

COUPLED BOUNDARY ELEMENT METHOD FOR MODELLING WAVE-SHIP HYDRODYNAMICS – RECENT DEVELOPMENTS AND RESULTS

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ABSTRACT

The provision of fast and accurate estimation of ship motions and resistance in waves represents the main criterion for the continued use of Boundary Element Methods in the maritime industry and research. In addition, it makes them well suited for maritime tools such as decision support and energy management systems. Therein, real-time evaluations of wave loads and power demands are often required, which makes the use of Boundary Element Methods in these tools advantageous compared to viscous CFD solvers. In this paper, we present strategies and results of a hydrodynamic coupled Boundary Element Method prototype designed for applications where accurate real-time predictions are crucial. The wave-ship interaction is modeled as a mixed boundary value problem in which the intersection of water and ship surface is treated explicitly, and spectral-like approaches are considered for fast simulation of wave dynamics. Moreover, the coupled approach ensures the simultaneous evaluation of wave and ship dynamics. The paper highlights the extension of the approach to boundary intersections on elements, presents results obtained for a set of test cases and outlines future development steps.

Keywords: Boundary Element Method, Hydrodynamics

1. INTRODUCTION

The behavior of ships operating in ocean waves is the result of complex dynamic processes interacting with each other. The analysis of these processes is challenging from three key perspectives: first, the nature of ocean waves, varying both over time and a wide range of conditions causing strongly different significant characteristics and wave loads [1, 2], second, the ship representing a technically sophisticated systems with non-trivial geometry and usually non-standardized properties affecting e.g. design [3], structural properties [4] and stability [5], and third, the interaction of waves and ships combining two dynamical systems with

nonlinear characteristics [6–8]. However, the accurate determination of the seakeeping behavior is important for a wide range of tools in ship design and operation. Among other things, the evaluation of designs in planning processes, the decision support during offshore operations, or the monitoring of energy flows requires such quantification of the ship performance in waves. Computer-based methods are used in these fields e.g., to derive seakeeping criteria [9, 10], to compute response amplitude operators [11, 12], or to provide time series of wave loads [13, 14]. Recently, the real-time capability of computational approaches has gained importance as it allows in-situ decision-making and processing which have applications in innovation fields of the maritime sector (e.g., autonomous [15] and emission-reduced ships [16, 17]).

Compared to other established physics-based numerical methods in ship hydrodynamics like Reynolds-Averaged Navier-Stokes Equation solvers, Large Eddy Simulations or Finite Element Methods, the derived mathematical problem in Boundary Element Methods (BEMs) is of lower modelling complexity (due to considering potential flow theory) and dimensionality (solving variables on two-dimensional surfaces for three-dimensional physics). This allows the computation of boundary value problems with large exterior domains and reduces resources and costs. The computational time can be further decreased by applying acceleration techniques suited specifically for BEMs (e.g. precorrected Fast Fourier Transform [18], the Fast Multipole Method [19, 20]). These characteristics qualifies BEM solvers for the aforementioned applications in which very fast solution evaluations are of relevance.

BEMs can be divided in the context of ship hydrodynamics into Free-surface Green function methods [e.g. 21–23], Rankine-source methods accounting for linear and nonlinear dynamics [e.g. 24–27] and more general BEMs for marine hydrodynamics, often using higher-order approximations for solution and geometry, as e.g. given in Refs. [28–30] – see also Refs. [31–33] for an overview. Two typical fields of applications are the eval-

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uation of ship motion in waves [e.g. 25, 34] and the computation of added wave resistance for ships in forward speed [e.g. 35–37]. Recently, the coupling of the High-Order Spectral (HOS) method [38, 39] with the BEM have been considered for nonlinear wave-wave and wave-ship interaction. Shi & Zhu [40] have introduced the SCBEM approach for analyzing nonlinear waves interacting with a fixed structure. The authors use the HOS approach and a Desingularized Rankine-source BEM [e.g. 41, 42] as well as different meshes to incorporate the HOS solution into the BEM framework. In Ref. [43], the method has been extended by automated meshing at the wave-ship intersection and applied to analyze the motions of the KRISO Container Ship in head seas. Seixas de Medeiros et al. [44] have considered a HOS-BEM coupling method in a numerical wave tank to generate focused waves and a soliton. Clearly, the consideration of potential flow limits the approaches: as viscosity, vorticity and compressibility are not considered, the viscous drag resistance, turbulence and vortex-induced drag are not inherently included and can only partly be considered by additional parametric models.

In addition, the determination of the waterline at the ship hull remains a challenge in hydrodynamic time-domain BEM solvers. Both ship hull position and surface elevation change in time, so that the boundary domain and related data have to be updated at every time step. This requires the inclusion of accurate, robust and efficient geometry modeling approaches favorably including spline function methods in the workflow of the used hydrodynamic solver. Moreover, standard singular integration techniques are not inherently feasible for elements at the ship-water intersection. These intersection elements are usually not fully included in the boundary domain. Consequently, the domain of integration changes compared to standard elements, which requires advanced regularization techniques, especially if hypersingular-order integral kernels are involved.

These challenges are addressed in the development of cBEM [33], a time-domain BEM approach that evaluates the coupled dynamics of ship-wave interaction. The techniques included are suited to model nonlinear phenomena and compute the hydrodynamic loads efficiently and accurately. The scope of this paper includes the development of the two-dimensional version of cBEM for linear wave-body dynamics. First, the theoretical foundation is shortly introduced, followed by methodological aspects with a focus on the implementation of routines and adaptation of methods that allow the intersection between elements (IonE case). Then, results of stationary simulations are compared to analytical solutions, and the effect of parameter variation is discussed. Finally, the conclusions are drawn and the future steps presented.

