

Sensitivity Analysis in Structural Health Assessment Using Generalized Polynomial Chaos Expansion: a Full-scale Bridge Test Case

17th World Congress on Computational Mechanics

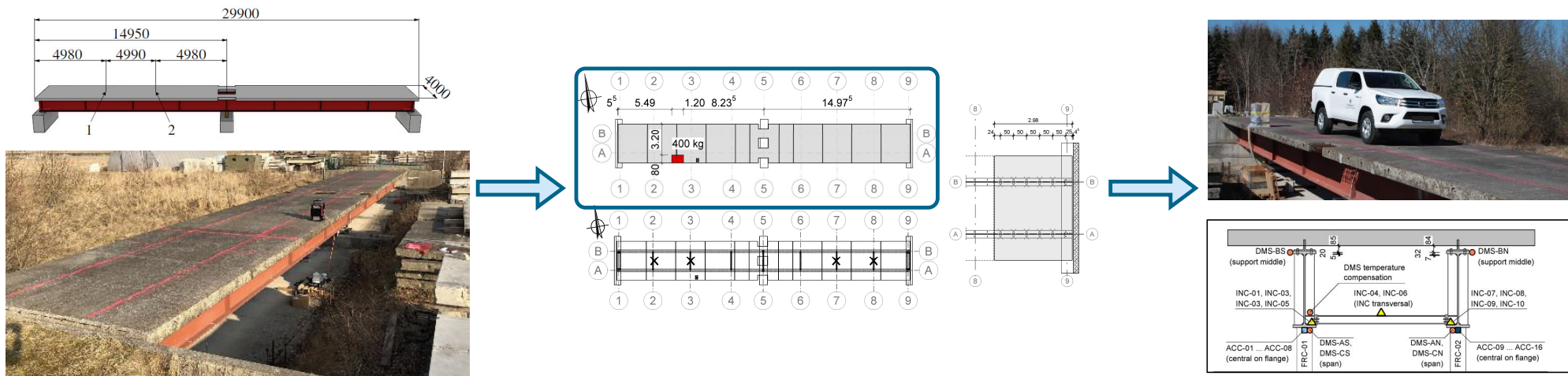
10th European Congress on Computational Methods in Applied Sciences and Engineering

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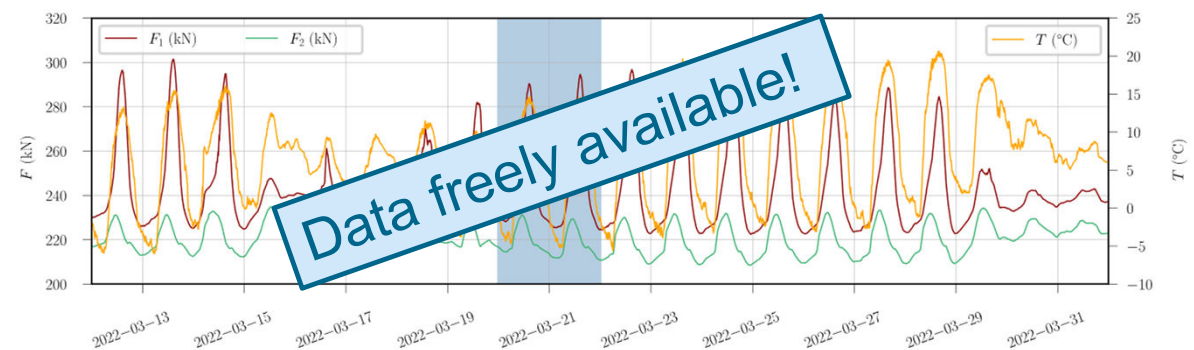
- 1. German Aerospace Center (DLR), Institute for the Protection of Terrestrial Infrastructures**
- 2. Helmut-Schmidt-University, Chair of Engineering Materials and Building Preservation**
- 3. University of the Bundeswehr Munich, Institute for Mathematics and Computer-Based Simulation**



A Benchmark Study for Bridge Monitoring

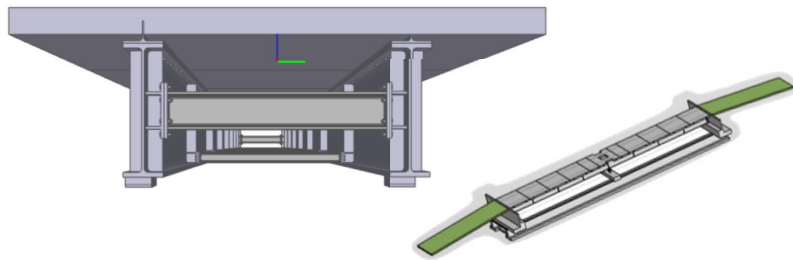


- Realized at the UniBW campus as testing facility;
- experimental data acquisition campaign [1];
- different design damage scenarios:
 - additional masses;
 - support settlements;
 - structural elements removal;
- ambient and operative loads conditions.



