Unobservables and Decoherence from Complexity

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With finite resources, time, precision, many measurements are out of reach. Can we give a rigorous argument? What are the consequences?

DLR

1 Large Sandwich-POVMs are Unobservable

Consider the propositional formula

$$F = \bigwedge_{m=1}^{M} \underbrace{\bigvee_{j=1}^{3} \pm v_{i_{mj}}}_{C_m} .$$

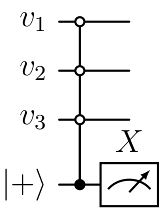
Associate a qubit with every variable v_1, v_2, ..., v_N. The following *valid* measurements would allow us to decide if F is satisfiable (NP-complete):

$$\mathcal{M}(C_m) = \{\Pi(C_m),\ \Pi(\neg C_m)\},$$

$$P_x(F) = \prod_{m=1}^{M} \Pi((-1)^{x_m} C_m),$$

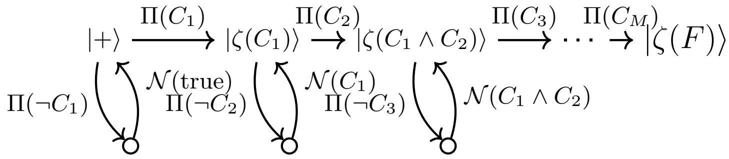
$$Q_y = \bigotimes_{i=1}^{N} \frac{\mathbb{1} + (-1)^{y_i} X}{2},$$

and $\mathcal{N}(F) = \{P_x(F) Q_y P_x(F)\}$.



The circuit implementation of $\mathcal{M}(v_1 \vee v_2 \vee v_3)$ illustrates the local nature of the measurement.

2 Algorithm



Idea: Measure until state satisfies all clauses. Undo unwanted outcomes.

Success probability of each step $> \frac{1}{2}$ is ensured by dilution $\mathcal{D}(F)$: Replace formula by equivalent larger formula, see paper[1].

1. Initialize $N + M$ qubits in $|+\rangle^{\otimes(N+M)}$.
2. Set $e =$ true.
3. For every clause C_i' in $\mathcal{D}(F)$:
   (a) Measure $\mathcal{M}(C_i')$.
   (b) If the outcome is $\Pi(C_i')$, then set $e = e \wedge C_i'$ and continue with next clause.
   (c) Else undo with $\mathcal{N}(e)$.
   (d) Try R times to get $\Pi(C_i')$.
   (e) Return "UNSAT".
4. Return "SAT".

With NP$\not\subseteq$BQP as axiom[2], $\mathcal{N}(F)$ is unobservable.

3 Consequences

3.1 Unobservable Paulis

- Naimark dilation of $\mathcal{N}(F)$:
- The isometry $V = \sum_{x,y} |x, y\rangle \langle y| H^{\otimes N} P_x(f)$ fulfills $V^\dagger |x, y\rangle\langle x, y| V = P_x(f) Q_y P_x(f)$.
- Extend V to a unitary U.
- POVM is U followed by Z-measurement.
- As Hermitian operator $\tilde{Z} = U^\dagger Z U$.
- Similarly, other rotated Pauli-operators are unobservable.

3.2 Unphysical Time Evolution

- Let $\mathcal{T}$ be the set of physical time evolutions.
- As $\tilde{Z}$ is unobservable and Z is observable, $U \notin \mathcal{T}$.
- The states $\rho = \sum_{z=0} p_z |z\rangle\langle z|$ and $\tilde{\rho} = U \rho U^\dagger$ with $p_i \neq p_j$ are not connected by any $U \in \mathcal{T}$.

3.3 Unobservable Coherence

- Let $|\psi_1\rangle$, $|\psi_2\rangle$ s.t. $|\psi_2\rangle \notin \mathcal{T}(|\psi_1\rangle)$.
- Any observable A has vanishing $|\langle\psi_1| A |\psi_2\rangle| \approx 0$.
- With $|+\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$, $\rho_{\text{coh}} = |+\rangle\langle+|$ and $\rho_{\text{incoh}} = \frac{1}{2}(|\psi_1\rangle \langle\psi_2| + |\psi_2\rangle \langle\psi_2|)$ are indistinguishable, as $\mathrm{tr}(A\rho_{\text{coh}}) - \mathrm{tr}(A\rho_{\text{incoh}})$ $=2\mathrm{Re}(\langle\psi_1| A |\psi_2\rangle) \approx 0$.

Classicality may partly arise because quantum phenomena are too complex to observe.

[1] M. Epping and J. Szangolies, arXiv:2606.20927
[2] S. Aaronson, "Guest Column: NP-complete problems and physical reality", ACM Sigact News 36, 30 (2005)