2. THEORY

In this section, basic theoretical aspects for the foundation of BEM methods are outlined.

2.1. Foundation of integral equations for Laplace equation

The reduction of dimensionality of the modelling problem traces back to the divergence theorem

$$\int_{\Omega} \operatorname{div}(\mathbf{f}) d\Omega = \int_{\Gamma} (\mathbf{f} \cdot \mathbf{n}) ds \quad (1)$$

that equates, under the assumption of a solenoidal vector field $\mathbf{f}$, the divergence of $\mathbf{f}$ integrated over a domain Ω with the integral of the flux of $\mathbf{f}$ over the domains' enclosing surface $\Gamma = \partial\Omega$. By considering generalized product rules for integration, and considering the Laplace equation,

$$\Delta\phi = \operatorname{div}(\operatorname{grad}(\phi)) = \nabla \cdot (\nabla\phi) = \sum_i^d \frac{\partial^2 \phi}{\partial x_i^2} = 0 \quad (2)$$

with the velocity potential ϕ , the Nabla-operator $\nabla = \sum_i^d e_i \partial/\partial x_i$ and the d -dimensional coordinates $\mathbf{x} = x_1, \dots, x_d$, the integral equation [45]

$$\phi(x) = \int_{\Gamma} G(x, y) \gamma_1 \phi(y) ds_y - \int_{\Gamma} \gamma_{1,y} G(x, y) \gamma_0 \phi(y) ds_y \quad \text{for } x \in \Omega, \quad (3)$$

allows to find a solution for the Laplace equation. The operators γ_0 and γ_1 , represent the trace and the conormal derivative and are associated to Dirichlet and Neumann data in the close neighborhood of the boundary Γ . The solvability of Eq. 3 requires (aside other assumption, see Refs. [45, 46]) the existence of the fundamental solution $G(x, y)$. For the Laplace equation, G , also referred as Green function, reads in two dimensions

$$G(x, y) = -1/(2\pi) \log(|\mathbf{x} - \mathbf{y}|) \quad \text{for } \Omega \in \mathbb{R}^2. \quad (4)$$

With Eq. (3), the boundary integral equations for the velocity potential and the normal velocity can be derived.

2.2. Boundary integral equations

In the scope of the BEM, boundary values are used to fulfill the PDE (here the Laplace equation) in the domain. To that end, the limit-to-the-boundary approach is considered and the resulting equations for $x \in \Gamma$ are [45]

$$\gamma_0 \phi(x) = (\mathcal{S}\gamma_1 \phi)(x) + (\mathcal{D}\gamma_0 \phi)(x) + [1 - \sigma(x)]\gamma_0 \phi(x) \quad (5)$$

and

$$\gamma_1 \phi(x) = (\mathcal{A}\gamma_1 \phi)(x) + (\mathcal{H}\gamma_0 \phi)(x) + \sigma(x)\gamma_1 \phi(x) \quad (6)$$

with σ depending on the regularity of the boundary, and the Single ($\mathcal{S}$), Double ($\mathcal{D}$), Adjoint ($\mathcal{A}$) and Hypersingular ($\mathcal{H}$) boundary integral operators. These operators are used to define corresponding boundary layer potentials, see Refs. [45, 46], by

$$\begin{aligned} (\mathcal{S}w)(x) &= \int_{\Gamma} G(x, y) w(y) ds_y \\ (\mathcal{D}v)(x) &= -\lim_{\varepsilon \rightarrow 0} \int_{y \in \Gamma: |y-x| \geq \varepsilon} [\gamma_{1,y} G(x, y)] v(y) ds_y \\ (\mathcal{A}w)(x) &= \lim_{\varepsilon \rightarrow 0} \int_{y \in \Gamma: |y-x| \geq \varepsilon} \gamma_{1,x} G(x, y) w(y) ds_y \\ (\mathcal{H}v)(x) &= -\int_{\Gamma} \gamma_{1,x} \gamma_{1,y} G(x, y) [v(y) - v(x)] ds_y \end{aligned} \quad (7)$$

for $x \in \Gamma$ and wherein w and v represent Neumann and Dirichlet data of the variable ϕ , respectively. The boundary layer potentials, Eq. (7), have certain mathematical properties, from which

the linear mapping of variables across function spaces (Sobolev trace spaces that contain w and v) is of importance in BEMs. The singular character of the kernel function G , Eq. (4), and its derivatives, applied in Eqs. (5) and (6), requires special treatment for the evaluation of the boundary integral equations in BEM simulations, see for methods and overview e.g. [47–50] and represents one of the major difficulties in BEM approaches.

2.3. Numerical evaluation

So far, the Eqs. (5) and (6) and variables therein have been considered in continuous space. For the numerical evaluation of an approximate solution, these equations have to be transferred into discrete space. This is done by discretizing the boundary into elements. On these elements, local basis functions are defined to represent given boundary data and enable the determination of the solution locally according to a specified accuracy order. Based on the discretization, the boundary integral operators for the considered problem can be assembled. Converting the boundary integral equation into the unknown variable and applying an appropriate solver for the resulting linear system yields a solution for the variable of interest.

3. METHOD

The specifications and recent developments of the coupled BEM (cBEM) solver are outlined in the following.

