Non-linear Pre-Filtering Techniques for MC-CDMA Downlink Transmissions

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Abstract— **We consider the downlink of a multi-carrier code-division multiple-access (MC-CDMA) system operating in a time division duplexing (TDD) mode and propose a non-linear pre-filtering scheme based on Tomlinson-Harashima pre-coding. In designing the pre-filtering matrices we adopt a minimum mean square error (MMSE) approach under a constraint on the overall transmit power since multiple access interference and multipath distortions are pre-compensated at the base station, low-complex receivers can be employed at the mobile units. Compared to other existing solutions, the proposed scheme provides better error rate performance.**

I. Introduction

The demand for high data rates in wireless transmissions has recently led to a strong interest in multi-carrier code-division multiple-access (MC-CDMA), which is an effective technique for combating multipath fading over highly dispersive wireless channels [1].

The main impairment in MC-CDMA systems is the multiple-access interference (MAI) which occurs in the presence of multipath propagation. Several advanced multi-user detection (MUD) techniques are available for interference mitigation [2]. Unfortunately, in spite of their effectiveness, these methods are not suited for downlink applications due to their relativity high computational complexity. As an alternative to MUD, interference mitigation can be accomplished at the transmit end using pre-filtering schemes. This requires channel state information (CSI) at the transmitter, which can be achieved in time division duplex (TDD) systems thanks to the channel reciprocity between alternative uplink and downlink transmissions.

Over the last years several linear pre-filtering schemes have been proposed to mitigate MAI and channel distortions in MC-CDMA downlink transmissions. References [3]- [7] provide a good sample of the results obtained in this area. In particular, in [3] the pre-filtering coefficients are designed according to the zero-forcing (ZF) criterion so as to completely eliminate the MAI at each mobile terminal (MT). The method discussed in [4] aims at maximizing a modified signal-to-interference-plus-noise ratio (SINR) at the MTs while in [5] the goal is the minimization of the bit-error-rate (BER) at the receivers. The latter performs remarkably better than the previous schemes but leads to a complicated non-linear optimization problem that cannot be solved in closed form. In [6], the pre-filtering coefficients are designed so as to minimize a proper cost function which is the sum of the inverse of the SINRs of all active users. Finally the method reported in [7] aims at minimizing the sum of the mean square error at all MTs.

Non-linear pre-filtering has recently been proposed as viable candidates to counteract the detrimental effects of MAI [8]- [10]. All these schemes are based on Tomlinson-Harashima (TH) precoding and significantly outperform linear pre-filtering solutions. Unfortunately, they are devised for single-user and/or multi-user transmissions over frequency-flat multiple-input multiple-output (MIMO) channels and cannot be easily extended to an MC-CDMA network. A TH-based scheme for MC-CDMA downlink transmissions has been proposed in [11]. This method relies on the QR-decomposition of the channel matrix and selects the pre-filtering coefficients under a ZF constraint. Albeit reasonable, the above solution is not based over any optimality criterion and does not exploit the advantages of multiple antennas at the transmitter.

In this work we propose a non-linear TH-based pre-filtering scheme for the downlink of a TDD MC-CDMA system equipped with multiple transmit antennas. The processing matrices are designed according to a minimum mean square error (MMSE) criterion under a constraint on the overall transmit power. The resulting technique allows the MTs to employ single-user detection (SUD) schemes and outperforms conventional (i.e., without pre-filtering) MC-CDMA transmissions. Computer simulations indicate that the proposed scheme achieves better error rate performance as compared to other existing solutions based on linear pre-filtering.

The remainder of the paper is organized as follows. Next section outlines the signal model and introduces basic notation. In Section III we derive the pre-filtering matrices using the MMSE criterion. Numerical results are provided in Section IV while conclusions are drawn in Section V.

