Ionospheric Distortion on Broadband BOC Signals

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1. Introduction

The purpose of this paper is to assess the effect of the Ionospheric incoherency on the S-Curve of a non-coherent DLL (Delay-Lock Loop) via time-domain simulation. In section 2, we derive the S-curve from the BOC signal definition, and present the relevant parameters: bias, correlation loss and detector gain, using a continuous model. Section 3 describes how this model can be discretized by applying the Sampling theorem. The sampling frequency is determined from the number of spectral lobes of the BOC signals that must lie inside the transmission bandwidth. Section 4 presents the simulation layout in which the first part is the time-domain simulator that produces the sequence of correlation samples, and the second part operates with this sequences in order to calculate the S-curve. Finally, section 5 contains the simulation results for the BOC(10,5) and BOC(14,2) signals.

2. Derivation of the S-curve Continuous Model

Let us assume a navigation signal $\mathrm{s}(t)$ with spreading code c_k, chip period T_c, data modulation d_k with rate f_d, and pulse shape $\mathrm{g}(t)$. This signal has the form

$$\mathrm{s}(t) = \sum_{k=-\infty}^{+\infty} d_k c_k \, \mathrm{g}(t - kT_c)\,, \qquad \text{Eq. 1}$$

where d_k depends only on $\lfloor kf_d \rfloor$ (greatest integer smaller than kf_d).

The BOC(n_1, n_2) is generated by multiplying a signal $\mathrm{s}(t)$ with chip rate $1/T_c = n_2 R_{\mathrm{CA}}$, and a rectangular pulse train $\mathrm{u}(t)$ with frequency $f_{\mathrm{sq}} = n_1 R_{\mathrm{CA}}$, being R_{CA} the C/A code chip rate. We will find convenient to write this train in terms of its spectrum, i.e.,

$$\mathrm{u}(t) = \sum_{\substack{k=-\infty \\ k \text{ odd}}}^{+\infty} \frac{1}{k} \exp(\mathrm{j}\, 2\pi\, k\, f_{\mathrm{sq}} \mathrm{t})\,. \qquad \text{Eq. 2}$$

Thus, the BOC signal is $\mathrm{r_o}(t) \equiv \mathrm{s}(t)\,\mathrm{u}(t)$. Let us call $\mathrm{h_I}(t)$ to the Ionospheric response. The received signal without noise would be $\mathrm{r}(t) \equiv \mathrm{r_o}(t) * \mathrm{h_I}(t)$, where "$*$" is the convolution operator. Using this definitions, the S-curve is

$$\varepsilon(\tau;\mathrm{r}) = \left|< \mathrm{r(t)}, \mathrm{r_o}(\mathrm{t} - \tau - \Delta/2) >\right|^2 - \left|< \mathrm{r}(t), \mathrm{r}_o(t - \tau + \Delta/2) >\right|^2, \qquad \text{Eq. 3}$$

where $< \mathrm{x}(t), \mathrm{y}(t) > \equiv \lim_{T_l \to +\infty} \frac{1}{T_l} \int_0^{T_l} \mathrm{x}(t)\, \mathrm{y}(t)^* \,\mathrm{d}t$, Δ is the early-late spacing, and τ is the delay of the locally-generated signal replica. We have used the notation $\varepsilon(\tau;\mathrm{r_o})$ to indicate that it is the S-curve with $\mathrm{r}(t)$ at the input.

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The main parameters of interest are the following:

- Bias: Delay τ_b that follows $\varepsilon(\tau_b;\mathrm{r})=0$.
- Gain: $\varepsilon'(\tau_b;\mathrm{r})/\varepsilon'(0;\mathrm{r}_o)$. (Detector gain normalised to the TEC=$0\cdot10^{16}\,(1/m^2)$ case.)
- Correlation loss due to the Ionosphere: $<\mathrm{r}(t),\mathrm{r}_o(t)>/<\mathrm{r}_o(t),\mathrm{r}_o(t)>$.

3. Discretization of the Signal Model

The time-domain simulation reproduces the derivation of the discrete equivalents of $\mathrm{r}_o(t)$ and $\mathrm{r}(t)$ for a given channel or transmission bandwidth BW. Figure 1 shows the BOC signal spectrum along with the relevant parameters. The sampling rate has to be greater than BW, in order to take into account the tails of the spectral replicas that touch but do not lie entirely inside the transmission band.

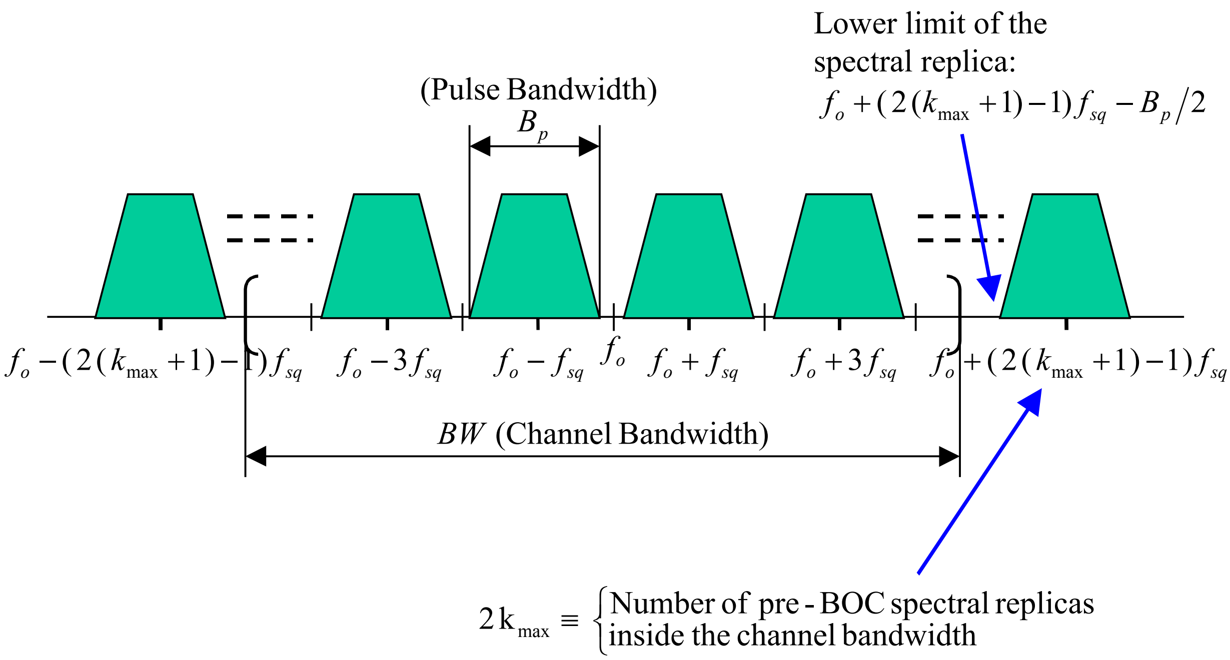

Figure 1. *BOC signal spectrum. The replicas of the navigation signal spectrum* $\mathrm{S}(f)$ *are in green and not to scale, (*$1/k$ *factor omitted).*