3.1. Overview

cBEM, introduced in Ref. [33], is a Galerkin-type BEM for the analysis of wave-ship interaction problems. The coupling of wave and ship dynamics, the consideration of nonlinear phenomena for accurate modeling of the underlying physics and the highly efficient computation are the main targets in the development of cBEM.

By incorporating the HOS approach in the BEM framework through the use of basis functions of compact support for the approximation of the sinus cardinalis function by Z-Splines [51], the efficient and accurate simulation of nonlinear wave dynamics under consideration of wave modulation can be achieved with cBEM. The consistency with the HOS approach requires the consideration of periodic boundary conditions (for open-ocean environment modeling) and, in general, the use of an equidistantly spaced grid for the free-surface domain.

The general model problem in ship hydrodynamics considers the ship floating in water (see e.g. sketch (1) in Fig. 1). For modeling ship hydrodynamics in cBEM, a mixed boundary value problem (MBVP) is used where the Dirichlet problem (Dirichlet data given, Neumann data searched for), considered on the water surface, is combined with the Neumann problem defined on the body surface. By the terms Neumann data and Dirichlet data, we refer to boundary values of a physical variable of interest (Dirichlet boundary condition) or its normal derivative (Neumann boundary condition), respectively.

For analyzing the loads on ships, the knowledge about the submerged part of the floating body is of relevance, as the submerged ship volume is used for the calculation of the buoyancy-driven restoring force and the area on the ship hull that is in contact with water, defines the integration domain of the pressure

to calculate the hydrodynamic forces. Hence, the approximation quality of the submerged hull section allows for increasing the accuracy for the prediction of loads on a ship in waves, especially in nonlinear hydrodynamic codes. To realize a high approximation quality in cBEM, the waterline at the ship hull, which marks the intersection of the floating body with the free surface and changes with time, is tracked and used to adapt the computational domain of the MBVP (change of geometry and boundary integral operators). For the integration of singular kernel functions, the direct method for two-dimensional Galerkin BEMs provided by Bonnet & Guiggiani [49] is used in cBEM. The computation of the hypersingular kernel in the self and adjacent regimes, detailed by the authors, is applied.

Currently, the solver is limited to two-dimensional physical space. The wave dynamics, governed by the definition of the boundary conditions at the free surface, are linearized. For the farfield solution, no acceleration technique is considered. In the scope of this paper, the body is assumed to be fixed, and the solution evaluation is performed at one time step. The basis functions have intra-element support, i.e. the support of the basis functions is restricted to one element.

3.2. Recent developments

The extension of the numerical routines to consider the intersection point to be somewhere on the intersection element (IonE-case) and the evaluation of the complete boundary domains are features recently implemented and detailed in this subsection.

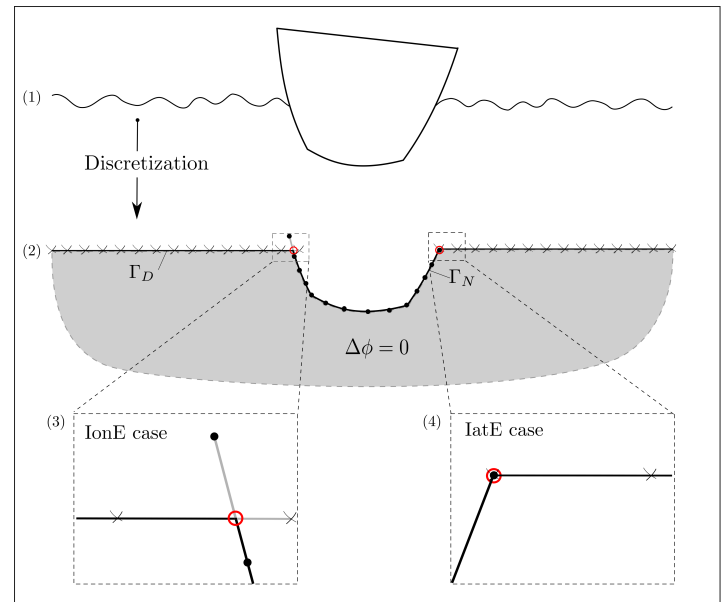


FIGURE 1: Sketch of the two-dimensional model problem showing a mid-section of a ship floating in waves (1) and its discretization into boundary elements (2). The element endpoints are represented by black crosses and dots for the free-surface boundary (Dirichlet data given, Γ_D) and the body boundary (Neumann data given, Γ_N), respectively; the intersection points are highlighted by red circles. A part of the water domain is presented and highlighted as gray area. Details at the intersection elements of ship and free surface are shown for the IonE (3) and latE (4) cases. The light gray parts of the intersection elements, shown in (3), are not included in the computational domain.

3.2.1. IonE-case As outlined above, the intersection between the ship and the water surface is explicitly considered in cBEM. The waterline reduces in two dimensions to two points, hereafter referred to as intersection points, see subfigure (2) in Fig. 1. In the development stages of the solver, different locations of the intersection point can be considered, shown in sketches (3) and (4) of Fig. 1. The extension of the two-dimensional cBEM solver for computing the IonE case is detailed in the following.

The basic idea presented here in the context of the IonE case is the treatment of intersection points as discontinuous element endpoints. This implies that elements of the boundary intersections are divided into two individual elements. From the two new elements, only one element falls in the region of the MBVP and is considered for computation. As a consequence of the approach, the geometries have to be adapted due to the insertion of new elements, which makes the intersection procedure less flexible than, e.g., when only the interval of the parametric coordinate would be changed on the intersection element. The data at the free surface, usually considered as equidistantly distributed, have to be evaluated with basis functions at the intersection points. The main advantage of this procedure is that the intersection elements are considered as complete so that the local support of the basis functions remains and the parametric coordinates still range over the interval $[0, 1]$.

