Human Centered Selection of Meeting Points in a Quantum and Classical Dial-a-Ride Problem: #806

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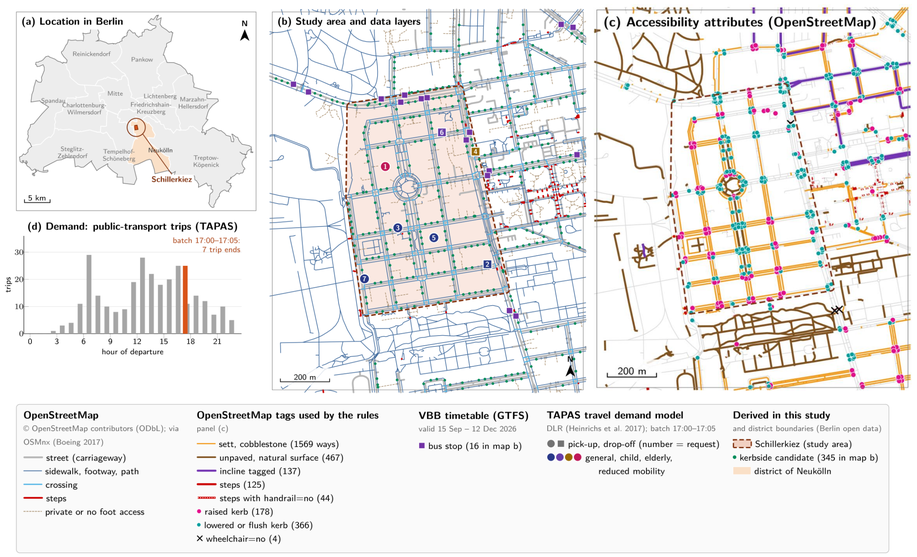
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Motivation & Background

- Shared stops make demand-responsive transport more efficient for the operator, but the passengers pay for it in extra walking and waiting (Stiglic et al. 2015; Czioska et al. 2019).
- Planning standards assume one walking distance for everyone, typically 300–400 m (CIHT 2018). People with reduced mobility, however, can manage only 150 m in a wheelchair, 100 m without a walking aid and 50 m with a stick (Inclusive Mobility 2021).
- The route matters as much as the distance. Steps, raised kerbs, rough surfaces, steep slopes and, for children, crossings of fast roads can make a nearby stop unusable (Wann et al. 2011).
- Choosing shared stops is a location problem that must be solved separately for each mobility class (Hakimi 1964; Church & ReVelle 1974).
- Quantum algorithms are increasingly proposed for such problems (Lucas 2014; Farhi et al. 2014; Grover 1996). So far, though, none have been shown to beat exact classical solvers (Abbas et al. 2024; Li et al. 2026).

Study Area & Data Sources



Research Questions

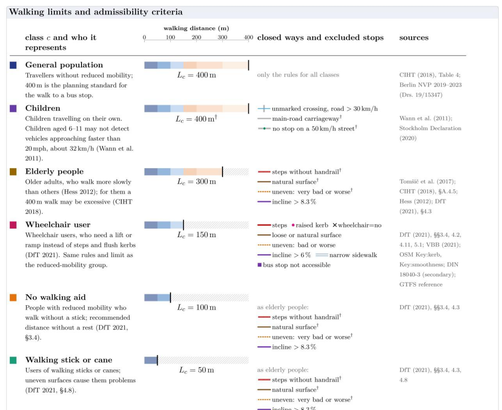
How can shared stops for a demand-responsive minibus be selected so that every traveler, including children, elderly people and people with reduced mobility, can walk to them, and can existing quantum algorithms solve this selection as well as exact classical methods?

(RQ1) Which stops are feasible of each user group reach, given the walking limit and the ways and stops that are admissible for the class?

(RQ2) Which set of shared stops is optimal for a batch of requests and what is the walking effort of each traveler?

(RQ3) How do the best classical and quantum algorithms compare in solution quality, computational effort and the outcome for the passengers?