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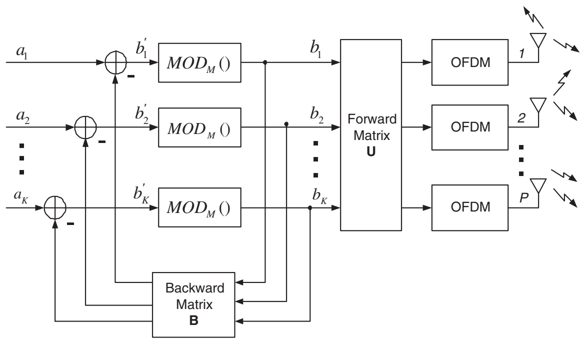

Fig. 1. Block diagram of the transmitter.

II. System Model

A. Transmitter Structure

We consider the downlink of an MC-CDMA network in which the BS is equipped with P antennas and the total number of subcarriers, N, is divided into smaller groups of Q elements. Without loss of generality, we concentrate on a single group and assume that the corresponding subcarriers are uniformly distribuited over the signal bandwidth so as to better exploit the channel frequency diversity. We denote $\{i_n\,;\ 1 \leq n \leq Q\}$ the subcarrier indexes in the considered group, with $i_n = 1 + (n-1)N/Q$. The BS employs the Q subcarriers to communicate with K active users ($K \leq Q$), which are separated by their specific spreading codes. The user data are collected into the K-dimensional vector $\mathbf{a} = [a_1, a_2, ..., a_K]^T$ (the notation $[\,\cdot\,]^T$ denotes the transpose operation) and are taken from an M-QAM constellation with energy $\sigma_a^2 = 2(M-1)/3$.

Figure 1 shows the transmit side of the system under investigation. The input symbols a_k are fed into the pre-filtering block, which consists of a *backward* matrix $\mathbf{B}$, K non-linear operators $MOD_M(\cdot)$ and a *forward* matrix $\mathbf{U}$. As discussed in [10], $\mathbf{B}$ must be *strictly* lower triangular to allow data pre-coding in a recursive fashion. The output of the forward matrix is finally passed to P OFDM modulators, each comprising an inverse discrete Fourier transform (IDFT) unit and the insertion of a cyclic prefix to avoid interference between adjacent blocks.

To explain the rationale behind the proposed transmit structure, we temporarily neglect the non-linear operation in Figure 1. In these circumstances, the pre-coded symbols $\mathbf{b} = [b_1, b_2, ..., b_K]^T$ are computed from $\mathbf{a}$ as follows

$$b_k = a_k - \sum_{\ell=1}^{k-1} [\mathbf{B}]_{k,\ell}\, b_\ell \tag{1}$$

where $[\mathbf{B}]_{k,\ell}$ is the (k,ℓ)th entry of $\mathbf{B}$. The above equation may be written in matrix form as

$$\mathbf{b} = \mathbf{C}^{-1}\mathbf{a} \tag{2}$$

where $\mathbf{C} = \mathbf{B} + \mathbf{I}_K$ is a *unit-diagonal* and lower triangular matrix while $\mathbf{I}_K$ denotes the identity matrix of order K.

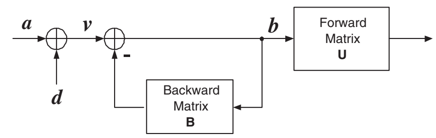

Fig. 2. Equivalent block diagram of the transmitter.

Inspection of (2) reveals that the energy of the pre-coded symbols $\mathbf{b}$ becomes very large when $\mathbf{C}$ is ill-conditioned, thereby leading to a significant increase of the transmit power. To overcome this problem, we adopt a method similar to the TH scheme [12] and introduce the following modulo operator

$$MOD_M(x) = x - 2\sqrt{M} \cdot \left\lfloor \frac{x - \sqrt{M}}{2\sqrt{M}} \right\rfloor \tag{3}$$

where x is a real number and $\lfloor c \rfloor$ represents the *smallest* integer larger than or equal to c. Note that for a complex-valued input the non-linear operation (3) is performed separately over the real and imaginary components.

A close observation of (3) reveals that the modulo operator performs a periodic mapping of the complex plane into the square region $\aleph = \{x^{(R)} + jx^{(I)} | x^{(R)}, x^{(I)} \in (-\sqrt{M}, \sqrt{M}]\}$. In this way the magnitude of the pre-coded symbols b_k is smaller than with linear pre-filtering and the transmit power is correspondingly reduced.

