SIDELOBE SUPPRESSION IN OFDM SYSTEMS

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Abstract In this contribution, we develop a method for reducing out-of-band emission caused by high sidelobes in OFDM systems. The method is termed multiple-choice sequences (MCS) and operates in the frequency domain of an OFDM system. The principle of MCS is to map the original transmission sequence onto a set of sequences and to choose, from this set, a sequence with the lowest power in sidelobes for the actual transmission. To enable successful signal detection, de-mapping of the received sequence onto the original sequence is required at the receiver. Hence, an index which uniquely identifies the selected sequence is signalled from the transmitter to receiver. From this generalized framework we derive several practical MCS algorithms. Simulation results show that the MCS method achieves a considerable sidelobe suppression which justifies the introduced signalling overhead.

1. Introduction

Orthogonal frequency-division multiplexing (OFDM) systems have gained a lot of popularity lately due to their high spectral efficiency and robustness to multi-path environments. OFDM has been chosen for many standards like ADSL, DAB, DVB, IEEE 802.11a [1]. One of the drawbacks of OFDM is the high out-of-band radiation caused by the high sidelobes of the OFDM transmission signal. The high sidelobes are particularly a critical issue in OFDM based overlay systems in which a broadband OFDM system is overlaid on top of existing narrowband systems [2]. As illustrated in Fig. 1, an overlay system exploits the unused parts of the spectrum assigned to the existing legacy systems, thus increasing the spectral efficiency. As this concept requires successful co-existence between the legacy system and the OFDM based overlay system, a crucial task in designing such an overlay system is the avoidance of interference towards the legacy system. Therefore, the reduction of out-of-band radiation becomes an essential topic, especially for the design of OFDM based overlay systems.

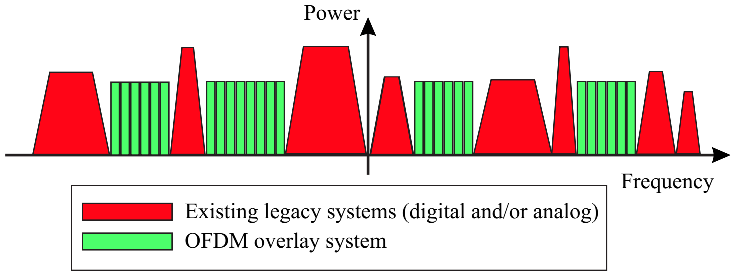

Figure 1. OFDM overlay concept - exploiting the frequency gaps in an existing frequency bandwidth.

The topic of sidelobe suppression in OFDM systems has not been extensively investigated so far. In [3], a multiplication of each OFDM symbol with a windowing function in time domain and insertion of empty guard bands are investigated. In [4] [5], insertion of a few dummy subcarriers at the edges of the used bands which are determined such that the sidelobes of the original OFDM signal are suppressed is presented. In [6], a technique in which the subcarriers are weighted so that the sidelobes of the transmission signal are minimized according to an optimization algorithm is proposed.

In this paper, a different method to significantly suppress the OFDM sidelobes is introduced. This technique, referred to as *multiple-choice sequences (MCS)*, performs mapping of the original transmission sequence onto a set of sequences. From this set, a sequence which offers maximum reduction of out-of-band radiation is chosen for the actual transmission. To enable successful signal detection, de-mapping of the received sequence onto the original sequence is required at the receiver. To this purpose, an index which uniquely identifies the selected sequence in the set of several MCS has to be signalled from the transmitter to receiver. This results in a slightly reduced data throughput. However, numerical results show that this moderate loss in throughput is justified by the significant sidelobe suppression achieved with this technique.

The paper is structured as follows. In Section 2 the signal model is introduced. The principle of MCS method is described and several MCS algorithms are proposed and analyzed in Section 3. The proposed MCS algorithms are compared by numerical simulations in Section 4. Finally, in Section 5 conclusions are drawn.