Mobility Classes & Criteria for Admissible Stops



criterion	OpenStreetMap tag	General population	Children	Elderly people	Wheelchair user[a]	No walking aid	Walking stick
Walking limit along the network	network distance	400 m	400 m†	300 m	150 m	100 m	50 m
Ways: closed for the class (✗) or usable (✓)							
Private or no foot access	access, foot = private, no	✗	✗	✗	✗	✗	✗
Steps	highway=steps	✓	✓	✓	✗	✓	✓
Steps without handrail†	handrail=no	✓	✓	✗	✗	✗	✗
Loose or natural surface[c]	surface=gravel, ground, dirt, grass, ...	✓	✓	✗ natural†	✗	✗ natural†	✗ natural†
Sett, cobblestone	surface=sett, cobblestone	✓	✓	see smoothness	see smoothness	see smoothness	see smoothness
Uneven surface[c]	smoothness	✓	✓	✗ very bad or worse†	✗ bad or worse	✗ very bad or worse†	✗ very bad or worse†
Steep incline	incline	✓	✓	✗ > 8.3 %	✗ > 6 %	✗ > 8.3 %	✗ > 8.3 %
Crossing at a raised kerb	kerb=raised	✓	✓	✓	✗	✓	✓
Crossing at a lowered or flush kerb	kerb=lowered, flush	✓	✓	✓	✓	✓	✓
Tagged not wheelchair accessible	wheelchair=no	✓	✓	✓	✗	✓	✓
Narrow sidewalk	width < 1.0 m, or < 1.5 m over > 6 m	✓	✓	✓	✗	✓	✓
Carriageway of a main road†	highway=primary, ..., tertiary	✓	✗	✓	✓	✓	✓
Stops: excluded for the class							
Barrier, station entrance or indoor way at the boarding point†	barrier, railway, indoor, level	✗	✗	✗	✗	✗	✗
Boarding way closed for the class	rules above	✗	✗	✗	✗	✗	✗
Kerbside stop on a street of 50 km/h or more†	maxspeed[d]	✓	✗	✓	✓	✓	✓
Bus stop not wheelchair accessible	wheelchair_boarding=2 (GTFS)	✓	✓	✓	✗	✓	✓

Problem Statement

The shared stop selection problem

1 The setting. A demand-responsive minibus has no fixed route: travellers book trips, the operator collects the bookings of a short time window (a *batch*, here five minutes) and plans one tour for them. The bus does not stop at every door; it halts at a few *shared stops*, and the travellers walk to and from them.

2 Requests and ends. A request $r \in R$ is one booked trip. Its *pick-up end* O_r is the address where the traveller starts, its *drop-off end* D_r the address where the trip ends. Only the ends inside the study area need a stop; they form the set E, and e denotes one of them.

3 Mobility classes. Travellers cannot all walk equally far, nor on the same ways. A *mobility class* $c \in \mathcal{C}$ groups travellers with the same walking ability, for example the general population, children, elderly people or wheelchair users (classes and sources: next page). Each end e has the class $c(e)$ of its traveller. A class fixes two things:

- the *walking limit* L_c, the longest walk to a stop that is acceptable for the class (for example 150 m for a wheelchair user);
- the *admissibility criteria*: the ways the traveller can use (for example no steps for a wheelchair user) and the stops excluded for the class, J_c^{ex}.

4 Candidate stops. J is the set of places where the bus could halt: the existing bus stops J^{bus} and new *kerbside stops* J^{kerb}, points along the kerb where a minibus can halt. The indicator $\kappa_j = 1$ marks a new kerbside stop, which has to be set up and checked on site, and $\kappa_j = 0$ an existing bus stop.

5 Walking distance. $d_e(j)$ is the length in metres of the shortest walk from end e to stop j along the pedestrian network, using only the ways admissible for the class $c(e)$; it is not a straight-line distance.

