

Applicability of force appropriation for mode tuning of aircraft

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Abstract

For the nonlinear characterization of modes, methods with harmonic excitation such as PLL and CBC have been proven useful. However, modal isolation for this purpose is required. This work investigates how modal isolation is achieved with appropriate shaker placement and force vectors. Different methods to compute optimal force pattern are revised and also one additional method based on the modal response is proposed in this work. The method discussed requires a modal model which is available from a linear identification of the structure or a finite element model. It is shown on a FE model of a glider how the first four modes are isolated by proper placement of shakers and choice of force vectors. The proposed methodology improves the preparation of nonlinear identification based on experimental continuation during a GVT.

1 Introduction

Ground Vibration Tests are conducted in order to determine the modal properties of an aircraft before first flight. These properties are required to assess the aeroelastic stability and also supports the analysis of the actual behavior in flight of the vehicle. Although numerical models are often available, they need to be validated with the dynamics of the actual aircraft.

Especially the nonlinear behavior is often not considered by computational models but are observed for real aircraft. A promising method which has been applied recently is phase locked loop control (PLL) in order to automate phase resonance testing (PRM) for fast tracking of backbone curves [1]. In this case harmonic excitation is applied to the structure. However, one requirement for the applicability of PRM is the possibility to isolate an individual mode through proper excitation. In the past, as PRM was broadly used, approaches have been discussed to find a proper force vector to isolate a mode of interest. [2, 3, 4] This work reviews the methods on force vector optimization to improve the identification with PLL on full-scale aircraft.

2 Theoretical background

This section provides an overview on the necessary equations for the approaches to find optimal force vectors. The equation of motion for harmonic excitation reads

$$(-\omega^2 [M] + i\omega [D] + [K]) \{x\} = \{F\} \quad (1)$$

$$\{x\} = [H(\omega)] \{F\} \quad , \quad (2)$$

where $[M]$ is the mass, $[D]$ is the damping and $[K]$ is the stiffness matrix. $\{x\}$ is the response and $\{F\}$ is the force vector with ω being the excitation frequency. $[H(\omega)]$ is the transfer matrix. Equation (1) is transformed into an eigenvalue problem when the external force is set zero and the internal damping is neglected:

$$([K] - \omega_n^2 [M]) \{\varphi\} = \{0\} \quad , \quad (3)$$

where $\{\varphi\}$ is the mode shape vector and ω_n the eigenfrequency. The solution of this equation allows the decomposition of Equation (1) if modal damping is assumed. For this the external force is transformed into modal force as $\{Q\} = [\varphi]^T \{F\}$ and the modal coordinate is defined as $\{q\} = [\varphi] \{x\}$. And $[\varphi]$ is the modal matrix, where the mode shape vectors $\{\varphi\}$ are arranged in a column matrix. In modal space, the equation transforms into simple single degree of freedom (SDOF) systems

$$q_n = \frac{1}{m_n (\omega_n^2 - \omega^2 + 2iD_n\omega_n\omega)} Q_n \quad . \quad (4)$$

First the force vector is transformed into modal space, then each mode is solved as a SDOF system, then the modal coordinate is transformed back into physical space as matrix multiplication.

The phase resonance method implements an experimental solution of the eigenvalue problem 3. An external harmonic excitation $\{F\}$ is applied which compensates for the internal damping forces and isolates one single mode of the structure. A successful identification is achieved when all responses are in phase and the response lags 90° after the excitation force. This is also called the phase resonance criterion. The resulting excitation frequency is then the eigenfrequency and the deflection shape of the structure is the corresponding mode shape. This is true provided that the mode of interest can be isolated with a given excitation setup and a proper chosen force vector. A spatial continuous excitation is not possible, due to the fact that some points are simply not reachable for shakers or some parts of the aircraft are not able to withstand the introduced load. Hence, not all modes can be identified properly with this methodology. The following paragraphs present approaches to find a proper force vector for mode isolation.

