

OCEAN SURFACE RECONSTRUCTION FROM DIRECT ASSIMILATION OF RADAR MEASUREMENTS USING THE ADJOINT METHOD

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ABSTRACT

We present an algorithm for phase-resolved reconstruction of ocean surfaces from marine radar measurements based on the adjoint method. No assumption is made about the spatio-temporal location of the radar measurements and they are directly processed in the adjoint model through the inclusion of the two main X-band radar modulation mechanisms, namely the tilt and shadowing modulations. This makes the proposed algorithm able to assimilate raw radar data of complex helix-shaped spatio-temporal structure. Using numerically generated radar measurements, we highlight the importance of the spatial discretization and show that the reconstruction accuracy converges with the measurement duration by mitigating the impacts of the strong measurements' spatio-temporal non-uniformity and of the tilt modulation. A detailed comparison between the reconstructed and reference surfaces reveals the local discrepancies, which can be explained by the amount of assimilated wave information and the different modulation mechanisms. The obtained accuracy is found to be high enough for detailed wave field analysis and phase-resolved predictions. Although linear waves are used in the presented applications, the chosen numerical framework allows a direct extension to nonlinear waves.

Keywords: ocean waves, marine radar, surface reconstruction, PDE-constrained optimization

1. INTRODUCTION

Phase-resolved predictions of incoming waves in the vicinity of non-stationary marine structures in the near future provide crucial information to predict, manage and control the structure's behavior in view of increasing the efficiency of offshore operations and maneuverability. For instance, such predictions can reduce the cost of offshore wind park installation and maintenance by extending operational windows, or increase the safety of aircraft take-offs and landings.

Such predictions rely on the wave-by-wave reconstruction of the surrounding wave field from measurements. Due to their

wide availability and capacity to measure wave features over a large area, X-band marine radars, traditionally used for navigational purposes, have been preferred over other types of sensors as the primary source of wave information for such surface reconstructions [1–3], even though other remote sensors have been considered, such as lidars [4, 5] or stereo camera systems [6, 7]. To apply fast processing algorithms like the widely used 3D-FFT approach [8, 9] and reach real-time capability, radar measurements are often treated as a sequence of spatial snapshots on a Cartesian coordinate system (referred to as “radar images”) rather than as the more complex spatio-temporal distribution that results from the rotation of the radar antenna and forms a helix in polar coordinates. This assumption has been shown to be an important source of errors in radar data interpretation [10]. Furthermore, radar measurements suffer from spatial gaps created by the so-called wave shadowing effects [11, 12]. We propose in this paper a surface reconstruction method that is adapted to such strongly nonuniform spatio-temporal distribution of measurements.

In addition to the assumption on data distribution, wave nonlinearities are typically disregarded in surface reconstruction algorithms to minimize the computational time even further [13–15]. Despite being applicable in configurations in which the wave steepness remains low, linear wave modeling leads to erroneous wave field descriptions by failing to capture important nonlinear phenomena observed in sea states of moderate to high wave steepness [16]. Among them, four-wave interactions (described by third-order wave models) have a strong impact on deterministic wave predictions through, e.g. the modeling of extreme events [17] and the correction of the dispersion relation of individual wave components [18]. In order to reconstruct/predict nonlinear wave fields, authors have studied weakly nonlinear wave models, for which analytical expressions can be found, such as the solution to the second-order wave equations [19, 20] or the improved choppy wave model [4, 5, 21]. However, these models cannot reach the accuracy of higher-order numerical models like the high-order spectral (HOS) method [22, 23]. The HOS method has been extensively used in deterministic wave prediction through

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its incorporation into diverse data assimilation techniques [24–28]. These techniques consist of minimizing a cost function that represents the misfit between observations (here, the wave measurements) and estimates of them from the solution of a physical model (here, the wave model). The approach chosen in this paper is based on the adjoint method, which was already used in Refs. [29, 30] for ocean surface reconstruction. The advantage of the adjoint method compared to other data assimilation techniques is the possibility to compute the gradient (and to some extent the Hessian) of the cost function w.r.t. the control parameters (here, the input parameters of the wave model) at the cost of one forward run of the physical model plus one backward run of the adjoint model. The minimization of the cost function can then be done efficiently with iterative algorithms.

Compared to the already proposed approaches [29, 30], which suppose that measurements are regularly distributed surface elevations, we extend here the adjoint method to *directly* assimilate radar data by (i) handling the complex spatio-temporal distribution of radar measurements and (ii) using a model of radar backscatter intensity. The first objective of this paper is to quantify the reachable reconstruction accuracy of our method, knowing that we introduce errors due to interpolation and radar model approximations. The second objective is to estimate the associated computational effort, characterized by the convergence rate of the reconstruction process (defining the required number of function evaluations) and the required measurement duration (impacting the computational cost of each function evaluation). Despite the fact that we have restricted this study to linear waves, the developed numerical framework can be directly extended to nonlinear wave models, and the obtained convergence rate and required measurement duration are expected not to be significantly impacted by nonlinear wave effects.