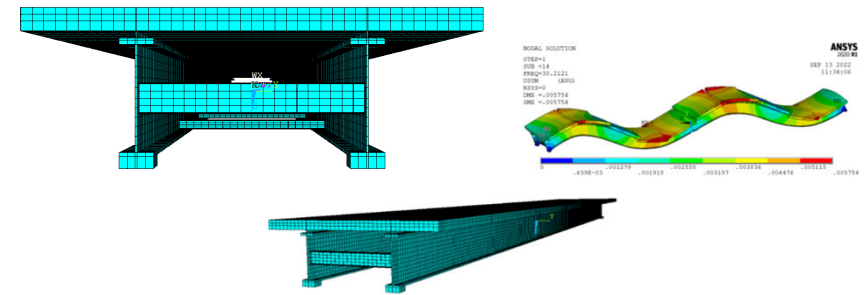
[1] Jaelani, Yogi, Alina Klemm, Johannes Wimmer, et al. 2023. Developing a Benchmark Study for Bridge Monitoring. *Steel Construction* 16 (4): 215–25. DOI: [10.1002/stco.202200037](https://doi.org/10.1002/stco.202200037).

Numerical Models of the Test-bridge...



Revit BIM Model [2]:

- **Purpose** is primarily for architectural design, documentation, and coordination. It captures geometric, material, and functional data but is not inherently analytical.
- **Use Case** is early-stage design, visualization, and integration with other BIM ecosystem tools.



Ansys FEM Model [3]:

- **Purpose** is Finite Element Analysis (FEA) for structural performance evaluation (e.g., stress, deformation, dynamic response). APDL (Ansys Parametric Design Language) allows scripting for parametric studies, automation, and complex simulations.
- **Use Cases** are advanced structural analysis, optimization, and research where fine control over meshing, solvers, and post-processing is needed.

[2] Janßen, S. (2018) *Entwicklung von Ansätzen zur messdatenbasierten BIM-Umsetzung eines digitalen Zwillings am Beispiel einer Testbrücke*, Helmut Schmidt Universität.

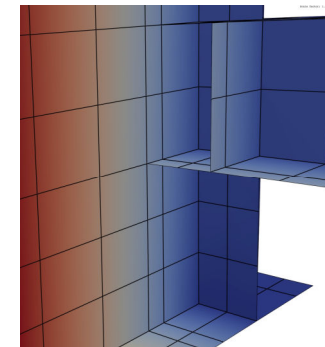
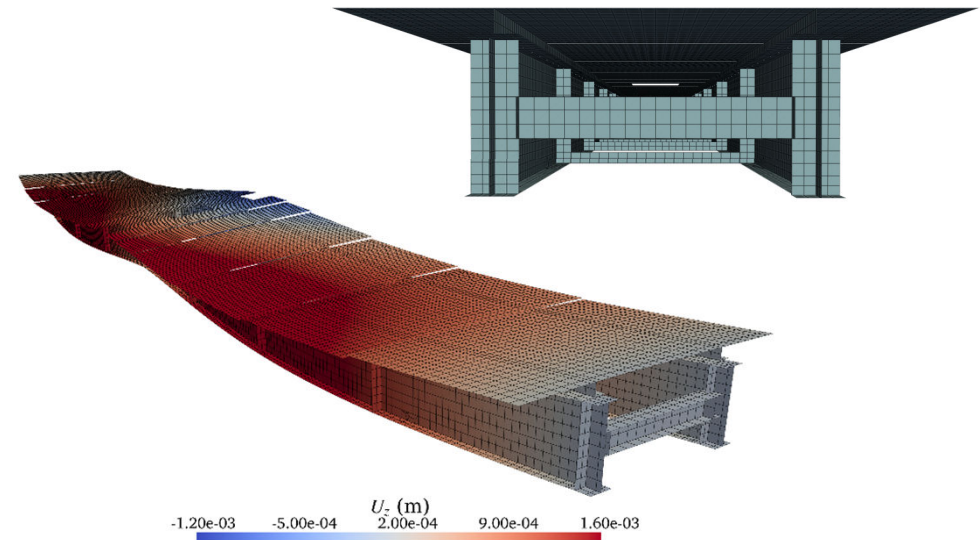
[3] Artola, S. (2022) *Developing a Benchmark Study for Structural Health Monitoring*, Technische Universität München.