2.1 Phase resonance criterion

For a harmonic excitation and response, all signals can be decomposed into real and imaginary parts. Assuming that the excitation signal is real, the response must be purely imaginary to fulfill the phase resonance criterion, i.e. 90° phase lag between excitation and response. The response of a structure due to an harmonic excitation at frequency ω is formulated in Equation (2). The phase resonance criterion is fulfilled if the real part of the response vanishes. As discussed earlier, an isolation is not always possible due to practical limitations of the exciter placement. Therefore an indicator γ , also called multivariate mode indicator function (MMIF) has been proposed by Williams et al. [2]

$$\gamma = \frac{\|\Re(\{x\})\|^2}{\|\{x\}\|^2} \quad (5)$$

This is zero if perfect modal isolation is seen. However, if this is not possible, the best possible solution results in the minimum value for γ . Therefore, an optimization problem is formulated, yielding an eigenvalue problem

$$([A] - \gamma [B]) \{F\} = \{0\} \quad . \quad (6)$$

Equation (2) is inserted into (5) and separated into real and imaginary parts to derive the matrices. The result follows as $[A] = \Re([H(\omega)])^T \Re([H(\omega)])$ and $[B] = \Re([H(\omega)])^T \Re([H(\omega)]) + \Im([H(\omega)])^T \Im([H(\omega)])$. The optimal force vector is computed as eigenvector with the corresponding lowest eigenvalue γ .

2.2 Modal excitation

Another approach is formulated by Chen [3], where the optimization parameter is not dependent on the phase but the modal force vector. The external excitation is transformed into the modal force vector Q as described above. Each entry of the modal force corresponds to the excitation of an individual mode. If all entries are zeros except for one, then modal isolation is achieved. This is again expressed as indicator

$$\gamma = \frac{|Q_n|^2}{\|\{Q\}\|^2} \quad . \quad (7)$$

After reformulating as an optimization problem, an eigenvalue problem is derived. The matrices are computed as $[A] = [\varphi]_n^T [\varphi]_n$, this is computed with the n th row of the modal matrix, and $[B] = [\varphi]^T [\varphi]$.

2.3 Modal response

While the MMIF optimization considers the phase or real part of the response for the optimization, the modal force vector optimization respects the modal isolation with the modal force vector. However, high modal participation is expected if the excitation frequency is close to the the respective eigenfrequency but low participation is expected for modes with eigenfrequency, which are far away from the excitation frequency. Hence, high modal excitation of a mode with an eigenfrequency far from the eigenfrequency of the mode of interest can be allowed, due to the fact that no significant contribution is expected from that mode. This is included in the indicator function when not the modal force is regarded but the modal response:

$$\gamma = \frac{|q_n|^2}{\|\{q\}\|^2} \quad , \quad (8)$$

where the participation for each mode is computed as

$$\hat{q}_n = \frac{\{\varphi_n\}^T}{m_n \sqrt{(\omega_n^2 - \omega^2)^2 + (2D_n \omega_n \omega)^2}} \{F\} = \sqrt{W_n} \{\varphi_n\}^T \{F\} \quad . \quad (9)$$

This is similar to the modal force vector optimization but weights it additionally with the distance to the eigenfrequency and the estimated damping. Reformulated again into an optimization problem, the best force vector is obtained for the highest indicator value. The matrices are computed as $[A] = [\varphi]_n^T W_n [\varphi]_n$ and $[B] = [\varphi]^T [W] [\varphi]$, i.e. this is a weighted version of the modal force isolation approach.

2.4 Optimal shaker placement

The methods above estimate the optimal force vector for given shaker positions. However, the positions are still to be defined. The quality of the modal isolation is easily obtained from the indicator function of the presented methods. To find the optimal shaker locations for a given mode, all possible positions are combined and the optimal force vector and the corresponding indicator function for the mode of interest is computed. From all possible combinations, the positions with the best indicator function is chosen. This is an optimization problem and can also be solved with sophisticated algorithm, such as a genetic algorithm as proposed by Wright et. al [4]. However, most often only certain exciter positions are available. If a subset of possible locations are defined beforehand, brute force trials are feasible and the best location is found from that. Furthermore, the mode indicator function proposed by Breitbach [5] is often used to determine the quality of the modal isolation. This is a variation of Equation (5) and computed as $1 - \Re(x)^T |x| / \|x\|$. With a known force vector, the response is readily computed from Equation (2) and the mode indicator function is applied to the response as quality indicator. From this approach, an optimal shaker location is found for each mode. However, multiple shaker positions are possible to isolate a given mode. It is best, when multiple modes can be isolated with one single shaker configuration by varying the force vector to reduce the testing time. Therefore, the results are ranked to assess if other shaker configurations are also able to isolate the mode of interest.