Accordingly, the paper is structured as follows. The first section describes the methods employed for the wave field modeling and the modulation mechanisms modeled to represent synthetic radar measurements. Then, the surface reconstruction algorithm is shortly explained, although the interested reader may refer to Refs. [29, 30] for a more detailed introduction to the adjoint method. The third section focuses on how the reference and radar data are generated, as well as on the numerical characteristics of the computational domain over which the surface is reconstructed. The fourth section presents the obtained results through the analysis of the convergence properties of the reconstruction algorithm, and then through a detailed comparison of surface profiles. The impacts of extending the algorithm to nonlinear waves are briefly introduced in the fifth section, before giving concluding remarks in the last section.

2. METHODS

We first describe in this section the theoretical background for the modeling of the wave field dynamics and synthetic X-band radar measurements, and then the surface reconstruction algorithm is presented.

2.1. Wave and radar modeling

The surface wave dynamics is described in a Cartesian coordinate system (x, y, z) with the z -axis pointing upwards and

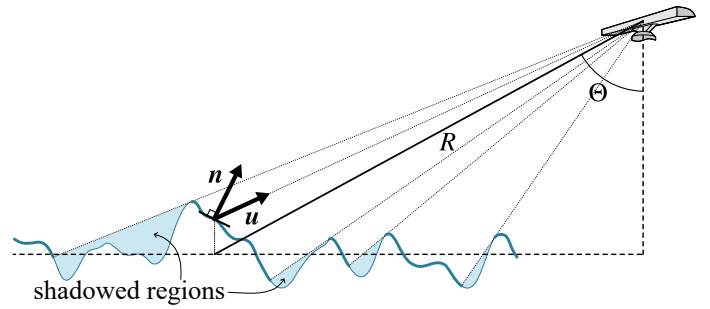


FIGURE 1: Geometrical description of shadowing and tilt modulation mechanisms.

$z = 0$ the plane depicted by the surface at rest. We consider a linear potential flow and two surface state variables, the velocity potential $\phi(x, y, t)$ and the elevation $\eta(x, y, t)$, fully describing the wave field evolution according to

$$\phi_t = -g\eta, \quad (1)$$

$$\eta_t = \mathcal{G}(\phi), \quad (2)$$

in which g is the gravitational acceleration. We assume infinite water depth and employ a modal approach so that $\mathcal{G}(\phi) = \mathcal{F}^{-1}(k\mathcal{F}(\phi))$ with $\mathcal{F}$ the spatial Fourier transform and k the corresponding wavenumber.

The radar backscatter is modeled using the shadowing and tilt modulation mechanisms [8] which rely on ray optics and can both be understood from a geometrical point of view (see Fig. 1). The shadowing modulation comes from the waves blocking the visibility of the surface from the radar. The tilt modulation represents the dependence of the intensity of the backscattered signal by the ocean surface on the incidence angle of the radar beam. Formally, the intensity reads

$$\sigma = \frac{\mathbf{u} \cdot \mathbf{n}}{\|\mathbf{u}\| \|\mathbf{n}\|}, \quad (3)$$

with $\mathbf{u}$ a vector pointing from the surface to the radar, $\mathbf{n}$ a vector normal to the surface and $\|\cdot\|$ the ℓ^2 norm. It is assumed that no information at all is available in the shadowed regions, creating spatial gaps in the measurements. In addition, the radar is scanning the surface with a specific rotation period, making the measurement describe a spatio-temporal helix instead of the usually assumed sequence of spatial snapshots [10].

Linearizing Eq. (3) w.r.t. to surface elevation and slope leads to [17]

$$\sigma = \cos \Theta + \eta_r \sin \Theta - \eta \frac{\sin^2 \Theta}{R}, \quad (4)$$

in which Θ is the nominal angle of incidence, R is the distance between the radar antenna and the horizontal plane for the considered measurement point and η_r is the surface slope along the line of sight. While Eq. (3) is used later to generate synthetic radar measurements from a reference wave field, Eq. (4) is used as part of a mapping operator (described in the next section) to reconstruct the wave field.

2.2. Surface reconstruction algorithm

Let us first assume that the ocean surface was measured by an X-band radar during a duration T , providing N_o spatio-temporal observations. Then, we consider a spatial domain made of N_s points to reconstruct the wave field, which we evaluate at N_t steps in time to compare with the observations. That leads to $N = N_s N_t$ total points for which optimal values of both ϕ and η are to be found, i.e. the problem has $2N$ unknowns.