...and yet Another One

PyANSYS (PyAPDL) Model:

- **Purpose** is same as APDL but scripted in Python. PyANSYS provides a Pythonic interface to Ansys solvers, enabling integration with state-of-the-art workflows: e.g., machine learning, data science, custom automation etc.
- **Use Case** when Python for scripting is needed to integrate Ansys with other Python tools and to leverage object-oriented programming (OOP).

✓ Python ecosystem integration, modern syntax and readability, OOP.	✗ Some advanced APDL functionalities may not be exposed in PyANSYS.
✓ Better debugging and IDE support.	✗ Limited backward compatibility.
✓ Automation and customization.	✗ No GUI integration.



Model Surrogation via Hoeffding-Sobol' Decomposition

Given a stochastic model $M(\mathbf{X})$, function of a random variable (RV) vector input $\mathbf{X}$ defined on a d – dimensional unit cube $\mathbf{K}^d$, the **Hoeffding-Sobol'** or **ANOVA** [4] decomposition allows to write it as a series of 2^d terms, one for each possible combination of the elements of $\mathbf{X}$:

$$\mathcal{M}(\mathbf{X}) = \mathcal{M}_0 + \sum_{i=1}^d \mathcal{M}_i(X_i) + \sum_{1 \leq i < j \leq d} \mathcal{M}_{ij}(X_{ij}) + \cdots + \mathcal{M}_{1,2,\dots,d}(\mathbf{X}).$$

If the integral of each summand term over any of its independent variable is null, i.e.:

$$\int_0^1 \mathcal{M}_{i_1,\dots,i_s}(X_{i_1},\dots,X_{i_s}) dX_{i_k} = 0 \quad \text{for } 1 \leq k \leq s,$$

then these properties follow:

1. the constant $\mathcal{M}_0$ is the mean value of the function:

$$\mathcal{M}_0 = \int_{\mathbf{K}^d} \mathcal{M}(\mathbf{X}) d\mathbf{X},$$

2. the mixed summands are orthogonal to each other:

$$\int_0^1 \mathcal{M}_{i_1,\dots,i_s}(X_{i_1},\dots,X_{i_s}) \mathcal{M}_{j_1,\dots,j_t}(X_{j_1},\dots,X_{j_t}) d\mathbf{X} = 0 \quad \text{for } \{i_1,\dots,i_s\} \neq \{j_1,\dots,j_t\}.$$

[4] Sobol', I.M. (2001). Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates, *Mathematics and Computers in Simulation*, 55(1): 271-280.
DOI: [10.1016/S0378-4754\(00\)00270-6](https://doi.org/10.1016/S0378-4754(00)00270-6).

Model Surrogation via Hoeffding-Sobol' Decomposition

$$\mathcal{M}(\mathbf{X}) = \mathcal{M}_0 + \sum_{i=1}^d \mathcal{M}_i(X_i) + \sum_{1 \leq i < j \leq d} \mathcal{M}_{ij}(X_{ij}) + \dots + \mathcal{M}_{1,2,\dots,d}(\mathbf{X})$$

The function $M(\mathbf{X})$ is required to be **square integrable**. In this case, by taking the square and integrating, the **total variance** can be partitioned in the sum of its **partial variance** contributions as:

$$\int_{\mathbf{K}^d} \mathcal{M}(\mathbf{X})^2 d\mathbf{X} - \mathcal{M}_0^2 = \sum_{i=1}^d \int_{\mathbf{K}^d} \mathcal{M}_i(X_i)^2 d\mathbf{X} + \sum_{1 \leq i_1 < \dots < i_s \leq d} \int_{\mathbf{K}^d} \mathcal{M}_{i_1, \dots, i_s}(X_{i_1}, \dots, X_{i_s})^2 d\mathbf{X}.$$

$$D = \sum_{i=1}^d D_i + \sum_{1 \leq i_1 < \dots < i_s \leq d} D_{i_1, \dots, i_s}, \quad 1 = \sum_{i=1}^d S_i + \sum_{1 \leq i_1 < \dots < i_s \leq d} S_{i_1, \dots, i_s}.$$

