

Research paper

Numerical approximation of the flexural properties for additively manufactured sandwich panels with a sheet-based TPMS core

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ABSTRACT

Triply Periodic Minimal Surface (TPMS) structures as core structure enable multifunctional sandwich panels with nearly isotropic mechanical properties and possible media transport due to their channel-like topology. A new approach to extend laminar flow on aircraft wings for a more sustainable and energy-efficient aviation uses such sandwich structures as suction panels. Usually exposed to wing bending, evaluating the feasibility of suction panels requires a method for the numerical approximation of the bending behavior of TPMS-based sandwich panels. This study uses a novel numerical bitmap-based approach to compute the flexural properties based on the second moment of inertia and two common theoretical approaches from existing studies. To validate the theoretical approaches, this study compares them to experimental determined flexural properties, obtained through four-point bending tests on additively manufactured TPMS-based sandwich panels. The results show great agreement of the numerical bitmap-based flexural properties matching the experimental values with high accuracy, outperforming the two more common approaches at high relative densities. With the validation being successful, the bitmap-based approach enables the numerical estimation of the flexural properties by determining the second moment of area directly from the geometry of arbitrarily structured sandwich panels, eliminating the need for prior experimental tests or complex simulations.

1. Introduction

Sandwich panels primarily find application in aerospace engineering and other lightweight use cases, due to their high stiffness-to-weight ratio [1]. Especially exposed to bending loads, their high second moment of area can be exploited [2]. For the core of the sandwich panel, the most commonly used structures include balsa wood, Honeycombs, and foams, which are typically referred to as cellular structures. Honeycombs represent a two-dimensional cellular structure with a closed-cell morphology that can be repeated periodically. Other cellular structures, such as foams, can occur with open cells and a stochastic pore distribution in all three dimensions [3]. Triply periodic minimal surface (TPMS)-based structures represent another group of cellular structures offering the additional benefit of functional integration. A recently shown example for the integration of additional functions is the concept of a TPMS core for the design of extended hybrid laminar flow control (xHLFC) suction panels [4,5]. Here, the porous upper facesheet of the sandwich panel and porous TPMS core allow the transport of air that is sucked

in for laminarisation for decreased fuel consumption in transport airplanes [4].

These xHLFC sandwich panels not only facilitate air suction but also serve a structural function, bearing aerodynamic loads as well as global bending and torsional loads imposed by the aircraft wing. Therefore, a comprehensive understanding for modeling the bending behavior is essential for further investigation of suction panels in the aviation industry and also for design purposes. The objective of this study is to approximate the bending behavior using a numerical approach from Lohuis et al. [6]. To validate the numerical approach, the bending behavior of TPMS-based sandwich panels is experimentally characterized and compared to the predictions from the numerical and additional semi-empirical models found in current research.

Minimal surfaces and their properties have been investigated by mathematicians for more than 150 years [7]. They include all surfaces that locally minimize the area by having a zero mean curvature. The shapes taken by soap films are minimal surfaces, where the films connect each other by the drive to minimise the potential energy of the

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resulting connection [8]. Some minimal surfaces allow the formation of a unit cell with the unit cell length l_{uc} , which can be continuously repeated in the three spatial directions [9], so-called triply periodic minimal surfaces (TPMS) [10]. One unique characteristic of TPMS is their single, continuously bent surface, separating space into two independent channel systems and allowing a fluid or gas to travel through the structure [11]. This channel-like system is suitable for fluid transport, such as air suction in the xHLFC application [4]. Next to this, TPMS structures find broad interest in heat exchanging research due to the mentioned separation into two domains [12,13] and their high wetted surface to fluid volume ratio. Typical TPMS are the Schwarz Primitive surface, described by H.A. Schwarz [14] in 1856, the Neovius surface [7] or the Gyroid surface, discovered by Alan H. Schoen [15]. In this study, the Gyroid will be considered in more detail as it shows suitable properties, both mechanical and aerodynamic, in current investigations on additively manufactured suction panels [4].

The TPMS itself describes a surface of zero thickness. A solid, volumetric cellular structure can be obtained from a TPMS by using two strategies. The first strategy uses the channel-like structure of the TPMS by defining one of the channels as solid. The volumetric body is then called a TPMS solid network. The second strategy includes thickening the surface itself to obtain a solid body, which is called a TPMS sheet network [11]. Yanéz et al. [16] and Al-Ketan and Rowshan [9] were able to prove experimentally and by simulations that TPMS sheet networks have both higher mechanical properties and better fatigue behavior compared to the TPMS solid networks, which is the reason why this study concentrates on the bending behavior of TPMS sheet networks.

The key parameter to characterize a cellular structure is its relative density RD describing the ratio of the cellular structure's density ρ^* and the solid's density ρ_s [3]. Gibson and Ashby initially described a relationship between the relative density of cellular solids and their tensile and compressive properties [3] in the form of scaling power laws. Based on this, several authors have transferred and adapted this relationship to TPMS structures. An overview of studies investigating these scaling power laws experimentally and numerically is given by Al-Ketan and Al-Rub [10].

Sandwich panels mainly consist of one, in this study TPMS-based, core and two facesheets attached to the upper and lower side of the core [17]. When characterizing the mechanical properties of a sandwich panel, the focus usually lies on the bending properties since the sandwich panels are generally a component to carry bending loads in lightweight applications. As wing design is dominated by bending loads, sandwich structures are an interesting approach for additional equipment like suction panels. The bending properties can be described by the flexural modulus E_b or the flexural stiffness EI , which contains the elastic modulus E and the second moment of area I . In the classical sandwich beam theory, it is generally assumed that the main load-bearing elements are the facesheets, whereas the weaker core acts as a support and carries shear loads. Therefore, the flexural stiffness of sandwich beams is often described only by the facesheet's elastic modulus E_f and their second moment of area I_f [18].

The common model described by Kuenzi [18] neglecting the core is also used in the standard ASTM 7250 [19] for determining the flexural stiffness of a sandwich panel. This assumption becomes questionable for the sandwich panel design used in this study, where high relative core densities are employed. In these cases, the mechanical performance of the core itself becomes relevant and influences the overall flexural stiffness of the panel. Therefore, a new approach is needed to accurately describe and predict the flexural behavior of sandwich beams and plates with high relative density cores.

To develop a new approach that includes the sandwich core, one way is to compute the second moment of area I of the whole panel and combine it with the elastic modulus E which is usually given by the manufacturer or can be determined with tensile tests [20]. Lohuis et al. [6] computed the second moment of area I of Honeycomb and Gyroid unit cells, which can be utilized to compute the flexural stiffness EI . The

cells are cut into binary 2D-bitmaps and a numerical algorithm calculates the second moment of area for each pixel of the Gyroid or Honeycomb bitmap with its Steiner part. It showed that the computed second moments of area for the Honeycomb and Gyroid structure remain the same for the same relative density at different unit cell lengths. Moreover, the second moment of area showed a linear trend depending on the relative density. Since the elastic modulus remains constant, the computed behavior would result in a linear behavior of the flexural stiffness EI . This bitmap-based algorithm applied for complete sandwich panels offers a new way to determine the flexural stiffness EI including the influence of the core structure which can be used in the design process instead of complex simulations or tests.

Validating such a bitmap-based approach requires a set of force-deformation data under bending stress over a wide range of relative densities. The experimental force-deformation data from bending tests allows to calculate the experimental flexural stiffness and subsequently to compare the results with the numerical computed flexural stiffness from the bitmap-based second moment of area. If the experimental values show the same linear behavior over a wide range of relative densities as the computed ones, this enables an instrument for approximating the bending behavior of TPMS core structures at any relative density by using the bitmap-based flexural stiffness EI .

For the characterization of TPMS structures, studies have been focusing on investigating the compressive properties of TPMS structures by testing several materials and relative densities to adjust the scaling power laws described by Gibson and Ashby [3,21]. For flexural properties, only a few studies investigate the behavior of TPMS-based sandwich panels and focus on specific topics like functional grading [22,23] or only consider a small range of relative densities [23,24]. Even fewer of these studies are trying to approximate the bending behavior with existing models that include the core structure like the sandwich beam theory [17] or developing new theoretical models. A broad overview of the existing studies is given in Table 1. It shows studies with a similar scope and links them by summarizing their manufacturing process, material, relative densities, and whether a theoretical approach has been considered. In the following, the focus lies on the used theoretical approach to approximate the bending behavior and the behavior of the flexural stiffness with an increasing relative density.

Peng et al. [26] conducted three-point bending tests on FDM-manufactured sandwich panels composed of three TPMS structures with relative densities between 0.05 to 0.5. The authors combined the sandwich beam theory [17] and power-scaling models proposed by Abueidda et al. [31] to describe the flexural stiffness EI . Over the whole range of relative density, the study showed an increasing experimental flexural stiffness with a linear trend. When comparing the experimental and theoretical flexural stiffness values expressed in N/mm at the relative densities $RD = 0.35$ and $RD = 0.5$, they observed an approximation with a mismatch of less than 10%. By using simulations as a comparative value for the flexural stiffness on two-dimensional cellular structures, Lin et al. [29] obtained a mismatch of 21% to the sandwich beam theory. The approach of combining the sandwich beam theory with scaling power laws gives another way of computing the flexural stiffness theoretically and is applied in this study to compare the results with the bitmap-based approach from Lohuis et al. [6].

Ejeh et al. [22] conducted four-point bending tests on TPMS-based beams without facesheets, focusing on the effects of relative density gradation and hybridization of different TPMS structures. For beams consisting of only one TPMS structure, the study presents scaling power laws for the flexural modulus between $RD = 0.1$ to 0.4, indicating a polynomial dependency between flexural modulus and relative density. In another study, Ejeh et al. [32] conducted impact tests on sandwich panels with a TPMS-core, observing a low difference in the impact performance for different unit cell lengths. Pelanconi and Ortona [30] used the Gyroid structure to rebuild and characterize the wings of butterflies, resulting in specimens with CFRP-reinforced struts at the top and bottom of the core as facesheets. The relative density was varied at three

Table 1
Overview on the existing studies investigating the flexural behavior of TPMS structures.

Source	Manufacturing Process		Material		Specimen	RD	Theoretical Approach
	Core	Facesheets	Core	Facesheets			
[22]	LPBF	<i>no facesheets</i>	AlSi10Mg	<i>no facesheets</i>	TPMS Beam	0.1–0.4	No
[25]	PolyJet	Cold Cure Vacc.	UV-Resin	CFRP-VE	Sandwich Panel	0.2–0.4	Yes
[26]	FDM	FDM	ABS	ABS	Sandwich Panel	0.05–0.5	Yes
[24]	SLM	OAA	304L	CFRP-EP	Sandwich Panel	0.15	No
[27]	DLP	DLP	UV-Resin	UV-Resin	Sandwich Panel	0.2–0.4	Yes
[28]	FDM	<i>no facesheets</i>	PEI	<i>no facesheets</i>	TPMS Beam	0.06–0.48	No
[29]	<i>Simulations</i>	<i>Simulations</i>	ABS	ABS	Sandwich Panel	0.1–0.3	Yes
[30]	SLA	Pultrusion	UV-Resin	CFRP-EP	Sandwich Panel	<i>not defined</i>	No

different values, showing an increasing bending stiffness expressed in N/mm . The specimen design could be considered as a sandwich panel, but is only applicable in specific use cases. These three studies represent cases, where the authors either focus on specific use cases, different test set-ups or investigate TPMS structures which differ from the Gyroid used in this study. All these factors mitigate the comparability of the obtained results for the force-deformation, flexural stiffness or flexural modulus due to the difference in structure or specimen design. Instead, the obtained trends of an increasing bending stiffness and the flexural modulus modeled with a scaling power law will be used as a comparative basis.

