

A Nonlinear Dynamics Phase Oscillator Model for the Simulation of Multistable Perception

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Abstract

A nonlinear dynamics model of multistable perception and numerical simulations of the quasiperiodic perception reversals due to ambiguous stimuli are presented. The perception state is formalized as the phase variable of a recursive cosinoidal map with the two control parameters: difference of meaning μ of ambiguous stimulus, and attention parameter G . G is assumed proportional to adaptive feedback gain g of a corresponding reentrant thalamocortical circuit (Edelman 2004). Coupling of the perception and attention dynamics with delayed phase feedback (delay T) enables transitions between chaotic and limit cycle attractors representing the perception states. Eigenfrequencies of the limit cycle oscillations are in the range of 10 – 100 Hz, in agreement with typical EEG frequencies. Quasiperiodic perceptual reversals are induced by adaptation (satiation) of attention. The coupled attention – perception dynamics reproduces the experimentally observed Γ -distribution of the reversal time statistics if a stochastic noise term is added to the attention equation. Mean reversal times of typically 3 – 5 s as reported in the literature, are correctly predicted if $T \approx 40$ ms, corresponding e.g. to the delay between stimulus onset and primary visual cortex (V1) response. Numerically determined perceptual transition times of ca. 5 T are in reasonable agreement with stimulus – conscious perception delay of 150 – 200 ms (Lamme 2003).

Introduction

Perceptual multistability is observed in different forms such as figure – ground reversal, rival – schemata reversal and perspective reversal, as induced by ambiguous pictures such as the Necker cube (Borsellino 1972). Another bistability effect is binocular rivalry as observed when both eyes are subject to a different stimulus (Blake & Logothetis 2002). Also dynamic stimuli may give rise to cognitive multistability, e.g. ambiguous motion displays such as plaids as induced by moving groups of crossed lines (Hupé & Rubin 2002). An overview and discussion in terms of the Praegnanz principle of Gestalt psychology was presented by Attneave (1971). He proposed the existence of a cognitive positive feedback loop with locking into the alternative schemata and fatigue of associated different neural structures. Multistability opens a unique perspective in consciousness research because with a constant visual input pattern switching between different consciously perceived interpretations can be investigated (e.g. Engel et.al. (1999), Leopold & Logothetis (1999), Srinivasan et.al. (1999)).

The present model contributes to the controversial discussion on the deterministic versus purely stochastic character of cognitive multistability. It provides a macroscopic approach for explaining the experimental

finding that deterministic (even chaotic) as well as stochastic dynamics contributes to measured reversal time statistics for different multistability phenomena (Richards et al. 1994). The model assumes a recursive perception dynamics (e.g. Lamme 2003) which is related to the reentrant interactions between distant neural groups of the dynamical core hypothesis of consciousness (Tononi, & Edelman 1998). The model is closely related to the mean field phase oscillator theory of coupled neuronal columns in the visual cortex (Schuster & Wagner 1990). The latter was used for modeling the synchronization of neuronal oscillations as the physiological basis of dynamic temporal binding which in turn is thought to be crucial for the selection of perceptually or behavioraly relevant information (Engel et al. 1999)((Engel et.al. 2001). Self oscillation of neuronal groups within columns and coupling between columns is excited when the external stimulus exceeds a certain threshold (Schuster & Wagner 1990). Single columns exhibit multistable characteristics of the neuronal mean field as function of the stimulus, similar to the present model. Within the phase synchronization theory phase locking between two different groups of neurons is described by means of the circle (sin) map. Phase oscillator dynamics is also the basis of Kelso's phase attractive circle map (deGuzman & Kelso 1991) which was used for describing human coordination dynamics as well as multistable perception (Kelso et.al. 1995). A continous bistable perception model of Ditzinger & Haken (1995) without delay is based on the 1st order 4th degree polynomial dynamics of two coupled perception states as order parameters which slave the neuronal microstates and which are coupled to the dynamics of two attention control parameters. The present approach models the recursive character of the neuronal processes (Lamme 2003) on a macroscopic basis by means of perception (v) - attention (G) coupling with delay T and attention fatigue (Orbach et al 1963) with time constant γ . Fatigue is assumed to be due to sensory gain control as described by Hillyard et.al. (1999) as mechanism for selective attention. Phase oscillator dynamics is realized by a recursive cosinoidal map as originating from mean field interference. Recently published experimental results of Nakatani & van Leeuwen (2005) support the perception – attention coupling approach.