The mentioned adaptation of geometry includes the definition of new elements and their inclusion in the geometry data structure. The size and coordinates of the new elements are computed using previously developed routines and geometry basis functions, respectively, and are written to the element-specific geometry data. This requires the extension of the point vector and the element-to-point map so that the new coordinates can be included and linked to the correct element ID. In the assembly of the boundary integral operators for Eqs. (5) and (6), the treatment of the singular integrals, specifically of the hypersingular kernel, is of relevance. For this, the direct approach of Bonnet & Guiggiani [49] is used, and the details of the adjacent routine at intersection elements are shown for the IonE and the IatE case in Fig. 2. The algorithm for the adjacent case of the hypersingular kernel contains two kernel evaluations that differ in the domain of integration: one includes the direct neighborhood of the common endpoint of the adjacent elements (singular point), and one considers a larger distance to the singular point. Numerically, the distinction between singular and non-singular computation is realized by switching the direction of the parametric coordinates.

3.2.2. MBVP-boundary The boundary accounted for the model problem considered in cBEM is composed of the submerged part of the body and of the surrounding free surface. For the subsequent processing, the boundary can be treated either as individual boundary parts of free surface and body or as a combined boundary, referred to as MBVP-boundary. The chosen treatment defines the region of validity of the boundary integral operators and changes the assembly operation. Using the MBVP-boundary, the operators are tailored directly to the model problem and the assembly loop is the same for all operators. When using the body and free-surface parts individually, the assembly routines differ for each boundary integral operator be-

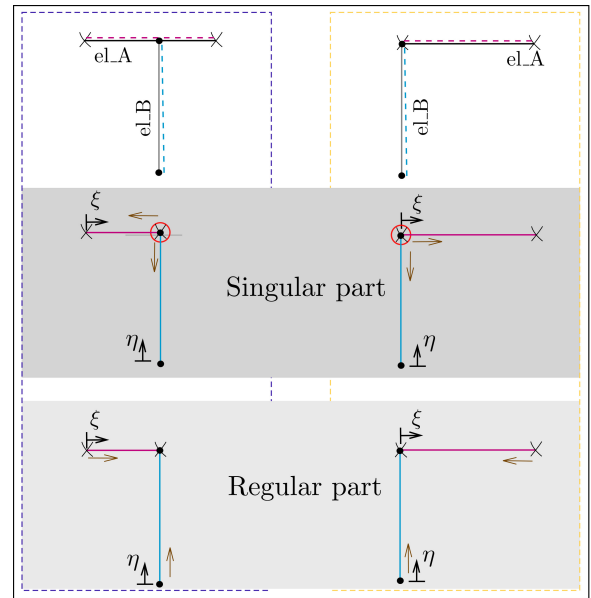


FIGURE 2: Schematic illustration of boundary elements at intersection location. Details for hypersingular adjacent case integration for IonE case (left panel) and IatE case. Parametric coordinates η and ξ for the regular and singular integration parts, see Ref. [49], are shown with coordinate directions indicated by brown arrows. The singular point locations are highlighted by red circles.

cause the operators represent the influence of different boundary part combinations.

The MBVP-boundary has recently been included in cBEM by adapting the way geometry data are stored. In general, geometry data contain the geometrical information (e.g., point position vectors, element-to-point maps) related to a certain discretization. They are stored in data containers that can be accessed by any function in cBEM.

For the setup of the MBVP-boundary, the parts of the water and body domain need to be assembled, the intersection performed, and the parts of the domains combined again. The last step is newly implemented in cBEM and requires a boundary sewing approach. The sewing function creates a new geometry data structure from given boundary parts (valid boundary domains, found after intersection). The new data structure is essentially a redefinition/reordering of the point vector and the element-to-point map. With the MBVP-geometry, the processing in cBEM can be continued: the integration points of the MBVP-domain can be set, and boundary integral operators can be computed. The resulting matrices represent the influence of all elements within the complete MBVP-domain. As the individual matrices (representing body-to-water, water-to-body, body-to-body and water-to-water influences of cBEM operators) are needed for the block matrix system, the operator-specific submatrices have to be selected from the MBVP-domain matrix. Therefore, assignment lists that assign MBVP-elements to either the water or body boundary domain are employed.

4. RESULTS AND DISCUSSION

Throughout this section, we investigate the error of numerical solutions with respect to analytical reference solutions. The

error behavior with increasing number of boundary elements (h-refinement) or increasing polynomial degree of the basis function (p-refinement) is of special interest as it allows to calculate the order of convergence. In general, the order of convergence and the order of basis functions should coincide, which allows to verify the implementation of the numerical method. After detailing the testing procedure and error norm, test cases are described, and results are shown and discussed.