From (3) it follows that

$$b_k = a_k - \sum_{l=1}^{k-1} [\mathbf{B}]_{k,l}\, b_l + d_k \quad k = 1, 2, \ldots, K \tag{4}$$

where $d_k = 2\sqrt{M}p_k$ and p_k is a complex-valued quantity whose real and imaginary parts are suitable integers that reduce b_k to the square region $\aleph$. Clearly, a unique p_k exists with such a property.

The above equation indicates that the modulo operation in (3) is equivalent to adding a vector $\mathbf{d} = [d_1,\, d_2, \ldots,\, d_K]^T$ to the input data $\mathbf{a}$. This results in the equivalent block diagram of Figure 2, from which it follows that

$$\mathbf{b} = \mathbf{C}^{-1}\mathbf{v} \tag{5}$$

where we have defined $\mathbf{v} = \mathbf{a} + \mathbf{d}$.

After non-linear pre-coding, each element of $\mathbf{b}$ is spread over Q chips using a unit-energy spreading sequence $\mathbf{c}_k = [c_k(1),\, c_k(2), \ldots,\, c_k(Q)]^T$, with $c_k(n) \in \{\pm 1/\sqrt{Q}\}$. The resulting vector $b_k\mathbf{c}_k$ is then fed to P linear pre-filtering units, one for each antenna branch. The output from the pth unit is

$$\mathbf{s}_{k,p} = b_k \mathbf{u}_{k,p} \qquad p = 1, 2, \ldots, P \tag{6}$$

where $\mathbf{u}_{k,p} = [u_{k,p}(1),\, u_{k,p}(2), \ldots,\, u_{k,p}(Q)]^T$ is a vector with entries

$$u_{k,p}(n) = c_k(n) t_{k,p}(n) \quad n = 1, 2, \ldots, Q \tag{7}$$

and $\mathbf{t}_{k,p} = [t_{k,p}(1),\, t_{k,p}(2), \ldots,\, t_{k,p}(Q)]^T$ are the pre-filtering coefficients of the kth user at the pth antenna. The

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contributions of the users are summed chip-by-chip to form the following transmit vector at the pth antenna

$$\mathbf{s}_p = \sum_{k=1}^{K} \mathbf{s}_{k,p} \qquad p = 1, 2, \ldots, P. \tag{8}$$

Finally, each $\mathbf{s}_p$ ($p = 1, 2, \ldots, P$) is mapped on Q subcarriers using an OFDM modulator for each antenna branch.

Collecting vectors $\mathbf{s}_p$ into a single PQ-dimensional vector $\mathbf{s} = \left[\mathbf{s}_1^T\, \mathbf{s}_2^T \ldots \mathbf{s}_P^T\right]^T$, produces

$$\mathbf{s} = \mathbf{U}\mathbf{b} \tag{9}$$

where $\mathbf{U} = [\mathbf{u}_1\ \ \mathbf{u}_2 \ldots \mathbf{u}_K]$ is the forward matrix in Figure 1 and $\mathbf{u}_k$=$[\mathbf{u}_{k,1}^T\ \ \mathbf{u}_{k,2}^T, ..., \mathbf{u}_{k,P}^T]^T$.

B. Receiver Structure

The signals transmitted by the BS array propagate through multipath channels. As shown in Figure 3, the P waveforms arriving at each receiver are implicitly combined by the single receive antenna and passed to an OFDM demodulator. Without loss of generality, we concentrate on the jth MT and write the DFT output as

$$\mathbf{X}_j = \mathbf{H}_j \mathbf{U}\mathbf{b} + \mathbf{n}_j \tag{10}$$

where $\mathbf{H}_j = [\mathbf{H}_{j,1}\mathbf{H}_{j,2}...\mathbf{H}_{j,P}]$ is a $Q \times QP$ channel matrix and $\mathbf{H}_{j,p} = \text{diag}\{H_{j,p}(i_1),\, H_{j,p}(i_2), ..., H_{j,p}(i_Q)\}$ represents the channel frequency response between the pth transmit antenna and the jth MT over the Q subcarriers. Also, $\mathbf{n}_j = [n_j(1),\, n_j(2), \ldots, n_j(Q)]^T$ accounts for thermal noise and it is Gaussian distribuited with zero-mean and covariance matrix $\sigma_n^2 \mathbf{I}_Q$.