In the following section 2 I describe the nonlinear dynamics model, followed by an analysis of the stationary behavior in section 3. Simulated time series are shown in section 4. A statistical analysis of the reversal time intervals in section 5 exhibits good agreement with published experimental data. A conclusion and outlook is presented in section 6.

The Recursive Mean Field Interference Model

As a kind of minimum architecture allowing for quasiperiodic perceptual reversals, I have proposed in previous papers coupling of the attention and perception dynamics via delayed phase feedback and attention satiation (Fürstenau 2003)(Fürstenau 2004). The feedback loop which couples the perception state $v(t)$ and attention parameter $G(t)$ may be compared with the reentrant thalamo-cortical and basal ganglia dynamics (Robinson 1998) as employed by Edelman (2004) within the dynamical core hypothesis of consciousness: corresponding to the thalamocortical loops, recursive interference between neuronal coherent perception fields creates alternative percepts through phase locking which in turn modulates the adaptive feedback gain. This picture is in agreement with Hillyard et.al. (1999) who find stimulus-evoked neuronal activity to be modified by an attentional induced additive bias or by sensory gain modulation. Increase of gain is physiologically correlated with increased blood flow through the respective cortical areas. Attention bias in the present model is determined by perception offset v_b . Attention induced gain modulation is modelled by adaptive feedback gain $g(t)$ which is assumed to be proportional to $G(t)$. Recent experimental evidence on perception – attention coupling with ambiguous stimuli was presented by Nakatani & van Leeuwen (2005) using EEG recording of frontal theta and occipital alpha bands and eye blink rate measurement. Accordingly in the present model, like in (Ditzinger & Haken 1995), the adaptive attention control parameter $G(t)$ induces the quasi - discontinuous transitions between the alternative stationary perception states P1 and P2, through attention fatigue (Orbach et.al. (1963)).

Formally this is achieved analogous to multistable optical systems (Watts & Fürstenau 1989)(Fürstenau 1991). Interference with contrast μ ($0 \leq \mu \leq 1$) is the superposition of (electromagnetic) fields $a_{0i} \exp\{j(\omega t + \Phi_i)\}$, $i = 1, 2$ with ω = frequency and Φ = phase. The superposition yields extinction or amplification of each other, depending on the relative phase shift $\Delta\Phi = \Phi_1 - \Phi_2$. It may be compared with the phase shift between the coupled self - oscillating neuronal columns of the mean field theory (Schuster 1990). A simplified block diagram is depicted in Figure 1.

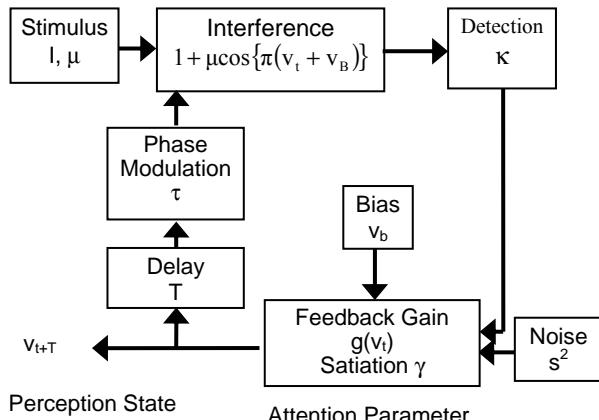


Figure 1: Simplified block diagram of perception – attention recursive interference model. Symbols explained in the text.