As seen, existing research studies have investigated TPMS structures as cores in sandwich panels and attempted to describe their bending behavior using theoretical approaches such as the sandwich beam theory combined with scaling power laws. These approaches have shown errors of 10% to 21%. Nevertheless, the studies focus on very specific design configurations and definitions. Only one identified study uses a broader range of relative densities while comparing experimental results to a theoretical model [26]. For the flexural stiffness as a parameter describing the bending behavior, the existing studies do not provide or use a standardized definition of flexural stiffness. As a result, terms such as flexural stiffness EI in Nmm^2 and spring stiffness in N/mm are used synonymously, which is why the cross-study comparability of the results is limited. In addition, the studies do not refer to established design rules provided by standards such as ASTM 7250 [19] or DIN 53293 [33], which would also enhance the comparability of results, since important bending parameters like the second moment of area depend on the dimensions of the sandwich panel.

Normalizing the bending properties by material, printer type and geometry allows further improvement of the cross-study comparability. A normalization of the bending properties of TPMS-based sandwich panels can be achieved by referencing the flexural stiffness and modulus with the corresponding values of the solid beam, printed with the same material and dimensions. The relative flexural stiffness and modulus then largely depend on the relative density, mostly independent of manufacturing process and material characteristics. Addressing this needs a theoretical model, validated at a wide range of relative densities with normalized flexural properties of standardized designed specimen. This new model significantly enhances the reliability and utility of comparative analyses across studies and is dependent only from the structure to predict the bending characteristics, for example of suction panels.

This study conducts a series of four-point bending tests on Gyroid sheet network-based sandwich panels with relative densities ranging from 0.1 to 0.7, using three different unit cell lengths. The aim of varying unit cell lengths is to investigate whether the bending properties differ at the same relative density but with different unit cell dimensions. For the experimental setup and calculations, ASTM 7250 [19] is utilized. Besides specified guidelines for the specimen design and an appropriate test set-up, it provides formulas for determining the flexural stiffness of sandwich panels with Honeycomb, balsa wood, and foam cores based on force-deformation data. Additionally, ASTM 6272 [34] offers guidelines for calculating the flexural modulus of beams tested under four-point bending.

To validate the findings and assumptions of Lohuis et al. [6], this study calculates the second moment of area I using the bitmap-based approach to compute the flexural stiffness EI and compares the obtained values to the results of the experimentally obtained flexural stiffnesses. To evaluate this approach, the sandwich beam theory from existing studies in combination with scaling power laws is used as a comparison for the prediction of the bending behavior. To approximate the actual force-deformation behavior, the modeled flexural stiffnesses are used in Bernoulli's beam theory to validate the quality of the different approximations.

Based on the findings from other studies, not only the facesheets but also the core are expected to have a major influence on the flexural stiffness. This motivates to use a model like the one of Lohuis et al. [6] that incorporates this into the calculation. For sandwich structures like this with a high relative density and a supporting core, existing models lack the possibility of considering that core. Most of them only consider the facesheets for the flexural stiffness, leaving a research gap in the field of more functional core structure. This study wants to fill this gap by using this new approximation approach that combines the impact of core and facesheets numerically. Especially in suction panels with a higher relative density either for implementing inserts or for adjusting pressure losses, this will be relevant. Therefore this approach is expected to give a good approximation of the flexural properties of TPMS sandwiches.

2. Materials and methods

To validate the bitmap-based numerical approach for approximating the flexural stiffness of sandwich panels with a TPMS core, comparative experimental values are needed. Therefore, test specimens need to be designed, manufactured and tested in a bending test set-up. The subsequent sections outline the methodology employed in the design of the specimens adhering to the procedures specified in ASTM 7250 [19] and DIN 53293 [33], and the principles behind the software-based design process of the Gyroid core. Given the completed design specifications for the test specimens, the section gives an insight into the manufacturing process employing selective laser sintering (SLS) along with the associated challenges encountered during fabrication to outline possible consequences on the results. Furthermore, the four-bending test setup and the procedure for calculating the flexural stiffness and flexural modulus from the experimental data are described according to the mentioned standard ASTM 7250 [19] and also ASTM 6272 [34].

2.1. Specimen design

Due to the complex Gyroid structure, the core and facesheets are designed separately. The section derives the outer dimensions of the sandwich panel's core and facesheets from the standard ASTM 7250 [19]. In addition, it consists of a description of the methodology and mathematical framework for designing the Gyroid core based on its design parameters unit cell length l_{uc} , wall thickness t_w and relative density RD . The described software-based design process combines the outer dimensions and the core to a test specimen forming a sandwich panel,

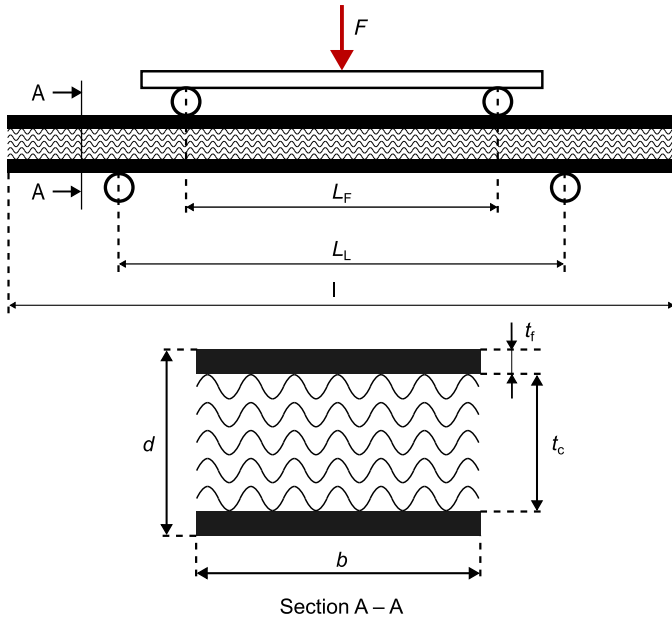


Fig. 1. Schematic drawing of a four-point bending test set-up along with the sandwich panel's outer dimensions.

Table 2
Dimensioning guidelines according to ASTM 7250 [19].

support span length L_L	width b	thickness
$L_L/d > 20$	$b/d > 2$	$t_f/t_c < 0.1$
$L_L/b > 2$	$b/l_{uc} > 3$	
$L_L - l \leq -50 \text{ mm}$		

which is varied by the relative density and the unit cell length l_{uc} , resulting in a test matrix in the Tables 3 and 4 representing all manufactured and tested specimens.

Dimension guidelines for sandwich panels

For defining the outer dimensions, Fig. 1 shows the parameters that need to be specified according to ASTM 7250 [19]. Here, the specimen dimensions length l , width b , facesheet thickness t_f , core thickness t_c and total thickness d are directly dependent on the four-point bending test set-up dimensions support span length L_L and load span length L_F .

While many of the mentioned studies in Table 1 design their panels according to individual dimensioning approaches, a standardized procedure following existing standards is advantageous for a comparable determination of the flexural properties, especially since the bending properties have a high dependency on the selected geometry. In the case of a four-point bending test set-up, the ASTM 7250 [19] and DIN 53293 [33] standards specify corresponding methods for determining the flexural stiffness of sandwich panels. Additionally, the ASTM 6272 standard provides guidelines for determining the flexural modulus. ASTM 7250 and DIN 53293 both provide design rules for a sandwich panel in the form of ratios, depending on the given test set-up dimensions. Comparing these two standards, the dimension rules for the ratio between the support span length L_L and the height d and the ratio between the width b and the height d are almost identical. ASTM 7250 [19] also specifies a ratio between the thickness of the cover layers t_f and the total thickness d of the sandwich panel, which is defined for adhering to the static principle of the sandwich beam theory. Since the scope of this paper is also to approximate the bending behavior with the use of the sandwich beam theory as a comparison to the newly developed numerical bitmap-based approach, the standard ASTM 7250 is chosen for dimensioning the specimen by using the guidelines given in Table 2.

Table 3

Specimen dimensions in mm.

L	L_L	b	d	t_c	t_f
294	250	40	14	12	1

The given guidelines of the ASTM 7250 [19] result in the outer dimensions of the specimens presented in Table 3. To reach a sufficient thickness for different unit cell lengths of the core, the support length-to-thickness ratio is set at approximately 18 instead of 20. According to the standard, the specimen length l should be set to 300 mm. Practical tests with the employed additive manufacturing machine and the used material have shown that the maximum achievable height of the specimens is limited to 294 mm. Since the additional 50 mm outside the stamps were originally intended to ensure sufficient overlap over the supports of the test setup, a reduction of 3 mm in overlap at each side is considered acceptable.

This study aims to characterize and approximate the bending behavior of sandwich panels with a TPMS-based core over a wide range of relative densities. While the defined outer dimensions of the panel remain static, a variation of the core's wall thickness t_w and unit cell length l_{uc} results in the intended variation of the relative density. The following section describes the design methodology of the Gyroid core, the approach for the relative density (RD) variation, and how it is linked to the unit cell length l_{uc} and wall thickness t_w .

Design methodology for TPMS sheet networks

The core of the sandwich panels in this study consists of a Gyroid sheet network structure, defined by the unit cell parameters length l_{uc} and wall thickness t_w . The unit cell is then repeated in all three spatial directions and limited by the specified outer dimensions of the sandwich panel. As already stated in the introduction, a TPMS is initially a surface without volume, which is transferred into a volumetric body by either thickening the surface (Fig. 2(a)) or defining one of the emerging channels as a solid (Fig. 2(b)). The initial surfaces are described by implicit functions, which exist for several types of TPMS, including the Gyroid, shown in Eq. (1) [35].

$$f_G = \sin(x) \cos(y) + \sin(y) \cos(z) + \sin(z) \cos(x) = c \quad (1)$$

To obtain a Gyroid sheet network, the minimal surface has to be extracted and finally thickened by the defined wall thickness t_w . In Eq. (1), the variables $x = 2\pi\alpha$, $y = 2\pi\beta$ and $z = 2\pi\gamma$ determine the unit cell length l_{uc} in the three spatial directions x , y and z , with the parameters α , β and γ being the inverse of the respective unit cell length [36]. The level parameter c determines which isosurface will be extracted from the equation. The minimal surface is approximated with the parameter c equal to zero [11]. The isosurface can be extracted using specialized algorithms like the marching cubes algorithm [37] or the dual contouring algorithm [38], transforming the implicit equation into a mesh with explicitly defined vertices and edges. Thickening the surface after modeling can either be done by modeling it with an iso value $c = 0$ and extruding it to a defined wall thickness t_w or by varying the constant c in the implicit equation and filling the space between the two resulting surfaces [36]. With the wall thickness t_w , the unit cell length l_{uc} , and the implicit function in Eq. (1), the core of the sandwich is fully defined.

