# High Precision Algorithm for Phase Estimates

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## BIOGRAPHY

Alexander Steingass graduated in 1996 from the University of Ulm/Germany with a degree in electrical engineering. He began working with the German Aerospace Center (DLR) in 1997 and is a Ph.D. candidate at the University of Essen/Germany. He took part in major navigation studies such as the Signal Design and Transmission Performance Study for GNSS-2 performed by the European Space Agency.

## ABSTRACT

High precision GPS receivers are using the estimation of the carrier phase to determine their position as precisely as possible. Since the GPS signal contains both the spreading code and the data signal this detection has to be performed incoherently. The result is a loss in accuracy, the so called squaring loss. This article presents a new technique to avoid this squaring loss. This is done by using a combination of a coherent and incoherent PLL. On using this concept the gain can reach values up to 3 dB in noise subpression performance.

## 1. INTRODUCTION

The method introduced in this article improves the accuracy of high precision GPS receivers. These receivers are using the estimation of the carrier phase to improve their accuracy of the position estimate. Usually the estimation of the carrier phase has to be performed incoherently due to the data stream which is modulated on the GPS signal. The incoherent detection uses a squaring operation to remove the BPSK-data signal from the received signal. Since squaring affects more the noise power than the signal power, the signal to noise ratio after the squaring operation decreases especially for bad channels (low $C/N_0$).

## 2. THE SIGNAL OF THE GLOBAL POSITIONING SYSTEM

From a signal processing point of view, GPS can be seen as a Direct Sequence Code Division Multiple Access (DS-CDMA) system. Each satellite broadcasts a $r_{\mathrm{d}}$ =50 bit/s data stream which is spread by a Gold code with length 1023 chips. $E_{cw}$ denotes the energy per code word, $E_{cp}$ the energy per chip. The chip rate is $r_{\mathrm{cp}}$=1 Mchip/s which results in a process gain of $E_{cw}/E_{cp}$ =30 dB. The occupied bandwidth is 20 MHz double side band.

Let $s_d(t)$ be the data signal, $s_c(t)$ the carrier and $s_{co}(t)$ the spreading code, then the GPS signal results in:

$$s_{gps}(t) = s_d(t) \cdot s_c(t) \cdot s_{co}(t) \qquad (1)$$

## 3. THE STRUCTURE OF A GPS RECEIVER

Ususally a GPS receiver estimates the distance from the satellite to the user by using two devices: A Delay Locked Loop (DLL) which is despreading the signal and estimating the group delay, and a Phase Locked Loop (PLL) for the phase estimate (see Figure 1). These work as follows: During the aquisition phase, the DLL is directed to tracking mode. The DLL is then despreading the signal, which results in a code

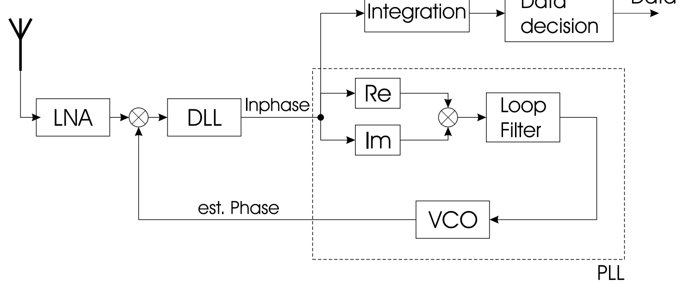


**Fig. 1**. Structure of a common GPS receiver

phase used for the group delay estimate and a despread signal termed "in-phase". On despreading, the DLL is removing the spreading code from the signal. In a next step an estimate of the carrier phase is performed. Since $s_{inphase}(t)$ still contains the data signal, an incoherent PLL must be used for the phase estimate. In figure 1, a Costas PLL [1] is chosen for this issue.