We formulate the surface reconstruction problem as a PDE-constrained optimization problem that is solved with the adjoint method. The PDEs are the wave Eqs. (1) and (2) and the cost function to minimize represents the discrepancy between the observations and their estimates from the state variables $\{\phi, \eta\}$ using a mapping operator based on a spatio-temporal interpolation and Eq. (4). With the adjoint method, the control parameters of the optimization problem correspond to the state variables at the initial time step, and the state variables at later time steps are obtained using a forward wave operator based on Eqs. (1) and (2). Accordingly, the problem becomes formally unconstrained, the number of unknowns reduces to $2N_s$ and the cost function $f: \mathbb{R}^{2N_s} \rightarrow \mathbb{R}$ writes

$$f(\mathbf{p}) = \frac{1}{2} \|\mathbf{C}(\mathbf{p}) - \boldsymbol{\zeta}\|^2 = \frac{1}{2} \|\mathbf{M} \circ \mathcal{W}(\mathbf{p}) - \boldsymbol{\zeta}\|^2,$$

where:

- $\boldsymbol{\zeta}$ is a set of N_o observations, consisting here of either observed surface elevations η^o or radar intensities σ^o . The observed elevations η^o are sampled by a virtual radar that considers only the shadowing modulation, while the observed intensities σ^o are computed according to the tilt-modulation Eq. (3).
- $\mathcal{W}: \mathbb{R}^{2N_s} \rightarrow \mathbb{R}^{2N}$ is the forward wave operator that provides $2N$ state variables $\{\phi, \eta\}$ at later time steps from $2N_s$ initial conditions $\mathbf{p} = \{\phi|_{t=0}, \eta|_{t=0}\}$ (i.e. the control parameters) using Eqs. (1) and (2). Because these equations are linear, the propagation can be done analytically, without numerical error.
- $\mathbf{M}: \mathbb{R}^{2N} \rightarrow \mathbb{R}^{N_o}$ is the mapping operator that provides N_o estimates of the observations by mapping the $2N$ state variables into the observation space. It consists of a cubic Lagrange interpolation when η^o is used, combined with Eq. (4) when σ^o is used.
- $\mathbf{C}: \mathbb{R}^{2N_s} \rightarrow \mathbb{R}^{N_o}$ is the composition of $\mathbf{M}$ after $\mathcal{W}$.

Then, according to the adjoint method framework [31], the gradient of the cost function w.r.t. the control parameters $\nabla f: \mathbb{R}^{2N_s} \rightarrow \mathbb{R}^{2N_s}$ reads

$$\nabla f(\mathbf{p}) = \mathbf{C}^* (\mathbf{C}(\mathbf{p}) - \boldsymbol{\zeta}),$$

with $\mathbf{C}^*$ the adjoint of $\mathbf{C}$. Because of the problem linearity, the adjoint operator reduces to the transpose: $\mathbf{C}^* = \mathbf{C}^T = \mathcal{W}^T \circ \mathbf{M}^T$. It propagates backwards in time and takes as sources the differences between the observations and their estimates $\mathbf{C}(\mathbf{p}) - \boldsymbol{\zeta}$.

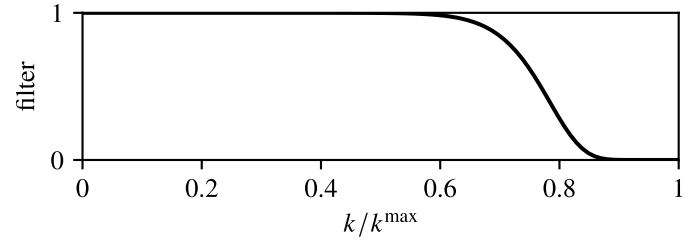


FIGURE 2: Shape of filter damping high-frequency components.

Further, the Hessian-vector product $H\mathbf{p}$ with H the $2N_s$ -by- $2N_s$ Hessian matrix of the cost function writes

$$H\mathbf{p} = \mathbf{C}^T \circ \mathbf{C}(\mathbf{p}).$$

Hence, the surface reconstruction is done by solving for $\mathbf{p}$ the linear system

$$H\mathbf{p} = -\nabla f(\mathbf{p}_0),$$

where $\mathbf{p}_0$ is an initial guess, here set to zero. Since $H = \mathbf{C}^T \circ \mathbf{C}$, it is solved using the MINRES algorithm [32], an efficient iterative solver for large symmetric linear systems. In the following applications, we stopped the MINRES iterations when the tolerance threshold $r_i/r_0 < \varepsilon$ is reached, with $r_i = \|\mathbf{H}\mathbf{p}_i + \nabla f(\mathbf{p}_0)\|$ the residual at the i^{th} iteration. As explained later, the tolerance ε is set either to 10^{-3} or to 10^{-4} depending on the considered case. Furthermore, we apply a high-frequency damping filter on the result of the adjoint operator, which was found to be beneficial to both remove spurious high-frequency components in the reconstructed surface and increase the convergence rate. Figure 2 shows the shape of the filter, which is applied along both x - and y -directions, as a function of the normalized wavenumber $k/k^{\max}$.

3. NUMERICAL SETUP

The characteristics of the reference wave field as well as of the synthetic radar measurements are detailed in this section, followed by those of the computational domain used for the surface reconstruction.