The wall thickness t_w and the unit cell length l_{uc} of the Gyroid core structure are used to set the relative density of the core at specific values. Eq. (2) describes the relative density RD as the quotient of the TPMS body's density ρ^* and the density ρ_s of the solid body with the same external dimensions [3]. Gibson and Ashby [3] provide a general approach for calculating the relative density of open-cell and closed-cell cellular structures with the Eqs. (3) and (4). These equations depend on the wall thickness t_w and the unit cell length l_{uc} , while the constants C_1 and C_2

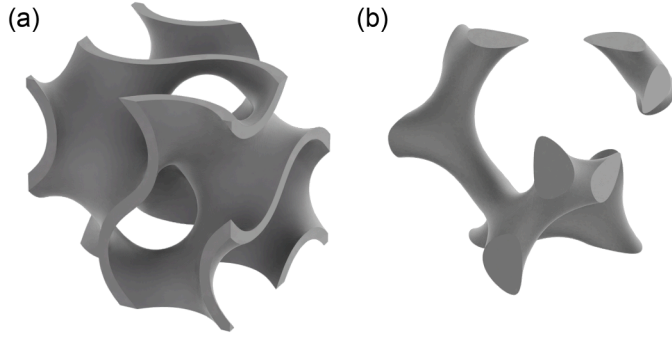


Fig. 2. Comparison of sheet (a) and solid (b) Gyroid networks.

have to be calculated individually for different cellular structures.

$$RD = \frac{\rho^*}{\rho_s} = \frac{m_s}{V^*} \frac{V_s}{m_s} = \frac{V_s}{V^*} \quad (2)$$

$$RD_{\text{open-cell}} = C_1 \left(\frac{t_w}{l_{uc}} \right)^2 \quad (3)$$

$$RD_{\text{closed-cell}} = C_2 \left(\frac{t_w}{l_{uc}} \right) \quad (4)$$

Even though the Gyroid sheet network could be considered to have an open-cell morphology, Traub et al. [11] were able to show that the relative density can be described with a high accuracy using the closed-cell approach in Eq. (2), especially for low relative densities. They modeled 13 unit cells with a length l_{uc} between 10 mm to 20 mm and wall thicknesses t_w between 0.5 mm to 2.0 mm in order to subsequently determine their relative density using the software Blender. Further evaluation including unit cells with higher relative densities shows that the relative density of Gyroid sheet networks can be described using a cubic approach in addition to a linear term, as shown in Eq. (5). This achieves an even higher accuracy and extends the region of applicable relative densities.

$$RD_{\text{Gyroid}} \approx 3.0918 \left(\frac{t_w}{l_{uc}} \right) - 4.1187 \left(\frac{t_w}{l_{uc}} \right)^3 \quad (5)$$

By using Eq. (5), the relative density RD , the unit cell length l_{uc} and the wall thickness t_w are added to the specimen dimensions. Table 4 shows the relative density being set at seven levels from 0.1 to 0.7 while the unit cell length will be set to three different values. Each combination of relative density RD and unit cell length l_{uc} results in a specific wall thickness t_w . Designing sandwich panels with different unit cell lengths l_{uc} at the same relative density RD results in a different number of unit cells n_{uc}^w and n_{uc}^h in the cross-section of the panel, which leads to a load-distribution over a different number of unit cells. This variation aims to determine the influence of the unit cell length l_{uc} on the bending behavior.

In the following, a specimen naming code in the form of G- l_{uc} - $RD \times 100$ is used to refer to specific specimen configurations in the following chapters. For example, the code G-12-10 refers to the specimen with a unit cell length of 12 mm and a relative density of 0.1 or 10%.

Implementation of the Specimen Design Process

While the previous section specified the dimensions of the test specimen, this chapter describes the methodology behind the software-based design process using the software Blender. Traub et al. [11] introduced this design methodology to create cubic Gyroid test specimens, while other authors also used different approaches with software solutions like MATLAB [39] or NTopology [40].

The main difference between the software design approaches is the procedure of thickening the minimal surface after generating it at an iso value $c = 0$ or to thicken the surface before the generation by varying

Table 4

Specification of each specimen configuration consisting of the unit cell length l_{uc} and relative density RD , along with the design wall thickness t_w and the number of unit cells n_{uc} in all three spatial dimensions.

Specimen	RD [%]	l_{uc} [mm]	t_w [mm]	n_{uc}^w [-]	n_{uc}^h [-]	n_{uc}^l [-]
G-6-10	10	6	0.19	6.67	2	49
G-8-10	10	8	0.26	5	1.5	37.5
G-12-10	10	12	0.39	3.33	1	24.5
G-6-15	15	6	0.29	6.67	2	49
G-8-15	15	8	0.39	5	1.5	37.5
G-12-15	15	12	0.58	3.33	1	24.5
G-6-20	20	6	0.39	6.67	2	49
G-8-20	20	8	0.52	5	1.5	37.5
G-12-20	20	12	0.78	3.33	1	24.5
G-6-30	30	6	0.59	6.67	2	49
G-8-30	30	8	0.79	5	1.5	37.5
G-12-30	30	12	1.18	3.33	1	24.5
G-6-45	45	6	0.90	6.67	2	49
G-8-45	45	8	1.20	5	1.5	37.5
G-12-45	45	12	1.80	3.33	1	24.5
G-6-60	60	6	1.24	6.67	2	49
G-8-60	60	8	1.65	5	1.5	37.5
G-12-60	60	12	2.47	3.33	1	24.5
G-6-70	70	6	1.48	6.67	2	49
G-8-70	70	8	1.97	5	1.5	37.5
G-12-70	70	12	2.96	3.33	1	24.5

the constant c as described in the previous section. Traub et al. [11] discovered that varying the constant c does not result in constant wall thicknesses t_w at relative densities above $RD \geq 0.3$. Since in this study the relative density is going to be set up until values of $RD = 0.7$, the first approach on thickening the minimal surface after the generation will be applied by using the software Blender.

Based on the cell parameter of the unit cell length l_{uc} from Table 4 and the outer dimensions of the core l , b and t_c from Table 3, the implicit function in Eq. (1) is calculated and modeled for the iso value $c = 0$ by utilizing the marching cubes surface extraction algorithm [37]. This procedure transfers the implicit function into an explicit mesh of the minimal surface for the entire sandwich panel core. The mesh is then symmetrically thickened by the individual wall thickness t_w by using Blender's solidify modifier, resulting in a sheet-based Gyroid network with the specified dimensions of the sandwich panel core. Finally, the facesheets are merged with the core using Blender's merge modifier to export the sandwich panel in the STL format for slicing and manufacturing.

2.2. Specimen manufacturing

This section describes the specimen manufacturing process and its influence on dimensional accuracy. The study focuses on the additive manufacturing process of selective laser sintering (SLS). The study used a Fuse 1 + 30W [41] production system from Formlabs, with polyamide PA11 [42] as the selected material. The layer thickness was set to 0.11 mm. The mixing ratio between old and new powder followed the manufacturer's recommendation, using 70% of new powder and 30% of old powder. During the manufacturing process, the specimens were oriented vertically on the building plate. To remove residual powder, post-processing was performed using the Sinterit Sandblaster [43], a sand-blasting system. Fig. 3 shows the post-processed specimens with a unit cell length of $l_{uc} = 12$ mm and a relative density from $RD = 0.1$ to 0.7.

To systematically investigate the manufacturing limits of the employed printing process, the test matrix included specimens with thin walls. This approach facilitated the identification of design thresholds beyond which fabrication was not feasible or the resulting parts did not meet the required quality standards. As a result, specimens with too

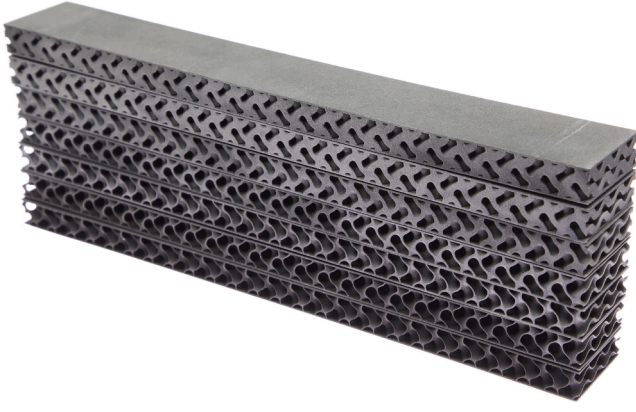


Fig. 3. Test specimens with relative densities $RD = 0.1$ to 0.7 and a unit cell length of $l_{uc} = 12$ mm.

thin walls, specifically those with $t_w \leq 0.39$ mm (G-6-10, G-8-10 and G-6-15) were not manufactured. Furthermore, specimens with high relative densities and small unit cell lengths, such as G-6-70 and G-8-70, exhibited channels that were not sufficiently large in diameter for effective mechanical cleaning using the sandblaster, and thus were also not manufactured. Therefore, only certain configurations where manufactured at very low or very high relative densities (G-12-10 and G-12-70). With these boundaries identified, the number of printable and tested specimens is reduced by the mentioned configurations, but still covering the whole range of relative density between 0.1 to 0.7 at three different unit cell lengths l_{uc} .

To evaluate the quality of the SLS process, the actual relative density of the specimens was measured after manufacturing and post-processing. Fig. 4 presents the mean values of the measured relative densities RD_m for all unit cell lengths over the designed relative density RD . The straight line represents the expected curve in the case of an identical measured relative density to the designed one. The manufactured specimens exhibit a deviation between the designed and the actual relative density, most likely due to residual powder that could not be effectively removed by sandblasting. The obtained results of force-deformation data, flexural stiffness and the approximation with Bernoulli's Beam Theory will show that the residual powder is not a factor influencing the bending behavior within the linear-elastic region. To compare the theoretical and experimental values of the flexural stiffness, flexural modulus and the force-deformation behavior with Bernoulli's Beam Theory, the focus lies on the linear-elastic region. At higher deformations outside of the linear-elastic region, the residual powder is likely to be compressed, leading to a possible falsification of the data. The further observations and results still refer to the designed relative density RD .