2. OFDM Signal Model

As illustrated in Fig.1, a real OFDM based overlay system might consist of several continuous transmission sub-bands in-between the legacy systems. The proposed algorithm can be applied to the OFDM transmission signal by considering all the sub-bands jointly or by considering each of the sub-bands separately. As we concentrate on the principle of MCS in this contribution, a simplified problem with a single continuous OFDM transmission band is considered in the following.

An OFDM system with a total number of N subcarriers is considered. The block diagram of the OFDM transmitter is illustrated in Fig. 2. The input bits are symbol-mapped and N complex-valued data symbols $d_n,\ n = 1, 2, \ldots, N$, are generated. These symbols are serial-to-parallel (S/P) converted resulting in an N-element data symbol array $\mathbf{d} = (d_1, d_2, \ldots, d_N)^{\mathrm{T}}$, where $(.)^{\mathrm{T}}$ denotes transposition. The array $\mathbf{d}$ is fed into the MCS sidelobe suppression unit which outputs the selected MCS, denoted with $\bar{\mathbf{d}} = (\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_N)^{\mathrm{T}}$, and the index of the chosen MCS, denoted with Q. The MCS algorithms that determine $\bar{\mathbf{d}}$ and Q are described in the next section. Finally, the selected MCS sequence $\bar{\mathbf{d}}$ is modulated onto the N subcarriers using the inverse discrete Fourier transform (IDFT). After that, parallel-to-serial (P/S) conversion is performed and a guard interval that exceeds the delay spread of the multipath channel is added as cyclic prefix. In addition, the index of the selected MCS sequence Q is coded in bits and transmitted over the corresponding signaling channel. Note that in the following, for simplicity, we assume that the cyclic prefix is considerably shorter than the useful part of an OFDM symbol.

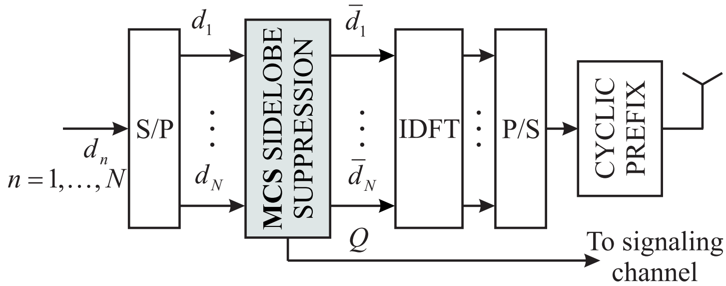

Figure 2. Block diagram of the OFDM transmitter with MCS sidelobe suppression.

3. Sidelobe Suppression by Multiple-Choice Sequences (MCS)

The Principle of MCS

The principle of MCS is illustrated in Fig. 3. A set of sequences $\mathbf{d}^{(p)} = (d_1^{(p)}, d_2^{(p)}, \ldots, d_N^{(p)})^{\mathrm{T}},\ p = 1, 2, \ldots, P$, is produced from the sequence $\mathbf{d}$. For each sequence $\mathbf{d}^{(p)}$ the average sidelobe power, denoted with $A^{(p)},\ p = 1, 2, \ldots, P$, is calculated. To determine $A^{(p)}$, a certain frequency range spanning several OFDM sidelobes, called optimization range, is considered using discrete frequency samples. Recalling that the spectrum of an individual subcarrier equals a si-function $\mathrm{si}(x) = \sin(x)/x$, $A^{(p)}$ is given by

$$A^{(p)} = \frac{1}{K} \sum_{k=1}^{K} \left| \sum_{n=1}^{N} d_n^{(p)} \mathrm{si}\left(\pi(y_k - x_n)\right) \right|^2,\ p=1,2,\ldots,P, \tag{1}$$

where $x_n,\ n = 1, 2, \ldots, N$, are the normalized subcarrier frequencies and $y_k,\ k = 1, 2, \ldots, K$, are normalized frequency samples within the optimization range. The index Q of the sequence with maximum sidelobe suppression is given by

$$Q = \arg\min_{p} A^{(p)},\ p = 1, 2, \ldots, P. \tag{2}$$

Thus, the sequence $\bar{\mathbf{d}}=\mathbf{d}^{(Q)}$ is chosen for transmission and output from the MCS unit.

