4 $w_* H$ for bottom-up diffusion (Schumann 1989). Here, w_* is the convective velocity scale and H denotes the depth of the CBL. As discussed by Wyngaard and Weil (1991), the transport asymmetry is consistent with the observed differences in the diffusion from point sources at various levels of a laboratory tank experiment, with results from field experiments (Eberhard et al. 1988), large-eddy simulations (LES), and stochastic dispersion theory.

In the CBL, the area fraction α of updrafts varies with altitude. It equals 0.5 at the bottom and near the top of the CBL but is smaller in the mixed layer of the CBL, reaching minimum values in between 0.3 and 0.45 (Young 1988; Schumann 1989; Schumann and Moeng 1991a). An area fraction α of updrafts less than 0.5 implies a positive skewness $S = \overline{w'^3}/(\overline{w'^2})^{3/2}$ of vertical velocity fluctuations w', so that S increases from zero near the surface to values of the order unity in the upper part of the CBL and returns to zero at the top of the CBL.

Wyngaard (1987) showed that transport asymmetry is exhibited by a simple kinematical model with a skewed but homogeneous distribution of vertical velocity fluctuations. Sawford and Guest (1987) and Weil (1990) have qualitatively reproduced the results from LES using one-dimensional stochastic dispersion models for vertically inhomogeneous and for Gaussian turbulence statistics, suggesting an alternative explanation for the transport asymmetry. Weil (1990) and Wyngaard and Weil (1991) also considered inhomogeneous skewed turbulence and showed that the asymmetry is strongest in inhomogeneous, large time-scale convective turbulence. They concluded that the transport asymmetry is a fundamental consequence of the interaction of skewed turbulence with a scalar flux gradient and exists even in homogeneous turbulence. Wyngaard (1987) and Wyngaard and Weil (1991) also showed that nonstationarity contributes to this asymmetry. These theories make use of a statistical description of vertical velocity fluctuations. They do not use an explicit model of the convective circulation with vertical and horizontal branches.

In this paper we identify the importance of the convective circulation structure for the transport asymmetry. For this purpose, two very simple kinematical models are introduced. Both use a skewed flow structure and a scalar flux gradient. They differ in the manner by which the species enters the updraft and downdraft regions with the consequence that the transport asymmetry shows up in only one of them.

2. Two simple models for bottom-up and top-down diffusion

a. Model 1

We consider a section of horizontally homogeneous CBL with a simple flow configuration as sketched in

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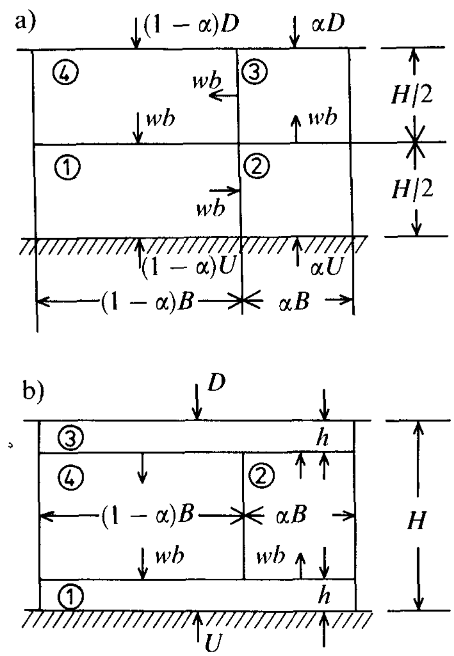

FIG. 1. Sketch of simple flow configurations for estimates of vertical diffusion.

Fig. 1a. The section moves with the mean wind horizontally so that advective changes are small. We assume that the flow is dominated by a steady circulation with updrafts and downdrafts extending over the whole boundary layer. The domain of constant height H and (arbitrary) width B is split into four subdomains. The bottom and top domains are of depth $H/2$: the right ones represent the updraft with a width αB, the left ones the downdraft with width $(1-\alpha)B$. The mean flow rate per unit length within the circulation is wb, where $b = B/2$. This flow rate from subdomain 1 to 2 is the same as that from 2 to 3, 3 to 4, and 4 to 1, because of continuity. For simplicity, we ignore any small-scale turbulent mixing between the subdomains but assume that the single domains become well mixed. A related model has been used in Schumann (1991) to explain the dynamics of the convective circulation. Its results agree qualitatively with observed properties of the CBL.