Assuming that we require $k_{\max}$ spectral replicas at each side of f_o, then from Figure 1, we can see that the lower limit of the first spectral replica that does not need to be simulated (the ($k_{\max}+1$)-th) must lie outside the channel band, i.e.

$$[2(k_{\max}+1)-1]f_{\mathrm{sq}}-Bp/2 > \mathrm{BW}/2 \qquad \text{Eq. 4}$$

Solving for $k_{\max}$ we obtain,

$$k_{\max}=\left\lceil \frac{1}{2}\left(\frac{\mathrm{BW}+B_p}{2f_{\mathrm{sq}}}-1\right)\right\rceil, \qquad \text{Eq. 5}$$

where $\lceil x \rceil$ calculates the smallest integer greater or equal than x. Thus, the simulation band must contain $k_{\max}$ spectral replicas at each side of f_o, i.e., the sampling frequency f_s must follow

$$f_s/2 > (2k_{\max} - 1) f_{\mathrm{sq}} + B_p/2 . \qquad \text{Eq. 6}$$

In the simulations, the sampling frequency has been $f_s = 2(2k_{\max} - 1) f_{\mathrm{sq}} + B_p \cdot 1.2$. The 1.2 factor avoids having any power close to the 0.5 normalised frequency.

Also the channel bandwidth has been selected to contain exactly an integer number of spectral replicas. Again, from Figure 1, we can see that the frequency band contains exactly k replicas if

$$\mathrm{BW} = 2\, R_{\mathrm{CA}} n_1 (2n_{\mathrm{L}} - 1) + B_{\mathrm{p}} . \qquad \text{Eq. 7}$$

4. Simulation Layout

Figure 2 shows the simulation layout. The first part produces the received signal without noise, and with and without the Ionosopheric effect, using a time-domain simulation. The second part calculates a fine grid of samples of the S-curve from the cross-correlation of the signals provided by the first part.

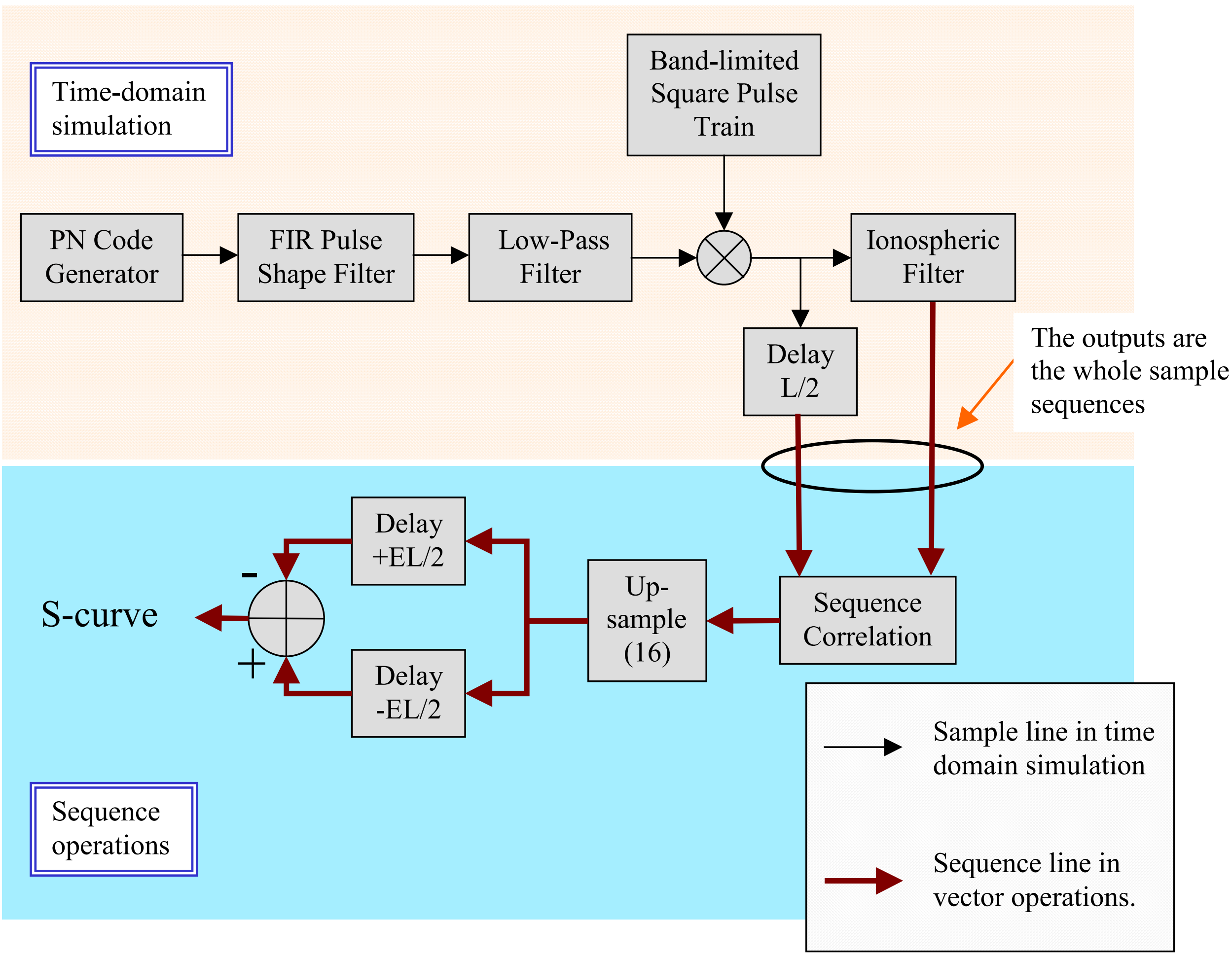
Time-domain simulation
Band-limited Square Pulse Train
PN Code Generator
FIR Pulse Shape Filter
Low-Pass Filter
Ionospheric Filter
Delay L/2
The outputs are the whole sample sequences
Delay +EL/2
Delay -EL/2
-
+
S-curve
Up-sample (16)
Sequence Correlation
Sample line in time domain simulation
Sequence line in vector operations.
Sequence operations

Figure 2. *Simulation Layout.*

The next subsections describe the operation of the relevant blocks.

