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To cite this article: Vladyslav Gapyak *et al* 2026 *Phys. Med. Biol.* **71** 195033

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OPEN ACCESS

RECEIVED
30 May 2026REVISED
13 August 2026ACCEPTED FOR PUBLICATION
9 September 2026PUBLISHED
9 October 2026

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MPISuperRes-PnP: a super-resolution zero-shot plug-and-play reconstruction algorithm for magnetic particle imaging

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E-mail: vladyslav.gapyak@dlr.de, thomas.maerz@h-da.de and andreas.weinmann@thws.de**Keywords:** magnetic particle imaging, superresolution, energy minimization inspired reconstruction, regularized reconstruction, plug-and-play, zero-shot denoiser, system matrixSupplementary material for this article is available [online](#)

Abstract

Objective. Magnetic particle imaging (MPI) is a promising, emerging medical imaging modality. MPI is based on the non-linear response of magnetic nanoparticles to an applied magnetic field and does not expose the specimen to ionizing radiation. The measured signal is the voltage induced in receive coils by the particles' response. Reconstructing the particle concentration from the signal constitutes the imaging task. Even using state-of-the-art measurement-based reconstruction, the associated spatial grid is very coarse, hence super-resolution techniques are important. In this work, we propose an approach for super-resolution in MPI inspired by energy minimization. **Approach.** Different methods have been proposed for super-resolution in MPI, ranging from upscaling of the associated system matrix to interpolation of the reconstruction. Here we incorporate super-resolution into the reconstruction task via an energy minimization formulation. Following the plug-and-play (PnP) approach to energy minimization we derive a splitting scheme where the arising Gaussian denoising task is treated with a pre-trained learned Gaussian denoiser. **Main results.** We derive a super-resolution method for MPI based on a PnP approach using a pre-trained denoiser in zero-shot fashion. This way, we incorporate benefits of deep learning without training and avoid the need of training data. Further, we provide a quantitative and qualitative evaluation of the proposed method on simulated data with realistic noise. Hyperparameters are selected via an extended parameter search. The found parameters are applied for reconstruction on real data. We qualitatively show the applicability of our method on real data (MPIData: EquilibriumModelWithAnisotropy and 2D-OpenMPI Data). **Significance.** The proposed method employs a deep-learning denoiser without training—thus it does not require presently scarcely available MPI training data. The denoiser behaves conservatively, i.e. no hallucination artifacts were observed. The super-resolution approach is generic such that it can be applied in future MPI contexts involving different regularizers or different imaging tasks.

1. Introduction

Magnetic particle imaging (MPI) is a medical imaging modality introduced in 2005 by Gleich and Weizenecker Gleich and Weizenecker (2005). It is a tracer-based modality, which uses superparamagnetic nanoparticles as a contrast agent. MPI aims at reconstructing the distribution of particles injected in the target specimen by exploiting the particles' non-linear response to dynamic magnetic fields. As of today, multiple medical applications for MPI have been proposed. Among such applications we mention

detection and cancer imaging Yu *et al* (2017), Song *et al* (2018), Du *et al* (2019), Tay *et al* (2021), tracing of stem cells Connell *et al* (2015), Jung *et al* (2018), Lemaster *et al* (2018), blood flow and cardiovascular imaging Vaalma *et al* (2017), Bakenecker *et al* (2018), Franke *et al* (2020), Tong *et al* (2021), safety measurements of medical implants Wegner *et al* (2025). MPI offers a series of benefits when compared with imaging modalities such as CT Buzug (2008), MRI Schmale *et al* (2015), PET Ter-Pogossian *et al* (1975) and SPECT Kuhl and Edwards (1963); for example, it does not employ ionizing radiation nor radioactive tracers, and offers shorter acquisition time as well as high spatial resolution Knopp and Buzug (2012). For a thorough comparison between MPI and other imaging modalities we refer the interested reader to Billings *et al* (2021), Yang *et al* (2022).

An MPI scan is usually performed by first applying a static selection field that magnetically saturates the particles everywhere in the specimen with the exception of a field-free region (FFR). Then, an additional dynamic drive field is superimposed to the static field to steer the FFR within the field of view (FoV) in the scanner. The magnetic nanoparticles exposed to the dynamic magnetic field induce a voltage in receive coils. The induced voltage constitutes the scan signal from which the particle concentration is to be reconstructed.

There are currently two main classes of approaches for the reconstruction of particle distributions: measurement-based approaches and model-based approaches. Beyond addressing theoretical understanding, model-based reconstruction techniques aim at significantly reducing calibration procedures; for details, see for instance Rahmer *et al* (2009), Goodwill and Conolly (2011), März and Weinmann (2016), Bringout *et al* (2020), Maass *et al* (2024), Gapyak *et al* (2025). In this work, we focus on the measurement-based approach. Measurement-based approaches typically acquire a system matrix obtained through the following calibration procedure: for each column of the matrix, one performs scanning of a delta probe located in a corresponding pixel or voxel of a chosen 2D pixel or 3D voxel grid Weizenecker *et al* (2007), Knopp *et al* (2011), Lampe *et al* (2012), Rahmer *et al* (2012). More precisely, a probe with a reference concentration of tracer is iteratively positioned and scanned at each cell (pixel / voxel) of the grid. Actually, several scans are performed and averaged for each position of a delta probe. This way the response of the scanning system to discrete delta impulses at each voxel position is collected and stored in the system matrix A . Assuming linearity, the acquired signal f of the scan of a specimen is then obtained as a superposition of the scans of the delta impulses. For reconstruction, one has to solve a corresponding system of equations $Au = f$, where the symbol u denotes the desired particle distribution; loosely speaking, one has to invert the system matrix for the measured signal f . Due to the presence of noise and the ill-conditioning of the system matrix (e.g. Knopp *et al* (2010), Storath *et al* (2017)), regularization techniques are needed for the inversion. We observe that—in a typical measurement-based reconstruction setup – the grid associated with the system matrix determines *a priori* the later spatial resolution of the reconstructions. The simple observation that doubling the resolution of the reconstruction would require performing new scans at 4 times the considered delta probes (2D), or 8 times the considered delta probes (3D), and that the calibration can take hours (e.g. Güngör *et al* (2022), Schrank *et al* (2022)), explains why super-resolution techniques are desirable in measure-based MPI reconstruction.

Related work. For super-resolution in measurement-based MPI reconstruction, most approaches proposed in the literature derive a higher resolved system matrix from a lower resolution measured version. Then they solve (a potentially regularized variant) of the corresponding linear system of Equations (as in standard reconstruction). The solution yields a higher spatially resolved reconstruction. Classical methods, e.g. Kluth *et al* (2020), as well as deep learning based methods have been proposed Baltruschat *et al* (2020), Schrank *et al* (2022). A transformer-based approach is the TranSMS Güngör *et al* (2022). There is also research which employs the measured system matrix and works on a super-resolved image domain. We refer in particular to the papers Timmermeyer *et al* (2013), Omer *et al* (2015) which employ classical (non-learning based) iterative schemes. An overview on super-resolution in MPI can be also found in the recent survey article Zhang *et al* (2024).

A particular class of deep learning-based reconstruction approaches are plug-and-play (PnP) approaches Venkatakrishnan *et al* (2013). PnP approaches have been used for a variety of imaging modalities, e.g. Ahmad *et al* (2020), as well as for computer vision tasks, e.g. Zhang *et al* (2022). For MPI reconstruction, PnP approaches have been proposed in Askin *et al* (2022) which is based on an ADMM scheme, as well as in Gapyak *et al* (2025a, 2025b) which uses a half-quadratic splitting and allows for a model-guided hyper-parameter choice strategy as well as an additional ℓ^1 -prior. PnP approaches typically result in iterative schemes which alternate between solving a classically regularized inversion problems such as classical Tikhonov reconstruction and applying Gaussian denoising. Gaussian denoising in turn is a standard computer vision task and many good deep learning denoising models are available which can be ‘plugged in’ to complete the iterative scheme.

Contributions. In this paper, we derive an energy-minimization inspired PnP-type method for super-resolution in MPI, and show its potential by application to real and synthetic data. More precisely, the contributions are:

- (i) Super-resolution and reconstruction are two distinct problems. In Gapyak *et al* (2025b), we developed the ZeroShot-PnP algorithm for reconstruction with system matrices. In this work, we address combined super-resolution with reconstruction as a new joint problem. The incorporation of super-resolution in PnP approaches is derived as a splitting scheme for a specifically defined cost function, providing mathematical motivation for the algorithm. Moreover, we observe that, as a consequence of the splitting scheme chosen, we are able to reuse many components developed in Gapyak *et al* (2025b). As a result, we introduce MPISuperRes-PnP, an algorithm for the super-resolved reconstruction in MPI leveraging a PnP approach and the denoising capabilities of a zero-shot denoiser, the *deep denoiser prior* Zhang *et al* (2022), akin to Gapyak *et al* (2025b).
- (ii) We apply the derived method to various public MPI datasets. In particular, we show reconstruction results on the MPI-MNIST dataset Iske *et al* (2025a) published with Iske *et al* (2025b), on the ‘MPIData: EquilibriumModelWithAnisotropy’ Knopp and Scheffler (2024) published with Maass *et al* (2024), and on the 2D OpenMPIData Knopp *et al* (2020).