- Each **Sobol' index** is a **sensitivity measure** describing which amount of the total variance is due to the uncertainties in the set of inputs;
- the **first** order indices give the influence of each parameter taken alone;
- The **higher** order indices account for possible mixed influence of various parameters.

Model Surrogation via Generalized Polynomial Chaos Expansion (gPCE)

Finally, the **gPCE** method allows to approximate $M(\mathbf{X})$ in the form of a truncated polynomial chaos expansion:

$$Y = \mathcal{M}(\mathbf{X}) \approx \sum_{\alpha \in \mathcal{A}} y_{\alpha} \Psi_{\alpha}(\mathbf{X}).$$

Multivariate polynomial:

$$\Psi_{\alpha}(\mathbf{X}) = \prod_{i=1}^M \Psi_{\alpha_i}^{(i)}(X_i), \quad |\alpha| = \sum_{i=1}^M \alpha_i,$$

where $\Psi_{\alpha_i}^{(i)}(X_i)$ is the univariate polynomial of degree α_i from the orthonormal family associated to X_i .

Distribution	Basis	Support
Gaussian	Hermite	$(-\infty, +\infty)$
Uniform	Legendre	$[-1, +1]$
Gamma	Laguerre	$[0, +\infty)$
Beta	Jacobi	$[-1, +1]$

Computation of coefficients:

$$y_{\alpha} = \mathcal{M}(\mathbf{X}) - \sum_{\alpha \in \mathcal{A}} y_{\alpha} \Psi_{\alpha}(\mathbf{X})$$

Model output **mean** and **variance**:

$$\hat{\mu}_Y = y_0, \quad \hat{\sigma}_Y^2 = \sum_{\alpha \in \mathcal{A} \setminus \mathbf{0}} y_{\alpha}^2$$

Sobol' indices are obtained analytically from the expansion coefficients:

$$S_u = D_u / D = \sum_{\alpha \in \mathcal{A}_u} y_{\alpha}^2 / \sum_{\alpha \in \mathcal{A} \setminus \mathbf{0}} y_{\alpha}^2$$

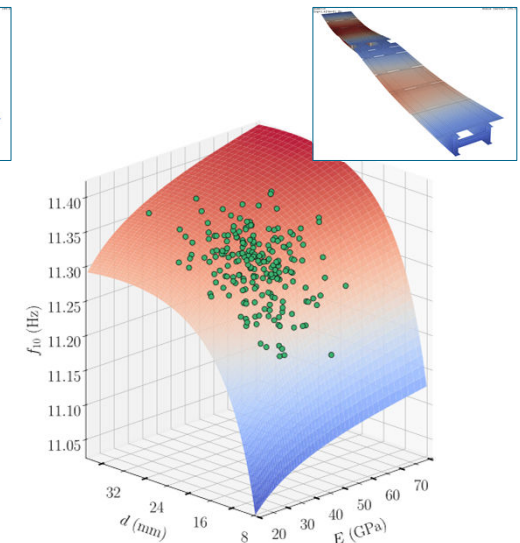
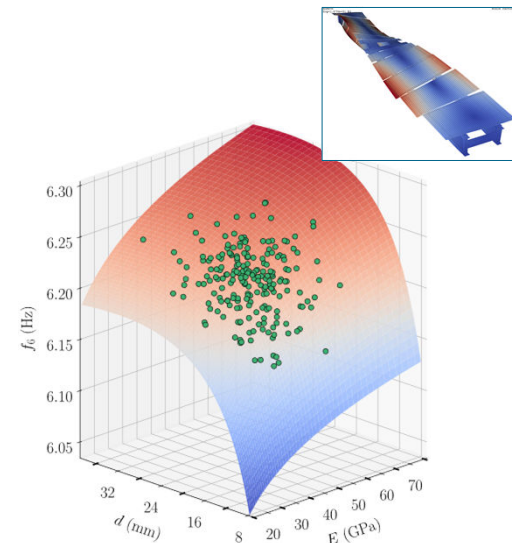
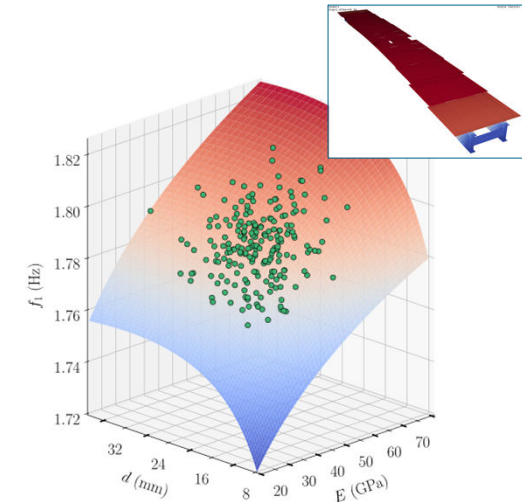
Application: Sensitivity Analysis of Selected Bridge Eigenfrequencies