4.1 Low-pass Filter

At the output of the pulse shape filter, a low-pass filter is applied in order to avoid aliasing after the BOC modulation. This is necessary because the S-curve biases to be measured are so small that they can be masked by any small aliasing.

4.2 Ionospheric Filter

The Ionospheric effect is implemented using an FIR filter that contains the Ionospheric impulse response. This is done in two steps:

- Apply a IDFT to a sequence of samples of the Ionospheric spectrum. This produces a sequence of impulse response samples if the number of samples is large enough.

- Truncate the sequence for a given truncation error, flip and conjugate. The result is the sequence of FIR coefficients. Given that the spectrum specification is very smooth, the number of samples left after truncation is small (7 to 20). The filter group delay is equal to the group delay given by the truncation plus the one given by the spectrum, the latter being zero in the current specification. If the truncation is symmetrical, then the group delay is $T_s(L-1)/2$, where L is the filter length.

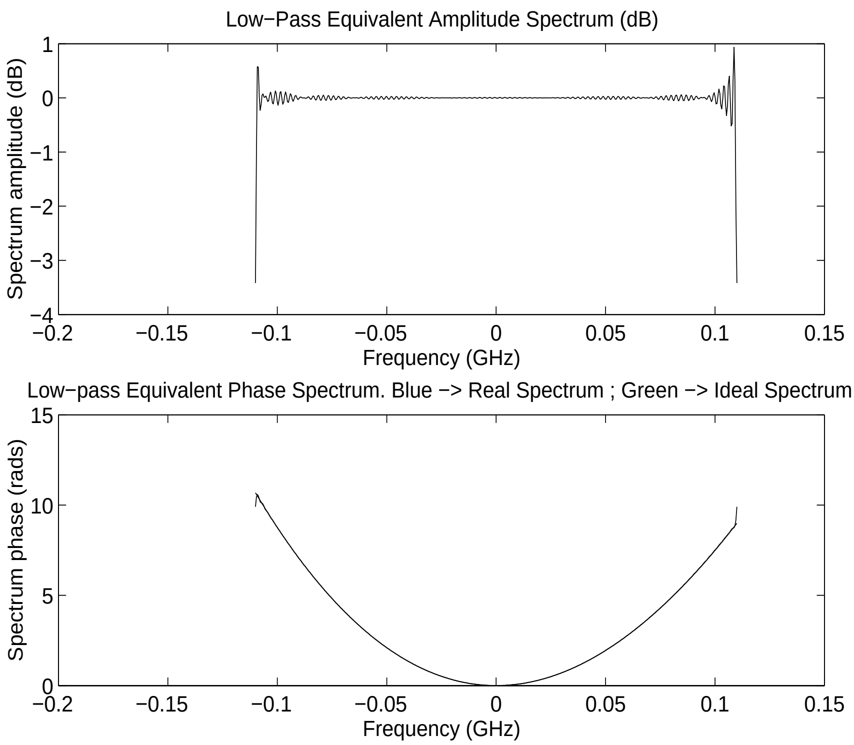

Figure 3. *Ionospheric spectrum and its approximation using an FIR filter with 191 coefficients, TEC* = $200\cdot 10^{16}\, 1/\mathrm{m}^3$.

The filter specification is as follows. Let f_n be the normalised frequency relative to the carrier frequency f_o (Hz) and the two-sided bandwidth B (Hz), i.e, $f = f_o + B f_n$. The Ionospheric response is

$$\mathrm{H}(f) = \exp\left(-\, j\, a\, \Delta w / (w_o^2 (w_o + \Delta w))\right), \qquad \text{Eq. 1.}$$

where $a = (2\pi)^2\, 40.3\, TEC / (3 \cdot 10^8)$ with TEC in $1/\mathrm{m}^3$, $\Delta w = 2\pi\, f_n B$, and $w_o = 2\pi\, f_o$. This formula does not take into account the phase and group delay at $f = f_o$.

4.3 Square Pulse Train

Since we are using a time-domain simulation, we can only employ band-limited square pulse trains. These have been generated using the Fourier series

$$\mathrm{u}(t) = \sum_{\substack{k=-\infty \\ k \text{ odd}}}^{+\infty} \frac{1}{k} \mathrm{e}^{\,\mathrm{j}2\pi k\, \mathrm{t}/(2\mathrm{T})}$$

Truncating at a given k, we obtain the band-limited pulse train. $2T$ is the period.

5. Simulation Results

Table 1 summarises the simulation results for the BOC(10,5) and BOC(14,2) signals. In each cell, a reference can be found to the corresponding figure. The transmission bandwidths has been chosen to contain an integer number of spectral replicas. For a BOC(n_1, n_2) signal, the bandwidth necessary to contain exactly n_{SR} spectral replicas is

$$\mathrm{BW} = 2\, R_{\mathrm{CA}} n_1 (2 n_{\mathrm{SR}} - 1) + B_{\mathrm{p}}\,. \qquad \text{Eq. 8}$$

Table 2 presents the gain factor and correlation loss for the minimum, mean, and maximum TEC values.

	Pulse Shape	N. of spectral replicas	S-Curve bias (nano-seconds)	S-Curve Gain (1/chip)	Correlation Loss (dB)
BOC(10,5) Signal	**Root-Raised Cosine, β=0.2**	2	Negligible (< 0.04 nano-sec)	Constant 1 (Figure 4)	Constant 0 dB (Figure 7)
		4	Negligible (< 0.04 nano-sec)	Decreases from 1 to 0.965 (Figure 4)	Increases from 0 to 0.027 dB (Figure 7)
	Rectangular, Bandwidth = 40 MHz	2	Negligible (< 0.06 nano-sec)	Decreases from 1 to 0.966 (Figure 5)	Increases from 0 to 0.011 dB (Figure 8)
		4	Negligible (< 0.06 nano-sec)	Decreases from 1 to 0.885 (Figure 5)	Increases from 0 to 0.56 dB (Figure 8)
	Rectangular, Bandwidth = 60 MHz	2	Negligible (< 0.06 nano-sec)	Decreases from 1 to 0.96 (Figure 6)	Increases from 0 to 0.22 dB (Figure 9)
		4	Negligible (< 0.06 nano-sec)	Decreases from 1 to 0.79 (Figure 6)	Increases from 0 to 0.116 dB (Figure 9)
BOC(14,2) Signal	**Root-Raised Cosine, β=0.2**	2	Negligible (< 0.01 nano-sec)	Constant 1 (Figure 10)	Smaller than 0.01 dB (Figure 13)
		4	Negligible (< 0.01 nano-sec)	Decreases from 1 to 0.87 (Figure 10)	Increases from 0 to 0.11 dB (Figure 13)