An implementation of MPISuperRes-PnP can be found on GitHub at: <https://github.com/VladyslavGapyak/MPISuperRes-PnP>.

Outline. Section 2 starts with a description of the proposed method. More precisely, in section 2.1, we review system-matrix based reconstruction in MPI, in section 2.2 we deal with data preprocessing, in section 2.3 we derive the proposed algorithm: we start out considering a suitable energy functional, discuss the choice of parameters, and provide a pseudocode. In section 3 we present the conducted experiments together with the results obtained. More precisely, in section 3.1 we describe the datasets used and in section 3.2 the image quality measures employed; then, in sections 3.3–3.6 we present the reconstructions on the four datasets described in section 3.1. Our conclusions follow in section 4.

2. Methods

In this section we introduce the MPI reconstruction problem based on a system matrix (section 2.1). We recall the main data preprocessing steps typically employed (section 2.2) and, finally, introduce the MPISuperRes-PnP algorithm in section 2.3.

2.1. System-matrix-based reconstruction in MPI

In MPI the target concentration of nanoparticles ρ_{GT} is to be reconstructed from the measured signal f . In the system-matrix-based approach, the problem is considered in a discrete (spatial) scenario. Consequently, in a two-dimensional setup, the FoV Ω is discretized by considering an $N_x \times N_y$ grid. We denote with $N = N_x \cdot N_y$ the total number of grid cells. With this discretization, the target concentration ρ_{GT} is approximated by a vector $x \in \mathbb{R}^N$ such that $\rho_{\text{GT}} \approx \tilde{x} \in \mathbb{R}^{N_x \times N_y}$ where $\tilde{\cdot}$ is the reshaping operator; additionally, the relation with the signal f is modeled via the relation

$$f = Ax + \eta \quad (1)$$

where the symbol $A \in \mathbb{R}^{M \times N}$ denotes the system matrix and η represents the inherent noise in the measurement Rahmer *et al* (2009). In the measurement-based approach, the system matrix A is acquired via a calibration procedure performed by iteratively scanning and repositioning a known concentration of nanoparticles (the δ -concentration) across each cell of the $N_x \times N_y$ grid. We point out that A is of size $M \times N$, where $N = N_x \times N_y$ as defined above, and M is the number of rows, which depends on the number of channels employed and on the preprocessing applied to the data before solving the linear system in (1) (cf section 2.2). As an example, in the 2D OpenMPI Knopp *et al* (2020) the calibration is performed on a 19×19 grid, yielding $N = 19^2 = 361$, and each scan is acquired at 1632 time points along two channels (with coils positioned along the x - and y -axis) and Fourier transformed, giving 817 complex-valued frequencies per channel; upon stacking of the real and imaginary parts per channel to obtain a real-valued matrix, the total amount of rows is $M = 2 \cdot 2 \cdot 817 = 3268$. Finally, given a scan f and a calibration matrix A , the objective is to retrieve x solving (1). Because both f and

A are affected by real noise, the inversion of (1) requires regularization techniques Kirsch *et al* (2011), Bertero *et al* (2021).

2.2. Data preprocessing

Usual preprocessing steps performed on MPI data affect the number of rows M of the linear system in (1). The reason is that these preprocessing steps are usually aimed at either improving the conditioning of the system in (1) or at discarding unreliable frequencies, that are deemed as such according to chosen criteria. We recall the main preprocessing steps as, for instance, outlined in Kluth and Jin (2019).

- (i) *Background correction*: according to the models, the measured data can be decomposed as $f = f_{cl.} + b_f + \eta_f$ and coherently the system matrix as $A = A_{cl.} + b_A + \eta_A$, where $f_{cl.}$, $S_{cl.}$ are the clean signals, b_f , b_A are the background signals and η_f , η_A are the arrays representing the noise. To exclude the background signals, which bear no information about the underlying particle concentrations, empty scans without any particles in the scanner are performed at the time of calibration as well as before or after the scan of the target specimen. These background scans can be subtracted from A and f to correct for the background terms.
- (ii) *Bandpass filtering*: some scanners (like the Bruker scanner) utilize an analog filter to suppress excitation crosstalk—at the price of decreasing the signal-to-noise ratio (SNR) – of frequencies below 80 kHz Rahmer *et al* (2012). For this reason, a high-pass filter at 80 kHz is usually necessary.
- (iii) *SNR-thresholding*: provided an estimation of the SNR of each frequency is available, SNR-based thresholding is usually performed Kluth and Jin (2019) by discarding those frequencies whose SNR is below a certain threshold Θ , e.g. below $\Theta = 1$ dB.
- (iv) *Whitening*: in Dittmer *et al* (2021) the authors have proposed to apply a whitening transformation, i.e. considering the whitened system $WAx = Wf$ where the matrix $W \in \mathbb{R}^{M \times M}$ is the diagonal covariance matrix obtained from background scans. More specifically, the diagonal entries of W are the terms $1/\sigma_b$ and σ_b^2 is the variance of the background scan.
- (v) *Low rank approximation*: following Kluth and Jin (2019), a target rank $K \leq \min\{M, n\}$ can be selected and a randomized singular value decomposition (rSVD) Halko *et al* (2011) is performed. The output of the rSVD is a triple of matrix (U_K, Σ_K, V_K^*) such that $A \approx U_K \Sigma_K V_K^*$. By employing U_K , the system with K rows, $U_K^* Ax = U_K^* f$, is then considered.

2.3. MPISuperRes-PnP : an algorithm for super-resolved regularized inversion with zero-shot learned-denoiser

In this section we introduce MPISuperRes-PnP, the proposed algorithm to incorporate super-resolution within the ZeroShot-PnP framework in Gapyak *et al* (2025b). We show that the proposed scheme arises from an energy minimization scheme which addresses (1) and incorporates regularization:

$$\hat{x} = \arg \min_x \left\{ \frac{1}{2} \|Ax - f\|_2^2 + \lambda \mathcal{R}(x) \right\}, \quad (2)$$

where $A \in \mathbb{R}^{M \times N}$, for $N = N_x \cdot N_y$ and M which depends on the chosen preprocessing applied to the system (see section 2.2). In equation (2) $\mathcal{R}$ is a chosen regularizer and $\lambda > 0$ is the regularization parameter regulating the strength of the regularization.

For the purpose of superresolution we need an upscaling operator $\mathcal{U}_s: \mathbb{R}^{N_x \times N_y} \rightarrow \mathbb{R}^{sN_x \times sN_y}$. The upscale factor $s \in \mathbb{N}$ is a parameter of the upscaling operator $\mathcal{U}_s$ which we define and implement via bilinear interpolation. Because we treat particle concentrations x both as 1-dimensional vectors in the solution of (5) as well as 2-dimensional images, we define the reshaping operator $\tilde{\cdot}: \mathbb{R}^{N_x \cdot N_y} \rightarrow \mathbb{R}^{N_x \times N_y}$ (turning 1d vectors into 2d images) and its inverse, the vectorization operator $\text{vec}: \mathbb{R}^{N_x \times N_y} \rightarrow \mathbb{R}^{N_x \cdot N_y}$ (turning 2d images into 1d vectors). The operator $U_s: \mathbb{R}^{N_x \cdot N_y} \rightarrow \mathbb{R}^{sN_x \cdot sN_y}$ is the equivalent of the upscaling operator $\mathcal{U}_s$, but operates on 1d vectors while $\mathcal{U}_s$ operates on 2d images. The two operators are related by the condition $\text{vec}(\mathcal{U}_s \tilde{x}) = U_s x$. The adjoint operator of U_s is the operator $U_s^*: \mathbb{R}^{sN_x \cdot sN_y} \rightarrow \mathbb{R}^{N_x \cdot N_y}$ defined by the property $\langle U_s x, y \rangle = \langle x, U_s^* y \rangle$ for all $x \in \mathbb{R}^{N_x \cdot N_y}$ and $y \in \mathbb{R}^{sN_x \cdot sN_y}$.

Having chosen the upscaling operator U_s , we consider an up-scale variable x_u defined on the higher resolved space $\mathbb{R}^{sN_x \cdot sN_y}$, and a down-scale variable x_d defined on the space $\mathbb{R}^{N_x \cdot N_y}$; in this way, $U_s x_d$ is the upscaled version of x_d and lives in the space as x_u , and vice versa, $U_s^* x_u$ lives in the same space as x_d .