We consider a species that is emitted from the surface with a uniform flux density U or likewise from the top downwards at rate D. Either U or D is zero. As a consequence, the mean concentration $\bar{c}$ in the whole layer increases continuously by

$$d\bar{c}/dt = (U + D)/H. \tag{1}$$

The mean vertical flux at midlevels in steady state is $(U - D)/2$. The mean concentration deviations from the volume mean $\bar{c}$ are c_i, $i = 1, 2, 3, 4$, in the four subdomains. Hence, the effective diffusivity K can be estimated from

$$\frac{U-D}{2} = -K\frac{\alpha(c_3 - c_2) + (1-\alpha)(c_4 - c_1)}{H/2}. \tag{2}$$

Integral balances of the changes in concentration deviations within the subdomains and the fluxes across its surfaces result in the following budget equations:

$$V_1 dc_1/dt = -wb(c_1 - c_4) + (1-\alpha)BU - V_1 d\bar{c}/dt, \tag{3}$$

$$V_2 dc_2/dt = -wb(c_2 - c_1) + \alpha BU - V_2 d\bar{c}/dt, \tag{4}$$

$$V_3 dc_3/dt = -wb(c_3 - c_2) + \alpha BD - V_3 d\bar{c}/dt, \tag{5}$$

$$V_4 dc_4/dt = -wb(c_4 - c_3) + (1-\alpha)BD - V_4 d\bar{c}/dt. \tag{6}$$

Here, $V_1 = V_4 = (1-\alpha)BH/2$, $V_2 = V_3 = \alpha BH/2$. The first term on the right-hand side of each of the preceding equations describes the advective fluxes from one subdomain to the next, the second term denotes the area sources, and the last the effective sink of concentration deviations due to mean concentration increase. The equations imply that the concentration deviations approach steady state after a time of order H/w while the mean concentration is increasing at constant rate. In this state, the required concentration differences can easily be found from the preceding equations:

$$wb(c_3 - c_2) = \alpha B(D - U)/2,$$
$$wb(c_4 - c_1) = (1-\alpha)B(D - U)/2. \tag{7}$$

Hence,

$$K = \frac{(D-U)H}{4[\alpha(c_3 - c_2) + (1-\alpha)(c_4 - c_1)]} = \frac{Hw}{4(1 - 2\alpha + 2\alpha^2)}, \tag{8}$$

independent of D and U. Thus, this model predicts equal diffusivities for bottom-up and top-down diffusion even for $\alpha \neq 1/2$. For $\alpha = 1/2$ the model gives $K = wH/2$, that is, about the right magnitude, $K \approx w_* H/4$, if $w = w_*/2$, which is a reasonable value; see Schumann (1989). The model predicts the diffusivity to decrease by up to a factor of 2 if α differs strongly from $1/2$. However, the model does not exhibit any transport asymmetry.

b. Model 2

In order to explain the asymmetry between bottom-up and top-down diffusion, we next consider the alternative simple flow configuration sketched in Fig. 1b. The domain of height H and width B is split again into four subdomains but with different topology. The bottom and top domains are of depth $h \ll H$ so that their volume can be ignored in comparison to subdomain 2, which represents the updraft region of width αB.

The downdraft in subdomain 4 is of width $(1-\alpha)B$. Otherwise, the model is as before. The model describes more realistically than the previous one the horizontal motion of fluid along the bottom surface by which any emitted species is carried directly into the updraft without direct diffusive fluxes into the downdraft. In this model, the effective diffusivity is estimated from

$$\frac{U-D}{2} = -K\frac{c_3-c_1}{H}. \tag{9}$$

Again, the concentration deviations from the mean $\bar{c}$ satisfy individual budget equations

$$V_1dc_1/dt = -wb(c_1-c_4) + UB - V_1d\bar{c}/dt, \tag{10}$$

$$V_2dc_2/dt = -wb(c_2-c_1) - V_2d\bar{c}/dt, \tag{11}$$

$$V_3dc_3/dt = -wb(c_3-c_2) + DB - V_3d\bar{c}/dt, \tag{12}$$

$$V_4dc_4/dt = -wb(c_4-c_3) - V_4d\bar{c}/dt. \tag{13}$$

Here, $V_1 = hB \ll V_2$, $V_2 = \alpha BH$, $V_3 = hB \ll V_2$, $V_4 = (1-\alpha)BH$. In steady state the required concentration difference can easily be found from the preceding equations:

$$wb(c_3-c_1) = (D-U)B/2 + (V_4-V_2) \times (1/2)d\bar{c}/dt = B[(1-\alpha)D-\alpha U]. \tag{14}$$