	Rectangular, Bandwidth = 40 MHz	2	Negligible (< 0.05 nano-sec)	Constant 1 (Figure 11)	Smaller than 0.01 dB (Figure 14)
		4	Negligible (< 0.05 nano-sec)	Decreases from 1 to 0.87 (Figure 11)	Increases from 0 to 0.11 dB (Figure 14)
	Rectangular, Bandwidth = 60 MHz	2	Negligible (< 0.05 nano-sec)	Decreases from 1 to 0.9 (Figure 12)	Increases from 0 to 0.07 dB (Figure 15)
		4	Negligible (< 0.05 nano-sec)	Decreases from 1 to 0.56 (Figure 12)	Increases from 0 to 0.24 dB (Figure 15)

Table 1. Simulation results for the BOC(10,5) and BOC(14,2) signals.

	Pulse Shape	**N. of spectral replicas**	**S-Curve Gain (1/chip) 10/30/150 TEC**			**Correlation Loss (dB) 10/30/150 TEC**		
BOC(10,5) Signal	**Root-Raised Cosine, β=0.2**	2	1	1	1	0	0	0
		4	1	1	0.98	0	0	0.015
	Rectangular, Bandwidth = 40 MHz	2	1	1	0.991	0	0	0.007
		4	1	0.997	0.936	0	0.001	0.032
	Rectangular, Bandwidth = 60 MHz	2	1	0.999	0.976	0	0.001	0.019
		4	0.999	0.995	0.877	0	0.003	0.066
BOC(14,2) Signal	**Root-Raised Cosine, β=0.2**	2	1	1	1	0	0	0
		4	1	0.997	0.929	0	0.002	0.058
	Rectangular, Bandwidth = 40 MHz	2	1	1	0.999	0	0	0.001
		4	1	0.997	0.923	0	0.002	0.06
	Rectangular, Bandwidth = 60 MHz	2	1	0.998	0.943	0	0.002	0.04
		4	0.999	0.986	0.709	0.001	0.007	0.151

Table 2. S-curve and correlation loss for the minimum, mean and maximum TEC, i.e., TEC = 10, 30, and 150 $\cdot 10^{16}(1/m^2)$.

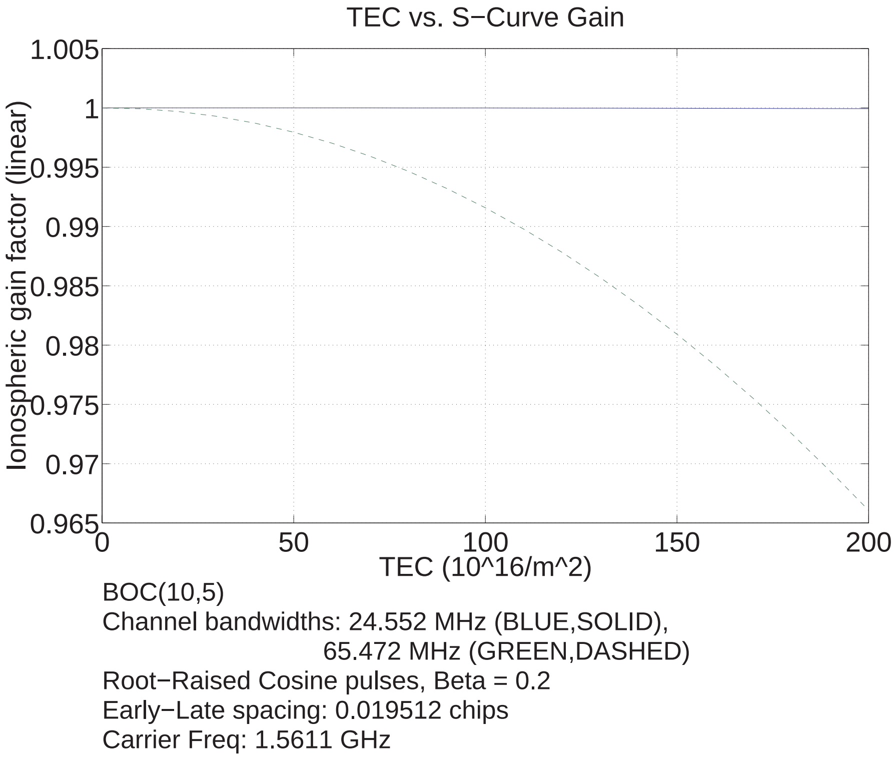

Figure 4. *Total Electronic Content versus S-curve gain for a BOC(10,5) signal modulated with Root-Raised Cosine pulses (β=0.2). The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively*.