- Sensitivity analysis of $M = 3$ mechanical parameters selected as inputs:
 - $[X_1, X_2, X_3] \leftrightarrow [E_c, d_b, E_m];$
- output quantities considered are relevant modes of vibration:
 - $[Y_1, \dots, Y_4] \leftrightarrow [f_1, f_6, f_{10}];$
- input RVs are log-normally distributed;
- different polynomial degrees have been tested, $p = 3$ and $p = 4$, results shown for $p = 4$;
- size N of experimental design chosen according to optimality criterion, $N = (p + 2)^M$;
- standard truncation scheme: $A^{M,p} = \{\alpha \in \mathbb{N}^M : |\alpha| \leq p\}; P = \frac{(M+p)!}{M!p!} = 35$ terms in the polynomial expansion.

RV	Parameter	μ	COV
X_1	E_c	35.0 GPa	20 %
X_2	d_b	18.0 mm	20 %
X_3	E_m	10 GPa	20 %

Application: Sensitivity Analysis of Selected Eigenfrequencies

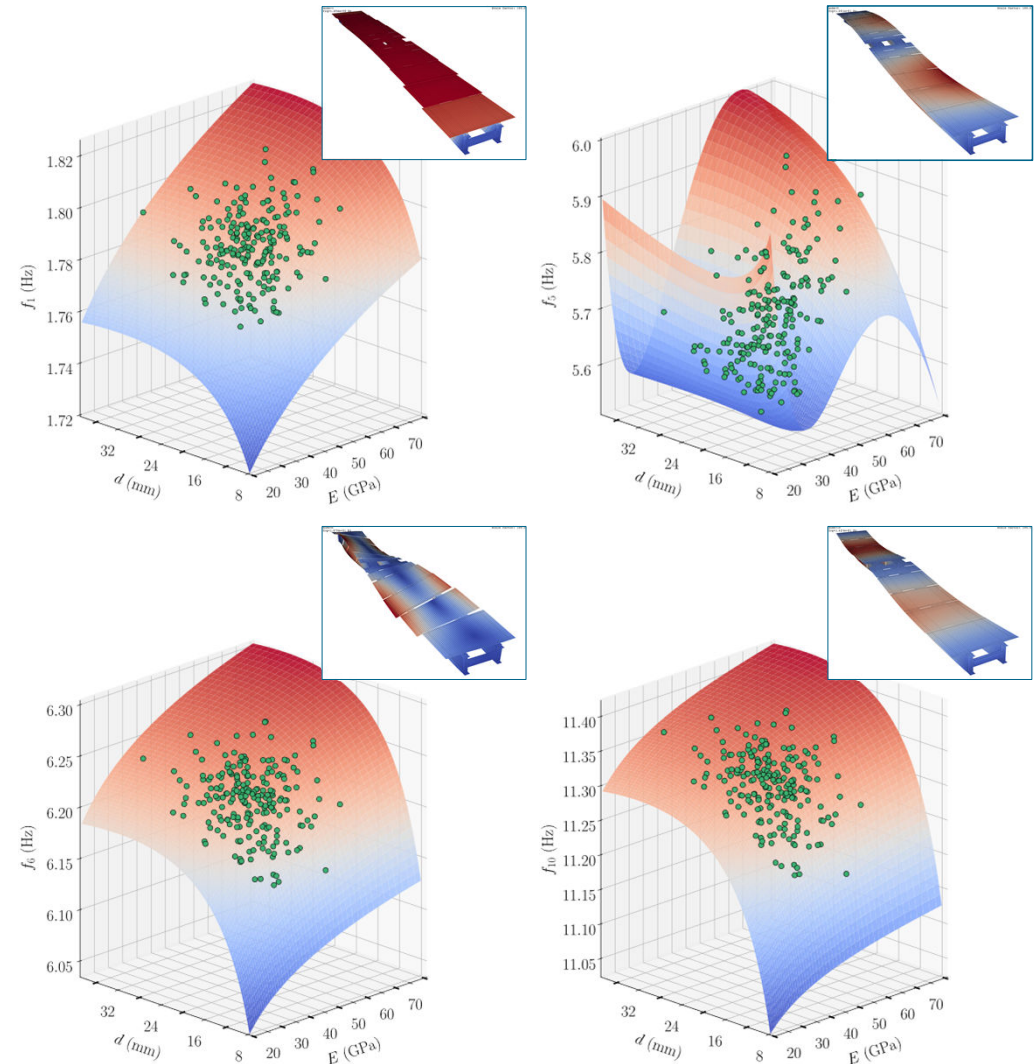
Index	Mode 1	Mode 6	Mode 10
	$p = 4$	$p = 4$	$p = 4$
S_1	0.89440	0.81868	0.17176
S_2	0.03798	0.06939	0.81185
S_3	0.06416	0.11139	0.01629
S_{12}	0.00236	0.00047	0.00006
S_{23}	0.00079	0.00005	0.00000
S_{13}	0.00013	0.00003	0.00003
S_{123}	0.00019	0.00000	0.00000
S_{T1}	0.89707	0.81917	0.17185
S_{T2}	0.04131	0.06991	0.81191
S_{T3}	0.06460	0.11144	0.01636