Another production-related deviation is observed in the distortion v of the specimen in the direction of the specimen height. To quantify this distortion, the height in the middle of each specimen is measured in an unloaded state and then under sufficient load to ensure it lays flat on a given surface, which results in a maximum distortion value for each specimen. Fig. 5 presents the mean distortion values of each specimen configuration over the designed relative density RD , with a maximum of about 1 mm. The mean distortion decreases with increasing relative density. The distortion occurred most likely because of the high gradients in temperature between the edges of the building space and its center during the SLS process. Elongated components are particularly susceptible to production-related distortions, as even small temperature differences can lead to deformations [44]. The specimen distortion is considered a potential factor influencing the results of the four-point bending tests, as the initial curvature changes the initial conditions for introducing the loads and the support stamp reactions. Consequently, it

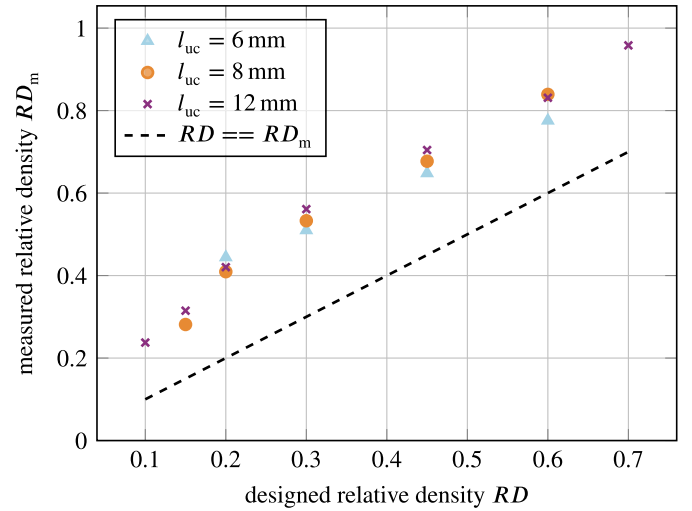


Fig. 4. Mean measured relative density RD_m for each unit cell length l_{uc} over the theoretically designed relative density RD .

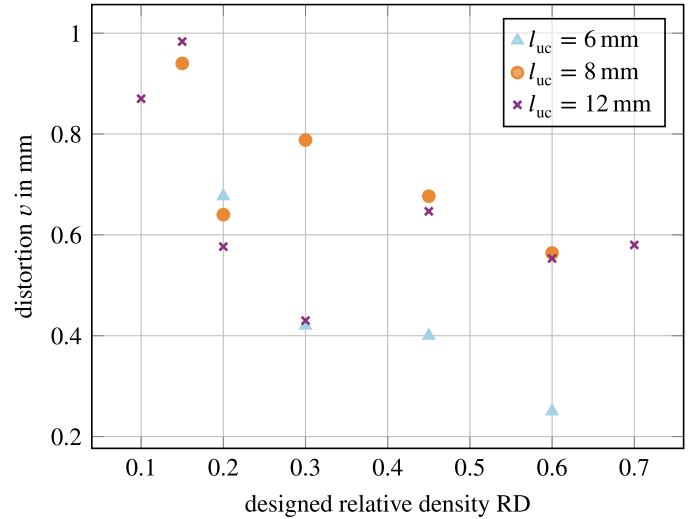


Fig. 5. Mean distortion of the test specimens over the relative density RD .

particularly influences the force-deformation data and is documented to identify possible correlations.

2.3. Test set-up

The four-point bending tests were performed on a Zwick/Roell universal test machine *zwickiLine* [45] with a maximum load capacity of 5 kN. The support stamp configuration enables a regular maximum length of $L_L = 250$ mm with both the supports and the force stamps having a width of 40 mm and a radius of 5 mm. Following the directives of ASTM 7250 [19] for calculating the flexural stiffness of sandwich panels, each specimen has to be tested in two different loading configurations. Fig. 6 shows these loading configurations which are defined by varying the distance L_B between the support stamp and the force stamp with one quarter (*Quarter-Point Loading* (4PL)) or one third (*Third-Point Loading* (3PL)) of the support stamp length L_L . For a support stamp length of $L_L = 250$ mm, the distance L_B has to be set at $L_{B,3PL} = 83.33$ mm and $L_{B,4PL} = 62.5$ mm. The variation of L_B is achieved by adjusting the distance between the force stamps L_F , leaving the support stamp length L_L at the same distance of 250 mm. While conducting the four-point bending tests, the deformation is measured using a digital displacement

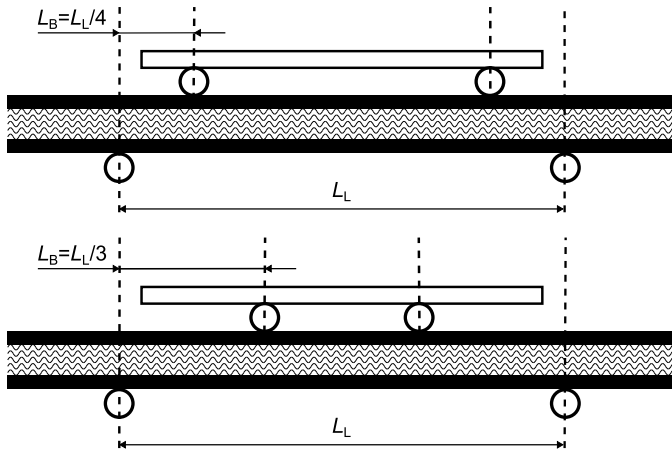


Fig. 6. The two testing configurations used in this study: Third-Point Loading (3PL) and Quarter-Point Loading (4PL).



Fig. 7. Third-Point Loading test set-up with a sandwich specimen before testing.

transducer with a range of 0.2 mm to 13.5 mm at a speed of 6 mm min⁻¹, following ASTM 7250.

The procedure of testing each specimen in two different loading configurations implies that the first test conducted has to occur in the linear-elastic region to avoid pre-damaged specimens returning biased or falsified results in the second configuration testing. To circumvent pre-damaging, a sixth specimens is printed for each specimen configuration and tested until destruction. A value of 20% of the monitored maximum force will be used as an individual threshold at which the first test will be stopped. Fig. 7 shows another limitation with the shape of the support stamp, where at a certain deformation the specimen leave stamp radius and lies only the stamp's flat inner side.

2.4. Calculation of flexural properties

Validating the flexural stiffness approximated with the bitmap-based second moment of area algorithm introduced by Lohuis et al. [6] requires determining the sandwich panel's flexural stiffness EI experimentally. In addition, the flexural modulus E_b can be taken into consideration as another key value determining the bending properties. It can also be approximated by using the bitmap-based second moment of area in connection with Bernoulli's Beam Theory. This gives two ways of validating the bitmap-based second moment of area. The flexural stiffness and flexural modulus can be determined experimentally from the experimental data.

Flexural stiffness following ASTM7250

To determine the flexural stiffness EI from the experiments, the standard ASTM 7250 [19] already used for the specimen design is employed. The standard distinguishes between the two cases of a known and unknown elastic modulus of the facesheets E_f . Here, the case with a known elastic modulus describes a calculation without considering the sandwich panel's core and without the need for experimental testing. Therefore, this study concentrates on using the second case, where experimental force-deformation and the influence of the TPMS core is included into the calculation.

For the calculation of the flexural stiffness of a sandwich panel with a known elastic modulus of the facesheets E_f , the flexural stiffness EI_F is calculated using Eq. (6). Examining Eq. (6) implies that the parameters of the cross-section given with the width b , height d and core height t_c along with the materials elastic modulus of the facesheets E_f are sufficient for determining the flexural stiffness EI_F . This is the case for classical sandwiches where the core does not contribute to withstanding bending loads, leading to a constant value independent of an increasing relative density.

$$EI_F = \frac{E_f(d^3 - t_c^3)b}{12} \quad (6)$$

For the case of an unknown elastic modulus of the facesheets E_f , the ASTM 7250 standard describes the determination of the equivalent flexural stiffness EI_{eq} by testing each specimen with the 3PL- and 4PL-Configurations illustrated in Fig. 6. After conducting this procedure, the flexural stiffness is then calculated with Eq. (7), which takes into account the deformations f_{4PL} and f_{3PL} of each loading configuration at the same level of force F .

$$EI_{eq} = \frac{99FL_L^3(1 - 92L_L^2/99L_1^2)}{6912f_{4PL}(1 - 3FL_Lf_{3PL}/4FL_Lf_{4PL})} \quad (7)$$

The sandwich panels manufactured and tested during this study are designed with cores with a relative density up to $RD = 0.7$, so it is assumed that the flexural stiffness will increase with increasing relative density and not remain constant. Furthermore, the objective of this study is the validation of the numerical flexural stiffness EI_N with the use of the bitmap-based second moment of area [6]. Since the bitmap-based second moment of area includes the sandwich panel's core, it is relevant that the whole sandwich panel, and therefore the force-deformation data at the different relative densities, is also taken into account during the experimental calculations. For taking the core into account and achieving effective comparability between the experimental and numerical flexural stiffness, this paper uses Eq. (7) to calculate the flexural stiffness EI_{eq} .

To achieve a better comparability across studies and materials, the flexural stiffness will be normalized by the maximum flexural stiffness EI_{max} of a solid beam with the same outer dimensions, computed with the use of Eq. (8).

$$EI_{max} = E_{PA11} \frac{bd^3}{12} \quad (8)$$

Flexural modulus according to ASTM D6272

The flexural modulus E_b is a characteristic value for describing the bending behavior of beams. An according calculation guideline to determine the flexural modulus experimentally is provided by the standard ASTM D6272 [34]. This guideline requires a conversion of experimentally recorded force-deformation values into the bending stress σ_b and the bending strain ϵ_b according to Eqs. (9) and (12). The basis for the conversion of the force values is the main bending theorem according to Eq. (9), where $M_{b,max}$ represents the maximum bending moment and W_x is defined as the quotient of the second moment of area and maximum distance between outer fiber and the neutral axis.

$$\sigma_b = \frac{M_{bmax}}{W_x} \quad (9)$$

By inserting the specimen cross-section dimensions as well as the parameter L_B for the corresponding loading configuration, Eq. (9) results in the Eqs. (10) and (11) for each loading configuration. The bending strain ϵ_b for the Third-Point Loading configuration is then described by Eq. (12).

$$\sigma_{b,3PL} = \frac{FL_L}{bh^2} \quad (10)$$

$$\sigma_{b,4PL} = \frac{3FL_L}{4bh^2} \quad (11)$$

$$\epsilon_{b,3PL} = \frac{4.7fh}{L_L^2} \quad (12)$$

After the conversion of the force-deformation data, the standard ASTM D6272 [34] describes the calculation of the flexural modulus by using Eq. (13). Here, the flexural modulus is defined by the ratio between the stress difference and the corresponding strain difference within the linear-elastic region, which is represented by a tangent plotted to the steepest region of the stress-strain curve, giving the elastic line of the stress-strain curve. With Eq. (13) and the elastic line, the flexural modulus is simply determined by the slope of the elastic line, depending on the used loading configuration.

$$E_{i,b} = \frac{\sigma_{b(i,2)} - \sigma_{b(i,1)}}{\epsilon_{b(i,2)} - \epsilon_{b(i,1)}} \quad i = 3PL, 4PL \quad (13)$$

The standard was originally developed for solid beams with a completely filled cross-sectional geometry, which is why the Eqs. (10)–(13) also only consider solid beams. Since the beams used in this study are sandwich panels and so do not have a solid cross-sectional geometry, the obtained results for the flexural modulus E_b would not be equally representative. This limitation is circumvented by determining the maximum possible flexural modulus $E_{b,max}$ and placing it into a relative relationship with the flexural modulus of the sandwich panels, calculated with Eq. (13). The relative view gains a qualitative statement about the flexural modulus. By being material-independent, it also improves the cross-study comparability in the future.