3 Numerical investigations

The proposed method is applied to a numerical model and the outcomes are discussed and investigated. Figure 1 shows the FE model of the glider. It consists mainly of bar elements and spring elements to represent the stiffness while the inertia properties are represented by discrete masses. For visualization purposes more grid points and lines have been introduced.

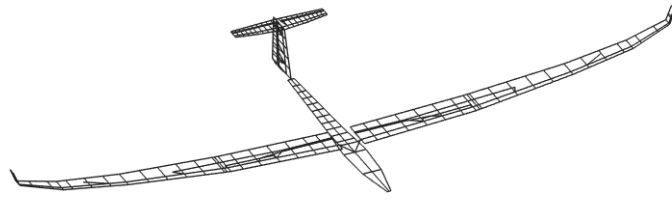


Figure 1: FE model of a glider

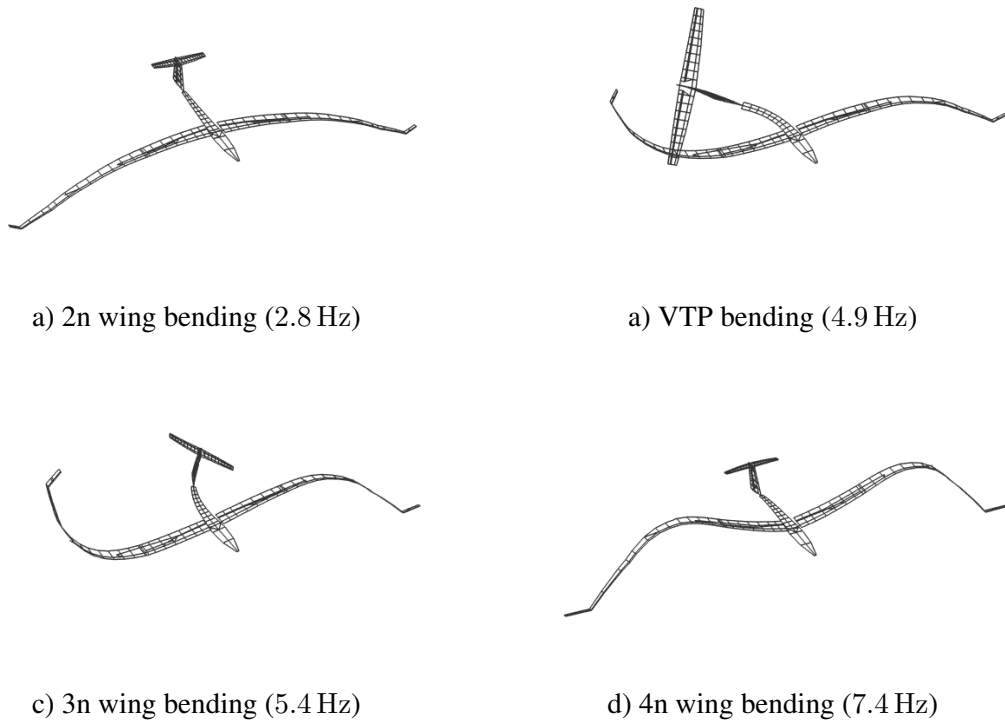


Figure 2: Mode shapes of the glider.

The first four flexible out of plane mode shapes are illustrated in Figure 2. Two symmetric mode shapes and two anti-symmetric mode shapes are presented. For the two symmetric modes, mainly wing deformation is seen and the mode with the higher frequency has more nodal lines. Furthermore, both modes are well separated in frequency. Regarding the two anti-symmetric modes, the tail deformation is seen in the mode shape as well. Once, the VTP is bending in phase with the wing at lower frequency and once, the tail is bending out of phase with the wing at a slightly higher frequency.