Hence,

$$K = \frac{Hw/2}{1+(1-2\alpha)\dfrac{H}{D-U}\dfrac{d\bar{c}}{dt}} = \frac{(D-U)Hw}{4[(1-\alpha)D-\alpha U]}. \tag{15}$$

For $D = 0$, $U \neq 0$, we obtain the bottom-up diffusivity, and for $U = 0$, $D \neq 0$, the top-down diffusivity,

$$K_{\rm up} = Hw/(4\alpha),\ K_{\rm down} = Hw/[4(1-\alpha)]. \tag{16}$$

The two values are equal, $K = wH/2$, for $\alpha = 1/2$. This is the same result as in the previous model; see (8). The ratio of diffusivities is

$$K_{\rm up}/K_{\rm down} = (1-\alpha)/\alpha. \tag{17}$$

Another measure of transport asymmetry is $\Delta_{\rm down}/\Delta_{\rm up}$, where $\Delta_{\rm down} = c_3 - c_1$ for downward flux and $\Delta_{\rm up} = c_1 - c_3$ for upward flux. Equation (9) shows that $\Delta_{\rm down}/\Delta_{\rm up} = K_{\rm up}/K_{\rm down}$ in this model. Incidentally, the ratio $\Delta_{\rm down}/\Delta_{\rm up} = (1-\alpha)/\alpha$ is identical to that given by Wyngaard (1987) but the present model is different. From LES, $\Delta_{\rm down}/\Delta_{\rm up}$ is found in between 2.5 (Wyngaard and Brost 1984) and 4 (Wyngaard 1987).

As shown by Wyngaard (1987) the skewness and updraft area fraction are simply related. If the vertical velocity would be horizontally uniform within the updrafts and within the downdrafts, then

TABLE 1. Influence of updraft area on skewness and diffusivities.

α	1/2	0.4	1/3	0.3	1/4	0.2
S	0	0.408	0.707	0.873	1.155	1.5
$K_{\rm up}/K_{\rm down}$	1	1.5	2	2.33	3	4

$$S = \frac{1-\alpha}{\alpha^{1/2}(1-\alpha)^{1/2}},\quad \alpha = \frac{4+S^2-S(4+S^2)^{1/2}}{2(4+S^2)}. \tag{18}$$

However, this relationship underestimates the skewness in the CBL for given values of $\alpha < 0.5$ because it ignores contributions from skewness within updrafts as discussed by Hunt et al. (1988).

Table 1 lists some typical results from these equations. Obviously, the model gives qualitatively the correct trend but the computed asymmetry reaches the observed values only for α smaller than the realistic value of about 0.4. This means that other effects enhance the transport asymmetry. Such effects may include small-scale skewed motions, vertical inhomogeneity, and asymmetry of the vertical velocity variance with respect to level $z = H/2$ (Sawford and Guest 1987; Weil 1990). Equation (15) shows that the difference between bottom-up and top-down diffusion originates from the asymmetry of the volumes of those regions into which the upward and downward diffusing flux is induced, and from the presence of a temporal change in the mean concentrations. The asymmetry of the flow is measured by $\alpha = V_2/(V_2+V_4)$, but lies also in the fact that all fluxes go from the surface through subdomain 1 into 2, without any part mixed directly into subdomain 4, and similar conditions for the top-down diffusing species. The latter fact is essential and was not noted before.

Hence, the transport asymmetry is caused in the CBL by the skewed turbulence structure composed of strong updrafts that shrink in mean cross section with altitude within the mixed layer, and are surrounded by weak and wide downdrafts. Different sizes of updrafts and downdrafts are necessary but not sufficient for the existence of a transport asymmetry; vertical variation in the width of updrafts is important too.