3.1. Reference wave field and radar observations

The linear wave field that is reconstructed in this study is generated from a JONSWAP frequency spectrum [33] with a peak period $T_p = 10$ s and a peak-enhancement factor $\gamma = 3.3$, that is multiplied by the directional spreading function $\cos^{2s}(\theta/2)$ where $s = 25$ and θ is the wave direction defined in such a way that $\theta = 0$ means propagation along the x -axis. The wave field is generated over a periodic domain of $40\lambda_p \times 40\lambda_p$ with 32 points per peak wavelength $\lambda_p \approx gT_p^2/(2\pi)$ along both x - and y -axes. The significant wave height H_s is about 2.5 m.

The virtual radar sensor is placed at an altitude of 20 m and is sampling measurements with a rotation period of 1.5 s, a range resolution of 7.5 m and an azimuth resolution of 1.5° . As explained in Sect. 2.2, two types of observations are considered: one is the surface elevation (denoted η^o) and the other one is the radar intensity (denoted σ^o) computed from Eq. (3). Both

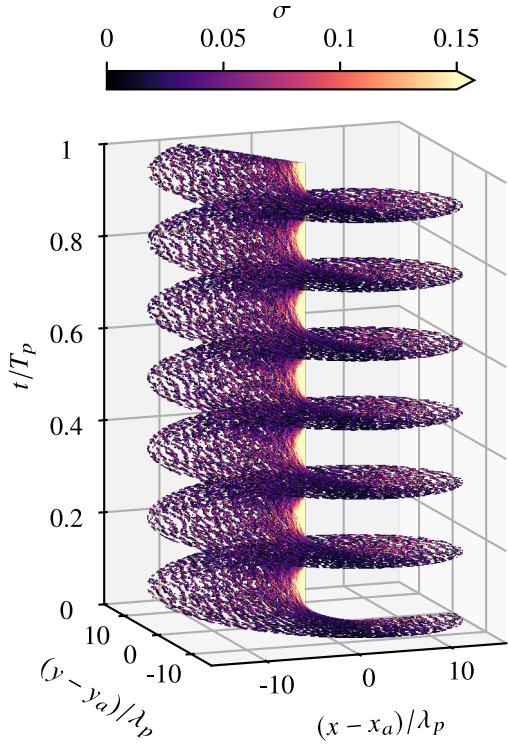


FIGURE 3: Synthetic radar observations obtained from a radar placed at coordinates (x_a, y_a) .

types have the same helix-shaped spatio-temporal distribution, an example of which using σ^o and a duration of one peak period is shown in Fig. 3.

3.2. Computational domain

The computational domain used for the surface reconstruction is defined w.r.t. the area over which the observations are used, which is smaller than the total radar footprint and referred to as the measured area (see Fig. 4). Since the assimilation is done during a finite measurement duration T , it is useful to leave some space between the boundaries of the measured area and the boundaries of the computational domain to (i) let the assimilated information propagate around the measured area and (ii) avoid the influence of the periodic boundary conditions on the relevant part of the reconstructed surface, i.e. over and around the measured area. In the presented results, the computational domain has dimensions $L_x = L_y = 4000 \text{ m} \approx 25\lambda_p$ and is discretized with N_x , N_y and N_t points along the x -, y - and t -axes, respectively. While $N_y = 256$ and N_t such that there are about 12.5 points per peak period (e.g. $N_t = 26$ for $T/T_p = 2$ and $N_t = 151$ for $T/T_p = 12$) were found to give a high enough resolution not to impact the results significantly, the value of N_x is discussed in Sect. 4.1.

4. RESULTS

The results section is structured as follows. First, the convergence properties of the reconstruction algorithm using both observation types η^o and σ^o are presented. Then, we focus on the physical aspects of the reconstructed surfaces by providing a detailed comparison with the reference surface.

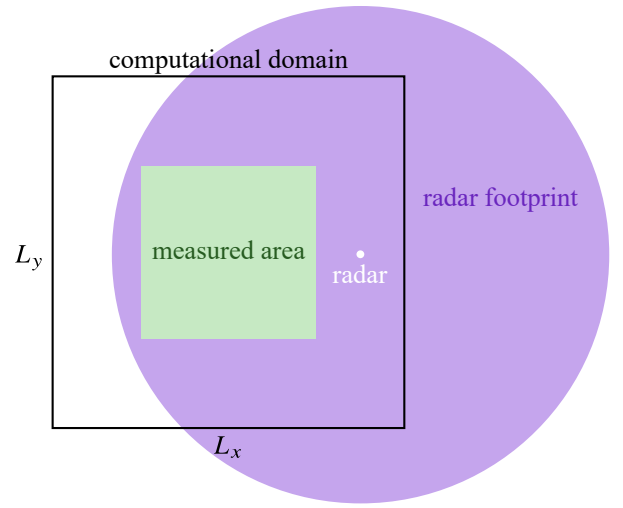


FIGURE 4: Computational domain used for the surface reconstruction.