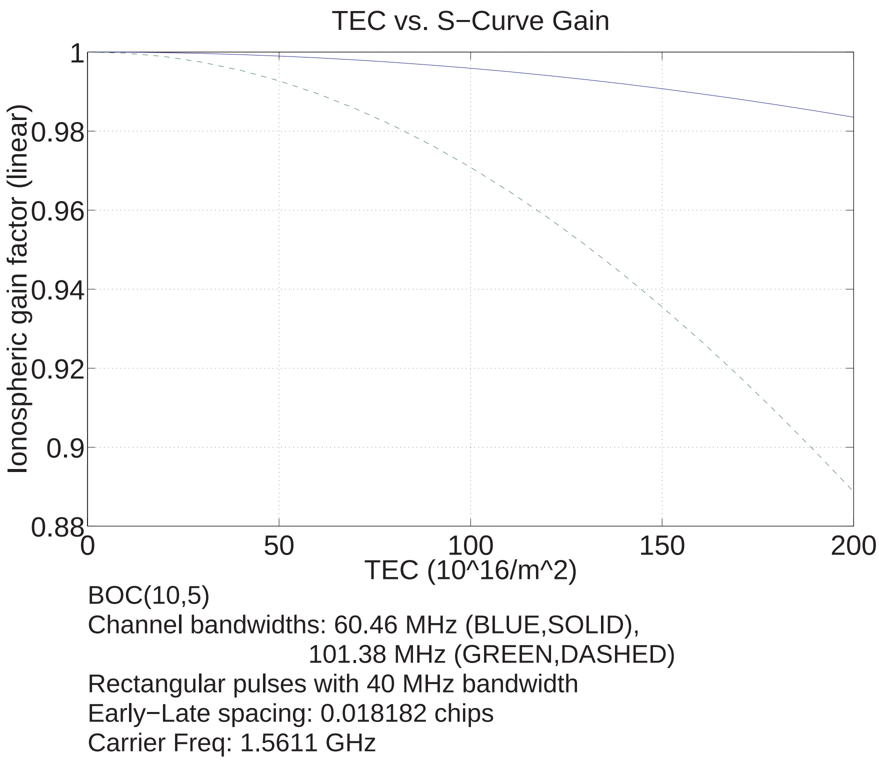

Figure 5. *Total Electronic Content versus S-curve gain for a BOC(10,5) signal modulated with rectangular pulses with 40 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

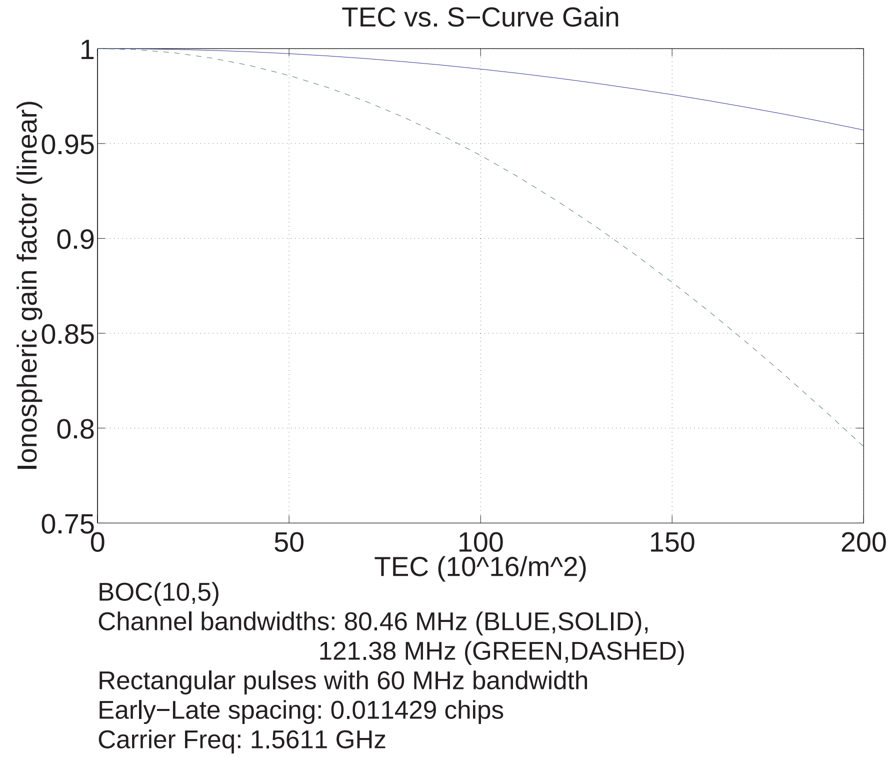

Figure 6. *Total Electronic Content versus S-curve gain for a BOC(10,5) signal modulated with rectangular pulses with 60 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

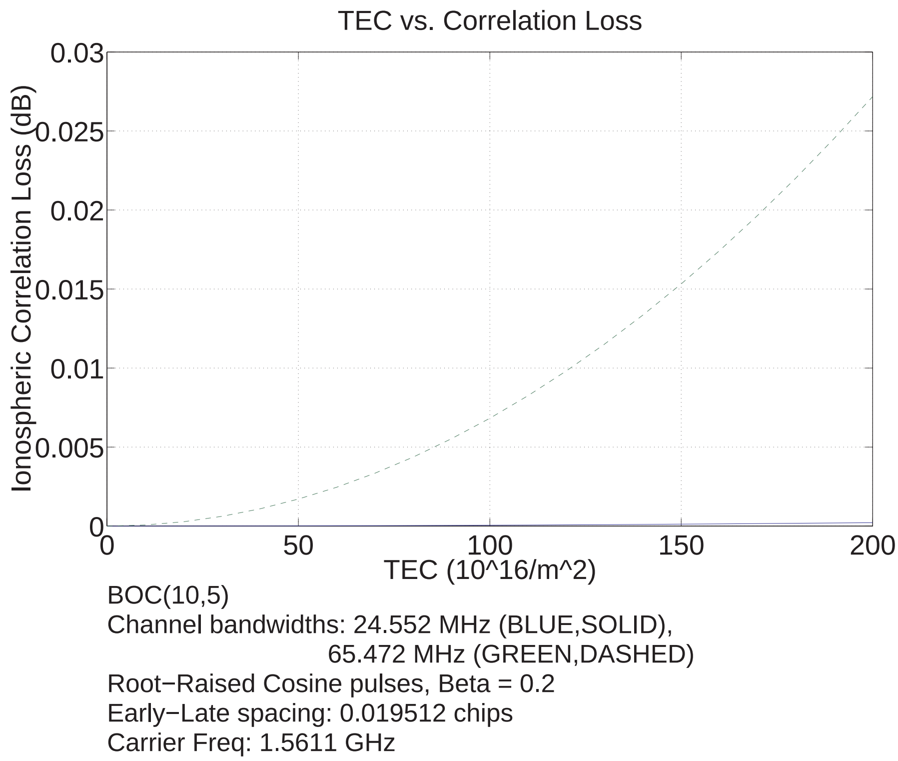

Figure 7.*Total Electronic Content versus S-curve correlation loss for a BOC(10,5) signal modulated signal modulated with Root-Raised Cosine pulses (β=0.2). The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