[5] Bonari, J., Marsili, F., von Danwitz, M., & Popp, A. (in press). Physics-Informed Sensitivity Analysis for Enhanced Structural Health Assessment: Test-Case for a Mixed Steel-Concrete Bridge. In Proceedings of the 10th Civil Structural Health Monitoring Workshop. arXiv: [2606.16308](https://arxiv.org/abs/2606.16308).

Application: Sensitivity Analysis of Selected Eigenfrequencies

Index	Mode 1	Mode 5	Mode 6	Mode 10
	$p = 4$	$p = 4$	$p = 4$	$p = 4$
S_1	0.89440	0.36075	0.81868	0.17176
S_2	0.03798	0.63309	0.06939	0.81185
S_3	0.06416	0.00592	0.11139	0.01629
S_{12}	0.00236	0.00022	0.00047	0.00006
S_{23}	0.00079	0.00001	0.00005	0.00000
S_{13}	0.00013	0.00002	0.00003	0.00003
S_{123}	0.00019	0.00000	0.00000	0.00000
S_{T1}	0.89707	0.36099	0.81917	0.17185
S_{T2}	0.04131	0.63331	0.06991	0.81191
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Sample-free Linear Bayesian Updating

Input parameters, **error** measure, and **measurable response** can also be written in **gPCE** format [6]:

$$\mathbf{Q} = \sum_{\alpha=0}^{L-1} \mathbf{Q}_{\alpha} \Phi_{\alpha}^{(Q)}(\mathbf{X}), \quad \varepsilon = \sum_{\alpha=0}^{M-1} \mathbf{E}_{\alpha} \Phi_{\alpha}^{(E)}(\mathbf{X}), \quad \mathbf{Y} = \sum_{\alpha=0}^{M-1} \mathbf{Y}_{\alpha} \Phi_{\alpha}^{(Y)}(\mathbf{X}).$$