4.1. Convergence properties

In order to study the performance of the presented method, we quantify the reconstruction accuracy in the form of the surface similarity parameter (SSP) defined as

$$\text{SSP} = \frac{\|\mathcal{F}(\eta^{\text{rec}}) - \mathcal{F}(\eta^{\text{ref}})\|}{\|\mathcal{F}(\eta^{\text{rec}})\| + \|\mathcal{F}(\eta^{\text{ref}})\|}, \quad (5)$$

with $\mathcal{F}(\eta)$ the spatial Fourier transform of η over the measured area. η^{rec} and η^{ref} refer to the reconstructed and reference surface elevation, respectively. As we are mostly interested in waves of high energy that can significantly impact the motion of a typical offshore structure, we only consider in Eq. (5) wavenumbers lower than $5k_p$ with $k_p = 2\pi/\lambda_p$ the peak wavenumber. For applications where extreme crest heights are critical (e.g. air-gap assessment during jacking operations), the performance of the surface reconstruction algorithm at higher wavenumbers would also be of significant interest, since they are responsible for the nonlinear superposition that leads to large surface deformations. A complete characterization of the proposed algorithm would thus necessitate analyzing the spectral reconstruction fidelity over a broader frequency band. In the following, we denote $\overline{\text{SSP}}$ the time-averaged SSP over the considered measurement duration.

The convergence of the optimization process, i.e. the progress of the MINRES algorithm, is tracked through the evaluation at each iteration i of the reconstruction accuracy $\overline{\text{SSP}}_i$, along with the ratio of the residual to its initial value r_i/r_0 and the ratio of the cost function to its initial value f_i/f_0 . These three convergence metrics are shown in Fig. 5 for a measurement duration $T/T_p = 12$. Results using both η^o and σ^o as well as two N_x values, 512 and 1024, are presented. We observe a much higher convergence rate for reconstructions using η^o (~ 90 iterations are needed to reach $\varepsilon = 10^{-4}$) than for those using σ^o (~ 340 iterations required). The lower convergence rate using σ^o is explained by the type of information contained in the radar intensity. While η^o is directly related to η through interpolation, σ^o is dominated by the radial derivative of the surface elevation η_r , thus mainly contains information about the surface slope.

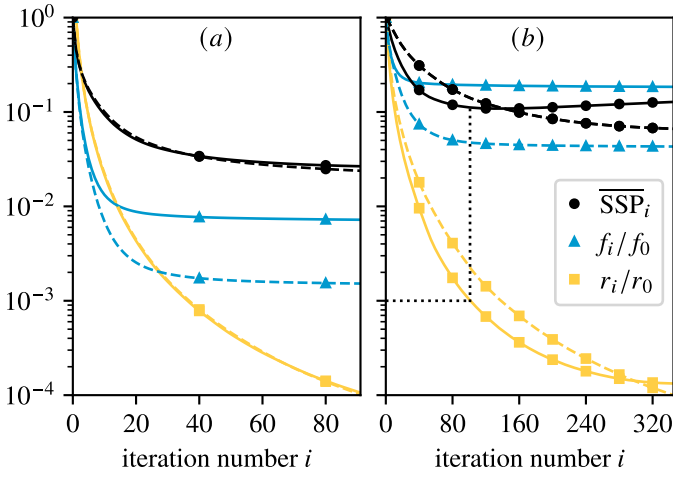


FIGURE 5: Evolution of convergence metrics $\overline{\text{SSP}}$, f_i/f_0 and r_i/r_0 as a function of the number of MINRES iterations using (a) η^o and (b) σ^o , for $N_x = 512$ (solid lines) and $N_x = 1024$ (dashed lines). The measurement duration is $T/T_p = 12$. The dotted lines indicate that over-optimization (i.e. increase of $\overline{\text{SSP}}$ with number of iterations) using σ^o and for $N_x = 512$ is prevented by setting $\varepsilon = 10^{-3}$.

Further, we see that despite allowing a better reduction of the cost function, increasing the value of N_x from 512 to 1024 using η^o has only a small impact on the accuracy. This is very different from using σ^o , for which the accuracy is clearly limited for low N_x values. Indeed, for $N_x = 512$, the value of $\overline{\text{SSP}}$ decreases rapidly and reaches a minimum after ~ 120 iterations, before slowly increasing during the following iterations. This comes from the fact that accurately extracting the surface slope information from σ^o requires the reconstructed wave field to properly represent shorter wave components, whose contribution in the surface slope spectrum are significantly more important than in the surface elevation spectrum. The required spatial resolution to prevent the solution from being over-optimized before the tolerance $\varepsilon = 10^{-4}$ is reached is thus higher using σ^o than using η^o . In the depicted configuration, N_x needs to be larger than N_y because the waves are mainly traveling along the x -direction.

The reconstruction accuracy as a function of the measurement duration, ranging from $T/T_p = 2$ to 12, is presented in Fig. 6. The same four configurations as in Fig. 5 are considered with a tolerance $\varepsilon = 10^{-4}$ except for the configuration using σ^o and $N_x = 512$ for which it is fixed to $\varepsilon = 10^{-3}$, preventing over-optimization from occurring (see Fig. 5b). Apart from the already noticed impact of increasing N_x from 512 to 1024 on the accuracy, Fig. 6 shows that $\overline{\text{SSP}}$ is converging rather rapidly with T . Using a longer measurement duration provides more information about the wave field dynamics and mitigates the impacts of the strong spatio-temporal non-uniformity in the distribution of the observations (i.e. helix-shaped structure and gaps due to the shadowing effect) as well as of the tilt modulation included in σ^o . We see that a measurement duration $T/T_p = 8$ already provides a decent estimate of the reachable accuracy.