In addition to the given procedures for the calculation of the flexural modulus and flexural stiffness from the experimental data, the next sections describe the computation of the theoretical values accordingly. In particular, the bitmap-based algorithm will be explained, and how the resulting second moment of area is used to calculate the numerical flexural stiffness EI_N and deformation. Furthermore, the section outlines the commonly used sandwich beam theory for approximating the semi-empirical flexural stiffness EI_{SE} .

3. Theoretical approaches for calculating the flexural stiffness

This paper aims to validate the numerical approach of computing the flexural stiffness EI_N using the bitmap-based algorithm [6]. Therefore, the authors examine the bending properties of TPMS-based sandwich panels such as the flexural stiffness EI_{eq} and the modulus E_b by four-point bending tests to compare the experimental and numerical results. In addition, the commonly used semi-empirical approach of the sandwich beam theory EI_{SE} and the model only considering the facesheets EI_F described in the standard ASTM 7250 [19] by Eq. (6) are taken into account to investigate which model reflects the real data most accurately. Since the flexural stiffness only considering the facesheets EI_F has already been examined, the following two sections will elaborate on the two numerical and semi-empirical approaches for the approximation of the flexural stiffness.

3.1. Semi-empirical sandwich beam theory

The standard ASTM 7250 defines the flexural stiffness of a sandwich panel with Eq. (6) by only considering the facesheets of the panel.

In contrast, Allen [46] and Plantema [47] describe the flexural stiffness of a sandwich panel with a rectangular cross-section according to Eq. (14) [48]. Here, the first term corresponds to the flexural stiffness of the facesheets bent around their own center of gravity, while the second term represents the stiffness of the facesheets around the center of gravity axis of the entire sandwich panel. The third term includes the bending stiffness of the core. The parameter E_f describes the elastic modulus of the facesheets, while E_c defines the elastic modulus of the core. It is assumed that the surface layers have a uniform thickness t_f . The distance between the centers of gravity of the two surface layers w , is represented by the thickness of the core t_c and the thickness of the surface layers t_f , such that $w = t_c + t_f$.

$$EI_{SE} = \frac{E_f b t_f^3}{6} + \frac{E_f b t_f w^2}{2} + \frac{E_c b t_c^3}{12} \quad (14)$$

Since the elastic modulus for the facesheets E_f is given by the manufacturer's data sheet [42], the values for the core's elastic modulus E_c remain to fully describe the flexural stiffness EI_{SE} according to Eq. (14). As stated in the introduction, the elastic modulus of TPMS structures can be modeled in relation to the relative density. Gibson and Ashby [3] initially provided basic scaling power models such as Eq. (15) for the elastic modulus of cellular structures depending on the relative density, where E_s corresponds to the elastic modulus of the material as a solid body.

Analogous to the determination of the relative density, the scaling power laws depend on the individual determination of the value for the parameter C , which depends on the respective cellular structure. For the determination of the parameter C experimental testing is needed, which is also why this approach is also called semi-empirical. Different authors obtained parameters and relationships accordingly [9,11,31]. However, the scaling power law by Abueidda et al. (index A) [31] is mostly used in the literature when it comes to modeling the flexural stiffness EI_{SE} of a TPMS-based sandwich panel using Eq. (14) as a basis. This law is presented in Eq. (16), obtained from simulations. In contrast, Traub et al. (index T) [11] derived a cubic scaling power law by carrying out experimental compressive tests on cubic specimens, resulting in Eq. (17). Both laws give different values for the elastic modulus of the core E_{cA} and E_{cT} which are substituted for E_c in Eq. (14), which results in the flexural stiffness EI_{SE-A} or EI_{SE-T} at a chosen relative density. Computing the semi-empirical flexural stiffness depending on relative densities between 0.1 to 0.7 represents another way of describing the bending behavior, which is compared with the newly developed numerical bitmap-based approach to investigate which model fits the experimental data most accurately. Moreover, by using two different scaling power laws, it is possible to examine which scaling power law works better with the semi-empirical approach.

$$\frac{E_c}{E_s} = C \left(\frac{\rho^*}{\rho_s} \right)^2 \quad (15)$$

$$\frac{E_{cA}}{E_s} = 0.555 \left(\frac{\rho^*}{\rho_s} \right)^{1.406} \quad (16)$$

$$\frac{E_{cT}}{E_s} = 1.1557 \left(\frac{\rho^*}{\rho_s} \right)^3 - 0.2720 \left(\frac{\rho^*}{\rho_s} \right)^2 + 0.1164 \left(\frac{\rho^*}{\rho_s} \right) \quad (17)$$

3.2. Numerical approximation of the flexural stiffness

The basis for the computation of the numerical flexural stiffness EI_N represents a bitmap-based algorithm which was first introduced by Lohuis et al. [6]. The algorithm has the function of computing the second moment of area for arbitrary structures and is not purely limited to TPMS. To determine the second moment of area of the sandwich panel with a Gyroid core tested in this study, each specimen configuration is designed with the length of one unit cell based on the Python SDF Library [49]. Only one unit cell length is needed since the Gyroid core is repeated periodically. In general, the algorithm is especially useful

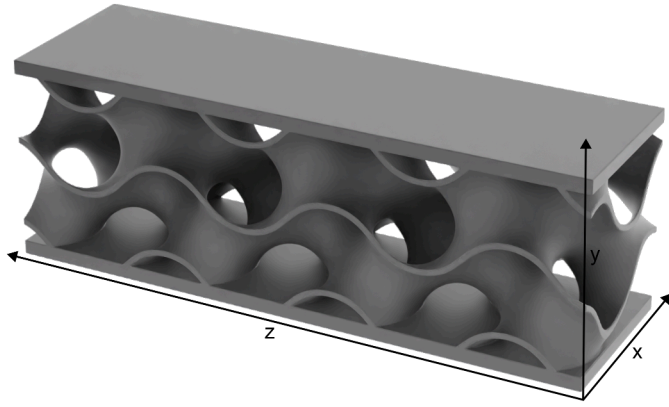


Fig. 8. Segment of SDF-modelled sandwich panel of configuration G-12-20 (One unit cell in x-direction).

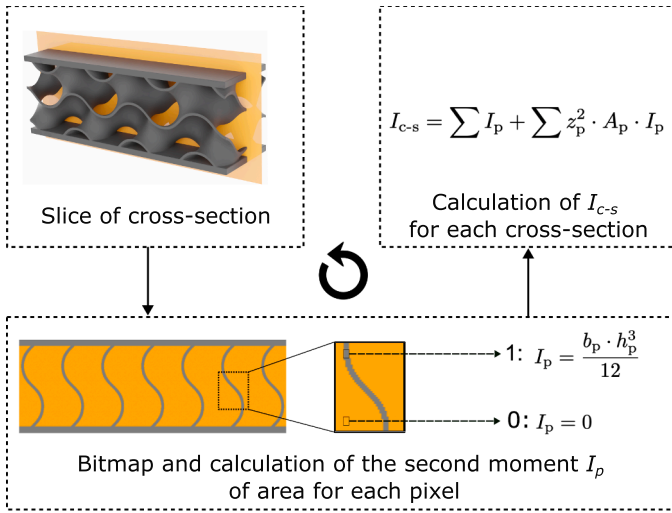


Fig. 9. Visualization of the bitmap algorithm to calculate the second moment of area for one single cross-section divided into three different steps.

for periodic structures as this reduces the length for which the Flexural stiffness has to be calculated. Symmetry towards the neutral axis can also reduce the computational effort. The main influences on the computational effort are the number of viewed cross sections, their size and the resolution of the bitmap images. Fig. 8 shows the sandwich panel configuration G-12-15, which is then divided by the algorithm into 150 equidistant cross-sections along the x-axis. For each cross-section, the bitmap algorithm follows the flowchart in Fig. 9. Here, the algorithm first generates a representative binary bitmap, where the gray pixels represent the structure and the orange pixels represent the voids in between. For each gray pixel it computes the second moment of area, along with its Steiner-part. All second moments of area are then summed up, resulting in a second moment of area for the whole bitmap. This procedure is repeated for each of the 150 cross-sections, resulting in a global distribution of the second moment of area along the x-axis. Fig. 10 shows the normalized distribution of the second moment of area over one unit cell, which varies along the x-axis due to the non-uniform geometry of the Gyroid unit cell.

Based on the varying second moment of area of all 150 cross-sections, Fig. 10 shows the varying second moment of area along the x-axis with the individual minimum, maximum, and mean value for the sandwich panel shown in Fig. 8. For the further computation of the flexural stiffness, the mean second moment of area of the 150 cross-sections is used as the global second moment of area of the entire sandwich panel. Repeating the computation of the mean second moment of area for each

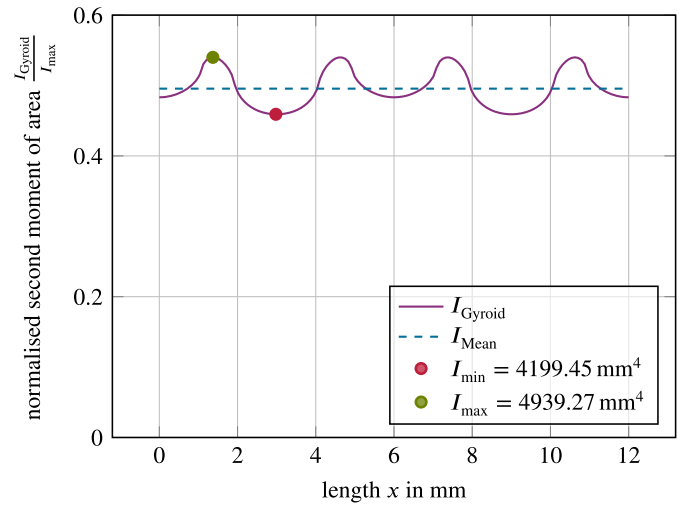


Fig. 10. Second moment of area curve of the specimen configuration G-12-20 over the length of one unit cell of the sandwich panel in Fig. 8.

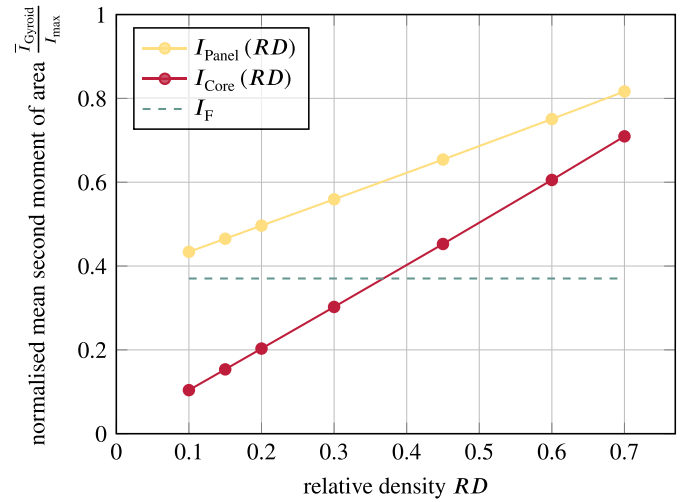


Fig. 11. Mean second moment of area of every specimen configuration for the sandwich panels and the sandwich core over the relative density RD along with the second moment of area of the facesheets I_F .

specimen configuration (see Table 4) results in functions for the second moment of area depending only on the relative density of the sandwich core and the sandwich panel.