With these definitions we propose to consider the following variation of (2)

$$\min_{x_d, x_u} \left\{ \frac{1}{2} \|Ax_d - f\|_2^2 + \lambda \mathcal{R}(x_u) \right\} \quad \text{s.t.} \quad x_u - U_s x_d = 0, \quad (3)$$

where we have decoupled the data fidelity (residual/least squares) term and the regularizer. In particular, the matrix A multiplies x_d , which is the variable living on the natural scale of A , whereas the regularizer $\mathcal{R}$ acts now on the variable x_u , which lives on the higher-resolution space; the constraint $x_u - U_s x_d = 0$ guarantees coherence between the two variables, i.e. that the argument of the regularizer x_u coincides with the upscaled version of the variable x_d in the data fidelity term.

We form the Lagrangian for the Half-Quadratic Splitting of (3) which reads

$$\mathcal{L}_\mu(x_d, x_u) = \frac{1}{2} \|Ax_d - f\|_2^2 + \lambda \mathcal{R}(x_u) + \frac{\mu}{2} \|x_u - U_s x_d\|_2^2 \quad (4)$$

with a (new) parameter $\mu > 0$. The minimization of the Lagrangian is performed by iterative alternating minimization w.r.t. x_d, x_u which yields the following iteration w.r.t. k :

$$x_d^{k+1} = \arg \min_{x_d} \left\{ \frac{1}{2} \|Ax_d - f\|_2^2 + \frac{\mu_k}{2} \|U_s x_d - x_u^k\|_2^2 \right\} \quad (5)$$

$$x_u^{k+1} = \arg \min_{x_u} \left\{ \lambda \mathcal{R}(x_u) + \frac{\mu_k}{2} \|x_u - U_s x_d^{k+1}\|_2^2 \right\}. \quad (6)$$

We observe that the problem in (5) is a Tikhonov-type problem whose Euler–Lagrange equations are

$$(A^* A + \mu_k U_s^* U_s) x_d = A^* f + \mu_k U_s^* x_u^k. \quad (7)$$

Because the operator $A^* A + \mu_k U_s^* U_s$ is symmetric positive definite, we may use the conjugate gradient method to solve the corresponding linear system.

Another observation is that the problem in (6) can be rewritten as

$$x_u^{k+1} = \arg \min_{x_u} \left\{ \frac{1}{2 \left(\sqrt{\lambda / \mu_k} \right)^2} \|x_u - U_s x_d^{k+1}\|_2^2 + \mathcal{R}(x_u) \right\} \quad (8)$$

which describes a Gaussian denoising of $U_s x_d^{k+1}$ with noise level $\sigma_{k+1} = \sqrt{\lambda / \mu_k}$. In the spirit of PnP algorithms Venkatakrishnan *et al* (2013), we substitute the classical Gaussian denoising step resulting from (6) with a machine-learning-based Gaussian denoiser to leverage its denoising capabilities. Finally, we arrive at the splitting scheme underlying the MPISuperRes-PnP algorithm:

$$x_d^{k+1} = \text{ConjGrad} \left(A^* A + \mu_k U_s^* U_s; A^* f + \mu_k U_s^* x_u^k \right) \quad (9)$$

$$x_u^{k+1} = \text{GaussDenoiser} \left(U_s x_d^{k+1}; \sqrt{\frac{\lambda}{\mu_k}} \right). \quad (10)$$

The obtained scheme has two important points of similarity with the ZeroShot-PnP algorithm proposed in Gapyak *et al* (2025b): (i) the performance of the scheme will depend on the choice of the Gaussian Denoiser chosen in (10); (ii) the noise level $\sigma_{k+1} = \sqrt{\lambda / \mu_k}$ of the iterate $U_s x_d^{k+1}$ is coupled with the Tikhonov parameter μ_k in (9). In view of these similarities, we employ the benchmark *deep denoiser prior* Zhang *et al* (2022) as denoiser in (10), because the *deep denoiser prior* can take noise level maps as additional input. We adapt also the automatic update strategy proposed in Gapyak *et al* (2025b) for μ_k .

We now provide more details to elucidate these statements. The backbone architecture of the *deep denoiser prior* Zhang *et al* (2022) is the DRUNet, a deep CNN architecture which combines a U-Net Ronneberger *et al* (2015) with the ResNet He *et al* (2016). The *deep denoiser prior* has been trained on a combination of various dataset such as the Waterloo Exploration Database Ma *et al* (2017), BSD Chen and Pock (2017), Flick2K Lim *et al* (2017) and DIV2K Agustsson and Timofte (2017). The denoiser is publicly available at: <https://github.com/csxn/DPIR> and was trained as follows: the authors of Zhang *et al* (2022) randomly cropped out 16 patches of size 128×128 , selected a random noise level σ chosen from $[0, 50]$ and added additive Gaussian noise with level σ . Additionally, a map filled uniformly with the value σ and of the same size as the image has been provided as noise level map. The range $[0, 50]$ has been chosen to account for large variations of the noise level. For completeness we also mention that the DRUNet consists of 32 638 656 trainable parameters. We remark that the *deep denoiser*

Algorithm 1. Pseudocode of the MPISuperRes-PnP algorithm.**Input:** data f , system matrix A , upscale factor s , interpolator U_s , n_{it} , μ_0 .**Output:** reconstructed $\tilde{x}_{rec}$.

```

1:  $x_d^0, x_u^0 \leftarrow 0$ ;
2:  $k \leftarrow 0$ ;
3: while  $k \leq n_{it}$  do
4:    $x_d^{k+1} \leftarrow \text{ConjGrad}(A^*A + \mu_k U_s^* U_s; A^*f + \mu_k U_s^* x_u^k)$ ;
5:    $\hat{\sigma}_{k+1} \leftarrow \text{Noise-Estimator}(x_d^{k+1})$ ;
6:   if  $k = 0$  then
7:      $\hat{\lambda} \leftarrow \mu_0 \cdot \hat{\sigma}_1^2$ 
8:   end if
9:    $\tilde{x}_u^{k+1} \leftarrow \text{Denoiser}(\mathcal{U}_s \tilde{x}_d^{k+1}, \hat{\sigma}_{k+1})$ ;           {ZeroShot-Denoiser}
10:   $x_u^{k+1} \leftarrow \text{vec}(\tilde{x}_u^{k+1})$ 
11:   $\mu_{k+1} \leftarrow \hat{\lambda} / \hat{\sigma}_{k+1}^2$ ;
12:   $k \leftarrow k + 1$ ;
13: end while
14: return  $\tilde{x}_u^{k+1}$ 

```

prior is used in MPISuperRes-PnP in a zero-shot fashion, that means it is used without further training nor fine-tuning on MPI-specific nor MPI-related data.

As mentioned above, the parameters in (9) and (10) are coupled and consequently, it is reasonable to take this into account when choosing the denoiser. Although the choice of the Gaussian denoiser that can be employed in (10) is free, we have seen that the *deep denoiser prior* has been designed in such a way that noise level maps can be set as inputs in the denoising, de facto allowing to perform denoising with a prescribed parameter in (10). In particular, inspired by the strategy proposed in Gapyak et al (2025b), we leverage this feature of the *deep denoiser prior* to devise an automatic parameter update during the iterations. More specifically, at the beginning we set μ_0 as a starting (hyper-) parameter; then, for each iterate $\tilde{x}_d^{k+1}$ obtained via (9) with μ_k , we estimate the noise level σ_{k+1}^2 of $\mathcal{U}_s \tilde{x}_d^{k+1}$ with the following estimation (which offers an upper bound on the real noise level):

$$\hat{\sigma}_{k+1}^2 := \text{Var}(\mathcal{U}_s \tilde{x}_d^{k+1}) = \mathbb{E} \left[\left(\mathcal{U}_s \tilde{x}_d^{k+1} - \overline{\mathcal{U}_s \tilde{x}_d^{k+1}} \right)^2 \right] \quad (11)$$

where $\overline{\mathcal{U}_s \tilde{x}_d^{k+1}}$ denotes the pixel average of $\mathcal{U}_s \tilde{x}_d^{k+1}$. We point out that we will use the hat sign $\hat{q}$ to denote an estimate of any given quantity q . In the first iteration ($k = 0$) we obtain the estimated noise level $\hat{\sigma}_1^2$ and consequently, can estimate the parameter λ , which mediates the noise level and the Tikhonov parameter, as $\hat{\lambda} = \mu_0 \cdot \hat{\sigma}_1^2$. We note that the parameter update scheme here considered makes sense in view of the fact that the parameters μ_k and σ_{k+1} are linked and that the *deep denoiser prior* can use this information for the denoising; hence, the *deep denoiser prior* operates synergetically with the splitting scheme in (9)-(10).

Finally, the proposed MPISuperRes-PnP method is summarized as pseudocode in algorithm 1. Hyper-parameters of the method are the starting value μ_0 and the number of iterations n_{it} .