This paragraph studies the response for a chosen shaker configuration. First, the frequency response for a single force excitation is presented, then the result for the response with two correlated forces is shown. Figure 3 plots the FRF at the excitation point for a single force acting at the left wing tip. Four peaks associated to the modes in Figure 2 are seen. The first one and the last one are the symmetric modes, whereas the two peaks in between are anti-symmetric as previously described. The solid blue line represents the response of the monitored point and the dashed lines represent the individual modes which participate to the response. It is seen that the first wing bending behaves almost like an SDOF, since the mode is well separated in frequency. One anti-resonance around 1.5 Hz is seen, which results from the rigid body modes, due to the free-free boundary conditions. The two anti-symmetric modes are close in frequency such that both are interacting so that isolated SDOF behavior is not seen. Finally, the mode with the highest eigenfrequency

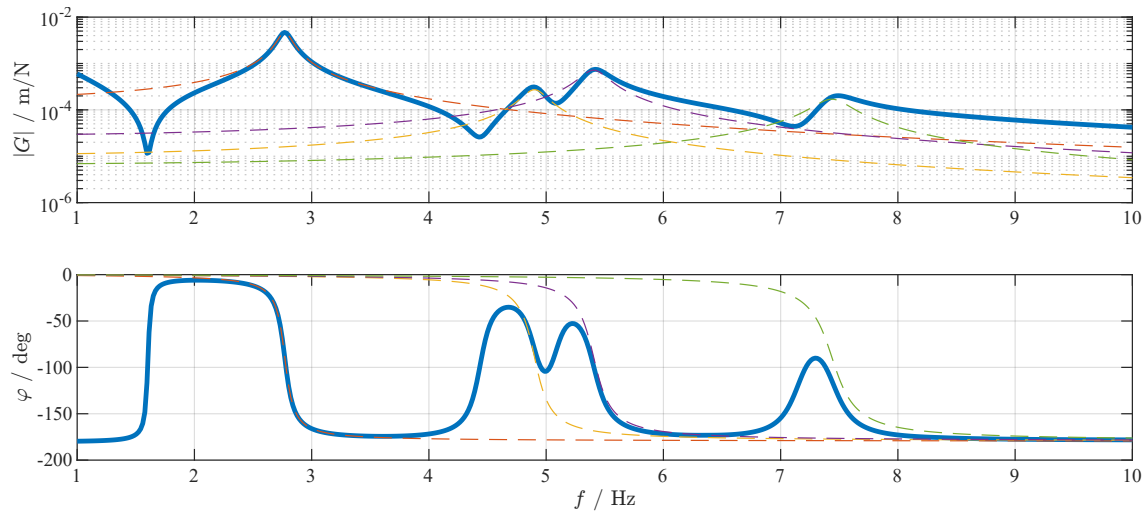


Figure 3: Response for single exciter

is the $4n$ symmetric wing bending which is well above the other modes in terms of frequency. The dashed green line is the corresponding SDOF response for this mode. It is seen that the actual response deviates from the individual mode response due to the modal interaction. Especially, an anti-resonance just before the actual mode is seen, which also modifies the phase response, such that the phase resonance criterion doesn't hold. This shows that only the first mode out of four can be isolated with a single shaker excitation.

3.1 Correlated excitation

Figure 4 shows the results for correlated force excitation. Additionally to the force at the left wing tip, an additional force at the right wing tip is introduced, as seen in Figure 5. The monitored response remains the same at the left wing tip. The left figure plots the result when both forces are in phase. As expected the two anti-symmetric modes are suppressed so that the response of those modes are not seen in the solid blue line anymore. The first mode remains perfectly isolated and the anti-resonance before that mode is not observed anymore. Nevertheless, the second symmetric wing bending still not perfectly isolated due to the anti-resonance just as before. The modal isolation still has improved in contrast to the single force excitation. The phase behavior of the actual response is also closer to the individual modal response represented by the dashed green line. However, the phase resonance criterion is not perfectly fulfilled and will result in a slightly shifted identified eigenfrequency with PRM. The right figure presents the result for a correlated excitation which is out of phase. The two symmetric mode shapes (dashed red and green) at lowest and highest frequency are well suppressed and nearly not seen in the actual response as solid blue line. The two anti-symmetric mode shapes are better excited but the modal interaction remain due to the small difference in eigenfrequency. As can be clearly seen, both modes are not isolated from this excitation.