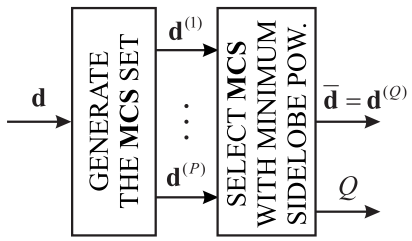

Figure 3. Block diagram of the MCS sidelobe suppression unit.

To enable successful data detection, the received sequence has to be de-mapped onto the original sequence at the receiver. The MCS set is constructed such that the knowledge about the index Q of the selected sequence is sufficient to perform this de-mapping. Thus, the index Q is coded in bits, passed from the MCS unit to the signalling channel, and sent to the receiver. For example, assuming an OFDM system with N subcarriers modulated with M-ary phase-shift-keying (M-PSK) or M-ary quadrature amplitude modulation (M-QAM) symbols, the overhead needed for the signalling information is

$$\lceil \log_2(P) \rceil / \big(\log_2(M) \cdot N + \lceil \log_2(P) \rceil \big), \tag{3}$$

which is negligible for large N and/or M. In (3), $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

At the receiver, an estimate $\tilde{\mathbf{d}}^{(Q)}$ of the transmitted sequence $\mathbf{d}^{(Q)}$ is obtained which is transformed into an estimate $\tilde{\mathbf{d}}$ of the original sequence $\mathbf{d}$ using the signalling information. Note that the signalling information is the index Q which indicates that the sequence $\mathbf{d}^{(Q)}$ out of the MCS set has been chosen for transmission.

In the following several computationally effective, but yet efficient algorithms to generate MCS sets are proposed and analyzed. The proposed methods do not degrade the bit-error rate performance at the receiver and require only a slightly increased signalling overhead.

Symbol Constellation Approach

This algorithm generates the set of MCS such that the elements $d_n^{(p)},\ n = 1, 2, \ldots, N$, of $\mathbf{d}^{(p)}$ belong to the same symbol constellation as the elements of $\mathbf{d}$. With this approach the fact that different symbol sequences have sidelobes with different powers is exploited.

Assume that the symbol constellation consists of M points that are numbered as $0, 1, \ldots, M{-}1$. To each symbol $d_n,\ n = 1, 2, \ldots, N$, an index $i_n \in \{0, 1, \ldots, M-1\}$ is assigned which corresponds to the number of the respective constellation point. Then, the index $i_n^{(p)}$ that corresponds to the MCS symbol $d_n^{(p)},\ n = 1, 2, \ldots, N,\ p = 1, 2, \ldots, P$, is given by

$$i_n^{(p)} = \Big((i_n + r_n^{(p)}) \bmod M \Big),\ n = 1, 2, \ldots, N,\ p = 1, 2, \ldots, P. \tag{4}$$

In (4), $r_n^{(p)}$ is an integer randomly chosen from the set $r_n^{(p)} \in \{0, 1, \ldots, M-1\}$. After determining P index vectors $\mathbf{i}^{(p)} = (i_1^{(p)}, i_2^{(p)}, \ldots, i_N^{(p)})^{\mathrm{T}}$ the MCS vectors $\mathbf{d}^{(p)},\ p = 1, 2, \ldots, P$, are obtained by taking the data symbols from the symbol constellation according to the vectors $\mathbf{i}^{(p)}$. We assume that the same random seed for generating $r_n^{(p)},\ n = 1, 2, \ldots, N,\ p = 1, 2, \ldots, P$, is used at both transmitter and receiver. Hence, the transformation of the received sequence back to the original sequence can be easily performed by exploiting the transmitted signalling information.