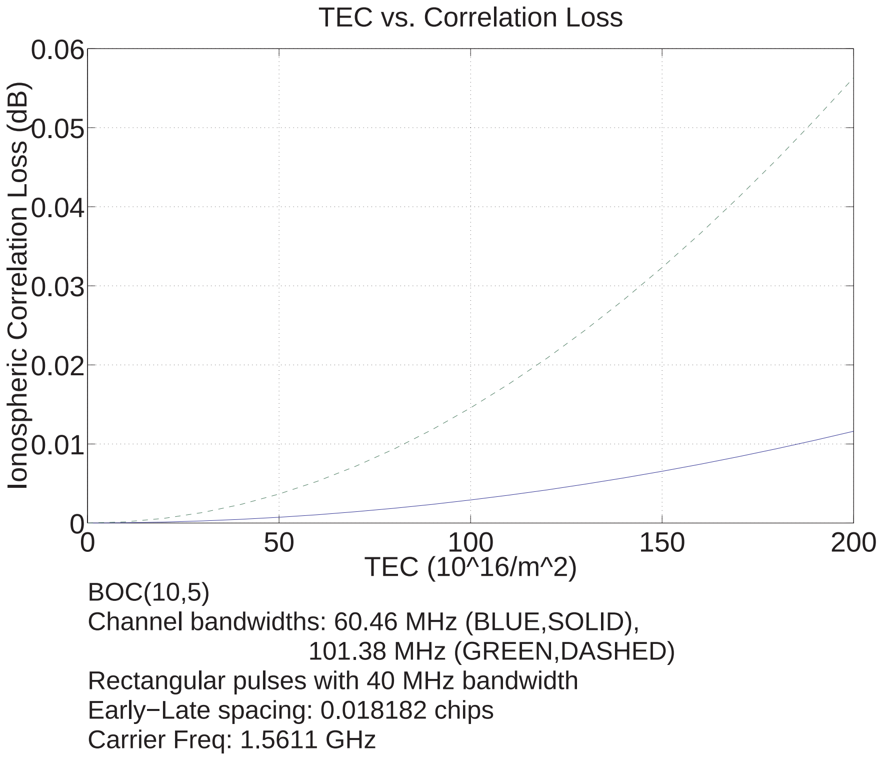

Figure 8. *Total Electronic Content versus S-curve correlation loss for a BOC(10,5) signal modulated with rectangular pulses with 40 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

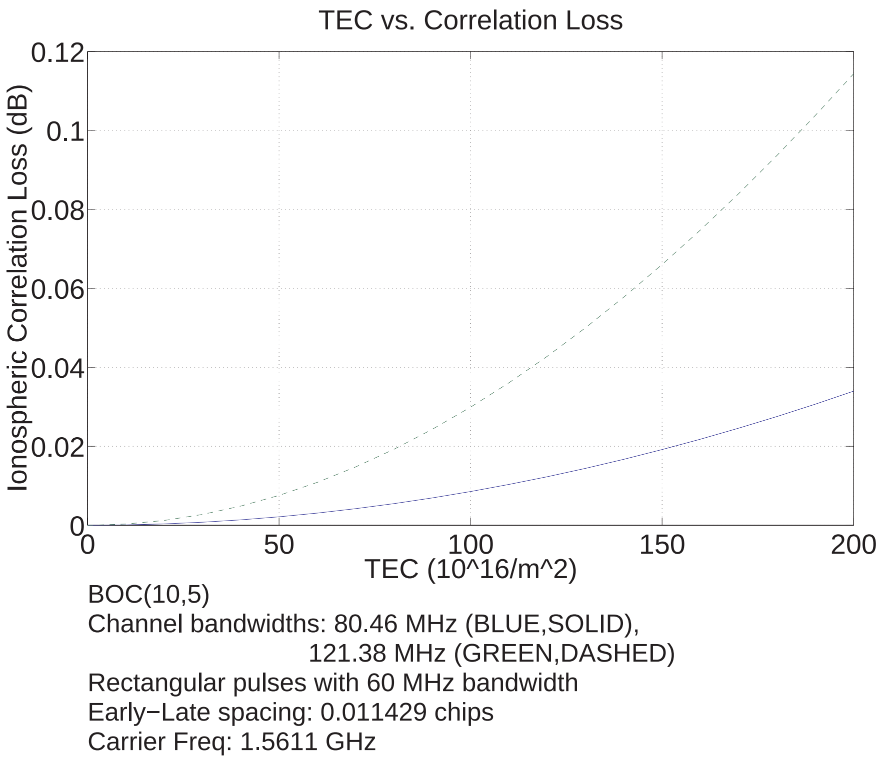

Figure 9. *Total Electronic Content versus S-curve correlation loss for a BOC(10,5) signal modulated with rectangular pulses with 60 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

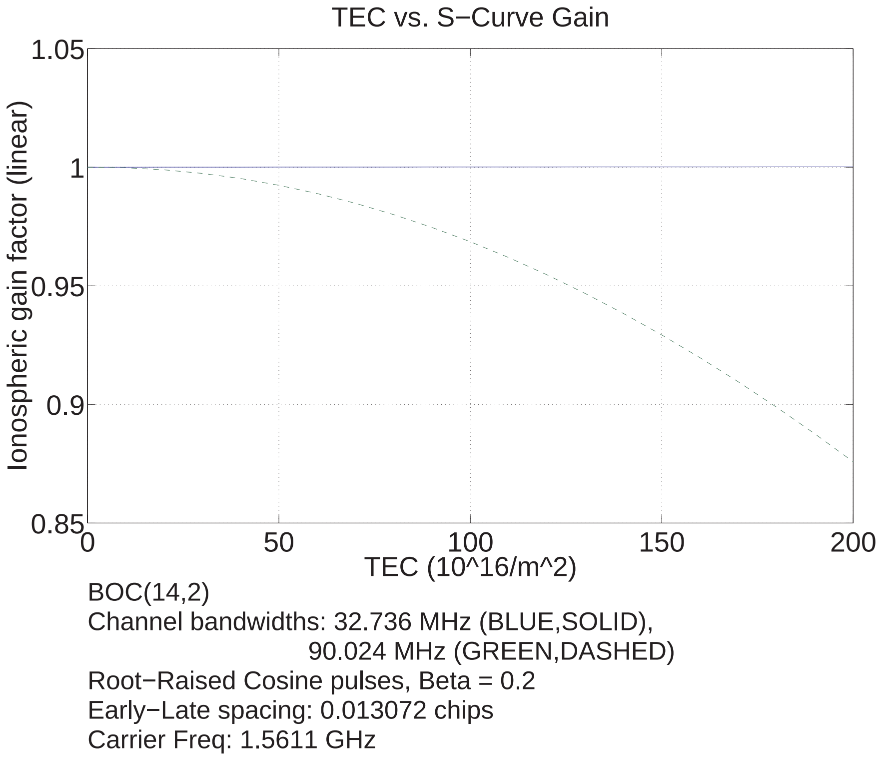

Figure 10. *Total Electronic Content versus S-curve gain for a BOC(14,2) signal modulated with Root-Raised Cosine pulses with β=0.2. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