Analytic solution. The testing of the numerical solution is done by considering the equations of linear wave theory (also known as Airy wave theory), see e.g. Refs. [33, 52]. The theory states that under the assumption of low wave steepness, the wave dynamics can be estimated by considering linearized equations [11]. The analytical equations in deep water of the velocity potential and its derivative in normal direction, ϕ and $\frac{\partial \phi}{\partial n}$, are given by

$$\phi(\mathbf{x}; t) = \frac{\hat{\eta}g}{\omega} \exp^{i(kx - \omega t)} \exp^{kz} = \hat{\eta}\Psi_{k,\omega}(\mathbf{x}, t) \quad (8)$$

$$\frac{\partial \phi}{\partial n}(\mathbf{x}; t) = \left(\left(\text{Re}(ik\hat{\eta}\Psi_{k,\omega}(\mathbf{x}, t)), \text{Re}(k\hat{\eta}\Psi_{k,\omega}(\mathbf{x}, t)) \right)^T, \mathbf{n} \right)$$

with the imaginary unit i , the normal vector $\mathbf{n}$, the two dimensional coordinate vector $\mathbf{x} = (x, z)^T$, the wave amplitude $\hat{\eta}$, the gravitational acceleration g , the wave number k , and the angular wave frequency ω , see e.g. Ref. [2]. The equation is used to derive the analytic values of both the reference solution and the boundary conditions at the linearized free surface ($z = 0$) and at the body surface at time step $t = 0$.

Error norm and estimated order of convergence. In the scope of this paper, the error of the numerical solution vector $\mathbf{x}$ with respect to the analytic reference solution vector $\mathbf{x}_{\text{ref}}$ is analyzed by the weighted relative error based on the L_2 -norm

$$\int (E_{L_2}) d\Gamma = E = \sqrt{\frac{\sum_i (|x_i - x_{i,\text{ref}}|)^2 w_i J_i}{\sum_i |x_{i,\text{ref}}|^2 w_i J_i}}. \quad (9)$$

Herein, the Jacobian J and the quadrature weights w of the particular element i are considered.

For the evaluation of the estimated order of convergence (eoc), the single convergence rates between two neighboring error-discretization pairs are evaluated by means of

$$eoc_i = -\frac{\log(E_i) - \log(E_{i-1})}{\log(nP_i) - \log(nP_{i-1})} \quad (10)$$

with the list index i , number of points nP , and the error E . To derive the convergence order of a considered parameter setting, the median is applied to the data set eoc_i . The median is chosen here as it is considered to be more robust to statistical outliers compared to the mean value.

The parameter study was conducted using automatized simulations, involving the modification of parameters and storing of data containers in folders created for the specific simulation test cases.

4.1. IonE box test case

The IonE box test case has been used to develop the routines for handling the IonE case in cBEM. Below, details to the test case setup and results of parameter variations are presented.

Description of test case. The considered MBVP-geometry for the test case is formed from two initially different geometry parts, namely a box opened upwards, and a top part. The box geometry corresponds to a semi-submerged square with an edge length of 2.3 m. The geometry parts are combined to the MBVP-geometry in such a way that the top part is cutted-off near the vertical bounds of the open box and a closed interior computational domain is formed. Thus, only the elements of the top part that are at least partly inside the boundary region limited by the vertical box edges remain, see also top left panel of Fig. 3(a). The intersection points of top and open box parts are considered to be somewhere on the intersection elements and defined as new element endpoints (IonE case).

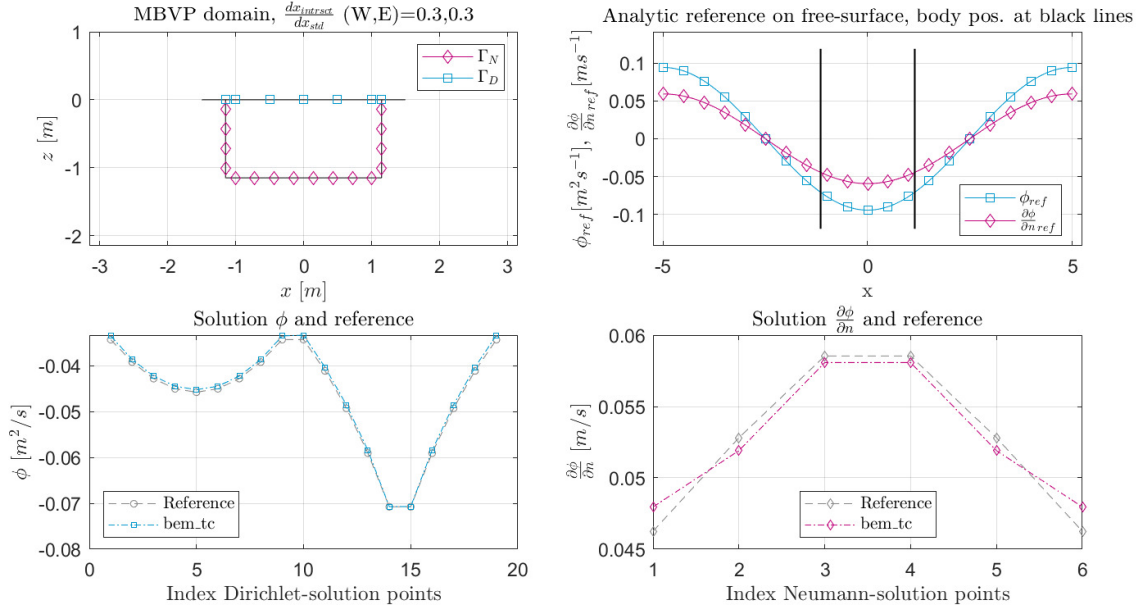
The procedure of tailoring the top part and creating the MBVP-geometry (referred to as sewing), is repeated when the discretization changes, e.g. if the number of boundary elements changes. The MBVP is defined by considering the top of the MBVP-geometry as Dirichlet problem and the rest as Neumann problem, representing the water boundary (i.e. the free surface) and body boundary, respectively. The normal vectors of the boundary parts point inside the closed domain. The relevant simulation parameters for the conducted test cases are given in Table 1 and the considered boundary conditions and reference solutions on Neumann and Dirichlet domains are derived from the analytic solution given in Eq. (8).