Fig. 11 shows the curves of the computed mean second moment of area for the whole sandwich panel and the core only, normalized by the corresponding second moment of area of the solid body. This normalization causes the difference in slope. The algorithm did not show a notable difference of the second moment of area of different unit cell lengths at the same relative density. Between the relative densities 0.1 to 0.7, the curves are described with the Eqs. (18) and (19), showing a linear relationship. In contrast, the semi-empirical model EI_{SE} follows a polynomial trend due to the scaling power laws, while the facesheet model EI_F considers a constant value. To validate which models fits the actual bending behavior the best and if the flexural stiffness follows a linear trend, the experimental flexural stiffness has to be compared to the theoretical models.

$$\frac{I_{Core}}{I_{max,Core}} \approx 1.00595 RD + 0.0009 \quad R^2 = 0.99995 \quad (18)$$

$$\frac{I_{\text{Panel}}}{I_{\text{max,Panel}}} \approx 0.63565 RD + 0.36865 \quad R^2 = 0.99994 \quad (19)$$

Besides the comparison of the theoretical and experimental flexural stiffnesses, the theoretical models for EI_N , EI_F , EI_{SE} can also be used in Bernoulli's beam Theory to approximate the force-deformation behavior of the test specimens in the linear-elastic region. This acts as a connection between experimental force-deformation and theoretical data, which allows a statement about the quality of the approximation gained by using Bernoulli's Beam Theory. According to Böge and Böge [50], the theoretical deformation f_B of a beam under a four-point bending load is described with Eq. (20) depending on the force F , the support spacing L_L , and the difference between the support stamp and force stamp L_B , depending on the used loading configuration. Most important, the denominator EI_i represents the flexural stiffness calculated using the semi-empirical (SE), facesheet (F) or numerical (N) model.

$$f_B = \frac{FL_L^3 L_B}{8EI_i L_L} \left(1 - \frac{4L_B^2}{3L_L^2}\right), \quad i = F, SE, N \quad (20)$$

With the defined specimen design methodology, the manufacturing, the test set-up, and the theoretical approximation of the flexural stiffness, the basis for conducting the four-point bending tests is set. To validate the newly developed bitmap-based second moment of area, the values for the experimental flexural stiffness are calculated from the obtained force-deformation data and compared to the ones calculated with the semi-empirical, facesheet and numerical models. The theoretical models allow then using Bernoulli's beam theory from Eq. (20) to estimate the sandwich beam's theoretical deformation by substituting EI_i with the flexural stiffness models. The theory's accuracy for TPMS-based sandwich panels is determined by comparing the estimated and experimental deformation, giving an insight into the applicability.

4. Results and discussion

This section presents and discusses the four-point bending test results, the comparison between the experimental and theoretical flexural stiffnesses, and the approximation of the force-deformation behavior using Bernoulli's Beam Theory in combination with the flexural stiffnesses from the numerical approach (EI_N), the semi-empirical approach (EI_{SE}) and only from the facesheets (EI_F). The results serve not only to validate the proposed bitmap-based approach but also to demonstrate its potential as a method for estimating the force-deformation behavior without relying on resource-intensive numerical simulations or expensive physical testing. The bitmap-based approach creates new possibilities for the design and analysis of large and geometrically complex core structures, such as functionally graded cores.

To determine the experimental flexural stiffness, the manufactured specimens were successively tested in the *Third-Point Loading* and *Quarter-Point Loading* configurations. During these tests, plastic deformation was avoided to ensure reliable data for calculating the equivalent flexural stiffness EI_{eq} as defined in Eq. (7). The force-deformation data obtained from the *Third-Point Loading* configurations was used to determine the flexural modulus as defined in Eq. (13). After that, destructive tests on the specimens were carried out to obtain the maximum load. From the beginning of the plastic region, the modelling approaches are not valid anymore.

Fig. 12 shows the maximum force measured for each specimen during the destructive tests, categorized by the unit cell length l_{uc} and the relative density RD . The error bars show the standard deviation of the mean maximum force within each specimen configuration per unit cell length l_{uc} . The overlapping standard deviations indicate an independence between the maximum force and the unit cell length. An increasing maximum force with a higher number of unit cells ($l_{uc} = 6$ mm) or larger unit cells ($l_{uc} = 12$ mm) is not observed. Both the mean force and the standard deviation remain on a similar level, increasing only with

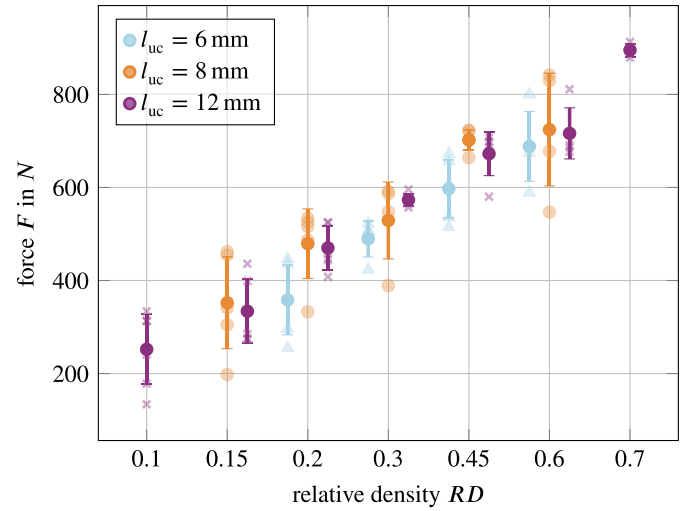


Fig. 12. Data for the maximum force obtained for every specimen, along with error bars representing the mean value and standard deviation of the maximum force.

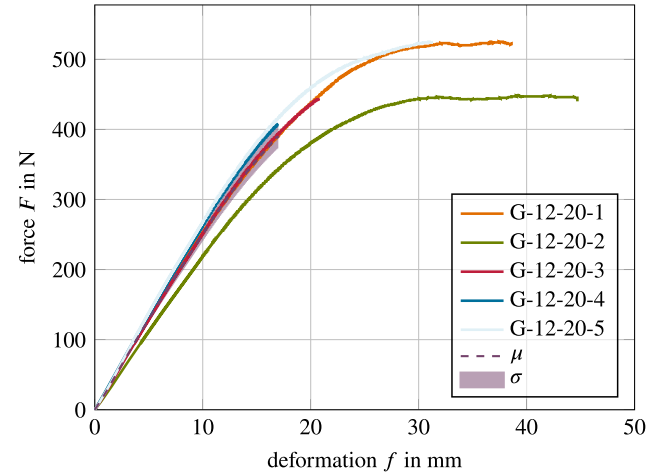


Fig. 13. Force-deformation curves, mean value μ and standard deviation σ of the specimen configuration G-12-20.

the relative density. This independency provides the basis to use all specimen configurations at one relative density for the calculation of the experimental flexural stiffness, regardless of the unit cell length. In addition, the models EI_N , EI_F and EI_{SE} are also valid to be only dependent on the relative density.

Next to the independence of maximum bending force and the number of unit cells, Fig. 12 shows a scatter in the maximum force within the individual specimen configurations. The force-deformation data in Fig. 13 allows a more detailed view of the configuration G-12-20 with the unit cell length $l_{uc} = 12$ mm as a representative example. The data shows a scatter in the maximum force, the maximum deformation as well as in the slope of the force-deformation curve. This might result from manufacturing-related effects in the SLS process like the obtained distortion. Since this study focuses on application-related findings, the optimization of manufacturing parameters and processes is beyond its scope. Other studies looked deeper into this field, like [51] and [52]. With the amount of specimens, the obtained data is considered sufficient for statistical evaluation of the real load capacity of the manufactured sandwich panels.

4.1. Flexural stiffness

This section presents the flexural stiffness EI_{eq} calculated from the experimental data obtained from the *Third-Point Loading* and *Quarter-Point Loading* configurations. The section then compares the experimental flexural stiffness to the numerical EI_N , facesheet EI_F , and semi-empirical EI_{SE} flexural stiffness models to validate the bitmap-based algorithm. Additionally, the flexural stiffness models are employed for modeling the bending behavior using Bernoulli's beam theory, and the errors between the different models and the actual force-deformation curve are compared. Based on the comparison, it can be assessed whether Bernoulli's theory is a suitable method for accurately representing the force-deformation behavior.

Experimental flexural stiffness

Fig. 14 shows the relative flexural stiffness for all specimen configurations over the relative density RD , accompanied by the confidence intervals for each relative density at a confidence level of $CI = 0.95$. The overlaid line represents the results from the numerical model of the flexural stiffness EI_N , which is used to compare the experimental flexural stiffness with the numerically computed one to validate the bitmap-based second moment of area approach by Lohuis et al. [6].

Notably, one specimen at the relative density in the configuration G-12-10 at $RD = 0.1$ exhibits a comparatively high flexural stiffness despite its low relative density, while one specimen at a relative density $RD = 0.45$ in the configuration G-6-45 shows a relative flexural stiffness $EI_{eq} > 1$. It is likely that the reason for this anomaly originates from the employed procedure of the standard ASTM 7250 [19] and the provided calculation guidelines. Examining the composition of Eq. (7) for the calculation of the flexural stiffness EI_{eq} reveals that the numerator does not depend on the deformation and increases proportionally with the force F . In contrast, the denominator incorporates the deformation ratios between the two different loading configurations f_1 and f_2 along with the force F , which is considered to have a higher influence on the calculated flexural stiffness. The ASTM 7250 standard mentions a procedure by modeling the force-deformation behavior as linear function using linear regression and using the linear equations obtained for the calculation of the flexural stiffness EI_{eq} . For a second-order linear equation system of force-deformation with the same intercept, the slope ratio equals the deformation ratio in Eq. (21). Examining the slope ratio of all tested specimens, it is observed that specimens G-12-10-3 and G-6-45-4 with significantly higher relative flexural stiffness and $EI > 1$ exhibit a slope ratio that lies outside the standard deviation range of the slope ratios of all specimens.

$$\frac{m_{3PL}}{m_{4PL}} = \frac{f_{3PL}}{f_{4PL}} \quad (21)$$

One reason for the slope ratios lying outside the standard deviations can be small damages that occurred during the 3PL test and led to a smaller slope during the 4PL test, resulting in a higher slope ratio than usual. This higher slope ratio lowers the denominator of Eq. (7) and leads to a higher calculated flexural stiffness EI_{eq} . However, during a physical inspection of the test specimens after the 3PL test, no visible damage could be observed. Therefore, only the specimen G-6-45-4 will be excluded from the following results since its relative flexural stiffness $EI_{eq} > 1$ cannot be explained by the principles of the underlying mechanical models.

In addition to this, Fig. 14 also shows relatively low values of the experimental bending stiffness, which lie outside the confidence intervals, especially at lower relative densities of $RD = 0.1 - 0.3$. These values correspond to the specimen with the lowest force-deformation slope and could be explained by the distortion in Fig. 5. The biggest distortion was measured at these lower relative densities.