3. Experiments and results

In this section we show the results obtained using the MPISuperRes-PnP algorithm on a variety of MPI datasets, both simulated and real. In particular, in section 3.1 we describe the datasets employed: the MPI-MNIST dataset Iske et al (2025a, 2025b), our own MPI-CHAR dataset, the OpenMPI dataset Knopp et al (2020), and the ‘Equilibrium Model with Anisotropy’ dataset Knopp and Scheffler (2024). In section 3.2 we describe the image quality measures employed to quantitatively evaluate the quality of the reconstructions. The reconstructions performed on the four datasets are described in the respective sections 3.3–3.6.

The MPISuperRes-PnP algorithm and the preprocessing of the data have been implemented in Python 3.9, using Numpy and PyTorch. The reconstructions were performed on a workstation with 13th Gen Intel(R) Core(TM) i9-13 900KS, 128 GB of RAM, an NVIDIA RTX A6000 GPU and Windows 11 Pro.

3.1. Datasets

In this work we test the MPISuperRes-PnP algorithm on a variety of simulated and real dataset, which we here describe:

1. The MPI-MNIST Iske *et al* (2025a, 2025b) is a recently published dataset of simulated MPI measurements obtained using state-of-the-art model-based system matrices. In addition, the dataset employs real noise measurements obtained from a real MPI scanner. In particular, the dataset contains simulated system matrices $A_{N_x \times N_y}$ for three different resolutions, namely, for (N_x, N_y) either $(15, 17)$, $(45, 51)$ or $(75, 85)$. For each of the (N_x, N_y) columns, the system matrix contains 817 Fourier complex-valued Fourier coefficients and 3 channels. Of these 3 channels we use the x and y channels coherently with a 2D scan, and stack the channels and the real and imaginary parts of the 817 coefficients to obtain a real-valued system matrix. The ground truths of this dataset have been generated in the following manner: the 28×28 images of the hand written digits in the MNIST dataset are first downsampled to a 11×11 pixel grid with the nearest neighbor interpolation scheme and padded with zeros to get to the size of 15×17 pixels. The resulting 15×17 -sized images are upsampled to the 75×85 grid using nearest neighbor interpolation. Additionally to simulated system matrices and ground truth, the MPI-MNIST dataset contains a set of real additive noise measurements that can be added to the simulations to produce realistic scans. Finally, the scan signal y is obtained for each ground truth x as

$$y = A_{75 \times 85} x + \eta \quad (12)$$

where η is an instance of real noise collected with a real scanner and included in the dataset.

2. We observe that in the MPI-MNIST dataset the ground truths are natively on a 15×17 grid and upsampled to the 75×85 grid using nearest neighbors interpolation. As a consequence, the features of the ground truth live on the 15×17 and no more fine-grained features are to be gained from reconstructing the phantom on a 75×85 grid. It follows that the MPI-MNIST data set is limited for testing super-resolution algorithms since it does not carry fine level details. For this reason, we have created the MPI-CHAR dataset. The MPI-CHAR dataset is a simulated dataset created using the data-generation methodology of the MPI-MNIST dataset, but using a set of ground truths with higher native resolution. More specifically, characters from an openly available font⁵ have been used to create ground truths on a 75×85 grid. These phantoms were generated by rendering TrueType character glyphs from the font and rotating them by a 30° angle using the Python Imaging Library, a process that relies on anti-aliasing and interpolation to represent sub-pixel glyph and edge geometry; as a result, the resulting phantoms are not strictly binary but exhibit intermediate gray-level values at the letter boundaries. These ground truths have been used to generate signals following the methodology of the MPI-MNIST dataset, i.e. applying the forward operator $A_{75 \times 85}$ and adding real noise as in (12). The matrix $A_{75 \times 85}$ and the noise instances are the one published in the MPI-MNIST dataset.
3. The OpenMPI dataset Knopp *et al* (2020) is a benchmark dataset, which contains real 1D, 2D and 3D scan data of 3 phantoms obtained with a Bruker scanner. Here, we use the 2D scan data and 2D system matrices. The dataset contains system matrices in two different resolutions: a 19×19 lower resolution and a 37×37 higher resolution system matrix. The presence of a higher resolution system matrix allows us to compare higher resolution reconstructions of real phantoms with the ones obtained with the MPISuperRes-PnP method.
4. The Equilibrium Model with Anisotropy dataset (EMWA dataset) of Knopp and Scheffler (2024) contains real 2D scan data of 6 phantoms obtained with a Bruker scanner. In this dataset, the system matrix provided has been calibrated on a 15×17 grid. The dataset does not offer a higher resolution system matrix. Nevertheless, we provide reconstructions on this dataset to further test the MPISuperRes-PnP algorithm on real MPI data.

The Open-MPI and the EMWA datasets contain real MPI scans and are used in this paper to provide a qualitative evaluation of the method on real data. The MPI-MNIST and the MPI-CHAR datasets are simulated (with real noise) and contain ground truths. These are used to compute quality metrics and provide a quantitative evaluation of the method. In the following Section we describe the metrics used.

⁵ Liberation Sans Font Regular is licensed under GNU general public license (GPL) and available at: www.1001fonts.com/liberation-sans-font.html.

3.2. Image quality measures

To quantitatively assess the quality of the reconstructions whenever the ground truths are available, we use peak SNR (PSNR) and the structural similarity index measure (SSIM) Wang *et al* (2003). Given two arrays f and g of size $N_x \times N_y$, the PSNR is defined as

$$\text{PSNR}(f, g) = 10 \cdot \log_{10} \left(\frac{R^2}{\text{MSE}(f, g)} \right) \quad (13)$$

where $R = \max\{f\}$ and MSE is the mean square error

$$\text{MSE}(f, g) = \frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (f_{ij} - g_{ij})^2, \quad (14)$$

and f_{ij} and g_{ij} are the (i, j) th pixel of f and g , respectively. The SSIM is defined as

$$\text{SSIM}(f, g) = [l(f, g)]^\alpha \cdot [c(f, g)]^\beta \cdot [s(f, g)]^\gamma \quad (15)$$

where l is the luminance, c the contrast and s the structure functions defined as

$$l(f, g) = \frac{2\mu_f \mu_g + C_1}{\mu_f^2 + \mu_g^2 + C_1}, \quad c(f, g) = \frac{2\sigma_f \sigma_g + C_2}{\sigma_f^2 + \sigma_g^2 + C_2}, \quad s(f, g) = \frac{2\sigma_{fg} + C_3}{\sigma_f^2 \sigma_g^2 + C_3} \quad (16)$$

in terms of the mean values μ_f , μ_g of f and g , the respective standard deviations σ_f and σ_g and their covariance σ_{fg} ; similarly to Dittmer *et al* (2021), we have set $\alpha = \beta = \gamma = 1$, and $C_1 = (0.01 \cdot R)^2$, $C_2 = (0.03 \cdot R)^2$ and $C_3 = 0.5 \cdot C_2$ where R is the range of the ground truth.

To account for the variation in range of the reconstructions, we compute PSNR and SSIM upon affine rescaling of the reconstructed images. More specifically, if x_{GT} is the ground truth and x is the reconstructed image, then we compute the quantity

$$\text{PSNR}_{\text{aff}}(x_{\text{GT}}, x) = \max_{a, b} \{ \text{PSNR}(x_{\text{GT}}, ax + b) \} \quad (17)$$

which can be computed simply as the PSNR between x_{GT} and $ax + b$ for the optimal parameters

$$a = \frac{\text{Cov}(x_{\text{GT}}, x)}{\text{Var}(x_{\text{GT}})}, \quad b = \bar{x} - a \cdot \bar{x}_{\text{GT}} \quad (18)$$

and $\bar{x}$ (resp. $\bar{x}_{\text{GT}}$) is the mean of x (resp. x_{GT}). Analogously, when computing the SSIM we will be in fact computing $\text{SSIM}(x_{\text{GT}}, ax + b)$ with a and b taken from (18).

3.3. Reconstructions on the MPI-MNIST dataset

The first reconstructions we present have been performed on the first 20 test phantoms of the MPI-MNIST dataset. The dataset offers a system matrix $A_{45 \times 51}$, calibrated on a 45×51 grid, which we use for reconstruction. We test the MPISuperRes-PnP algorithm with a super-resolution factor of $s = 2$ (see section 2.3) and we compare it with a baseline method. The baseline method Güngör and Barıştop (2020) for comparison can be described in the following way: the reconstruction is performed using a matrix $A_{sN_x \times sN_y}$ which is obtained from the $A_{N_x \times N_y}$ matrix by interpolating its columns by a factor of $s = 2$, using bilinear interpolation; the super-resolved system matrix $A_{sN_x \times sN_y}$ is used as forward operator in conjunction with a solver of linear systems. In this experiment we have chosen to use the ZeroShot-PnP algorithm as a regularized linear system solver for the system matrix $A_{sN_x \times sN_y}$. To differentiate between the reconstruction algorithms, we denote them with the following notation: we write $\text{Super}_s(A_{N_x \times N_y})$ to refer to the MPISuperRes-PnP algorithm that performs an upscaling of factor s ; coherently with the notation of the upscaling operator in section 2.3. In contrast we write $U_s(A_{N_x \times N_y})$ to denote reconstructions obtained by using the super-resolved system matrix $A_{sN_x \times sN_y}$, i.e. the baseline method. In particular, in figure 2 we show reconstructions obtained with upscaling the system matrix $A_{45 \times 51}$ by a factor of 2, and in accordance with our notation, we denote the method with the symbol $U_2(A_{45 \times 51})$.