3.2 Exciter configuration

In order to identify the two anti-symmetric modes, more exciter are required to suppress the respective other anti-symmetric mode. Typical exciter positions are chosen to find an optimal placement and the required force vector to successfully isolate the modes. Figure 5 shows the shaker locations which have been chosen for this example. At the wing are four shaker positions allowing excitation in plane and out of plane. The fuselage has two exciter points for excitation in wing span direction and wing out of plane direction. The VTP has one shaker position for excitation in wing span direction. The HTP has four points prepared following the same concept as the wing for in plane and out of plane excitation. This results in 11 excitation points.

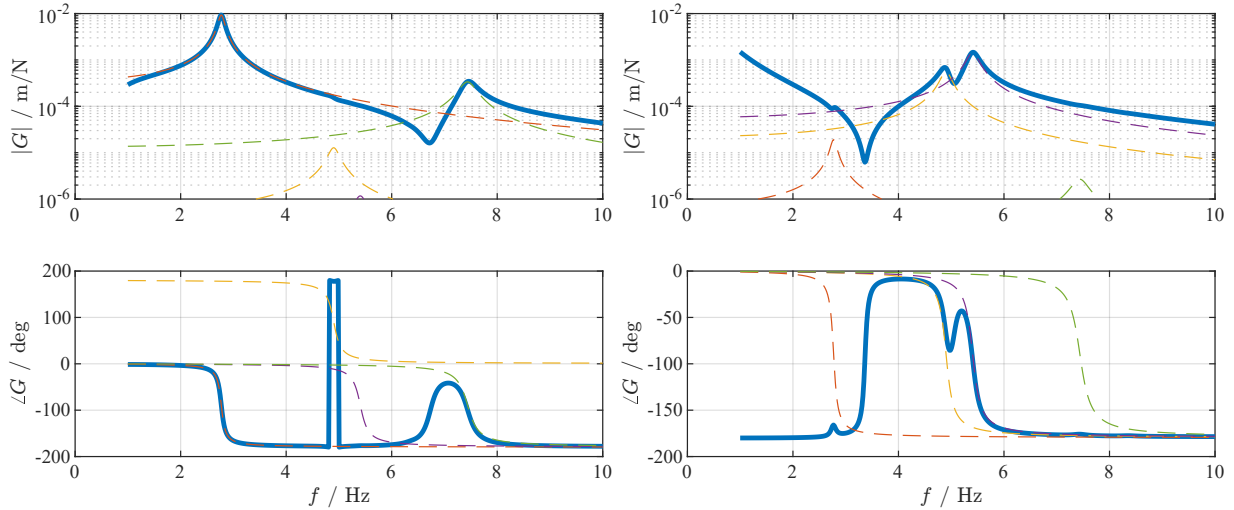


Figure 4: Response for two shakers left: symmetric; right: anti-symmetric excitation

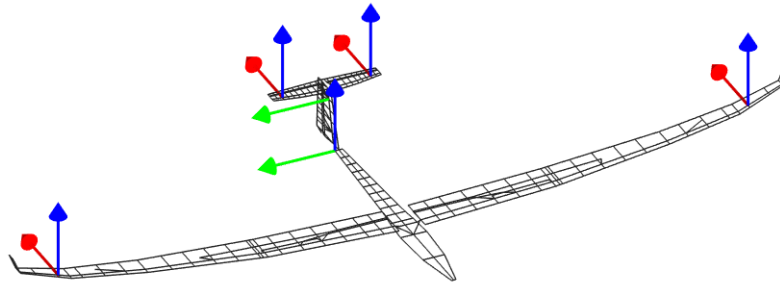


Figure 5: Exciter position

To find the best shaker positions and the corresponding force vectors, first the number of shakers are selected. From multiple trials, three are found to be reasonable to isolate the modes of interest. If more are chosen, the methods discussed reduces one force component to almost zero, so that the position can be neglected. All possible combinations for three shakers are tested and the best is selected. The methods presented above resulted in very similar results, so that only the results of the modal response indicator is presented hereafter.