With the confidence intervals for each relative density, Fig. 14 shows the quality of the newly developed bitmap-based numerical model EI_N

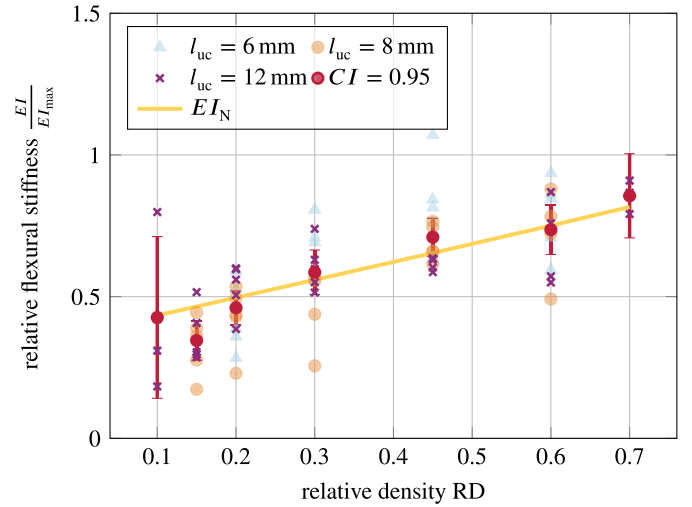


Fig. 14. Relative flexural stiffness of all specimen configurations depending on the relative density along with confidence interval per RD and the theoretical model EI_N .

for flexural stiffness. The model EI_N matches all of the confidence intervals except for the one at a relative density of $RD = 0.15$. The confidence intervals at relative densities of $RD = 0.1$ and $RD = 0.7$ are notably larger compared to the others, which can be explained by a lower number of statistical specimens at these relative densities. This increases the uncertainty in the region of these relative densities. Nevertheless, the data that is available from the specimens that were manufacturable shows that the model seems to fit even at these boundary regions. Matching all confidence intervals except for $RD = 0.15$ shows that the bitmap-based numerical model for the flexural stiffness EI_N delivers a good fit for approximating the flexural stiffness of sandwich panels for a Gyroid core with its linear trend.

Semi-empirical and numerical flexural stiffness

Fig. 15 extends the comparison between the experimental and theoretical flexural stiffness by showing the experimental flexural stiffness, the numerical model EI_N based on the second moment of area and the semi-empirical models EI_{SE-T} and EI_{SE-A} by using the scaling laws of Traub (T) [11] and Abueidda et al. (A) [31] to compare the different models to each other. Moreover, a straight line EI_F shows the facesheet flexural stiffness calculated by Eq. (6) in the case of a known flexural modulus according to the standard ASTM 7250 [19]. Comparing all these models to the experimental data shows that the bitmap-based numerical model EI_N provides the most accurate fit for higher relative densities $RD = 0.2$ to 0.7 . Only at relative densities from $RD = 0.1$ to 0.15 the models EI_{SE-T} , EI_{SE-A} and EI_F provide a higher accuracy, so that the sandwich core could be neglected here for the approximation of the flexural stiffness.

While the numerical approximation shows a linear relationship between relative density and flexural stiffness, the semi-empirical models display a non-linear relationship. This non-linearity comes from power laws for the core's elastic modulus E_c in Eq. (14). The constant flexural stiffness of the facesheets EI_F matches with the confidence intervals of the experimental values for relative densities in the range of $RD = 0.1$ to 0.15 . A few values of the experimental flexural stiffness are higher than the one for EI_F . This indicates that at these low relative densities, the core has a low but recognizable impact on the flexural stiffness. For higher relative densities $RD > 0.15$, only considering EI_F leads to an increasing deviation from the experimental values, which indicates that the core contributes to the overall flexural stiffness of the sandwich panel. In this range, Eq. (6) becomes inadequate for accurately capturing the bending behavior.

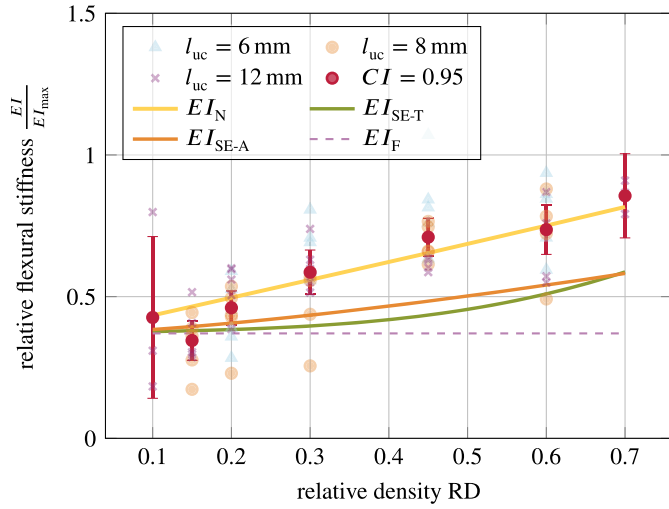


Fig. 15. Experimental flexural stiffness for all unit cell lengths along with the numerical model EI_N and semi-empirical model EI_{SE} with the models of Traub et al. EI_{SE-T} and Abueidda et al. EI_{SE-A} .

The semi-empirical models match the confidence interval at a relative density of $RD = 0.15$. Beyond this range, the predicted flexural stiffness becomes lower than the experimentally calculated values and the numerical model provides a higher accuracy, matching all confidence intervals. The model employing the scaling power law of Abueidda et al. [31] exhibits a higher slope than Traub [11], which matches the confidence interval at $RD = 0.2$. The differences in slope may be explained by the methodologies used to compute the scaling power laws: while Abueidda et al. [31] used simulation data, Traub [11] developed his model based on experimental compression tests on SLA-manufactured cubic specimens. As in this study, the load capacity may be affected by production-induced factors, resulting in lower values for the elastic modulus E_c of the core.

Summarizing the examination of the flexural stiffness curves for the different models and the way they match the experimental data, rating the bitmap-based approach of computing the second moment of area as a reliable approach to calculate the flexural stiffness EI_N is valid. However, at low relative densities, the bitmap-based approach tends to overestimate the core's influence on the overall flexural stiffness. As the specimens are manufactured from AM, defects from layer adhesion or also from the melting or sintering process are expected like they are also known from similar processes. Because of this, overestimations are a known issue when considering AM structures [53]. Especially the thin walls at low relative densities areas are prone to these defects as less supporting material is present that could transfer the loads. Therefore, the core is not as supporting as expected from calculations and models not considering the core are closer to the real values.

Quality of approximation

By using the different models of the theoretical flexural stiffnesses to describe the force-deformation behavior of the sandwich panels with Bernoulli's beam theory, an alternative viewpoint on the same underlying mechanical behavior is provided. This procedure aims to evaluate how well each model approximates the experimental force-deformation response across different relative densities to validate if Bernoulli's Beam Theory can be used for this type of applications.

For a better comparison between the Bernoulli-based approximations, which are only valid in the linear-elastic region, and the experimental data, the mean curve of every individual configuration test series is approximated by a straight line applied to the area of the steepest slope to get an elastic line of the force-deformation curves. This elastic line is a reference for comparing the approximated models. According to that, Fig. 16 shows the mean curve μ and the corresponding stan-

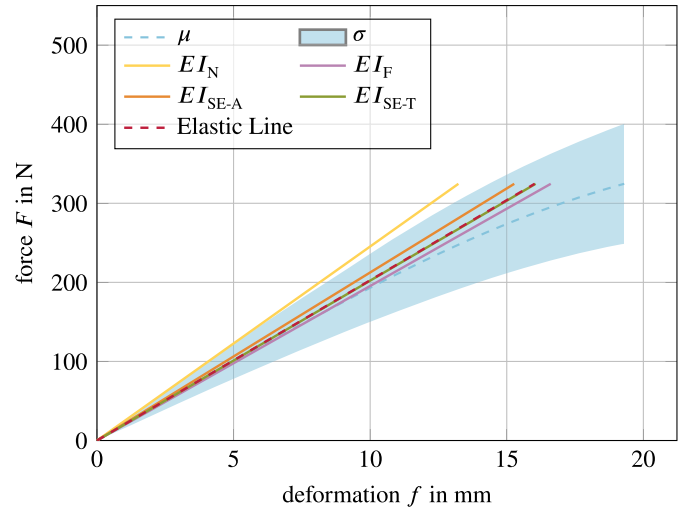


Fig. 16. Mean deformation μ of the series configuration G-8-15 along with the elastic line and the approximations using the Bernoulli Beam Theory.

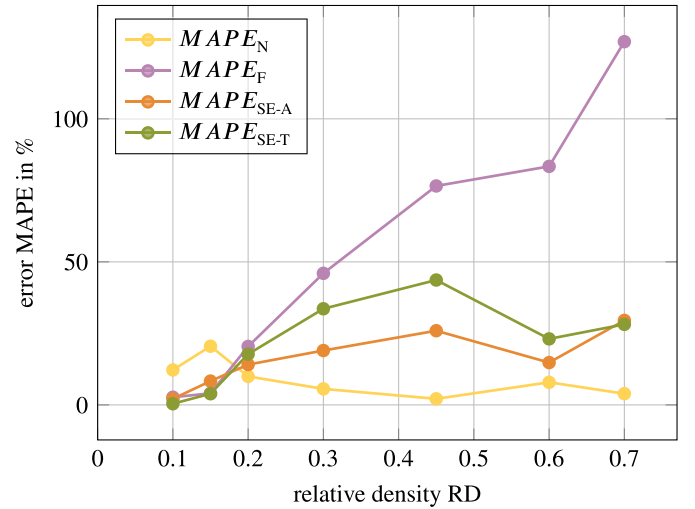


Fig. 17. Error MAPE for the numerical and semi-empirical models over the relative density RD.

dard deviation σ of the test series for the configuration G-8-15 along with the elastic line at the steepest slope. Four further lines describe the theoretical deformation of Bernoulli's beam theory resulting from the numerical, semi-empirical, and facesheet flexural stiffness of the models EI_N , EI_{SE-A} , EI_{SE-T} , and EI_F .

The Mean Absolute Percentage Error (MAPE) allows a qualitative comparison of the individual approximations. The MAPE provides a relative value to evaluate the quality of the approximation. In the context of the present study, the MAPE is calculated using Eq. (22), where f_i represents the actual deformation of the elastic line at the data point i , and $\hat{f}_i$ denotes the approximated value at the same data point [54].

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{f_i - \hat{f}_i}{f_i} \right| \times 100 \quad (22)$$

Fig. 17 shows the progression of the mean MAPE of all unit cell lengths over the relative density RD for each of the individual models, while Table 5 presents the corresponding actual values for the MAPE.

A direct comparison of the approximation models reveals that the models EI_{SE-T} , EI_{SE-A} , and EI_F initially describe the actual deformation for low relative densities $RD = 0.1$ to 0.15 more accurately than the numerical model EI_N , which is consistent with the observation from

Table 5

Actual values of the MAPE in % for each of the models and relative densities, visible in Fig. 17.