We have performed reconstructions with starting parameters $\mu_0 = 10^i$ for $i = -16, \dots, -4$ for 100 iterations and computed PSNR and SSIM of all iterations. Additionally, we have computed the relative L^2 -norm $\frac{\|x_u^{k+1} - x_u^k\|_2}{\|x_u^{k+1}\|_2}$ for each phantom, parameter and iteration k . The stopping criterion is the following: we have considered the iteration k^* and parameter μ_0^* to be optimal if the average PSNR is

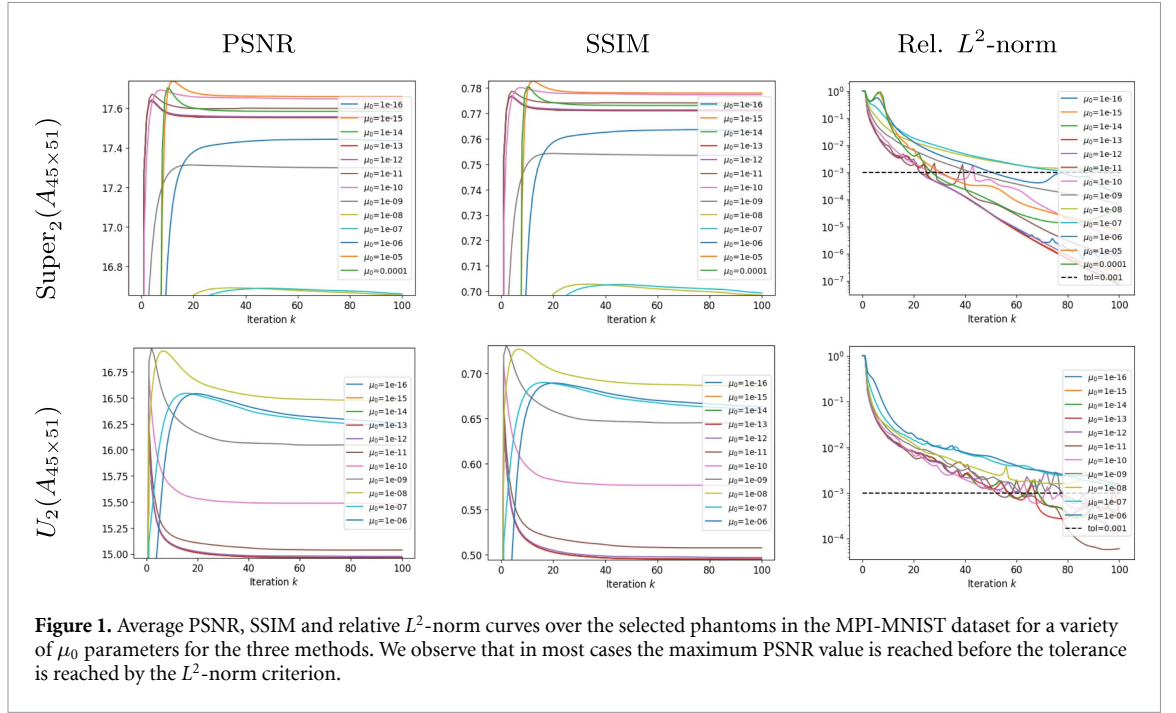
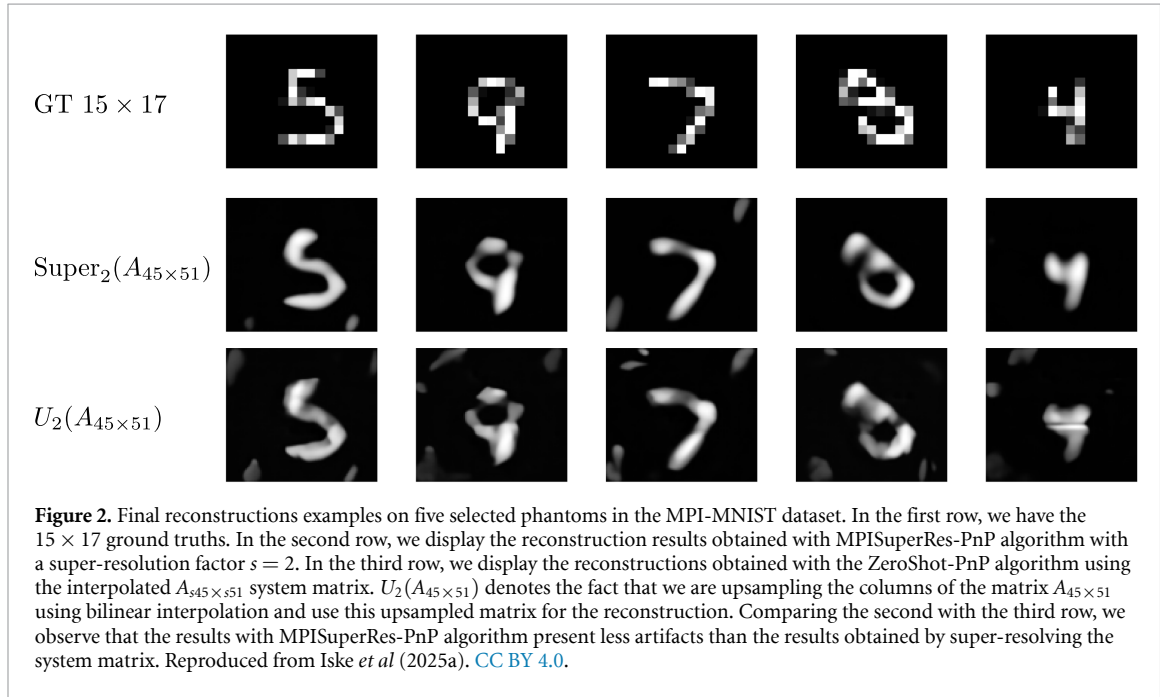


Figure 1. Average PSNR, SSIM and relative L^2 -norm curves over the selected phantoms in the MPI-MNIST dataset for a variety of μ_0 parameters for the three methods. We observe that in most cases the maximum PSNR value is reached before the tolerance is reached by the L^2 -norm criterion.

maximal and the average relative L^2 -norm is below 10^{-3} . In other words, we have computed the average PSNR curve over the dataset and the average relative L^2 -error curve and denoted with n_{PSNR} the iteration at which the average PSNR curve is maximal; analogously, we have extracted n_{tol} the iteration at which the relative L^2 -error curve falls below the tolerance; then, the iteration number $k^* = \min\{n_{\text{tol}}, n_{\text{PSNR}}\}$ is chosen as the stopping iteration for all reconstructions in the dataset. The average PSNR curves, the relative average SSIM curves and the relative L^2 -errors are displayed in figure 1. The optimal parameters as well as the number of iterations selected with this validation and the relative average PSNR and SSIM scores are displayed in table 1. From table 1 we observe that the PSNR and the SSIM values obtained with MPISuperRes-PnP method are higher than the ones obtained with the baseline method that uses a super-resolved system matrix ($U_2(A_{45 \times 51})$). This result shows quantitatively that the MPISuperRes-PnP algorithm produces improved reconstruction results compared with more standard super-resolution techniques. For a qualitative evaluation of the results, we display the final reconstructions as well as the ground truths of 5 phantoms in figure 2. In particular, we observe that, from a visual standpoint, the result with the super-resolved matrix $U_2(A_{45 \times 51})$ contained a series of reconstruction artifacts that are not present in the reconstructions with the MPISuperRes-PnP algorithm. In section 3.1 we argue that the MPI-MNIST dataset is not properly suitable to test super-resolution because of the way the dataset is produced. We recall briefly that the ground truths in the MPI-MNIST dataset live natively on a 15×17 grid and are upsampled by nearest neighbors when producing the data scans. However, the nearest neighbor interpolation preserves the native 15×17 scale of the features of the phantom. Consequently, the ground truths thus produced have in principle no fine(r) features living on the 45×51 or 90×102 scales on which we reconstruct. In particular, it is reasonable to expect that the employment of bilinear interpolation results in a smoothing out of the 15×17 features, when applied to recover potential features on more finely resolved grids. Coherently, it is reasonable to expect that, if there are no higher-level features to be extracted, the PSNR and SSIM score of super-resolved reconstruction could be lower than reconstruction without super-resolution. To verify this fact and corroborate our argument in favor of using the MPI-CHAR dataset, we have performed the reconstructions without super-resolution, i.e. with $s = 1$ and for which the methods $\text{Super}_1(A_{45 \times 51})$ and $U_1(A_{45 \times 51})$ coincide. For this non super-resolved reconstruction we have obtained the validated parameters $\mu_0 = 10^{-4}$, $n_{\text{it}} = 10$ and PSNR values of 18.18 ± 1.20 , whereas the SSIM are 0.800 ± 0.0356 . Comparing these scores with the one in table 1, we observe that neither MPISuperRes-PnP algorithm nor the baseline method $U_2(A_{45 \times 51})$ achieve PSNR and SSIM score higher than the non-super-resolved reconstruction on the 45×51 grid. This confirms our hypothesis that this dataset—which lacks higher resolution features by construction—is not fully suitable to investigate super-resolution methods. Given this observation, we have employed the data provided with the MPI-MNIST dataset to produce a higher resolution set of ground truths to test super-resolution, the MPI-CHAR dataset. In the next experiment we perform reconstructions on the