Let p_α be the probability that a sequence at the input of the MCS unit has an average power in the optimization range above a certain threshold α. With the symbol constellation approach the P generated MCS sequences belong to the same symbol constellation as the sequence input to the MCS unit. Therefore, the corresponding probability for each of the P generated sequences is also p_α, whereas for the output MCS sequence this probability is

$$\bar{p}_\alpha = (p_\alpha)^P, \tag{5}$$

i.e., the probability is reduced from p_α to $(p_\alpha)^P$, proving the benefits of the proposed approach.

Interleaving Approach

The interleaving approach produces P MCS sequences by permutating the input sequence in a pseudorandom order. As a result, the resulting MCS symbols equal

$$d_n^{(p)} = d_{\Pi_n^{(p)}}, \quad n = 1, 2, \ldots, N, \quad p = 1, 2, \ldots, P, \tag{6}$$

where $\Pi_n^{(p)}$ are permutation indices stored at both transmitter and receiver. The permutation indices $\Pi_n^{(p)}$ take values from the set $\Pi_n^{(p)} \in \{0, 1, \ldots, N-1\}$ such that $\Pi_n^{(p)} \neq \Pi_m^{(p)}$ if $n \neq m$.

Similar to the symbol constellation approach, the MCS symbols $d_n^{(p)}$ produced by the interleaving approach stay in the same symbol constellation as the original symbols d_n. However, unlike the symbol constellation approach, the number of different MCS $\mathbf{d}^{(p)}$ possible with the interleaving approach decreases when the original sequence $\mathbf{d}$ contains reoccurring data symbols. For example, if $\mathbf{d} = (1, 1, \ldots, 1)^{\mathrm{T}}$ the interleaving approach always produces $\mathbf{d}^{(p)} = (1, 1, \ldots, 1)^{\mathrm{T}},\ p = 1, 2, \ldots, P$, irrespective of the selected permutation indices. As a consequence, the probability $\tilde{p}_\alpha$ that an output MCS sequence has an average power in the optimization range above a certain threshold α satisfies the condition

$$(p_\alpha)^P \leq \tilde{p}_\alpha \leq p_\alpha. \tag{7}$$

Note that the equality in (7) is valid only if $P = 1$.

Phase Approach

In this approach the MCS symbols are obtained by applying random phase shifts to the original symbols. Hence, the resulting MCS symbols are formed as

$$d_n^{(p)} = d_n \exp\left(j\varphi_n^{(p)}\right), \quad n = 1, 2, \ldots, N, \quad p = 1, 2, \ldots, P, \tag{8}$$

where the phase shifts $\varphi_n^{(p)}$ lie in the interval $[0, 2\pi)$ and are generated as

$$\varphi_n^{(p)} = 2\pi \left(\frac{\overline{r}_n^{(p)}}{\overline{M}} \right). \tag{9}$$

In (9), $\overline{M}$ is a constant integer and $\overline{r}_n^{(p)}$ is an integer randomly chosen from the set $\overline{r}_n^{(p)} \in \{0, 1, \ldots, \overline{M}-1\}$. Thus, $\varphi_n^{(p)}$ can take one of the $\overline{M}$ discrete phase values. Again, the same random seeds are used at the transmitter and receiver. Note that assuming a BPSK system and $\overline{M} = 2$, this approach becomes equivalent to the corresponding symbol constellation approach.

In the phase approach, the resulting MCS symbols do not necessarily belong to the same symbol constellation as the original symbols. Hence, a property similar to those described in (5) and (7) cannot be easily derived except for some special cases, e.g., $\overline{M} = 2$.

4. Simulation Results

In this section, several numerical results are given that illustrate the effectiveness of the proposed MCS methods.