An ambiguous stimulus with strength I and difference of meaning μ of the two possible percepts P1, P2, excites two corresponding hypothetical mean fields with phase difference $\Delta\Phi = \pi v_t$. Difference of meaning as quantified by contrast μ corresponds to the degree of coherence of the two fields. For ambiguous stimuli $\mu > 0.18$ (see below) Superposition creates the typical sinusoidal dependence of the output (= squared modulus of the sum of the field amplitudes, with detection conversion factor κ) on the phase difference $\Delta\Phi$ as mapping function. A recurrent process is established by feedback of the output after amplification (feedback gain g , attention bias v_b , satiation (fatigue) and recovery time constants γ and τ_G) with delay T and low pass filtering (time constant τ) into $\Delta\Phi$, via a phase modulation mechanism. As a quantitative estimate for T I chose the stimulus – primary visual cortex response delay (≈ 40 ms, (Lamme 2003)). One possibility for phase feedback is frequency modulation of the input field (Fürstenau 1991), comparable to the stimulus induced modulation of the neuronal mean field limit cycle oscillations (Schuster & Wagner 1990). The normalized output $v_t = U_t / U_\pi$ of the feedback circuit with $d\Phi/dU = \pi/U_\pi$ and $U \sim$ percept intensity = |superimposed percept field strength|² defines the phase variable v as perception state and synergetic order parameter. A similar recursive (discrete) phase oscillator mapping function (phase attractive circle map) was proposed by Kelso et.al. (1995), however without using feedback gain as control parameter.

Formally the model is described by coupling a 1st order differential delay equation for $v(t)$ (assuming a strongly damped feedback system) with a linear equation for the control parameter dynamics $G(t)$. Random disturbances due to dissipative processes are modelled by adding a stochastic force $L(t)$ with Gaussian white noise (variance s^2) to the attention equation $G(t)$, similar to (Ditzinger & Haken 1995).

$$\tau \dot{v}_{t+T} + v_{t+T} = G[1 + \mu \cos(\pi(v_t + v_B))] \quad (1a)$$

$$\dot{G}_t = (v_b - v_t)/\gamma + (G_{off} - G_t)/\tau_G + L_t \quad (1b)$$

The rhs. of equ. (1a) describes the conventional interference between two coherent fields. In what follows I assume the phase bias $v_B = 0 \bmod 2$. The attention parameter $G(t) = \kappa I g(t)/U_\pi$ with phase – voltage modulation factor $d\Phi/dU = \pi/U_\pi$ is the product of feedback gain $g(t)$ and input (stimulus) I . The attention dynamics is determined by perception bias v_b (determining the relative preference of P1 and P2), satiation speed $1/\gamma$, recovery time τ_G and G_{off} = attention (gain) parameter with stimulus off, defined by $\mu = \mu_{off} < 0.18$ (see below).

Stationary Solutions

Two types of instabilities are observed with recursive systems described by equation (1a): period doubling and node bifurcation. Figure 2 depicts the stationary solutions including period doubling up to period 8, $v_{t+iT} = v_t = v^*$, $i = 1, 2, 4, 8$.

Period doubling pitchfork bifurcations are observed on both positive slope regions. The graph yields the control parameter values at the first three bifurcation points providing a first approximation to the Feigenbaum constant

$\delta_\infty = 4.6692$ via $\delta^l_\infty \approx (G_2 - G_1)/(G_3 - G_2)$. The period doubling behavior proves that within certain parameter ranges (μ, τ) any system noise has chaotic contributions (Fürstenau 2004).

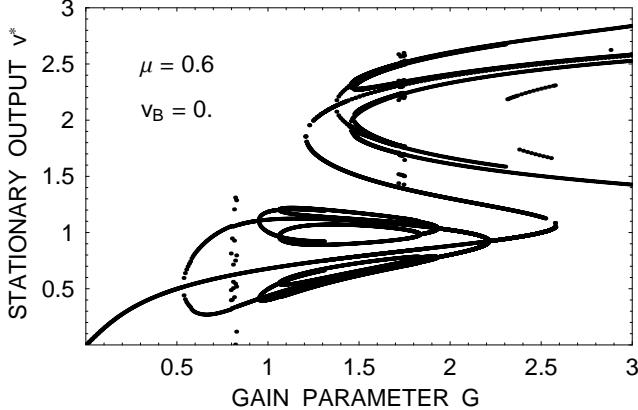


Figure 2: Stationary solutions of equation (1a) with period doubling bifurcations. P1 = lower v^* -level; P2 = higher level.