Table 1. Results of the validation on the MPI-MNIST dataset displayed in figure 1.

	s	Reco. grid	μ_0	n_{it}	PSNR ($\uparrow$)	SSIM ($\uparrow$)
$\text{Super}_2(A_{45 \times 51})$	2	90×102	10^{-5}	12	17.74 ± 1.19	0.7827 ± 0.0311
$U_2(A_{45 \times 51})$	2	90×102	10^{-9}	2	16.98 ± 1.18	0.7305 ± 0.0356



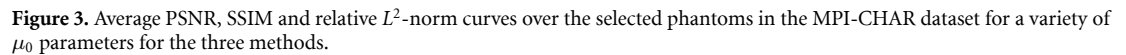
MPI-CHAR dataset and obtain higher PSNR and SSIM scores than without super-resolution, confirming in hindsight that the MPI-MNIST dataset is not fully suitable to test super-resolution.

3.4. Reconstructions on the MPI-CHAR dataset

In the previous experiment we have shown that, compared with the baseline method (super-resolving the system matrix), the MPISuperRes-PnP algorithm is capable to yield reconstructions with higher PSNR and SSIM values as well as with less reconstruction artifacts, proving the advantages of using MPISuperRes-PnP. We have also observed that the MPI-MNIST dataset contains ground truths that natively contain features that live on a 15×17 grid and that consequently, is not suitable to properly test the results of super-resolution. In this experiment we show that the MPISuperRes-PnP is capable of yielding improved reconstruction results from coarse system matrix in a simulated scenario. These improvements are demonstrated both quantitatively and qualitatively. The dataset utilized has been obtained in the following way: a set of 94 ground truths of size 75×85 is obtained using the letters from an openly available font dataset described in 3.1; from the MPI-MNIST dataset Iske et al (2025a, 2025b) we have considered the clean system matrix $A_{75 \times 85}$ and real noise samples η provided in the same dataset; the simulated scans are then obtained for each ground truth $x \in \mathbb{R}^{75 \times 85}$ by

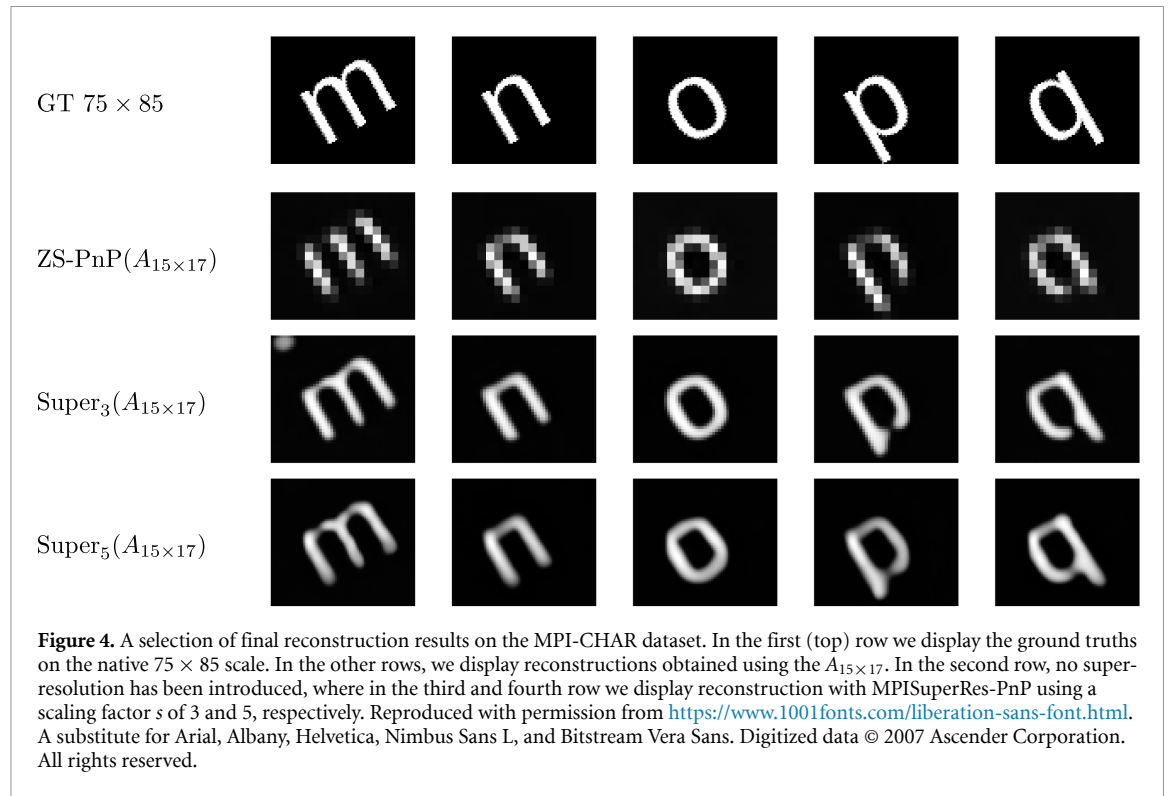
$$y = A_{75 \times 85}x + \eta. \quad (19)$$

To test the super-resolution reconstruction we have considered the coarse system matrix $A_{15 \times 17}$ for the reconstruction. First, we simply reconstruct with $s = 1$, i.e. with the ZeroShot-PnP algorithm (we denote the results coherently with section 3.3 as ZS-PnP). The outputs of size 15×17 are upsampled with zero-order upscaling to 75×85 to compute the PSNR. Subsequently, we test the MPISuperRes-PnP algorithm with $s = 3$ and $s = 5$, and compare it upon upscaling with the ground truths to compute the PSNR. The reconstruction with the MPISuperRes-PnP algorithm are denoted with Super_s for the upscaling factor s . We have plotted the average PSNR, SSIM and L^2 -norm curves for each of the starting parameter $\mu_0 = 10^i$ with $i = -16, \dots, -4$ in figure 3. Coherently with the previous experiment, we have chosen the optimal parameters that maximize the PSNR value before the relative L^2 -norm reaches a tolerance of 10^{-4} . The corresponding validated parameters and final average PSNR and SSIM values are displayed in table 2. From the results in table 2 we observe that, when the underlying ground truth has features