As shown earlier, the first bending mode is easily isolated. One single shaker at the wing tip is enough to enforce SDOF behavior. From a practical point of view, two shakers are better to distribute the load and additionally to suppress the anti-symmetric modes around the first wing bending, although not required. For the 3n wing bending, two shakers have been tried as well and the best excitation points were at the wing tip in out of plane direction and at the VTP in wing span direction. This is neither a symmetric or anti-symmetric excitation but results in a better isolation of the targeted mode. It is seen in Figure 2 b) that the VTP motion contributes to the mode and hence the excitation increases the response and further also suppresses the anti-symmetric wing bending. In order to consider the symmetry of the structure, three shakers are now selected to find the best force vector and the best shaker positions. This results in the wing tip out of plane direction with the VTP excitation in wing span direction. The estimated force patterns are presented in Figure 6. The left plot a) shows that the wing tip forces are small, so that VTP excitation only might be enough to isolate the targeted mode. However, the little contribution at the wing tip has a significant impact in suppressing the wing bending. The resulting response at the right wing tip is presented in Figure 7 on the left side. Although contributions from the other modes are still seen, in the vicinity of the resonance, a SDOF behavior is observed. The half power bandwidth is shaded in gray, where clearly a SDOF response is

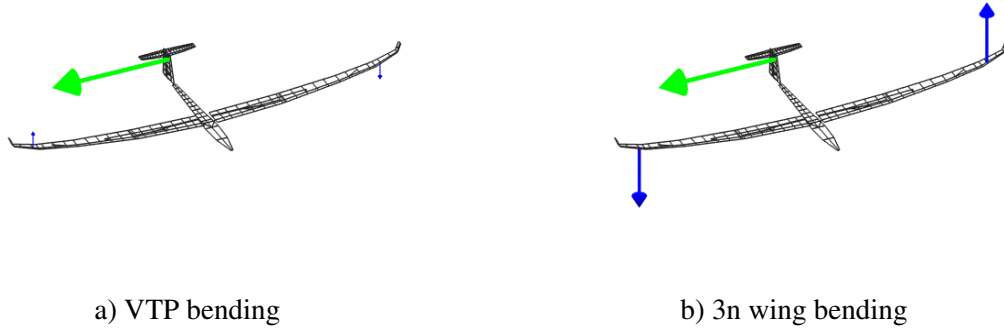


Figure 6: Force pattern for modal isolation.