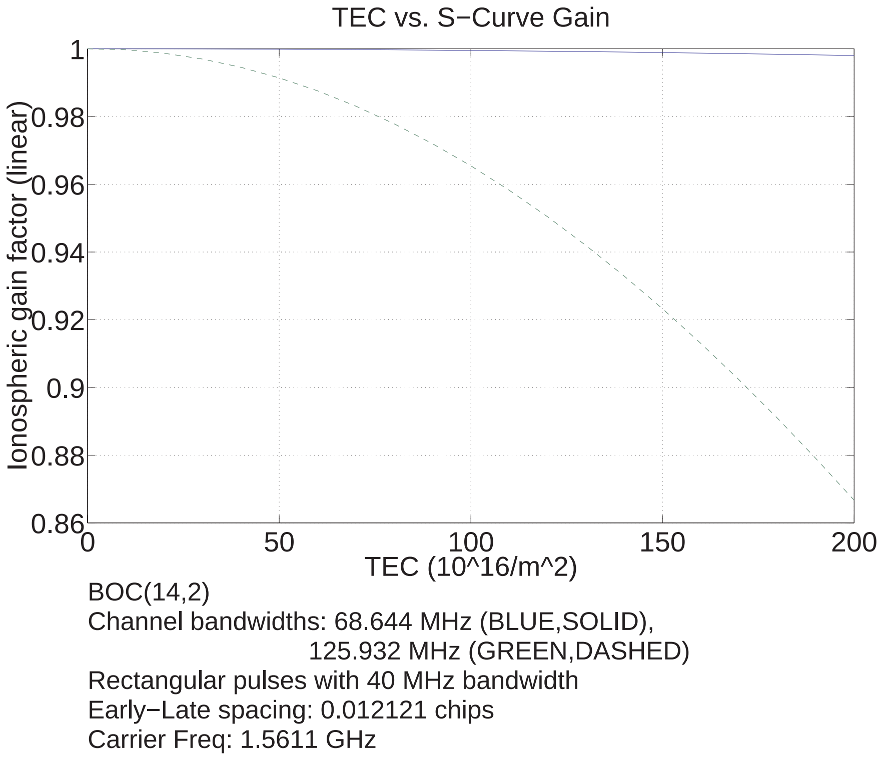

Figure 11. *Total Electronic Content versus S-curve gain for a BOC(14,2) signal modulated with with Rectangular pulses with 40 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

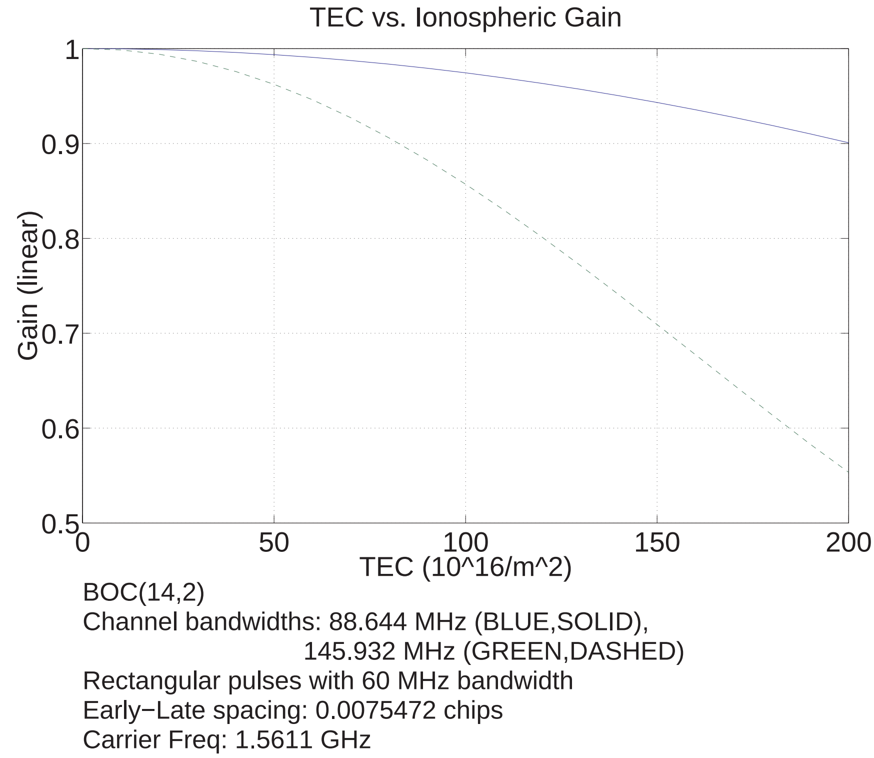

Figure 12. *Total Electronic Content versus S-curve gain for a BOC(14,2) signal modulated with with Rectangular pulses with 60 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

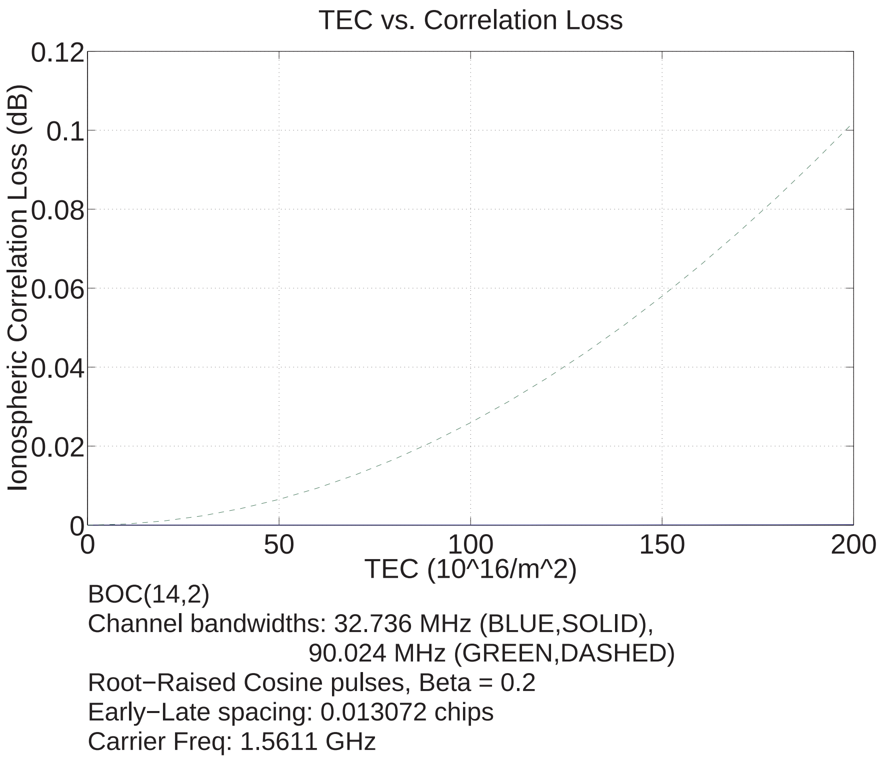

Figure 13. *Total Electronic Content versus S-curve correlation loss for a BOC(14,2) signal modulated with Root-Raised Cosine pulses with β=0.2. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