In (Fürstenau 2003) I have shown the stationary solution of $v^*(\mu, G)$ to exhibit a cusp catastrophe topology. This finding agrees with a proposal of Poston & Stewart (1978) who developed a qualitative deterministic model of cognitive bistability based on catastrophe theory. At the critical value, $\mu_n = 0.18$, node bifurcation is observed and the slope of the stationary system state v^* as function of G becomes infinite. For $\mu < \mu_n$ both percepts are fused into a single meaning. For $\mu > \mu_n$ hysteresis with negative slope sections of the stationary perception state curve $v^*(G)$ is observed and $v^*(G)$ becomes multivalued corresponding to ambiguous perception. For maximum contrast $\mu = 1$ the horizontal slope $(dG / dv)^{-1} = 0$ yields $v_i^\infty = 2i - 1$, $i = 1, 2, 3, \dots$ as stable perception levels in the limit $G \rightarrow \infty$. Node bifurcation is required for explaining the existence of ambiguous perception within the present model. Under increasing stimulus strength I or feedback gain g the stationary (1st order) perception state v^* jumps discontinuously from P1 to P2 at the turning points of the S-shaped hysteresis curve. The transition of P2 back to P1 occurs at a lower stimulus or gain parameter g ($\sim G$) value due to the hysteresis. The width of the instable negative slope section and the multivalued G – range increases with μ . A similar hysteresis is observed for the coupling constants of columns of the visual cortex within the neuronal mean field theory (Schuster & Wagner 1990). For comparison with experimental reversal time measurements, μ – values ($> \mu_n = 0.18$) have to be guessed for the computer simulations by trial and error with the present state of model development. Qualitatively it is predicted that mean reversal times increase with μ (increasing width of the hysteresis) i.e. the difference of meaning of P1, P2.

Simulated Perception – Attention Dynamics

In this section I present numerical solutions of the the coupled differential – delay equations (1a, b) as obtained with the dynamical systems tool Matlab – Simulink (solver

ode23tb for stiff problems). Figure 4 shows results for $I = 1$, $\mu = 0.6$, $\tau/T = 0.1$, $\gamma = 60$, $\tau_G = 500$, attention bias $v_b = 1.5$, noise variance $s^2 = 0.001$, and time scale in units of the simulation interval $T_S = T/2 = 20$ ms, with stimulus – off sections ($\mu_{\text{off}} = 0.1$, $G_{\text{off}} = 1.5$) at the beginning and end of the time series.