Variation of basis function order. The results of the test case for varying basis function order are shown in Figs. 3(b) and (c). The setup of the test case is presented in Fig. 3(a) where the box with the tailored top part and the corresponding boundary conditions, Dirichlet data in blue and Neumann data in red, are highlighted.

The results for the eoc, Figs. 3(b) and (c), are in the expected range for the chosen setup of basis functions and quadrature rules. The variations of convergence order, given at higher number of solution points, can be explained by an error increase for very small sizes of the intersection elements. Usually, the error fluctuation is within a small range, but for the highest chosen basis function, a more extreme case is shown at the second highest grid resolution. As the error in this case is very small, it is regarded as acceptable. The merging of the intersection and its adjacent element to increase the element size might be considered in critical cases, although the error due to the lower grid resolution might be similar.

Variation of wave length. Figure 4 presents the analytic solutions and the error-convergence plots for different wave lengths λ . With the wave length, the wave number $k = 2\pi/\lambda$ changes and new analytic reference solutions are obtained.

In Figs. 4(b) and (c) it can be seen that the eoc is changing very little over the different wave lengths. But the error level for the number of solution points changes. This can be explained by the approximation properties of the basis functions with respect to the analytic reference. As for decreasing wave length, the curvature of the analytic solution over one element increases, the approximation error can be expected to increase to a certain amount. In the range of the considered wave lengths and grid resolutions, the error is in an acceptable range. Hence, the con-



(a) Test case setup (top left) and analytic reference and boundary condition at water boundary for wave length, $\lambda = 10$ m (top right). A visual comparison of the solution and reference is presented for Dirichlet values (bottom left) and Neumann values for the discretization $nW_{total} = 20$ and $nB/edge = 8$.

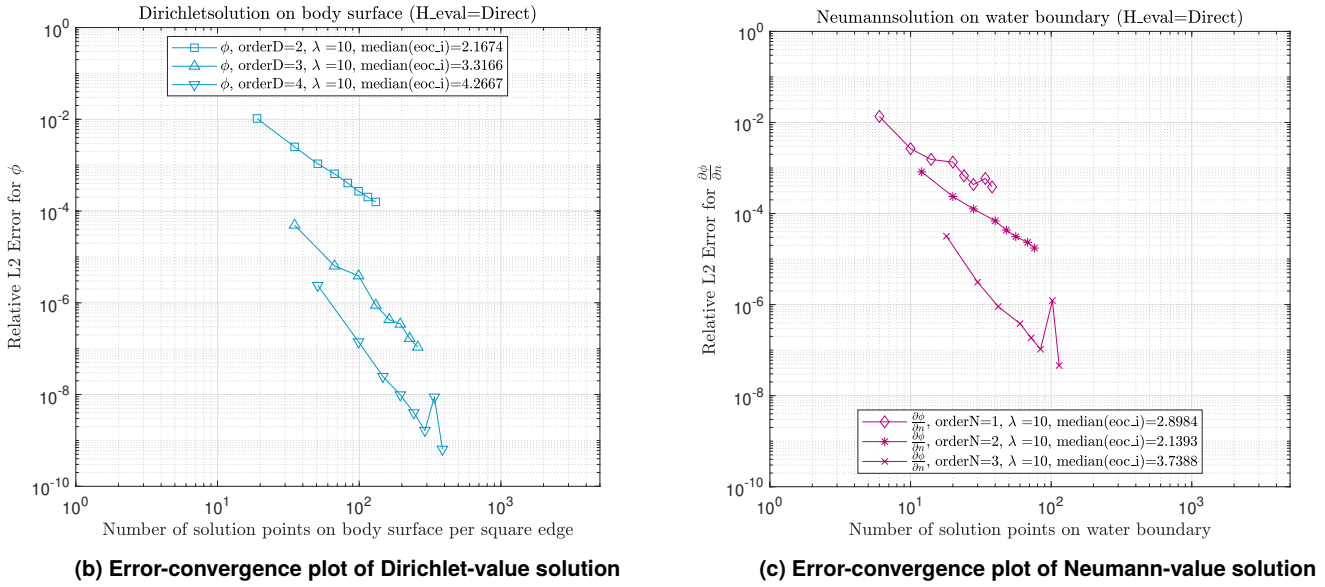
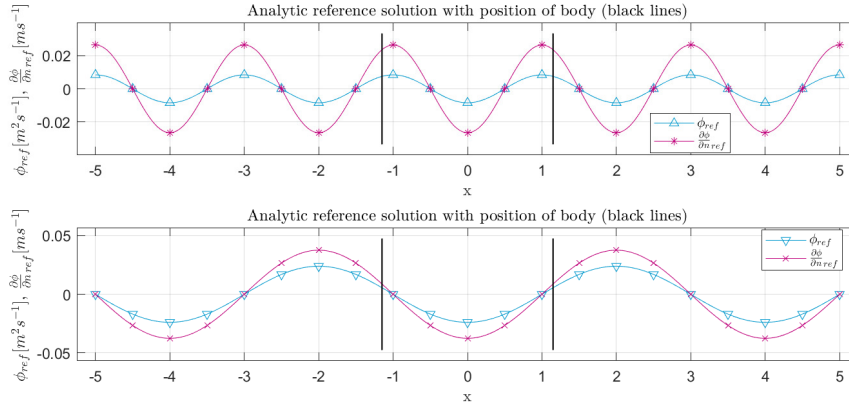