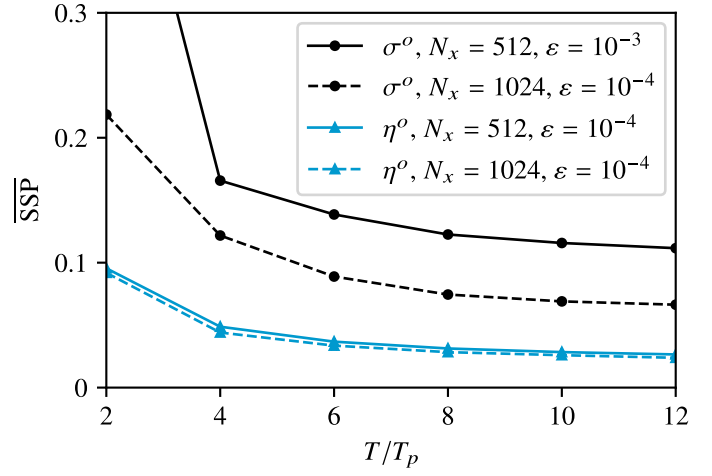


FIGURE 6: Convergence of reconstruction accuracy with longer measurement duration.

4.2. Surface profiles

This section aims at providing a more detailed comparison between the reconstructed and reference solutions by looking at the local discrepancies. To simplify the analysis, we introduce the dimensionless time $t^* = [t - (t_r - T)] / T_p$ that ranges in the interval $[0, T/T_p]$ for every value of T and in which t_r represents the end of the reconstruction (i.e. the measurement interval is $[t_r - T, t_r]$).

Figures 7 and 8 present the reconstructed surface elevation using η^o and σ^o , respectively, for $T/T_p = 12$, $N_x = 1024$ and $\varepsilon = 10^{-4}$. Using both η^o and σ^o , the reconstructed surface matches the reference inside and – due to the propagation of the assimilated information during the measurement interval – around the measured area. At the beginning (end) of the measurement interval, $t^* = 0$ (12), the assimilated information extends upstream (downstream), i.e. left (right) side of the measured area. This effect is more pronounced the longer the measurement duration. The exact boundaries of the extended area of accurately reconstructed surface are not trivial to define, although recent efforts to give a reliable description of the accessible prediction horizon from a set of wave measurements should be noted [16, 34, 35].

Following the same reasoning based on the availability of information in the measurements, the boundary effects can be explained. At $t^* = 0$, the reconstruction accuracy on the right part of the measured area is affected by the small amount of wave information measured in this region (see Figs. 7c and 8c). Similarly, at $t^* = 12$, the reconstruction accuracy on the left part is affected (see Figs. 7i and 8i). These effects are also observable in Fig. 9, where the time evolution of the SSP gets higher at the beginning and end of the measurement interval. The stronger decrease of accuracy at the end of the measurement interval than at the beginning is due to the stronger shadowing effect on the left part of the measured area – it is further away from the radar – than on the right side. At the beginning of the measurement interval, the decrease of accuracy is stronger using σ^o than using η^o because the observations on the right part of the measured area are made at a large viewing angle with the waves' propagation