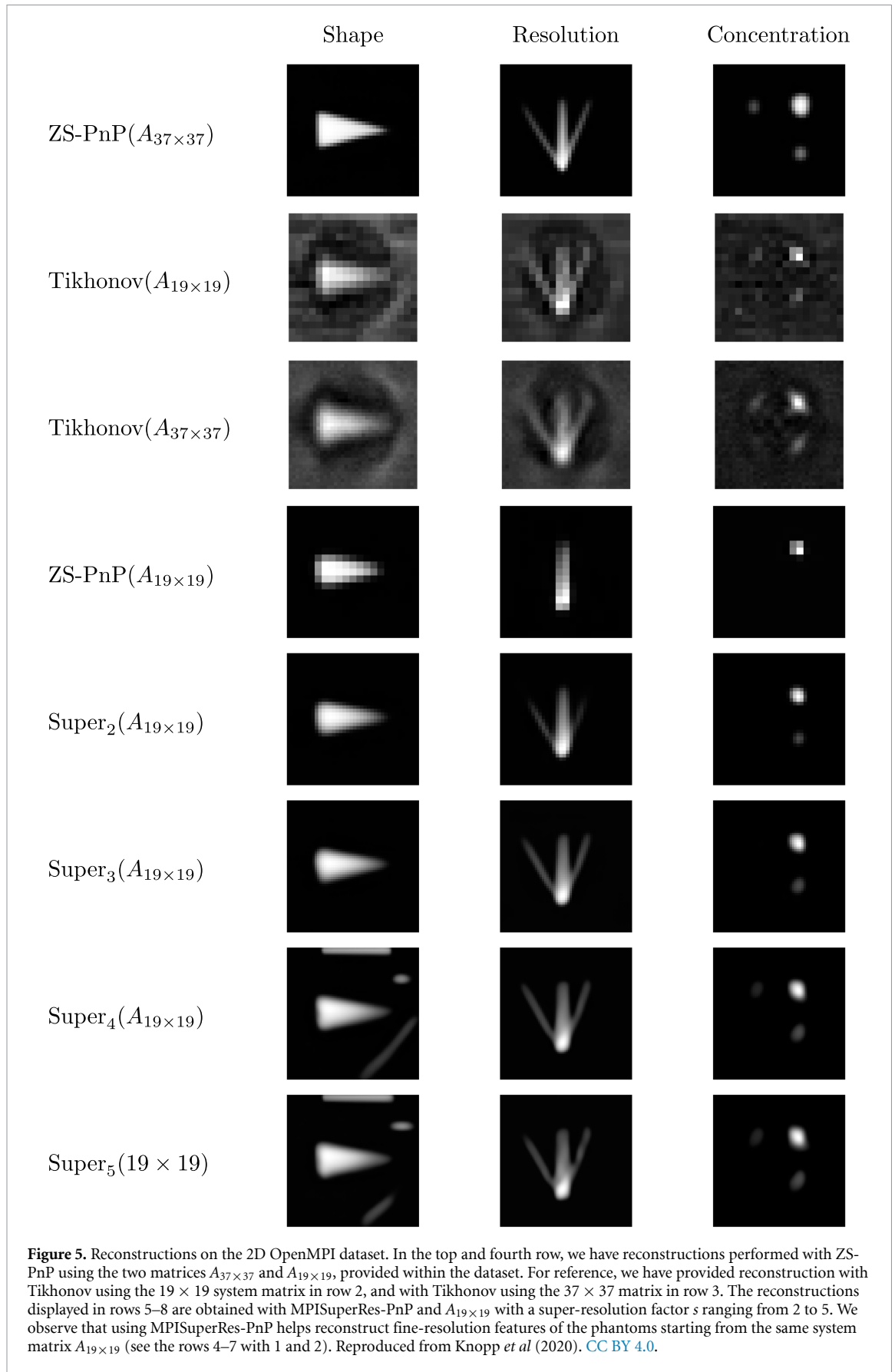
	s	Reco. grid	μ_0	n_{it}	PSNR ($\uparrow$)	SSIM ($\uparrow$)	Time
ZS-PnP($A_{15 \times 17}$)	1	15×17	10^{-10}	12	16.15 ± 2.76	0.6996 ± 0.0858	1.97 ± 0.15 s
Super ₃ ($A_{15 \times 17}$)	3	45×51	10^{-16}	11	18.48 ± 2.30	0.8354 ± 0.0686	5.43 ± 0.63 s
Super ₅ ($A_{15 \times 17}$)	5	75×85	10^{-10}	11	18.72 ± 2.26	0.8449 ± 0.0671	3.12 ± 0.13 s

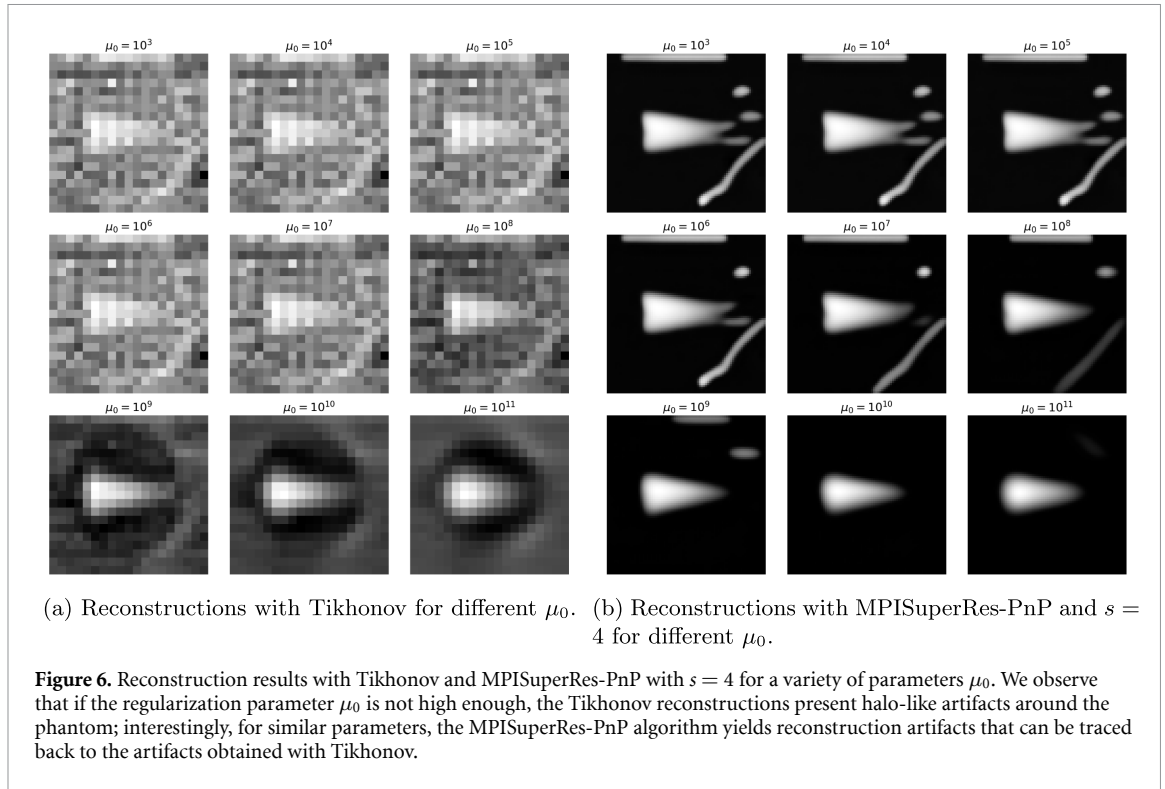
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3.5. Reconstructions on the 2D OpenMPI dataset

In this section we perform reconstructions on the openly available OpenMPI dataset. The OpenMPI dataset contains real data scans of three phantoms: the shape, resolution and concentration phantoms. In addition to the scan data, two different system matrices are available: a system matrix $A_{19 \times 19}$ calibrated on a 19×19 grid and a higher resolution system matrix $A_{37 \times 37}$ calibrated on a 37×37 grid. The presence of the higher-resolution system matrix $A_{37 \times 37}$ is important because in the real-data scenario ground truths are not available. We therefore use $A_{37 \times 37}$ to reconstruct the phantom from the scan data and obtain an approximation of the underlying real phantom on a 37×37 grid. These reconstructions are used as a comparison basis for the reconstructions obtained with the MPISuperRes-PnP algorithm. In fact, in this experiment we compare the reconstructions on the 37×37 grid with reconstructions that use $A_{19 \times 19}$ but produce super-resolved reconstruction using MPISuperRes-PnP. We produce reconstruction results for scaling factors $s = 2, 3, 4, 5$. For comparison an analysis of artifacts, we additionally provide reconstructions obtained with the Tikhonov method with both $A_{19 \times 19}$ and $A_{37 \times 37}$. Concerning the preprocessing steps described in section 2.2, all reconstructions have been performed discarding the frequencies below the 80 kHz, as they are notoriously unreliable Maass *et al* (2024). Additionally, we use the SNR estimation available with the dataset and cut all frequencies whose SNR is below 1 for the reconstructions with $A_{37 \times 37}$. No SNR thresholding has been applied to the reconstructions that use $A_{19 \times 19}$. Finally, a low rank approximation is performed using rSVD and a target rank K as described in section 2.2. This rank has been set to $K = \frac{N_x \cdot N_y}{2}$ when the system matrix $A_{N_x \times N_y}$ is employed. For all reconstructions, the number of iterations has been set to $n_{it} = 10$. The reconstruction parameter μ_0 has been set to 10^7 for all reconstructions with $A_{37 \times 37}$. For the reconstructions with $A_{19 \times 19}$ we have set μ_0 to be 10^8 for the shape and the resolution phantoms, and 10^{10} for the concentration phantom. For the Tikhonov parameter, we have selected as regularization parameter 10^9 when using $A_{19 \times 19}$ and 10^7 when using $A_{37 \times 37}$. These parameters have been selected by visual inspection of the final results. A certain degree of dependency of μ_0 on the specific phantom when working with real data has also been observed in Gapyak *et al* (2025b). The final reconstruction are displayed in figure 5. As mentioned, we use the reconstructions obtained with $A_{37 \times 37}$ as a comparison reconstruction in which the real features of the underlying phantoms are reconstructed and visible (top row). We observe that, when using $A_{19 \times 19}$, finely-resolved features of the phantoms are not being reconstructed (e.g. the side branches of the resolution phantom or the missing diagonal dots in the concentration phantom in figure 5). However, when using MPISuperRes-PnP with $s \geq 2$ we observe that the features of the phantoms that were missing in the 19×19 reconstruction are now visible. We remark once more that both