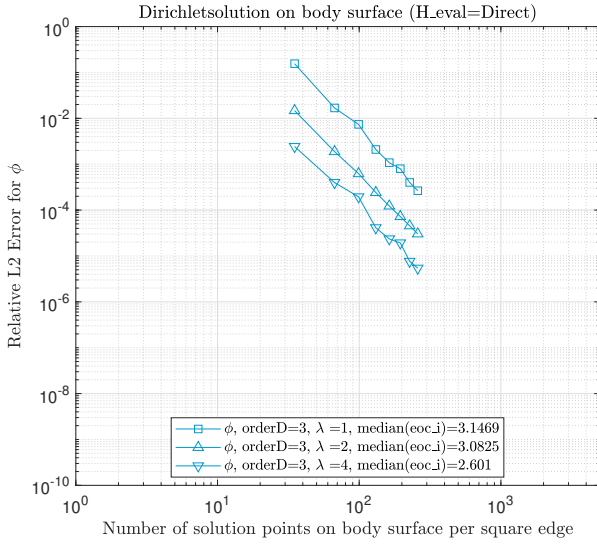
FIGURE 3: Setup and results of the lonE box test case for different orders of basis functions.

TABLE 1: Overview on control parameters for simulation of the lonE box test case. OrderD and orderN refer to the expected order of convergence for Dirichlet and Neumann data, based on the chosen configuration of basis functions and quadrature rules. Abbreviations used refer to GL=Gauss-Lobatto quadrature, NC = closed Newton Cotes quadrature, cont.= continuous- and disc.= discontinuous end-point bindings, and Nim= number of images.

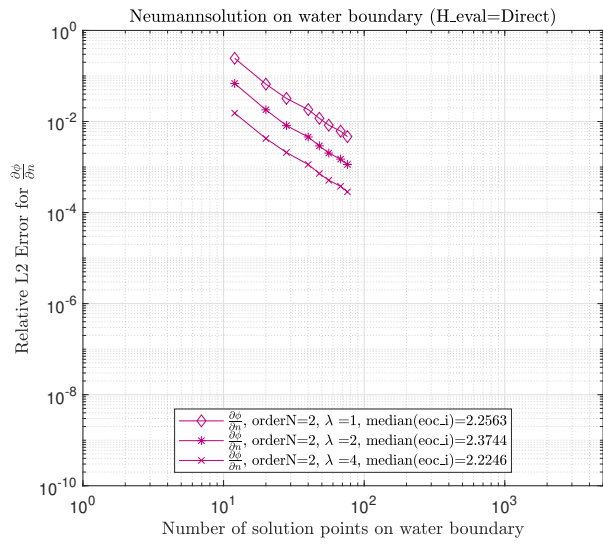
Parameter	wave length [m]	orderD	typeQuadD	orderN	typeQuadN	near/adj., self-order	Nim
VarOrder	10	2,3,4	NC, conti.	1,2,3	GL, disc.	20,20	-
VarWaveL	1,2,4	3	NC, conti.	2	GL, disc.	20,20	-
cBEM tc	10	2	NC, conti.	1	GL, disc.	20,20	10



(a) Analytic reference and boundary condition at water boundary for wave lengths $\lambda = 2$ m (top) and $\lambda = 4$ m.



(b) Error-convergence plot of Dirichlet-value solution



(c) Error-convergence plot of Neumann-value solution

FIGURE 4: Setup and results of the IonE box test case for different wave lengths.

sidered order of basis functions can be seen to approximate the solution function well in that parameter setup. Nevertheless, this analysis implies that for smaller wave lengths the grid resolution should be increased to avoid unacceptable errors, specifically for lower order basis functions.

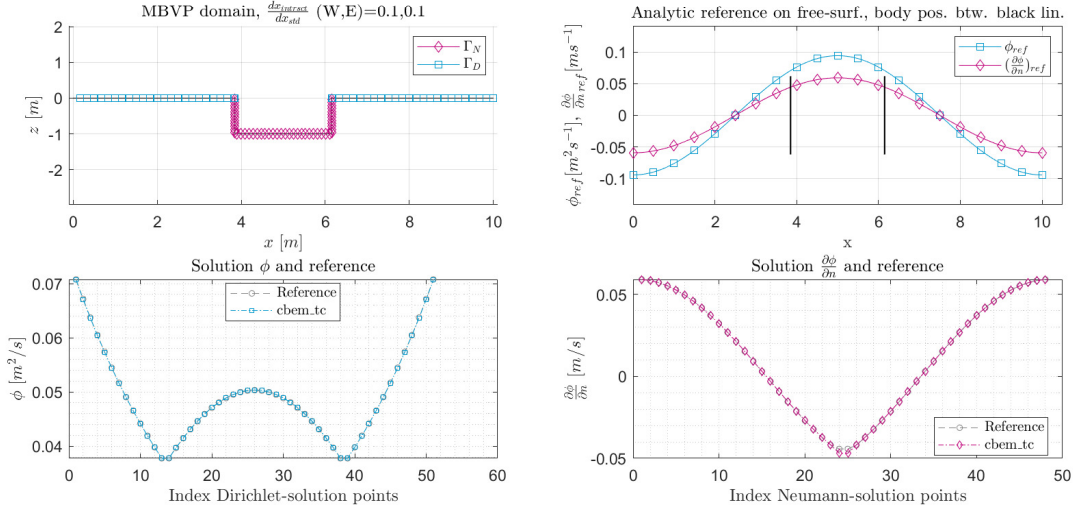
4.2. cBEM test case

Based on the results for the IonE box test case, the described methods are implemented in the cBEM framework and related test case results are presented hereafter.