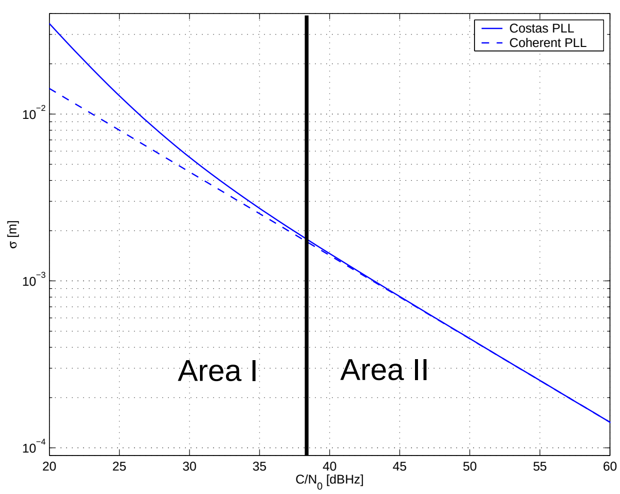


**Fig. 2**. Performance of a Costas PLL

The variance of a phase locked loop (PLL) is strongly dependent on the carrier to noise density power ratio. Let $G_{\text{PLL}}(f)$ be the transfer funktion of a PLL. The loop bandwidth $B_{\text{l}}$ [3] can be calculated by

$$B_{\text{l}} = \int_0^\infty G_{\text{PLL}}(f)\, df.$$

given by [2]

$$\sigma = \sqrt{\frac{B_{\text{l}}}{C/N_0}},$$

where the received power is denoted as $C$ and the noise power density as $N_0$. Using an incoherent PLL with a correlator length of $T_I$, the standard deviation of the error is given by

$$\sigma = \sqrt{\frac{B_{\text{l}}}{C/N_0}} \cdot \left(1 + \frac{1}{2 \cdot T_I \cdot C/N_0}\right).$$

These standard deviations are plotted in Figure 2. In area II a linear decrease of the tracking jitter for increasing $C/N_0$-ratio can be noticed. In area I the squaring loss [1] becomes greater as the $C/N_0$ decreases, degrading the performance of the incoherent PLL in comparison to the coherent PLL. This leads to a worse than linear increase of the phase error variance for decreasing $C/N_0$ (see Figure 2).

Under good receiving conditions GPS reaches a $C/N_0$ of 45 dBHz. For AWGN transmission conditions the DLL can synchronise the signal down to 25 dBHz. This results in a dynamic range from 25 to 45 dBHz. For this reason the squaring loss degrades the phase estimation significantly for noisy channels.

## 4. GPS DATA TRANSMISSION

In this section we calculate the performance of the GPS data transmission. Let us denote $E_b$ to the bit energy. In that case at the upper edge of the dynamic range ($C/N_0$=45 dBHz) an

$$E_b/N_0 = 45 \text{ dBHz} - 10 \cdot \log_{10}(50 \text{ Hz}) = 28 \text{ dB}$$

is reachable which allows a nearly error free transmission [5]. At the lower edge we reach

$$E_b/N_0 = 25 \text{ dBHz} - 10 \cdot \log_{10}(50 \text{ Hz}) = 8 \text{ dB}.$$

Figure 3 gives the BER of the uncoded transmission as well as the union bound [4] of the exended (32,26) Hamming code used for GPS. This union bound is a lower bound for the BER. It can be reached if the code is maximum likelihood decoded which is possible for

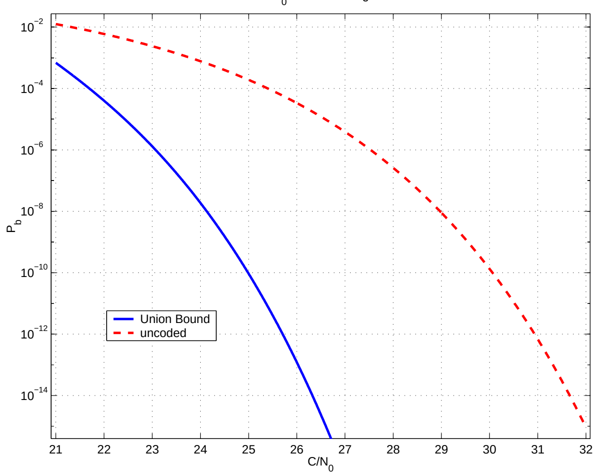


**Fig. 3**. Bit error Rates for GPS with and without forward error correction

the code used in GPS. Taking into account the bit rate of 50 bits/s to a BER of $10^{-10}$ only one bit error every seven years will occur. That clearly states that there are practically no errors within the $C/N_0$ range of $[25 \ldots 45]$ dBHz.