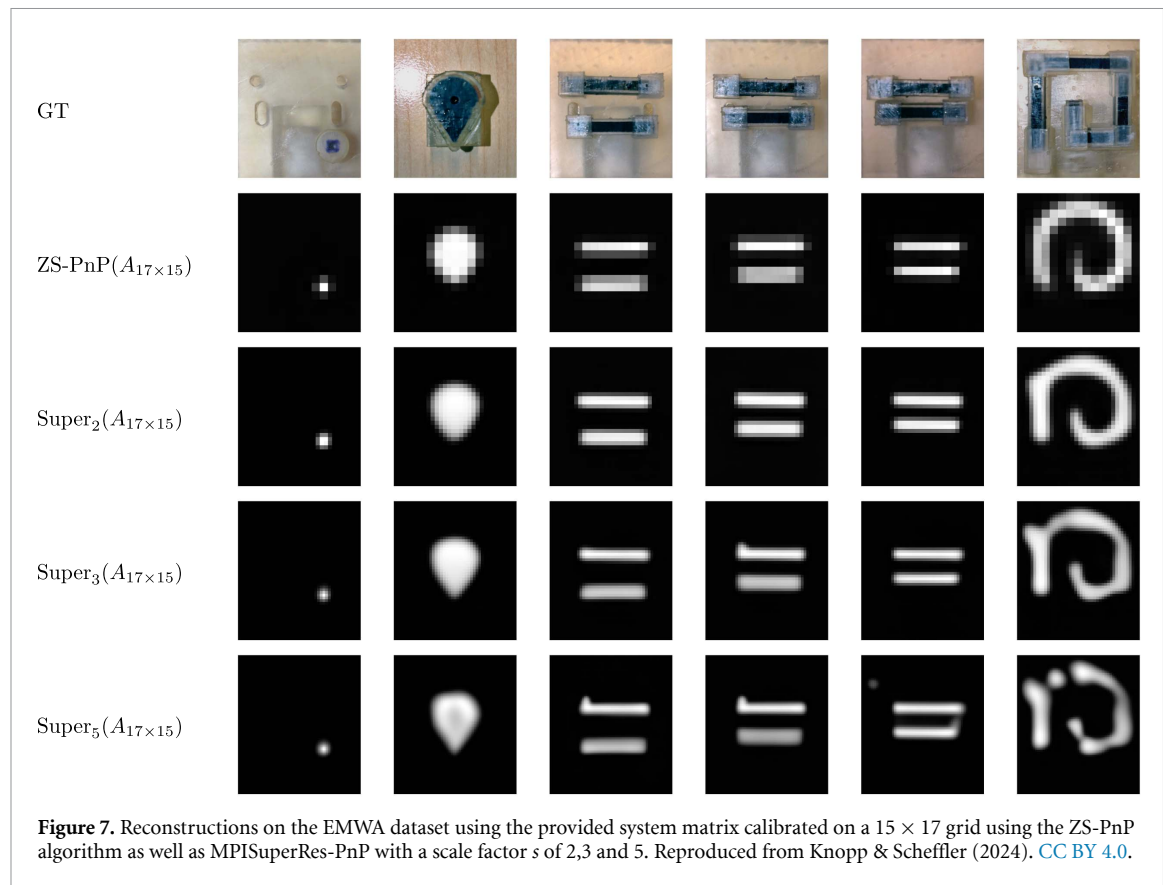
the reconstruction on the 19×19 grid and the reconstructions using MPISuperRes-PnP algorithm both use $A_{19 \times 19}$, i.e. the reconstructed feature with MPISuperRes-PnP are due to MPISuperRes-PnP itself and not due to an increase in the amount of input information. These results underpin the usefulness of MPISuperRes-PnP to reconstruct super-resolved images starting from lower-resolution system matrices in real MPI scenarios. We observe that although increasing s from 2 to 4 can help reconstructing the features of the concentration phantom, it can also introduce reconstruction artifacts as in the case of the shape phantom. A potential intuition for this can be the following: if $s = 1$ and we initialize the variable $x_d^0 = 0$, then the Tikhonov problem in (5) is precisely

$$x_d^1 = \arg \min_{x_d} \left\{ \frac{1}{2} \|Ax_d - f\|_2^2 + \frac{\mu_0}{2} \|x_d\|_2^2 \right\}, \quad (20)$$

which yields the solution obtained with Tikhonov regularization with regularization parameter μ_0 ; in particular, the Tikhonov reconstruction serves as starting point for our iterative scheme. Consequently, artifacts arising in the Tikhonov reconstruction due to noise in the data or any other effect are fed into the Gaussian denoiser and in subsequent iterations. In support of this we have produced a comparison between the Tikhonov results and the reconstructions with MPISuperRes-PnP (with $s = 4$) in figure 6. We point out that the artifacts in the shape phantom in figure 5 for $s = 4$ or 5 coincide with the halo-shaped artifact in the Tikhonov reconstruction. The reason why the Tikhonov artifacts are suppressed when $s = 2$ or 3 but appear if s is higher, can be due to the fact that, when we operate on a finer pixel resolution, the Gaussian denoiser might interpret the artifacts as fine structures of the underlying particle distribution and attempt to bring them to the foreground.

3.6. Reconstructions on the EMWA dataset

In the previous experiment we have shown reconstructions on the OpenMPI dataset, which contains 3 phantoms. In order to further show the super-resolution capabilities of MPISuperRes-PnP across multiple instances of real data, we additionally perform reconstructions on the EMWA dataset. This dataset contains scans of 6 phantoms and a system matrix calibrated on a 15×17 grid. The dataset does neither contain ground truths nor a higher-resolution system matrix, but pictures of the phantoms. Consequently, we only qualitatively evaluate the super-resolution capabilities of MPISuperRes-PnP. The data has been processed by cutting frequencies below 80 kHz, by performing SNR thresholding with a threshold of 1 and low-rank approximation with rSVD with target rank $K = \frac{15 \cdot 17 - 1}{2}$. Reconstructions



have been performed with $\mu_0 = 10^9$ for all phantoms and resolution factors s . Concerning s , as done for the OpenMPI dataset, we perform reconstructions for $s = 2, 3, 5$. The final reconstructions and the provided pictures with the phantoms are displayed in figure 7. We observe that, compared with the non-super-resolved reconstruction on the 15×17 grid, MPISuperRes-PnP algorithm can help in reconstructing more sharply certain features of the phantoms (e.g. the tip of the ice-cream phantom and a clearer structure of the snail phantom). In this experiment we also observe the presence of reconstruction artifacts when the super-resolution factor $s \geq 3$. For example, we observe the shape artifacts in the snail phantom for $s = 5$; however, this observation is in line with the behavior of super-resolution methods when the upscaling factor is high enough reported in MPI literature (e.g. in Güngör *et al* (2022)). Moreover, the presence of artifacts for resolution parameters s high enough has been observed also for the OpenMPI dataset in section 3.5.

4. Conclusions

In this paper, we have derived a PnP-type method for super-resolved MPI reconstruction. More specifically, we have introduced MPISuperRes-PnP, which leverages the denoising capabilities of a zero-shot denoiser: the *deep denoiser prior*. The incorporation of super-resolution was derived using a splitting scheme that minimizes a specifically defined cost function. We have provided mathematical motivation for the proposed algorithm. Further, we have discussed the choice of parameters, and we have provided a pseudocode of the algorithm.

We have shown the proposed method's potential by application to real and synthetic data. More precisely, we have first applied the derived method to the synthetic MPI-MNIST dataset, and derived quantitative and qualitative results. We observed that the dataset lacks finer level details. This observation led us to consider a self-designed synthetic dataset which is inspired by the MPI-MNIST dataset but provides higher-level details. On this dataset, we derived quantitative and qualitative results as well. In particular, we could observe the algorithm's capabilities concerning the reconstruction of higher-level details. Finally, we have applied the algorithm to real data from the MPIData: EquilibriumModelWithAnisotropy, and the 2D-OpenMPIData datasets. Due to the lack of ground truths for the real data, we have performed only a qualitative evaluation of the reconstruction

results on those two real datasets. In summary, our experiments on real and synthetic data, have shown the proposed method's potential for super-resolved MPI reconstruction.

Among the limitation of the proposed method and its current implementation we mention the reconstruction time which is in the order of a couple of seconds but not in a sub-second regime; however, methods that can reconstruct many times per second are important in real-time MPI applications, and consequently, the optimization for speed of the current framework and implementation is an important future direction of research. Further topics of future research include the incorporation of more recent zero-shot denoisers (based on the transformer architecture). Additionally, examining the effect of other splitting and interpolation schemes, constitute a further research direction.

Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: An implementation of MPISuperRes-PnP can be found on GitHub at: <https://github.com/VladyslavGapyak/MPISuperRes-PnP> Gapyak et al (2026).

Funding

This work was supported by the Hessian Ministry of Higher Education, Research, Science and the Arts within the Framework of the 'Programm zum Aufbau eines akademischen Mittelbaus an hessischen Hochschulen' and by the German Science Fonds DFG under grant INST 168/4-1.

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