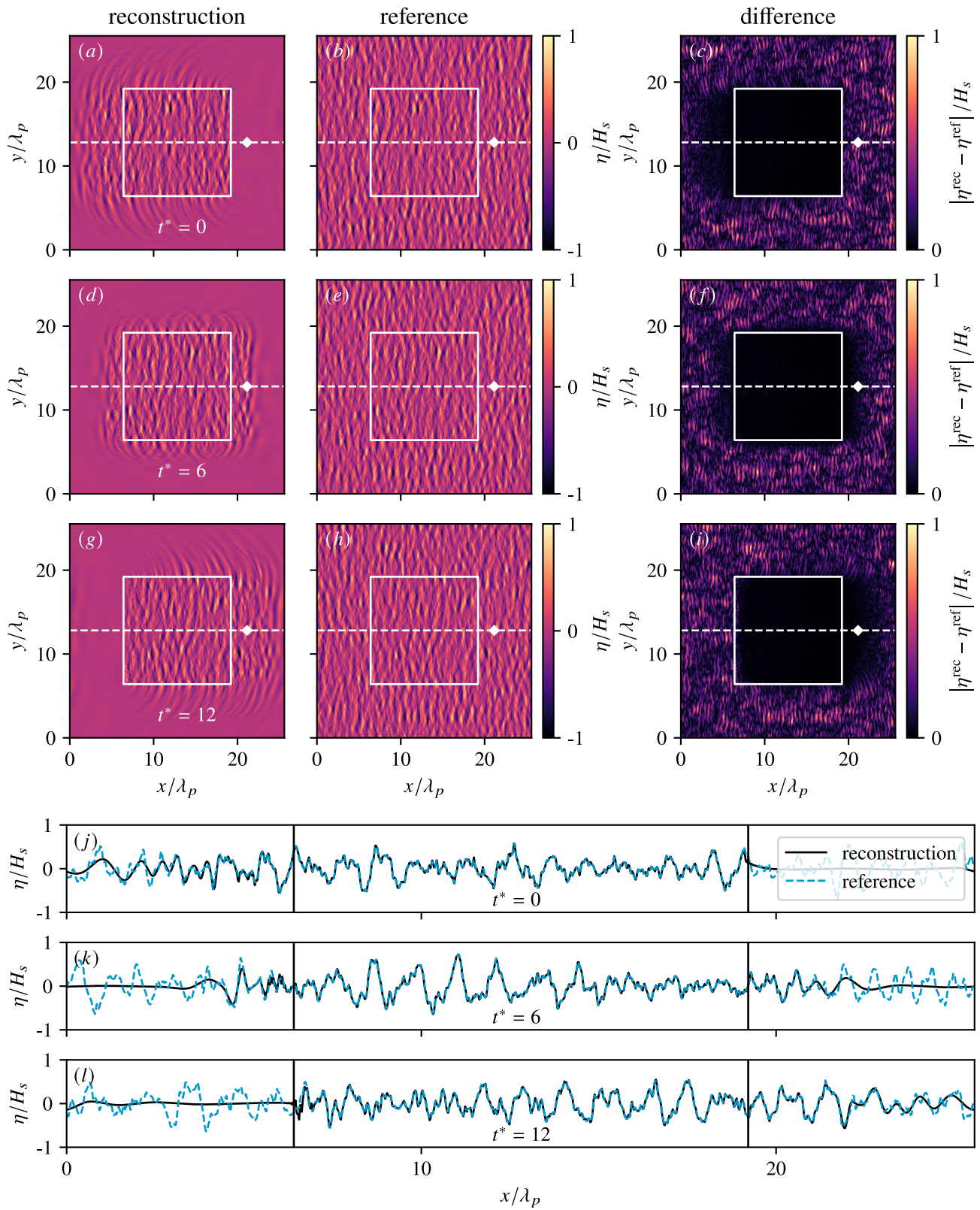


FIGURE 7: Comparison of reconstructed and reference solutions at the beginning ($t^* = 0$), middle ($t^* = 6$) and end ($t^* = 12$) of the measurement duration using η^o and for $T/T_p = 12$, $N_x = 1024$ and $\varepsilon = 10^{-4}$. The square and the diamond on the (x, y) plots mark the boundaries of the measured area and the location of the radar, respectively. The surface elevation along the dashed line crossing the domain is plotted on the bottom plots, in which the vertical lines mark the boundaries of the measured area.