RD	EI_N	EI_F	EI_{SE-A}	EI_{SE-T}
0.1	12.22	2.69	2.11	0.41
0.15	20.51	4.04	8.33	3.91
0.2	9.96	20.43	14.10	17.75
0.3	5.61	45.99	19.10	33.61
0.45	2.13	76.53	25.93	43.65
0.6	7.90	83.38	14.80	23.08
0.7	3.92	126.97	29.54	28.16

before that the numerical model EI_N does not match the confidence interval at these relative densities.

Between the models of EI_{SE-T} , EI_{SE-A} , and EI_F , the latter shows a slightly higher error for the relative densities $RD = 0.1$ – 0.15 than the semi-empirical models. The corresponding plot in Fig. 16 of the configuration G-8-15 shows that the curve of the model EI_F is lying under the mean elastic line while the curves for the models EI_{SE-A} and EI_{SE-T} are lying over and nearly on the mean elastic line. This observation is following Fig. 15 where the calculated flexural stiffness at a relative density of $RD = 0.1$ to 0.15 is slightly higher for the models using the sandwich beam theory with Eq. (14) and therefore including the core, compared to the model EI_F which is only considering the facesheets.

While the model EI_F does not take the sandwich panel core into account, the models EI_{SE-T} and EI_{SE-A} incorporate the core via scaling power laws. However, at these low relative densities, as shown in Fig. 15, the impact of the core on the flexural stiffness is very low. Considering the whole core leads to a higher flexural stiffness for the relative densities $RD = 0.1$ to 0.15 , which cannot be represented by the available experimental data, causing a lower accuracy using the numerical model EI_N .

For relative densities $RD \geq 0.15$, the numerical model shows a higher accuracy than the semi-empirical and facesheet models. The model EI_N achieves consistently high accuracy for relative densities in the range $RD = 0.2$ to 0.7 , with a MAPE between 2.13% and 9.96%. The models EI_{SE-T} , EI_{SE-A} , and EI_F exhibit an increasing deviation between the theoretically calculated and the actual deformations with a continuously increasing MAPE, which reaches up to $MAPE = 126.97\%$. The high MAPE results from Eq. (6) neglecting the sandwich panel core's contribution to the overall flexural stiffness since it is likely that the core affects the flexural stiffness significantly at high relative densities. Consequently, for $RD \geq 0.2$, the model EI_F is not considered a valid option for approximating the bending behavior of TPMS-based sandwich panels at these high relative densities.

In contrast to the continuously increasing error of the model EI_F , $MAPE_{SE-T}$ and $MAPE_{SE-A}$ remain consistently lower than the values from only considering the facesheets, caused by the sandwich beam theory, which explicitly considers the core, providing a more accurate approximation of the bending behavior. Due to the higher slope of the model EI_{SE-A} than EI_{SE-T} , its error is continuously lower. Even though with values from 14.10% to 29.54% at relative densities $RD = 0.2$ to 0.7 the $MAPE_{SE-A}$ remains relatively low, the $MAPE_N$ provides continuously higher accuracy. Therefore, the bitmap-based numerical model EI_N with a linear trend is considered as the one which is approximating the force-deformation behavior in the most accurate way.

For higher relative densities, the model EI_N demonstrates significant advantages over the classical models, and even at lower relative densities, its performance remains reasonable with a $MAPE \leq 21\%$, suggesting its applicability across the whole range of relative densities. Especially at higher relative densities $RD \geq 0.2$, EI_N achieves consistently higher accuracy than the commonly used models, resulting in increased certainty regarding the structural behavior. At these high relative densities, the channel size led to more issues with cleaning the residual powder from

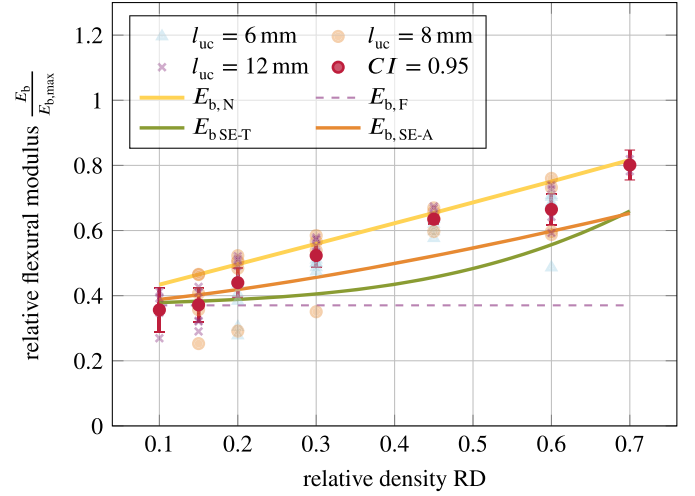


Fig. 18. Experimental flexural modulus for all unit cell lengths along with the numerical model $E_{b,N}$ and semi-empirical models $E_{b,SE-A}$ and $E_{b,SE-T}$ and the facesheet model $E_{b,F}$.

the specimens as mentioned before. The good fit of the numerical model with the experimental data supports the assumption, that the powder in the channel does not influence the mechanical behavior significantly, as the calculations only consider the area designed as solid.

Independent of the relative density used, this approach requires no experimental data, unlike the scaling power laws employed in the models based on sandwich beam theory, enabling the estimation of the structural behavior of any sandwich panel with minimal effort. The bitmap algorithm is theoretically able to transfer any cross-section into a bitmap to calculate the second moment of area, offering high transferability to sandwich panels with individual core structures and dimensions.

Flexural modulus

Another important parameter that is derived from bending tests is the flexural modulus. Using Eqs. (10) and (12), the flexural stress and strain are calculated from the force-deformation data. Then, the slope of a straight line fitted to the region of the steepest ascent of the stress-strain curve represents the flexural modulus according to Eq. (13).

Modeling the theoretical deformation using Bernoulli's beam theory and the numerical and semi-empirical flexural stiffness enables the transformation of theoretical force-deformation data into theoretical flexural stress and strain data, yielding a theoretical flexural modulus.

Fig. 18 shows the experimental flexural modulus for each specimen configuration in dependency of the relative density, along with the corresponding confidence intervals and the numerical, semi-empirical and facesheet flexural moduli $E_{b,N}$, $E_{b,SE-A}$, $E_{b,SE-T}$, and $E_{b,F}$. The experimental flexural modulus increases linearly with the relative density. The theoretical curve based on the model EI_N matches the confidence intervals at relative densities $RD = 0.3$, $RD = 0.45$, and $RD = 0.7$, although it shows a continuous offset relative to the experimental data. In contrast, the semi-empirical models $E_{b,SE-A}$, $E_{b,SE-T}$, and the facesheet model $E_{b,F}$ align with the confidence intervals at lower relative densities ($RD \leq 0.2$). A similar trend appeared in the flexural stiffness results. Notably, Fig. 18 also demonstrates that the model EI_{SE-A} achieves higher accuracy at elevated relative densities in comparison to EI_{SE-T} , a result attributable to its use of a higher scaling power law, which enhances the consideration of the sandwich panel core.

In general, the flexural modulus is a material parameter. In contrast to this, this study evaluates structures, not solid materials. The benefit from this point of view is that complex structures like the Gyroid can be considered as surrogate materials with their bending properties calculated by the proposed bitmap approach. The trend shown in Fig. 18 leads to the assumption that also the flexural modulus of TPMS-based sand-

wich panels shows a linear behavior with an increasing relative density in both the experimental data and the bitmap-based calculations. This good fit makes it possible to use the latter for predicting bending properties instead of extensive testing or complex simulations.

For the overall context of suction panel design and calculation, both the viewpoints flexural stiffness and flexural modulus show the capability of the bitmap-based approach for approximating the bending behavior. For the presented case of the Gyroid structure, both parameters were predicted with high accuracy. Especially for regions with higher relative density, the approximation results were good where classical approaches for sandwich calculation failed. This paves the way for using it in the design of TPMS sandwich structures like the mentioned suction panels, but also in simulations as complex TPMS structures can be simplified as surrogate materials. In addition, the working principle of the algorithm this approach is based on enables the transfer to other structures as it computes the moment of area from sectional views and pixel positions. Based on the working principle, the algorithm is efficient for periodic structures, as for them only a smaller section has to be evaluated. This can be tested on the TPMS structures, but also on structures with a completely different topology. As there are many newly developed cellular structures, especially from bionics like honeycombs or specific ones like cedarwood-inspired cells [55], this can be of interest in new applications.

5. Conclusion

In this study, a novel bitmap-based approach for calculating the flexural stiffness of sandwich panels with a complex core structure was validated. Extensive four-point bending tests of sandwich panels with a TPMS core were conducted, varying the relative density between 0.1 and 0.7 to determine the experimental flexural stiffness and compare it to the bitmap-based flexural stiffness. The specimens were manufactured using the additive manufacturing process selective laser sintering. With a successful validation, the bitmap-based approach enables the numerical estimation of the flexural stiffness by determining the second moment of area directly from the geometry of arbitrarily structured sandwich panels, eliminating the need for prior experimental tests.

To validate the bitmap-based numerical model, two more approaches for approximating the flexural stiffness used in studies with a similar scope were introduced. These approaches include one only considering the facesheets and a semi-empirical one which are referred to as the facesheet and sandwich beam theory model. The comparison of experimental results with these three theoretical models showed that the bitmap-based approach provides more accurate predictions, particularly at higher relative densities, outperforming the sandwich beam theory and facesheet model. With this, the proposed method can take into consideration sandwich core structures with high density, where the core contributes to carrying bending loads. This is normally left out in current sandwich theories, making the bitmap approach a possibility to bridge that gap. Its working principle based on slicing the structure and basic mechanical laws enables the applicability of the bitmap-based calculation of the second moment of area for predicting the bending behavior of sandwich panels with different core structures over a wide range of relative densities as in functionally graded TPMS sandwich panels, but not limited to these. At lower relative densities, the core exhibited a reduced contribution to the flexural properties, where the two additional approaches provided a higher accuracy, showing some limitation of the numerical bitmap-based model at lower relative densities.

Next to the theoretical computation of the flexural stiffness and the comparison to experimental values, this allows for direct application within Bernoulli's Beam Theory to approximate the deformation behavior of sandwich panels across the linear-elastic region. This opens the possibility of using the approach for efficient FEM-based material modeling with a surrogate for the complex TPMS structure, early-stage design evaluations, and custom core structure optimization in lightweight applications.

Future work can extend the applicability of the bitmap-based approach to other TPMS structures or multi-material composite structures, therefore validating its performance across different manufacturing techniques. The algorithms performance for these structures will have to be tested to investigate the limitations. Especially for less periodic structures the calculation of the mean flexural stiffness may be a challenge. However, for periodic structures or symmetric cross sections, the approach is expected to work efficiently. Furthermore, a normalization of bending properties, independent of material and process parameters, provides the basis for significant improvement in comparability between studies and enables robust, transferable material data for structural investigations. Regarding the suction panel, the section-wise calculation can be applied to their more complex composition, with curvature, functionally graded core structures and perforations in the skin.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT to suggest improvements in language and readability. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

CRediT authorship contribution statement

Jann Schöneich: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization; **Jan Kube:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Project administration, Methodology, Investigation, Conceptualization; **Hendrik Traub:** Writing – review & editing, Visualization, Validation, Supervision, Software, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization; **Christian Hühne:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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