All these quantities can be expressed by a common polynomial expansion $\hat{\Phi}$, same applies to experimental data:

$$\mathbf{Q} = \sum_{\alpha=0}^{L-1} \hat{\mathbf{Q}}_{\alpha} \hat{\Phi}_{\alpha}(\mathbf{X}) = \hat{\mathbf{Q}} \hat{\Phi}, \quad \varepsilon = \sum_{\alpha=0}^{M-1} \hat{\mathbf{E}}_{\alpha} \hat{\Phi}_{\alpha}(\mathbf{X}) = \hat{\mathbf{E}} \hat{\Phi}, \quad \mathbf{Y} = \sum_{\alpha=0}^{M-1} \hat{\mathbf{Y}}_{\alpha} \hat{\Phi}_{\alpha}(\mathbf{X}) = \hat{\mathbf{Y}} \hat{\Phi}, \quad \mathbf{Z}_m = \sum_{\alpha=0}^{M-1} \hat{\mathbf{Z}}_{\alpha} \hat{\Phi}_{\alpha}(\mathbf{X}) = \hat{\mathbf{Z}} \hat{\Phi}.$$

With the new extended PCE expansion $\hat{\Phi}$, a Kalman step can be written, without the need of sampling from the model:

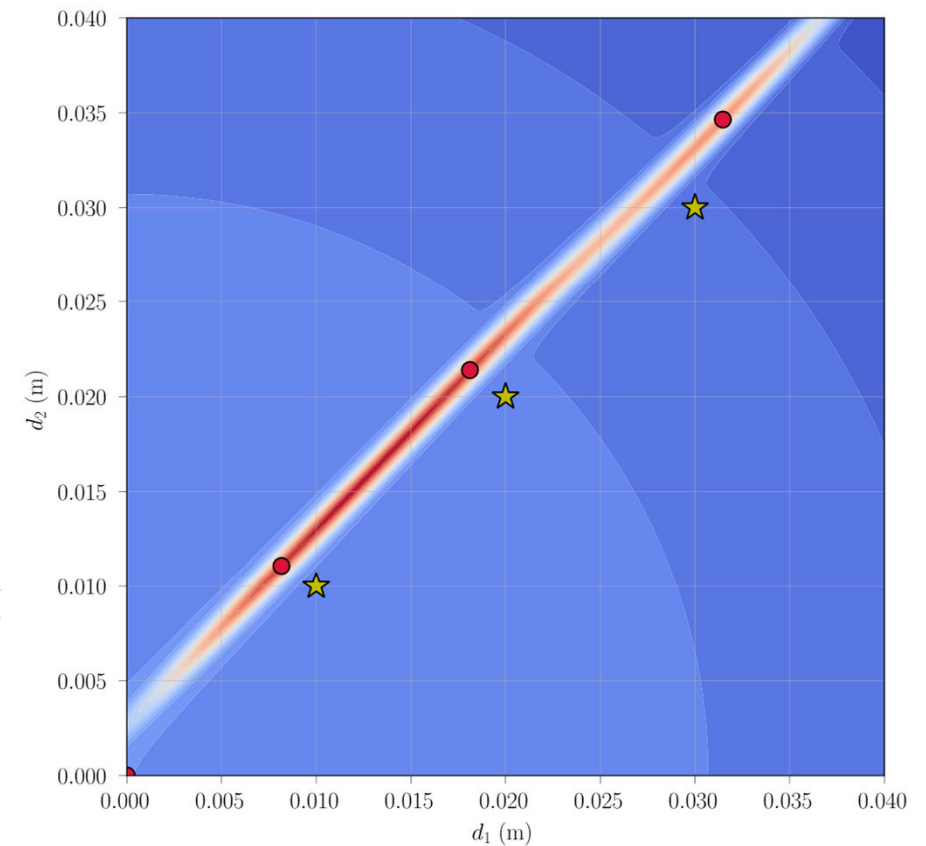
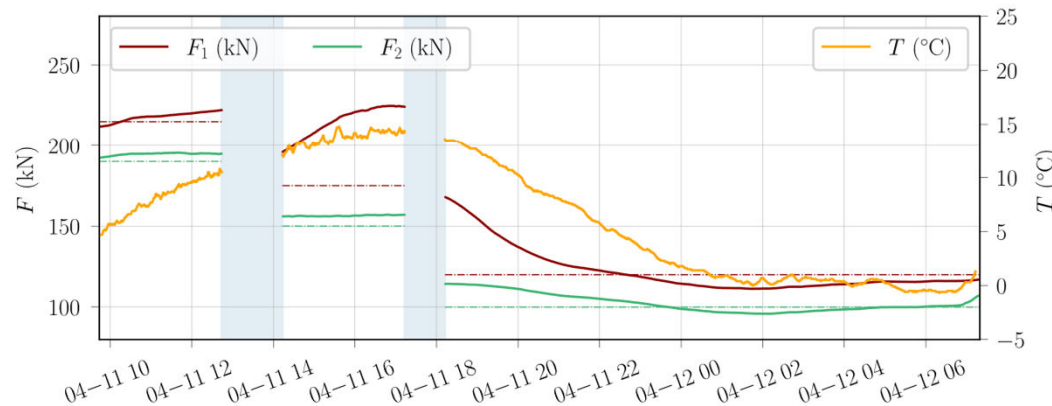
$$\hat{\mathbf{Q}}' = \hat{\mathbf{Q}} + \mathbf{K}(\hat{\mathbf{Z}} - \hat{\mathbf{Y}} - \hat{\mathbf{E}}).$$