To keep the complexity of the MT at a reasonable level, the decision statistic for b_j is obtained by feeding $\mathbf{X}_j$ to a linear single-user detector. This produces

$$y_j = e \cdot \mathbf{q}_j^{\mathrm{H}} \mathbf{X}_j \tag{11}$$

where e is a real positive factor that can be thought of as being part of the automatic gain control (AGC) unit, while $\mathbf{q}_j = [q_j(1), q_j(2), ..., q_j(Q)]^T$ is a *unit-norm* vector that performs both channel equalization and signal despreading. Substituting (10) into (11) yields

$$y_j = e \cdot \mathbf{g}_j^H \mathbf{U}\mathbf{b} + e \cdot w_j \tag{12}$$

where $w_j = \mathbf{q}_j^H \mathbf{n}_j$ is the noise contribution and $\mathbf{g}_j = \mathbf{H}_j^H \mathbf{q}_j$ is a QP-dimensional vector that depends on the channel coefficients and data detection strategy. In this contribution, several well-known SUD techniques are considered, including pure despreading (PD), equal gain combining (EGC) and maximum ratio combining (MRC) [1]. As shown in Figure 3, the decision statistic y_j is then fed to the same modulo operator employed at the transmitter. The output is finally passed to a threshold device which provides an estimate of a_j.

Stacking the decision statistics of all users into a single vector $\mathbf{y} = [y_1, y_2, ..., y_K]^T$, we get

$$\mathbf{y} = \mathbf{G}^H \mathbf{F}\mathbf{b} + e \cdot \mathbf{w} \tag{13}$$

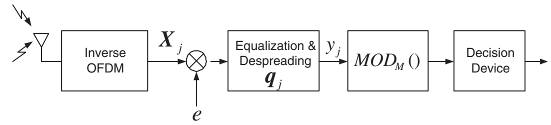

Fig. 3. Block diagram of the jth receiver.

where $\mathbf{F} = e \cdot \mathbf{U}$, $\mathbf{w} = [w_1, w_2, ..., w_K]^T$ is a Gaussian vector with zero-mean and covariance matrix $\sigma_n^2 \mathbf{I}_Q$ and $\mathbf{G}$ is the following $PQ \times K$ matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{H}_{1,1}^H \mathbf{q}_1 & \mathbf{H}_{2,1}^H \mathbf{q}_2 & \cdots & \mathbf{H}_{K,1}^H \mathbf{q}_K \\ \mathbf{H}_{1,2}^H \mathbf{q}_1 & \mathbf{H}_{2,2}^H \mathbf{q}_2 & \cdots & \mathbf{H}_{K,2}^H \mathbf{q}_K \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{1,P}^H \mathbf{q}_1 & \mathbf{H}_{2,P}^H \mathbf{q}_2 & \cdots & \mathbf{H}_{K,P}^H \mathbf{q}_K \end{bmatrix}. \tag{14}$$

III. Design of The Backward and Forward Matrices

The optimality criterion for the design of $\mathbf{U}$ and $\mathbf{C}$ is the minimization of the following mean square error (MSE)

$$J = E\left\{\ \|\mathbf{y} - \mathbf{v}\|^2\right\} \tag{15}$$

where the statistical expectation must be computed with respect to data symbols and thermal noise. In doing so, we impose a constraint on the overall transmit power and assume that the pre-coded symbols b_k are statistically independent with zero-mean and power σ_a^2 as the user data. Although not rigourously satisfied, this assumption is reasonable for large M-QAM constellations with size $M \geq 16$ [10]. In the above hypothesis, the power constraint can be formulated as $\text{tr}\left\{\mathbf{U}^H\mathbf{U}\right\} = K$ or, equivalently,