The analysis shows further the importance of the time dependence of the concentrations that lead to the final sink terms in the preceding budget equations. We expect similar changes in the diffusivities if the concentration changes not because of unsteadiness but because of other source or sink terms such as chemical production or destruction. Such effects have been, in fact, observed in LES of the transport of chemically reacting components (Schumann 1989). Moreover, Wyngaard and Weil (1987) show that the transport asymmetry exists in flows without unsteadiness but with a flux gradient that balances an internal source of material. As implied by Eq. (1), flux gradients and nonstationarity are equivalent if other sources or sinks are absent. Hence, we cannot decide which is more important.

3. Conclusions

We have deduced two kinematical models to explain qualitatively the transport asymmetry that is reflected by differences between bottom-up and top-down diffusivities and their dependence on the skewness of the vertical velocity fluctuations. The models resolve the basic convective circulation in a very simple manner but do not account for small-scale turbulence. Hence, they provide insight into the basic transport properties only.

The first simple model shows that the transport asymmetry may be absent even if the flow is skewed. The transport asymmetry is present, however, in the second of the two simple models where the fluid enters the updrafts and downdrafts by horizontal motions mainly near the bottom and top boundaries and less by lateral mixing between updrafts and downdrafts within the CBL. Such a flow structure is more realistic than that of the first model (Schumann and Moeng 1991b). Therefore, several conditions must come together to explain the differences between bottom-up and top-down diffusion. These include nonstationarity (or sink terms, e.g., from chemical reactions or from flux divergence), skewness, and the inhomogeneous manner by which the species is introduced into the flow.

In a related study (Schumann 1991) it has been shown that the transport asymmetry may be present even if updrafts and downdrafts occupy the same area fraction but if the surface is undulated with updrafts rising from crests and downdrafts sinking into valleys of a wavy surface. In this case, for bottom-up diffusion the species is transported first with the shorter updraft while for top-down diffusion the species is transported first with the longer downdraft, causing larger bottom-up than top-down diffusivities.

These results suggest that a transport asymmetry is generally to be expected in flows with asymmetric structure.

REFERENCES

Eberhard, W. L., W. R. Moninger, and G. A. Briggs, 1988: Plume dispersion in the convective boundary layer. Part I: CONDORS field experiment and example measurements. *J. Appl. Meteor.*, **27**, 599–616.

Hunt, J. C. R., J. C. Kaimal, and J. E. Gaynor, 1988: Eddy structure in the convective boundary layer—new measurements and new concepts. *Quart. J. Roy. Meteor. Soc.*, **114**, 827–858.

Moeng, C.-H., and J. C. Wyngaard, 1984: Statistics of conservative scalars in the convective boundary layer. *J. Atmos. Sci.*, **41**, 3161–3169.

Sawford, B. L., and F. M. Guest, 1987: Lagrangian stochastic analysis of flux-gradient relationships in the convective boundary layer. *J. Atmos. Sci.*, **44**, 1152–1165.

Schumann, U., 1989: Large-eddy simulation of turbulent diffusion with chemical reactions in the convective boundary layer. *Atmos. Environ.*, **23**, 1713–1727.

——, 1991: A simple model of the convective boundary layer over wavy terrain with variable heat flux. *Beitr. Phys. Atmosph.*, **64**, 169–184.

——, and C.-H. Moeng, 1991a: Plume fluxes in clear and cloudy convective boundary layers. *J. Atmos. Sci.*, **48**, 1746–1757.

——, and ——, 1991b: Plume budgets in clear and cloudy convective boundary layers. *J. Atmos. Sci.*, **48**, 1758–1770.

Weil, J. C., 1990: A diagnosis of the asymmetry in top-down and bottom-up diffusion using a Lagrangian stochastic model. *J. Atmos. Sci.*, **47**, 501–515.

Wyngaard, J. C., 1987: A physical mechanism for the asymmetry in top-down and bottom-up diffusion. *J. Atmos. Sci.*, **44**, 1083–1087.

——, and R. A. Brost, 1984: Top-down and bottom-up diffusion of a scalar in the convective boundary layer. *J. Atmos. Sci.*, **41**, 102–112.

——, and J. C. Weil, 1991: Transport asymmetry in skewed turbulence. *Phys. Fluids*, **A3**, 155–162.

Young, G. S, 1988: Turbulence structure of the convective boundary layer. Part II: Phoenix 78 aircraft observations of thermals and their environment. *J. Atmos. Sci.*, **45**, 727–735.