This near perfect knowledge of the data sequence will now be used to avoid the squaring loss for the phase estimate.

## 5. USING DECISION FEEDBACK TO AVOID THE SQUARING LOSS

The key issue is now to perform decision feedback for the phase estimate:

1. Despread the signal using a DLL
2. Estimate the carrier phase using a Costas PLL
3. Detect the data bits
4. Decode the Hamming code
5. Remove the data bits from the inphase signal.
6. Use a coherent PLL for the accurate phase estimate.

Figure 4 shows the block diagram of the "Decision feedback PLL". First the signal is despread by a DLL using the GPS C/A code.

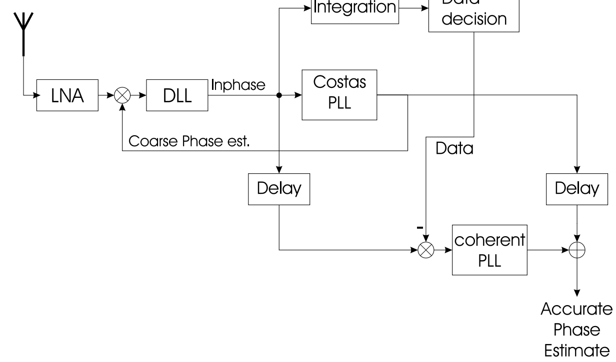


**Fig. 4**. Structure of the decision feedback GPS receiver

The bandwidth of such a loop is typically set to 1-2 Hz. A coarse estimate of the carriere signal is then performed on using a Costas PLL which has as well a very low bandwidth, e.g. 2 Hz. This necessarity occurs due to the low bit rate of the data stream. The coarse estimate is used to correct the phase and frequency before using the DLL.
After synchronising the bit stream, an integrator is detecting the data bits by integrating over 20 ms. The forward error correction hamming code is then decoded. The data stream is now removed from the inphase output of the DLL. Thus a data free signal which contains the carrier phase is achieved.

This allows the use of a coherent PLL for the phase estimation in a next step. To obtain the correct absolute phase measurement the (delayed) coarse estimated phase must be added to the coherent phase estimate.

## 6. SIMULATION RESULTS

Figure 5 shows the performance on using the decision feedback principle for a GPS phase estimate. The more the $C/N_0$ decreases, the higher is the gain of the decision feedback PLL in comparison to the standard approach. A gain of up to 3 dB is possible.

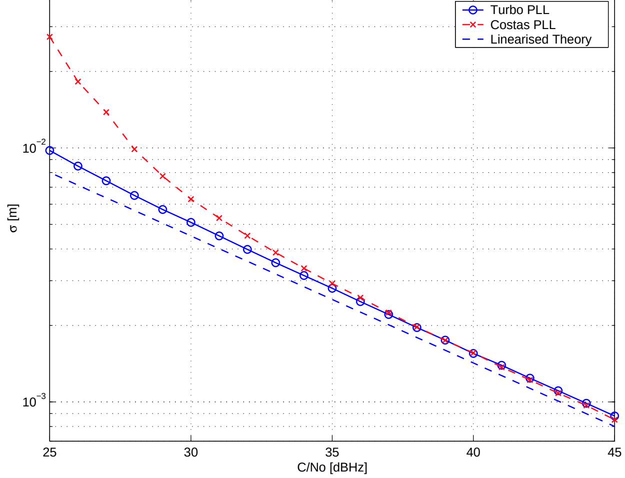


**Fig. 5**. Performance of the standard approach and the decision feedback PLL without decoding the ext. Hamming code.

## 7. CONCLUSIONS

It has been shown that the decision feedback PLL principle clearly outperforms the standard PLL approach. Since this principle causes not much computational complexity it should be no problem to integrate this concept into existing receiver structures.

## 8. ACKNOWLEDGEMENTS

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