[6] Friedman, N., Zoccarato, C., Zander, E., & Matthies, H. G. (2021). A Worked-out Example of Surrogate-based Bayesian Parameter and Field Identification Methods. In J. Chiachío-Ruano, M. Chiachío-Ruano, & S. Sankararaman, *Bayesian Inverse Problems* (1st ed., pp. 155–203). CRC Press. DOI: [10.1201/b22018-10](https://doi.org/10.1201/b22018-10).

Application: Bridge Foundation Settlements under Design Damage

Sample-Free Bayesian updating of foundation settlements:

- **input:** acquired reaction forces F_1 (kN) and F_2 (kN);
- **output:** assimilated value of settlements, d_1 (m) and d_2 (m);
- **damage scenario:** artificial settlement of 0.01 m, 0.02 m, and 0.03 m, directly applied to central support via hydraulic jacks;
- **temperature influence:** still to be taken into account [7].



[7] Bonari, J., Marsili, F., von Danwitz, M., & Popp, A. (in press). High-Fidelity Numerical Modeling for the Mechanical Characterization of a Full-Scale Test Bridge. In Proceedings of 4th International Conference on Natural Hazards & Infrastructure. arXiv: [2606.16308](https://arxiv.org/abs/2606.16308).

Future Developments



QUEENS [8] (**Q**uantification of **U**ncertain **E**ffects in **E**ngineering **S**ystems) is a Python framework for solver-independent multi-query analyses of large-scale computational models.

Its features are comprehensive of:

- parameter studies and i
- sensitivity an
- surrogate mod
- uncertainty quantification;
- Bayesian inverse analysis.

15:00 - 16:30, ICM 14a
MS031F - Open-Source Software in Mechanics VI



<https://queens-py.github.io/queens/index.html>

[8] Biehler, J., Nitzler, J., Brandstaeter, S., Dinkel, M., Gravemeier, V., Haeusel, L. J., Robalo Rei, G., Willmann, H., Wirthl, B., & Wall, W. A. (2025). *QUEENS: An Open-Source Python Framework for Solver-Independent Analyses of Large-Scale Computational Models*. arXiv: [2508.16316](https://arxiv.org/abs/2508.16316).

Conclusions

- A comprehensive approach for structural analysis and condition characterization of a representative bridge structure has been presented.
- The study features a full analysis pipeline that goes through:
 1. data collection and analysis;
 2. high-fidelity and surrogate model definition;
 3. model synthesis through real experimental data.
- Every step is seamlessly integrated in within the same software ecosystem.

This demonstrator is a valuable testbed for the development and analysis of new techniques applied to the protection of **Terrestrial Infrastructures** and, finally, offers a sound testbed for the development of functional hybrid **Digital Twins**.

Thank you!



Institute for the Protection
of Terrestrial Infrastructures



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References:

- [1] Jaelani, Yogi, Alina Klemm, Johannes Wimmer, et al. 2023. Developing a Benchmark Study for Bridge Monitoring. *Steel Construction* 16 (4): 215–25. DOI: [10.1002/stco.202200037](https://doi.org/10.1002/stco.202200037).
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- [6] Friedman, N., Zoccarato, C., Zander, E., & Matthies, H. G. (2021). A Worked-out Example of Surrogate-based Bayesian Parameter and Field Identification Methods. In J. Chiachío-Ruano, M. Chiachío-Ruano, & S. Sankararaman, *Bayesian Inverse Problems* (1st ed., pp. 155–203). CRC Press. DOI: [10.1201/b22018-10](https://doi.org/10.1201/b22018-10).
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Jacopo Bonari, July 23rd 2026, Munich