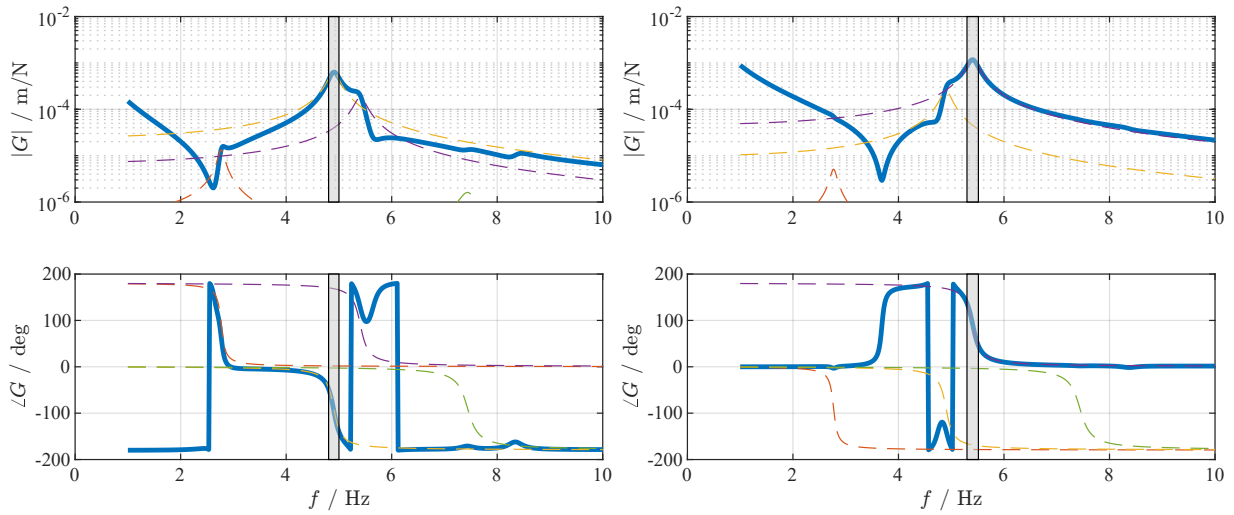


Figure 7: Response for three shaker. Left: isolation of VTP bending; right: wing bending

seen. In PRM testing, the excitation frequency is not varied a lot and hence only the frequencies close to the excitation frequency is of interest. The resulting force vector for the 3n wing bending is seen in Figure 6 b). More contribution from the shakers at the wing is seen here. Additionally, the forces on the wing are out of phase to the force vector on the left side for the VTP bending. The resulting response on the monitor point on the right wing tip is illustrated in Figure 7 on the right side. Again participation of the other modes are still seen but within the half power bandwidth of the resonance frequency a SDOF behavior is observed such that the PRM method is applicable.

From the presented four modes, three are successfully isolated. However, the second symmetric wing bending is still not perfectly isolated although an attempt might be considered successful if this mode was targeted with two shakers at the wing tip only. Nevertheless, from the mode shapes illustrated in Figure 2 it is seen that the deflection of the inner part of the wing is contributing more for the 4n wing bending mode. In a further trial two more excitation points are included in the inner part of the wing in order to properly isolate the second symmetric wing bending. Figure 8 shows the shaker locations on the left side. Both force components are the same resulting in a symmetric force pattern. The response at the left wing tip is presented on the right plot. This time, the anti-resonance is not seen anymore, hence a SDOF behavior is clearly seen for both symmetric modes. Having a look in Figure 2 on the first (a) and second symmetric bending (d) it is seen that the wing tip are both in phase, whereas the inner part are out of phase, i.e. in the first wing bending the inner wing moves down, whereas the inner part of the second symmetric bending moves up. Due to the out of phase movement and the observation on a non driving point at the wing tip, the anti-resonance is not seen

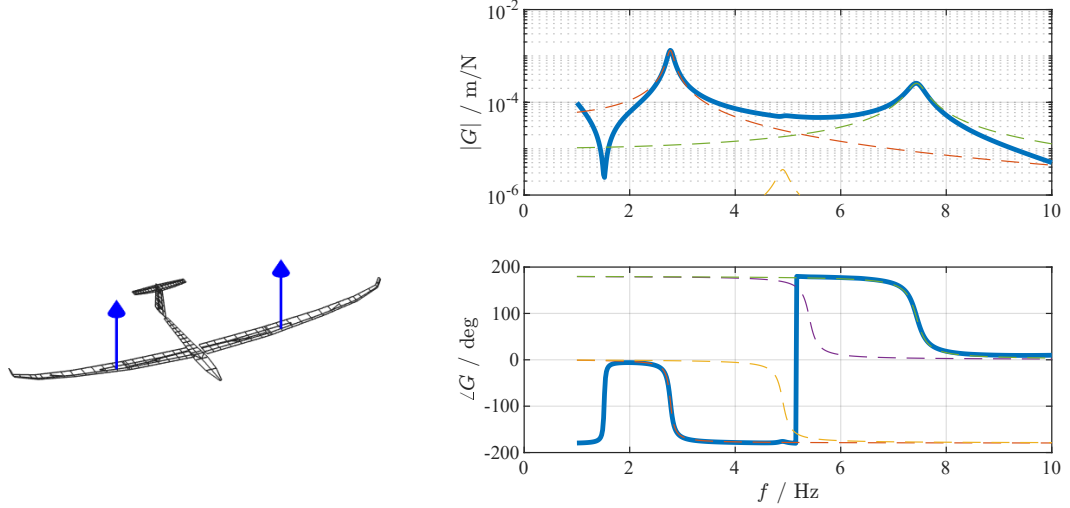


Figure 8: 4n wing mode excitation on the left and response on the right.

anymore. Therefore both modes present a SDOF behavior resulting in a successful modal isolation with one shaker configuration.