6 Choice set. The stops to which the traveller at e may be sent are those that are not excluded for the class and lie within the walking limit:

$$C_e = \{\, j \in J \setminus J^{\text{ex}}_{c(e)} \;:\; d_e(j) \le L_{c(e)} \,\}.$$

If C_e is empty, no shared stop is acceptable for that traveller, who is left out of the stop selection.

7 Plans. A *plan* assigns to every end $e \in E$ a stop $j(e)$. It is feasible if

(i) $j(e) \in C_e$ for every end e, — every traveller can walk to the assigned stop;
(ii) $j(O_r) \neq j(D_r)$ for every request r, — nobody is picked up and dropped off at the same stop.

Different travellers may be assigned the same stop: this sharing is what saves halts. The *open stops* $X = j(E)$ are the stops where the bus actually halts; $|X|$ is their number.

8 Objective. Feasible plans are compared in two steps, *lexicographically*:

1. **fewest open stops**, $k^* = \min |X|$: every halt costs time to all passengers on board, so the plan must halt as rarely as possible;
2. among the plans with k^* stops, the **smallest cost**

$$F \;=\; \gamma \sum_{j \in X} \kappa_j \;+\; \sum_{e \in E} d_e(j(e)), \qquad \gamma = 50 \text{ m},$$

the total walking of all travellers in metres plus 50 m for every new kerbside stop. An existing bus stop is therefore preferred unless it adds more than 50 m of walking in total.

Because F ranks only plans that already have k^* stops, a shorter walk never justifies an extra halt. The optimal value is written F^*.

From Classical to Quantum Approach

Elderly person, drop-off at 17:00–17:05 in the Schillerkiez; walking limit 300 m; same map frame (810 m wide) in every step; symbols of the admissibility table.