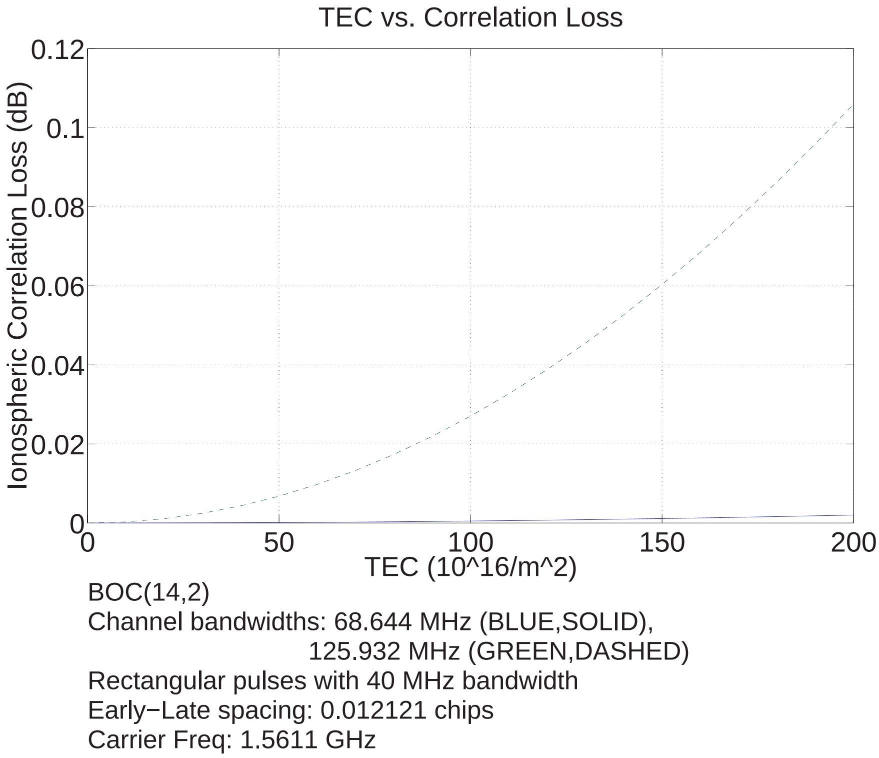

Figure 14. *Total Electronic Content versus S-curve correlation loss for a BOC(14,2) signal modulated with rectangular pulses with 40 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

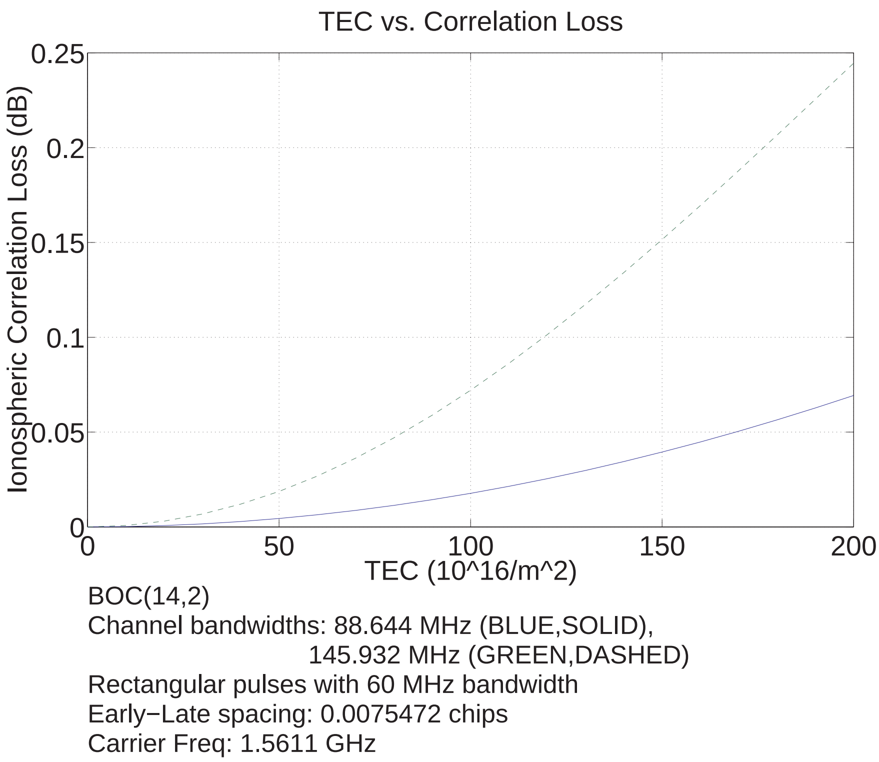

Figure 15. *Total Electronic Content versus S-curve correlation loss for a BOC(14,2) signal modulated with rectangular pulses with 60 MHz bandwidth. The blue and green curves correspond to channel bandwidths that contain 2 and 4 spectral replicas respectively.*

6. Conclusions

We have simulated the effect of the Ionospheric incoherency on the DLL S-curve. The relevant errors analysed have been the S-curve bias, gain and correlation loss. Their values have been calculated versus the Total Electronic Content of the Ionosphere, the channel bandwidth, and the pulse or CDMA-only signal bandwidth. *The results show that the Ionospheric effects are negligible except in the case of pulses with high bandwidth (Rectangular 60 Mhz) and high channel bandwidth (4 CDMA spectral replicas), in which a small correlation loss, (up to 0.15 dB), and a small gain reduction (*0.709 factor) *may occur* . In all results, the correlation loss increases with the TEC, and the gain factor decreases.

7. References

[1] Stefan Schlüter, Dr. Evelin Engler, "Modelling the ionospheric distortion effect", Technical Note, Institute of Communications and Navigation, German Aerospace Centre, 2001.