4 Discussion

It has been investigated how a chosen mode is isolated with dedicated excitation patterns in order to identify those with PRM. This technique has been proven useful as additional measurement in order to characterize the nonlinear behavior. Three procedures to calculate the optimal force vector for a given mode is reviewed. One is based on the phase, trying to minimize the real part, meaning that a 90° criterion is enforced. One is based on the modal force vector where the modal force maximized for one mode and minimized for the others. One more optimization has been proposed in this work where the modal response is regarded, where one modal participation is maximized. For the application of the methods, a modal model is required which is identified from an experiment or derived from a computational model. All have in common that the location of excitation is chosen a priori. Thus a brute force method has been applied additionally with a fixed number of shakers where a subset of excitation points is chosen beforehand. The excitation locations must be compliant to withstand the introduced force and also reachable, hence not all excitation location points are feasible.

The approach has been tested on a FE model of a glider, where first a modal analysis has been conducted and the resulting modal model is used for further investigations. Four modes of interest are investigated, where two are symmetric and two anti-symmetric which are closely spaced in frequency resulting in modal interaction. With the proposed method the two anti-symmetric modes were successfully separated and isolated. Furthermore the effect of anti-resonance on a mode which is well separated in frequency is seen for the two symmetric modes. Unfortunately with the chosen excitation points, the second symmetric bending mode was not properly isolated. With two new excitation points, which are chosen with engineering judgment, all four modes in this example are successfully isolated with the computed force vector and two optimal shaker configurations.

Furthermore, modal synthesis has been used as a tool to compute the response and also predict the applicability of the proposed excitation pattern to isolate the mode of interest. Often the response over a broader frequency range is not regarded but only the resonance frequency in nonlinear modal tuning. This work provides a tool for use in GVT, where a modal model is identified first. In a second stage, when nonlinear characterization of the aircraft is required, the identified modal model is used to predict the best shaker locations and force patterns to characterize the nonlinear behavior for mode of interest.

5 Summary

Harmonic excitation has been implemented successfully in the past to identify nonlinear modes, e.g. with CBC or PLL. However, it is required that the mode of interest can be isolated with a proper force vector. It has been shown in this work that one shaker is not able to properly isolate all modes. Sometimes it might be even possible that a mode cannot be isolated at all, since not all locations are accessible or are able to withstand the excitation force. Nevertheless an approach has been presented which is easily implemented within a modal survey to support the modal identification of modes with nonlinear characteristics. The modal model required for this methodology can be identified from the test, so that no further numerical model is required.

References

- [1] M. Tang, C. Stephan, and M. Böswald, “Phase resonance method for nonlinear mechanical structures with phase locked loop control,” in *Proceedings of ISMA*, Leuven, Belgium, 2020. [Online]. Available: <https://hal.archives-ouvertes.fr/hal-03082517/document>
- [2] R. Williams, J. Crowley, and H. Vold, “The multivariate mode indicator function in modal analysis,” in *International Modal Analysis Conference*, 1985, pp. 66–70.
- [3] W. Chen, M. Jin, and H. Song, “Optimal configuration of shakers for phase resonance testing using modal parameters,” *Journal of Mechanical Engineering Science*, vol. 233, no. 16, pp. 5676–5690, 2019.
- [4] J. R. Wright, P. S. Holmes, M. J. Desforges, and J. E. Cooper, “Optimum exciter placement for normal mode force appropriation,” *Proceedings of the Institution of Mechanical Engineers Part G-Journal of Aerospace Engineering*, vol. 212, no. 5, pp. 325–338, 1998.
- [5] E. Breitbach, “A semi-automatic modal-survey test technique for complex aircraft and spacecraft structures,” in *Proceedings of ESA Testing Symposium*, Frascati, Italy, 1974, pp. 519–528.