$$\frac{\text{tr}\left\{\mathbf{F}^H\mathbf{F}\right\}}{K} = e^2. \tag{16}$$

In minimizing the right-hand-side (RHS) of (15), we follow an approach similar to that employed in [13]. To this purpose, we substitute (5) and (13) into (15) and, bearing in mind that $\mathbf{b}$ and $\mathbf{w}$ are statistically independent, we obtain

$$J = \sigma_a^2 \cdot \text{tr}\left\{(\mathbf{G}^H\mathbf{F} - \mathbf{C})(\mathbf{G}^H\mathbf{F} - \mathbf{C})^H\right\} + e^2 \cdot K\sigma_n^2. \tag{17}$$

Next, substituting (16) into (17) yields

$$J = \sigma_a^2 \cdot \text{tr}\left\{\left(\mathbf{G}^H\mathbf{F} - \mathbf{C}\right)\left(\mathbf{G}^H\mathbf{F} - \mathbf{C}\right)^H + \rho \cdot \mathbf{F}^H\mathbf{F}\right\} \tag{18}$$

where we have defined $\rho = \sigma_n^2/\sigma_a^2$. Our objective is to determine the matrices $\mathbf{F}$ and $\mathbf{C}$ that minimize the RHS of (18). We begin by keeping $\mathbf{C}$ fixed and setting to zero the gradient of J with respect to $\mathbf{F}$. This produces

$$\mathbf{F} = \mathbf{G}(\mathbf{G}^H\mathbf{G} + \rho\mathbf{I}_K)^{-1}\mathbf{C}. \tag{19}$$

Next, we substitute (19) into (18) and use standard calculations to obtain

$$J = \sigma_n^2 \cdot \text{tr}\left\{\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C}\right\} \tag{20}$$

where

$$\mathbf{R} = \mathbf{G}^H\mathbf{G} + \rho\mathbf{I}_K. \tag{21}$$

TABLE I
MC-CDMA SYSTEM PARAMETERS

Carrier Frequency	5.2 GHz
Transmission Bandwidth	25 MHz
Number of Subcarriers	$N = 64$
Useful MC-CDMA Symbol Duration	3μs
Guard Length	0.44μs
Spreading Length	$Q = 8$
Signal Constellation	16-QAM

In Appendix A it is shown that the unit-diagonal and lower triangular matrix $\mathbf{C}$ minimizing the RHS of (20) is given by

$$\mathbf{C} = \mathbf{LD} \tag{22}$$

where $\mathbf{LL}^H$ is the Cholesky factorization of $\mathbf{R}$ while $\mathbf{D}$ is a $K \times K$ diagonal matrix which scales the elements on the main diagonal of $\mathbf{C}$ to unity, i.e., $\mathbf{D} = \mathrm{diag}\left\{1/[\mathbf{L}]_{k,k}; 1 \leq k \leq K\right\}$.

Substituting (22) into (19) produces

$$\mathbf{F} = \mathbf{G}\left(\mathbf{L}^{-1}\right)^H \mathbf{D} \tag{23}$$

and recalling that $\mathbf{U} = (1/e) \cdot \mathbf{F}$, from (16) we have

$$\mathbf{U} = \sqrt{\frac{K}{tr\left\{\mathbf{F}^H\mathbf{F}\right\}}} \cdot \mathbf{F}. \tag{24}$$

Finally, collecting (20) and (22) yields the minimum of J in the form

$$J_{\min} = \sigma_n^2 \sum_{k=1}^{K} \frac{1}{[\mathbf{L}]_{k,k}^2}. \tag{25}$$

In the sequel, the scheme that employs the matrices $\mathbf{U}$ and $\mathbf{C}$ given in (22) and (24) is referred to as the non-linear transmit Wiener filter (NL-TWF).