BPSK modulation is applied and no channel coding is considered. The number of used subcarriers is set to $N = 12$. The optimization range consists of 16 sidelobes at each side of the spectrum and starts from the first sidelobe outside the OFDM transmission bandwidth. Different MCS methods are considered assuming different sizes of the MCS set P.

In Fig. 4, the normalized power spectrum of the OFDM signals averaged over all possible symbol vectors, i.e., 2^N symbol vectors, prior and after the MCS unit are compared. The symbol constellation approach is applied and the size of the MCS set is fixed to $P = 4$. The benefits of the MCS technique are clearly visible. In comparison to OFDM without MCS the sidelobes are suppressed by around 6.1 dB on average. In addition, from (3) it follows that these results are related to a reduction in system throughput of 14% for the chosen system parameters. This signalling overhead reduces if more subcarriers and/or higher modulation schemes are applied.

In Fig. 5, the sidelobe suppression averaged over all possible symbol vectors for different sizes P of the MCS set and different MCS methods is given. To calculate the average sidelobe suppression, standard OFDM without MCS block is taken as a reference. It can be seen that the symbol constellation approach outperforms the other techniques. In particular, the interleaving approach is outperformed as it offers less degrees of freedom in construction of the MCS set than the symbol constellation approach. The performance of the phase approach depends on the number of possible random phases $\overline{M}$. To obtain these simulation results $\overline{M}$ has been set to $\overline{M} = 64$. As already noted, setting $\overline{M} = 2$ would lead to the same sidelobe suppression results as obtainable by the symbol constellation approach. As expected, in all considered MCS approaches,

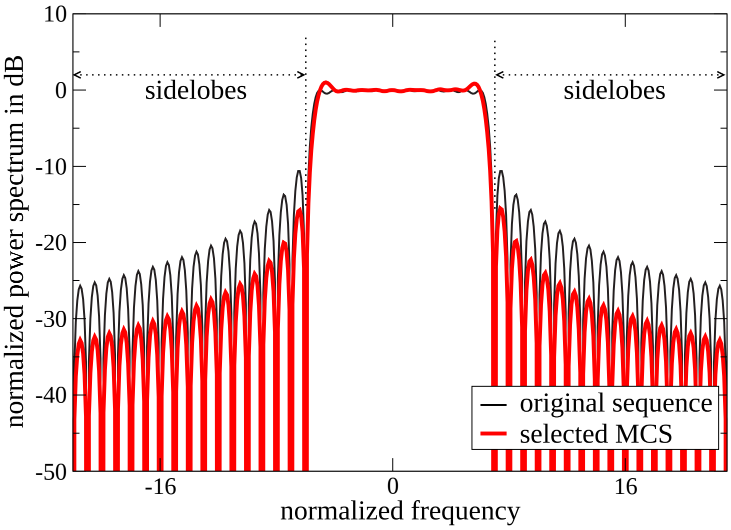

Figure 4. OFDM spectrum of the original transmission sequence and of the transmission sequence after the MCS unit averaged over all possible data sequences; symbol constellation approach; BPSK, $N = 12$, $P = 4$.

an increase in size of the MCS set improves sidelobe suppression, but simultaneously leads to a further increase in signalling overhead. As a consequence, there is a trade-off between the additional sidelobe suppression obtained by enlarging the set size P and the increased signalling overhead. Setting $P = 2$, 4, or 8 seems to be a good compromise. A further increase of P appears to be unjustified as it leads to a relatively high signalling overhead with only moderate further improvement in sidelobe suppression.

The probability that average power in the considered sidelobes of the chosen MCS exceeds the threshold α is presented in Fig. 6. Simulation results are given for $P = 4$ and $P = 16$ assuming different MCS algorithms. As reference, corresponding probability for standard OFDM without MCS block is given. As it can be seen, the symbol constellation approach with $P = 16$ performs better than other considered alternatives. Moreover, for $P = 4$, there is almost no difference in performance between the symbol constellation and interleaving approach, whereas the phase approach performs considerably worse. Again, for the phase approach $\overline{M}$ has been set to $\overline{M} = 64$. Finally, we note that the presented numerical results agree with the analytical results given in (5) and (7).