Feasibility Networks for Distinct User Groups

DARP-MP Results

a Door to door
7 halts route 3,164 m no walking
b With meeting points
3 halts route 2,449 m (−23 %) walking 1,280 m, longest 333 m (traveller 7)
entry → 3 door of 4 (.4) → 2 door of 2 (.2) → 3 door of 5 (!5) → 4 door of 7 (!7) → 5 door of 3 (!3) → 6 door of 1 (!1) → 7 door of 6 (.6) → exit
entry → 1 stop 372 (.2, 4, 6) → 2 stop 150 (.2 ! 3, 5, 7) → 3 door of 1 (!1) → exit
pick-up drop-off class reduced mobility elderly child general minibus route walk meeting point (visit order) door halt (visit order)
Solution for all passengers: every valid plan, its passenger quality, and what each method returns
plan
stop and walk (m) of each traveller
passenger quality, all travellers
how often each method returns this plan (%)
open stops
1 2 3 4 5 6 7
served
total walk
longest walk
extra vs walk
cost
classical
QAOA, noise-free
Heron r3 noise
Grover
stops
no walk, 1m
ground 4d
ground 2d
stairs 9m
ground 4d
stairs 6d
ground 2d
exact
HiGHS
exact
# F. HiGHS
# F. HiGHS
p = 1
p = 3
p = 1
p = 3
1 372+150 + door 54 308 232 90 141 120 333 7 1,280.3 333.3 +156.7 1,380.3 100 0.02 / 0 1.36 / 0.85 5.96 / 3.60 83.2 96.1
2 372+216 + door 54 351 188 90 187 120 288 7 1,282.6 351.2 +157.0 1,382.6 0 0.02 / 0.03 1.36 / 0.80 4.80 / 1.85 7.35 1.50
3 187+150 + door 56 306 232 10 141 218 333 7 1,298.1 333.3 +159.3 1,448.1 0 0.02 / 0 1.12 / 1.00 3.18 / 1.80 2.40 0.65
4 187+216 + door 56 351 188 10 187 218 288 7 1,300.4 351.2 +159.6 1,450.4 0 0.02 / 0.03 1.78 / 1.30 8.73 / 3.95 3.15 1.15
5 372+150 + door 54 306 90 141 120 333 7 1,373.5 333.3 +170.0 1,473.5 0 0.01 / 0 1.76 / 1.25 5.45 / 3.35 0 0
6 372+216 + door 54 351 90 187 120 288 7 1,419.9 351.2 +176.7 1,519.9 0 0.01 / 0.03 0.35 / 0.30 2.02 / 0.95 0 0
7 187+150 + door 56 306 326 10 141 218 333 7 1,391.1 333.3 +172.6 1,541.1 0 0.01 / 0 1.04 / 0.30 3.74 / 1.05 0 0
8 187+216 + door 56 351 326 10 187 218 288 7 1,437.7 351.2 +179.2 1,587.7 0 0.01 / 0 0.69 / 0.60 3.66 / 2.00 0 0
Invalid: a construct is broken, e.g. a traveller without a stop 0 99.9 / 99.9 99.6 / 99.5 63.5 / 81.5 1.50 0.65
Optimal plan (.4!1) : join 372 (bus stop), 150 and the door, $F^* = 1{,}380.3$ m, numbers: walk of each traveller on its route, onward (on).
method
condition
open stops (plan #)
plan the passengers get: served
plan the passengers get: transfer %
plan the passengers get: total walk
plan the passengers get: longest walk
plan the passengers get: extra vs min. walk
cost F
reliability (%): optimal
reliability (%): valid
reliability (%): stable
offset: CZ (Heron r3)
Classical exact (HiGHS) 372+150 + door (#1) 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 100 (proven) 100 59 hrs — — —
QAOA standard, p = 1 noise-free, best of 20,000 shots no valid plan among the shots 0.03 13.55 17 —
QAOA standard, p = 2 noise-free, best of 4,000 shots 372+216 + door (#2) 7 of 7 372, 90 1,282.6 351.2 +157.0 1,382.6 0 0.03 17 199
noise-free, best of 20,000 shots 372+216 + door (#2) 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 0.02 0.13 17 —
Heron r3 noise, best of 4,000 shots 372+216 + door (#2) 7 of 7 372, 90 1,282.6 351.2 +157.0 1,382.6 0 0.07 17 221
QAOA XY mixer, p = 1 noise-free, best of 20,000 shots 372+150 + door (#1) 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 1.34 9.44 17 —
Heron r3 noise, best of 4,000 shots 372+150 + door (#1) 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 0.85 6.45 17 —
QAOA XY mixer, p = 2 noise-free, best of 20,000 shots 372+150 + door (#1) 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 5.96 37.5 17 —
Heron r3 noise, best of 1,000 shots 372+150 + door (#1) 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 3.60 19.4 17 161
Grover adaptive search stop after 5 failures 372+150 + door (#1) in 81% of runs 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 83.2 96.5 18 —
stop after 9 failures 372+150 + door (#1) in 96% of runs 7 of 7 372, 90 1,280.3 333.3 +156.7 1,380.3 96.1 99.4 18 —
one Grover stop, Heron r3 noise optimal pair measured in 6 % of 100 shots (noise-free 90.1 %) no usable search — — 18 5,182
From the passengers' set, stops in the chosen plan (#1 or #2) are shown as ■ bus stop, ● kerbside stop, ✕ excluded, · other candidate. Reliability: probability that one shot (5,000 shots) or one run (Grover) gives the optimal (+ valid pair, ! 99 percent); results on simulators or hardware, not fault-tolerant.

Zoom on Door-to-Door

Discussion, Limitations & Outlook

- Each user group reaches different stops. Steps, rough surfaces and raised kerbs exclude many nearby stops for wheelchair and walking-stick users.
- The classical method finds the proven optimum quickly. Grover search needs fewer steps than QAOA, but only in simulation and no quantum advantage is demonstrated yet.
- The admissibility criteria are simplified rules inferred from literature review. The quality OpenStreetMap tags and its completeness depend on crowdsourced data annotation.
- Next steps include validation the admissibility criteria with real users, testing the algorithms on real quantum hardware and scaling up the number of passenger requests.

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More information about the project on our website

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