It is interesting to note that NL-TWF reduces to the linear TWF (L-TWF) solution discussed in [7] when $\mathbf{C} = \mathbf{I}$ (corresponding to $\mathbf{B} = \mathbf{0}$). In these circumstances there is no TH pre-coding and the minimum MSE reads $J'_{\min} = \sigma_n^2 \cdot \mathrm{tr}\left\{\left(\mathbf{G}^H\mathbf{G} + \mathbf{I}_K\right)\right\}$ or, equivalently,

$$J'_{\min} = \sigma_n^2 \sum_{k=1}^{K} \frac{1}{[\mathbf{L}]_{k,k}^2} + \sigma_n^2 \sum_{k=2}^{K}\sum_{i=1}^{k-1} \left|\left[\mathbf{L}^{-1}\right]_{k,i}\right|^2. \tag{26}$$

Inspection of (26) and (25) indicates that $J'_{\min} \geq J_{\min}$, meaning that L-TWF cannot perform better than NL-TWF.

IV. Simulation results

A. System parameters

Computer simulations have been run to assess the performance of NL-TWF. The main system parameters are summarized in Table I. The underlying channel model is based on the channel A defined in the European BRAN Hiperlan/2 standardization project. We extend this model to the case of multiple transmit antennas by considering P independent channels for each active user. The channels have the same

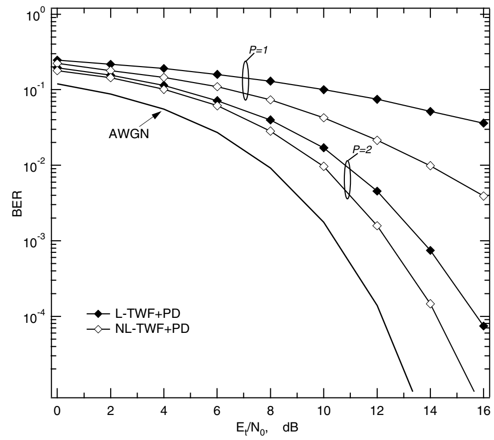

Fig. 4. BER performance vs. E_t/N_0 with $K = 8$ and P=1 or 2 using PD as a detection strategy.

average energy for each user and are kept fixed over the downlink time-slot (slow fading), but vary from slot to slot. Also, perfect channel knowledge is assumed at both the transmitter and receiver ends.

B. Performance assessment

The system performance is assessed in terms of average BER computed over all active users versus E_t/N_0, where E_t is the *transmitted* energy per bit and $N_0/2$ is the two-sided noise power spectral density at the receiver side. The number of active users is $K = 8$ (fully-loaded system) and the transmitted symbols are uncoded. For a fair comparison with a variable number of transmit antennas, E_t/N_0 (in dB) has been scaled by 10log(P) so as to cancel out the corresponding array gain.

Figure 4 compares NL-TWF with L-TWF when only a PD operation is performed at the receiver. The number of transmit antennas is P=1 or 2 and the performance over the AWGN channel is also shown for comparison. Dramatic performance improvements are observed with both schemes in passing from $P = 1$ to $P = 2$. As expected, NL-TWF outperforms linear pre-filtering, even though the gain reduces as the number of transmit antennas increases.

Figure 5 shows the performance of NL-TWF in conjunction with different SUD schemes and $P = 1$. The curve labeled MMSE has been obtained using a conventional MMSE single-user receiver [1] and it is shown for comparison. We see that NL-TWF+EGC gives the best results for E_t/N_0 >10 dB and achieves a gain of approximately 6 dB with respect to the conventional MMSE receiver at an error rate of 10^{-2}. At first sight, the fact that NL-TWF+MRC performs worse than NL-TWF+EGC may appear surprising. A possible explanation is that MRC is effective against thermal noise, but it enhances the MAI. The latter can be mitigated by NL-TWF at the price of some power boosting, which degrades the BER performance.