Note that the MCS technique can be easily combined with other sidelobe suppression methods, e.g., methods from [3]- [6]. However, due to the space limitation of this paper we skip details of such analysis.

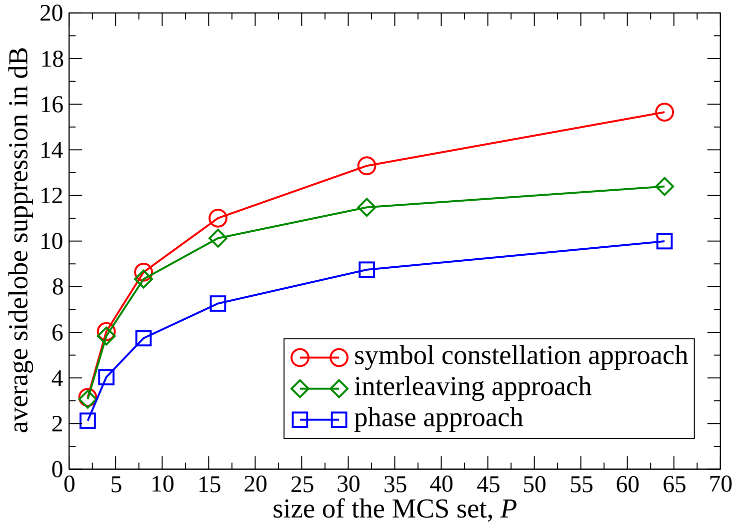

Figure 5. Average sidelobe suppression for different sizes of the MCS set P and for different MCS methods; BPSK, $N = 12$.

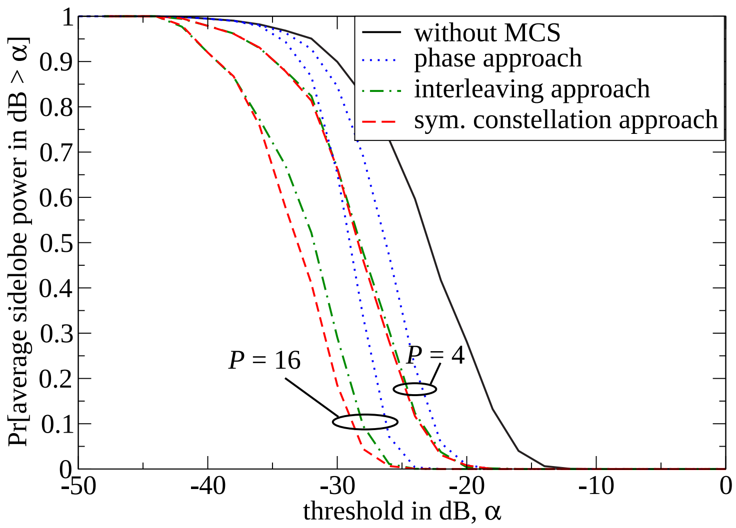

Figure 6. Probability that average power in the considered sidelobes of the chosen MCS exceeds the threshold α; BPSK, $N = 12$.

5. Conclusions

In this paper, we have introduced a new technique, termed multiple-choice sequences (MCS), to suppress sidelobes of OFDM transmission signals. The MCS technique can be used to improve the spectral efficiency of OFDM based transmission systems and/or to reduce interference of OFDM based overlay systems towards the legacy systems sharing the same frequency band. The proposed sidelobe suppression scheme is capable of easily reducing the sidelobes of OFDM transmission signals by several dB. The price to pay for this achievement is a moderate reduction in system throughput, since the transmission of additional signalling information is required.

Acknowledgment

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