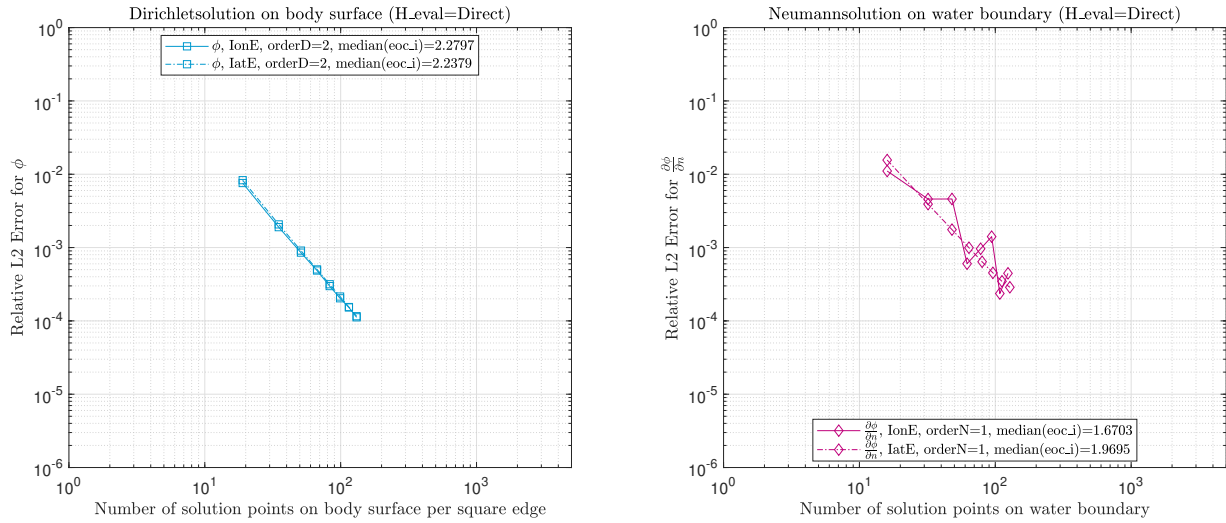
Description of test case. In contrast to the IonE box test case, the parts of the water boundary outside the box are retained, while the parts inside are omitted for the construction of the MBVP-geometry shown in the top left part of Fig. 5(a). The Laplace equation is considered below the free surface and outside the body, physically describing the water domain. The normal vectors of the free surface direct in negative z -direction and the body normal vectors point outside. For the approximation of the periodic boundary conditions at the left and right borders of the free-surface boundary, the farfield domain is extended by ten

domain images at both ends according to the Method of Farfield Extension described in Ref. [33]. For the remaining settings, the configuration of the IonE box case is considered and Table 1 details the simulation parameters.

Convergence test. For piecewise linear basis functions on the Dirichlet boundary and constant basis functions on the Neumann boundary, the errors for varying grid spacing are shown in Fig. 5(b) for the IonE and IatE cases. As in the IonE box test case, the error for the Neumann solution in the IonE case varies from the linear convergence trend about an acceptable range due to the different sizes of the intersection elements. The influence of the intersection element size can be demonstrated by the results of the IatE case in a very similar setup (body width 2 m) in which the intersections lie strictly at the element endpoints and all elements have the same size for each grid spacing. Here, the errors of the Neumann solutions do not contain any fluctuations and the convergence follows the linear trend. The observation that the convergence rate for the constant basis functions setting is higher than the expected order of $O(h)$, also visible in Fig. 3(c), might be explained by the use of the L_2 -norm for the error analysis



(a) cBEM test case setup (top left) and analytic reference and boundary condition at water boundary (top right) for the IonE case. A comparison of the solution and reference is presented for Dirichlet values (bottom left) and Neumann values for the discretization $nW_{total} = 60$ and $nB/edge = 24$.



(b) Error-convergence plot of Dirichlet- and Neumann-value solutions for the IonE (solid) and latE (body width=2 m, dash-dotted) case.

FIGURE 5: Setup and results of the cBEM test case for IonE and latE case for expected orders of convergence orderD=2 and orderN=1.

and by the smoothing of the solution by the boundary integral operators, a phenomenon known for constant basis functions, see e.g. Ref. [45].

5. CONCLUSIONS AND FUTURE OUTLOOK

In this article, we outlined the solution evaluation in cBEM for the IonE case by including a new element endpoint at the location of the boundary intersection. The validity of the approach has been tested by using the IonE box test case. The results indicated convergence rates that are in line with the order of the basis functions used. Additionally, the direct use of the MBVP-boundary as geometry for the cBEM computations was explained, and the advantage in terms of operator assembly was outlined.

Future work includes the validation of two-dimensional

wave-body interaction in the time domain and the method expansion to a three-dimensional model problem. For testing time-domain hydrodynamics, freely-floating bodies are considered. This combines time integration of wave and body variables, the creation of the MBVP-geometry, and the reassembly of the boundary integral operators. The extension of cBEM to three dimensions in space involves the consideration of an additional spatial dimension for geometries, boundary integral operators, and singular integration techniques. Of specific numerical interest are the accurate and efficient evaluation of intersection curves of free-surface and ship geometry and the integration over incomplete boundary elements, including desingularization. Based on the three-dimensional cBEM implementation, the implementation in a higher-programming language framework, and the inclusion of both nonlinear approaches and acceleration techniques will be

addressed.

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