The results of Figure 6 have been obtained in the same operating conditions of Figure 5, except that two transmit antennas are now employed. In this case NL-TWF+MRC outperforms the other schemes. The reason is that setting

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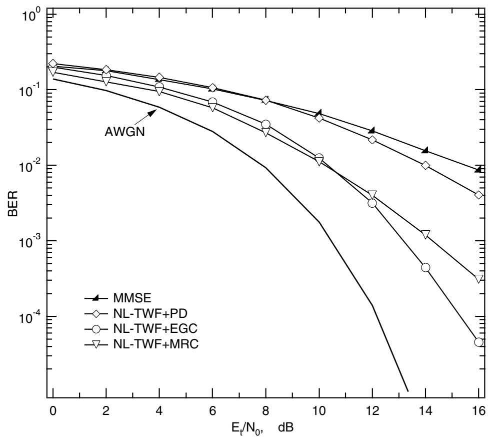

Fig. 5. BER performance vs. E_t/N_0 with $K = 8$ and P=1 using different SUD schemes.

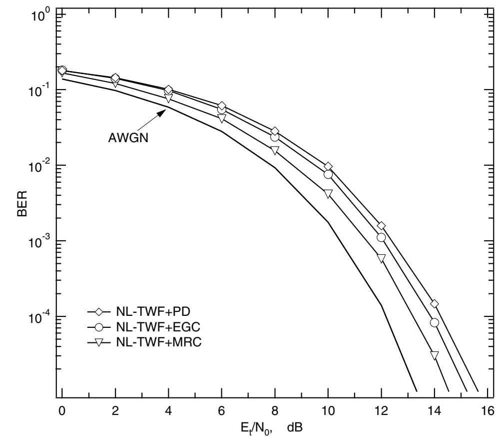

Fig. 6. BER performance vs. E_t/N_0 with $K = 8$ and P=2 using different SUD schemes.

$P = 2$ provides the transmitter with some degrees of freedom that can be effectively exploited by NL-TWF to mitigate the MAI without excessive power boosting. In these circumstances the main impairment at the receive side is represented by thermal noise and MRC becomes the best detection strategy.

V. Conclusions

We have discussed non-linear pre-filtering for TDD MC-CDMA downlink transmissions, where channel state information is obtained at the BS by exploiting the channel reciprocity between uplink and downlink transmissions. The proposed scheme exploits knowledge of the SUD scheme employed at the MTs to minimize the sum of the mean square errors at all MTs. Simulation results indicate that the system performance is significantly improved with respect to other existing solutions based on linear pre-filtering.

Appendix A

In this Appendix we highlight the major steps leading to (22). We begin by considering the Cholesky factorization of the matrix $\mathbf{R}$ in (21), i.e.,

$$\mathbf{R} = \mathbf{L}\mathbf{L}^H \tag{27}$$

where $\mathbf{L}$ is a $K \times K$ lower triangular matrix with real positive elements on the main diagonal. Substituting (27) into (20) produces $J = \sigma_n^2 \cdot \mathrm{tr}\left\{\mathbf{C}^H \left(\mathbf{L}^H\right)^{-1} \mathbf{L}^{-1}\mathbf{C}\right\}$ or, equivalently,

$$J = \sigma_n^2 \cdot \mathrm{tr}\left\{\left(\mathbf{L}^{-1}\mathbf{C}\right)^H \mathbf{L}^{-1}\mathbf{C}\right\}. \tag{28}$$

Next, we observe that $\mathbf{L}^{-1}\mathbf{C}$ is still a lower triangular matrix and, in consequence, equation (28) can be rewritten as

$$J = \sigma_n^2 \sum_{k=1}^{K} \frac{1}{\left|[\mathbf{L}]_{k,k}\right|^2} + \sigma_n^2 \sum_{k=2}^{K} \sum_{j=1}^{k-1} \left|\left[\mathbf{L}^{-1}\mathbf{C}\right]_{k,j}\right|^2 \tag{29}$$

where we have born in mind that $\mathbf{C}$ is unit-diagonal and lower triangular. From (29) it follows that the minimum of J is achieved when $\mathbf{L}^{-1}\mathbf{C}$ is diagonal, i.e.,

$$\mathbf{L}^{-1}\mathbf{C} = \mathbf{D}. \tag{30}$$

Finally, premultiplying both sides of (30) by $\mathbf{L}$ produces the result (22) in the text.

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