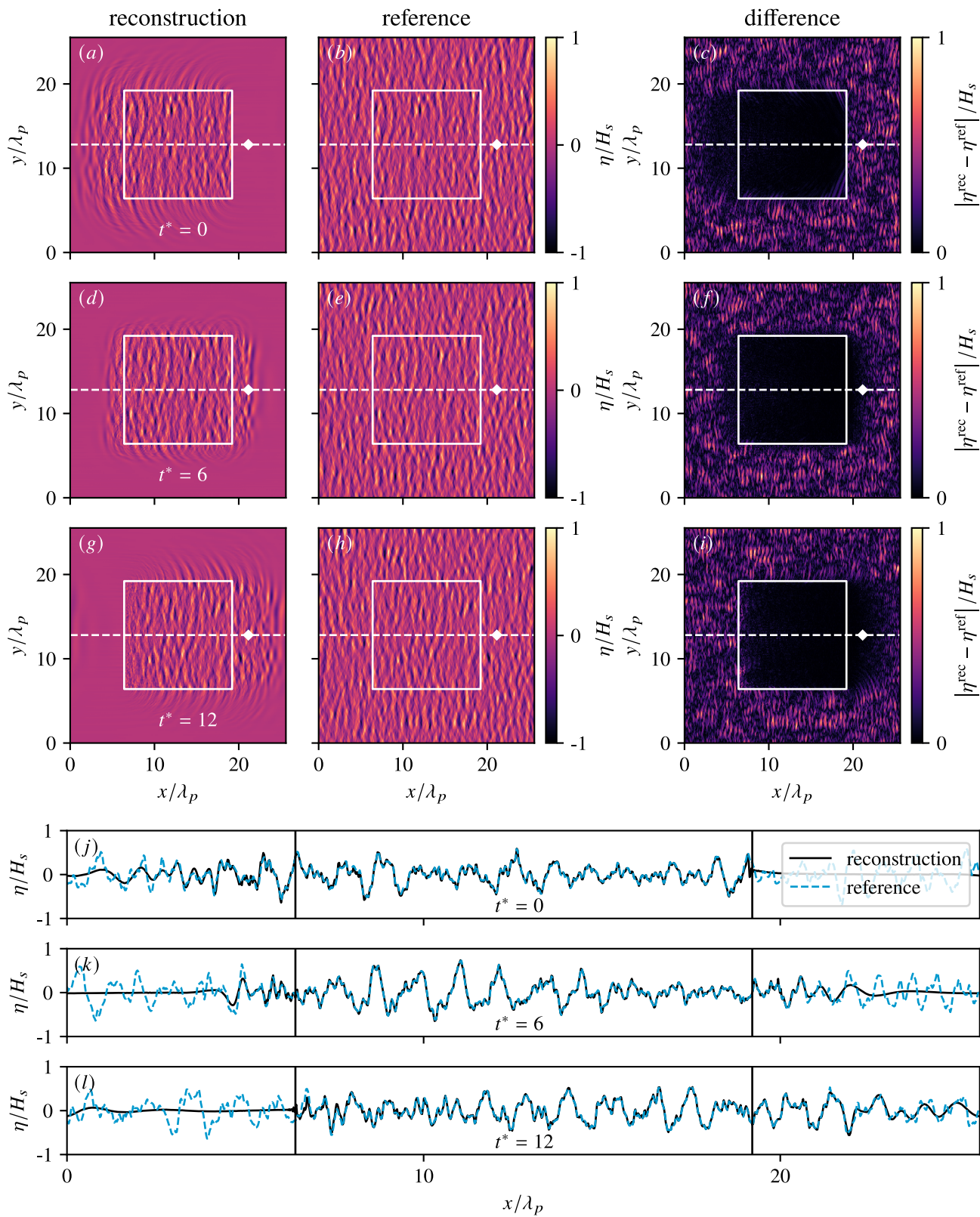


FIGURE 8: Similar to Fig. 7 but using σ° .

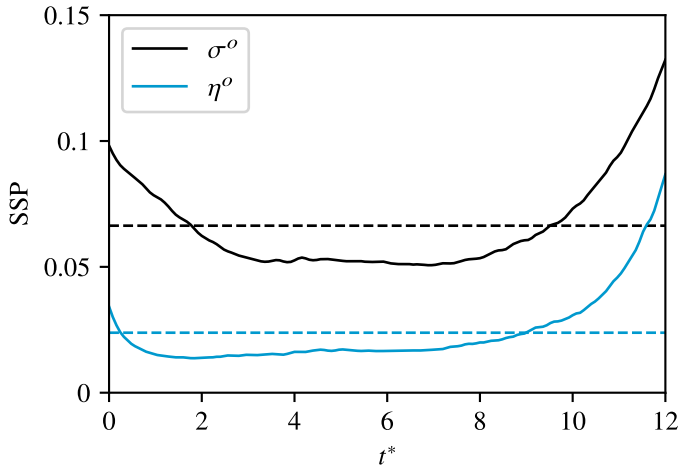


FIGURE 9: Time evolution of the SSP during the measurement interval for $T/T_p = 12$, $N_x = 1024$ and $\varepsilon = 10^{-4}$. The horizontal dashed lines mark the values of SSP.

direction, making radial slope information hold only little content about the surface dynamics.

5. NOTES ON THE NONLINEAR EXTENSION

In the context of nonlinear waves, the wave operator $\mathcal{W}$ becomes nonlinear, which has the following implications. First, the time integration of the equations cannot be fully analytical anymore. For the nonlinear parts, a time-stepping scheme has to be used, which implies that the time step size of the computational domain is no longer defined by the accuracy of the time interpolation only, but also by the accuracy of the time integration, which is expected to impose a stronger limitation. Second, the adjoint operator is no longer equal to the transpose of C : it has to be computed from the nonlinear adjoint wave operator, i.e. $C^* = \mathcal{W}^* \circ \mathcal{M}^T = \mathcal{W}'^T \circ \mathcal{M}^T$ with $\mathcal{W}'$ the tangent linear wave operator. Expressions of both $\mathcal{W}'$ and $\mathcal{W}^*$ at third order can be found in the literature [29, 30]. Third, the Hessian-vector product can be evaluated using, e.g. the Gauss-Newton approximation

$$Hp \approx C^* \circ C'(p),$$

where $C' = \mathcal{M} \circ \mathcal{W}'$ is the tangent linear operator. Then, iterations using either gradient information only, such as L-BFGS, or both gradient and Hessian information, such as Gauss-Newton, could be employed to solve the nonlinear problem [36].

6. CONCLUSIONS

We proposed a surface reconstruction algorithm from measurements that mimic the outputs of a marine radar. Through the analysis of the convergence properties of the algorithm, based on the adjoint method and limited here to linear waves, we highlighted the need for a larger number of iterations and a higher spatial resolution in the case of ocean surfaces sampled by radar intensities, compared to using surface elevation samples, due to the embedded information being mainly related to the surface slope. Nevertheless, we showed that it is possible, from a measurement duration of approximately 8 peak periods, to achieve an accuracy that is high enough for ocean surface analysis and for

providing initial conditions to a wave model for phase-resolved predictions. As explained in Sect. 5, the nonlinear extension of the method is direct and will be the object of further work.

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