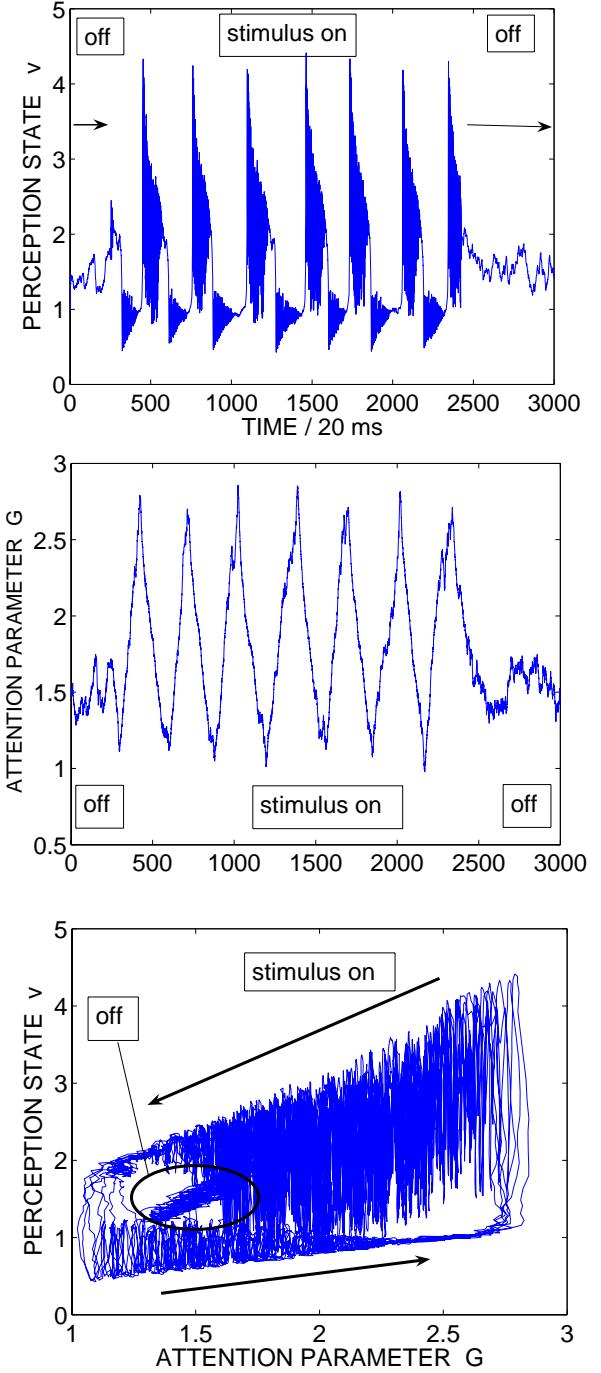


Figure 4: Numerical solution of equ. (1). From top to bottom: time series $v(t)$; $G(t)$; phase space trajectories v vs. G . Stimulus off ($\mu = 0.1$) during initial and final simulation phases. See text for simulation parameters.

The time series of the perception state $v(t)$ shows the spontaneous transitions between stationary states near $v^* \approx 1$ (P1) and $v^* \approx 2$ (P2) with the expected superimposed limit cycle and chaotic oscillations. The transition time between P1 and P2 is of the order of $5 - 10 T_s \approx 100 - 200$ ms, in reasonable agreement with the time interval between (visual) stimulus onset and the beginning of conscious perception (Lamme 2003). The phase space plot v vs. G exhibits separated regions of the stimulus – off and stimulus – on (P1 and P2) states with trajectories of fast oscillations superimposed on the slow satiation (fatigue) dynamics.

The linear stability analysis of equ. (1a) yields eigenfrequencies of limit cycle oscillations $f \approx i / (2T(1-t/T))$, $i = 1, 2, \dots$ in the typical EEG range (< 100 Hz). The evaluation of the Lyapunov coefficient confirms chaotic contributions for $\tau < T$ (Fürstenau 2003)(Fürstenau 2004).

The reversal time period is determined by the satiation and recovery time constants γ, τ_G with an absolute scale given by $T_s = \text{simulation time intervall} = T/2$. Limit cycle oscillations and deterministic chaos within P1, P2 is a characteristic of the individual perception states and has its origin in the finite delay time T . The effect of decreasing T within the recursive perception – attention dynamics is depicted in Figure 5.

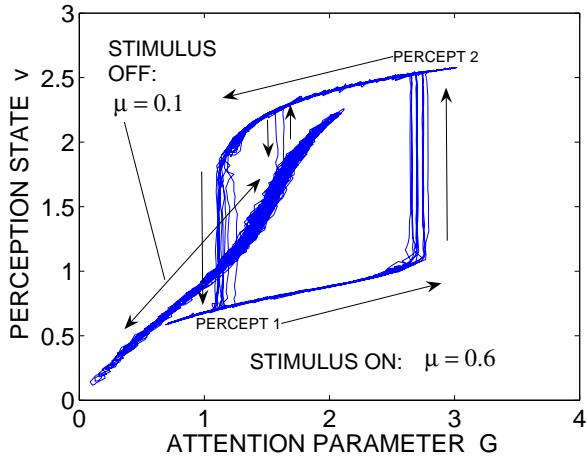


Figure 5: Phase space plot of perception – attention dynamics with zero feedback delay ($T = 0$).

The phase space plot of Fig. 5 clearly shows that with zero delay ($T = 0$) the limit cycle and chaotic oscillations vanish which are superimposed on the stationary perception states of the hysteresis loop of Fig. 4. This leads to a clear separation of stimulus – off ($\mu = 0.1$) and – on ($\mu = 0.6$) states in v – G – phase space. The scattering of the reversal time period, however as indicated by the scattering of the P1 – P2 transitions, appears not to be significantly effected. A quantitative comparison is presented in Table 1 of the following section.

Figure 6 shows how the perception bias v_b determines the relative dominance of one of the two percepts. In this example, after stimulus on ($\mu = 0.1$ is switched to $\mu = 0.6$) the offset $v_b = 0.9$ forces the perception to the lower perception state P1 with suppression of P2. Noise variance

($s^2 = 0.001$). Other parameters are the same as in Figure 4. The relative durations of the dominant and suppressed phase of a percept are determined by the bias parameter v_b . It may be used as a means to qualitatively model experimental results with perception biased towards one of the two percepts as reviewed in (Engel et al 1999).

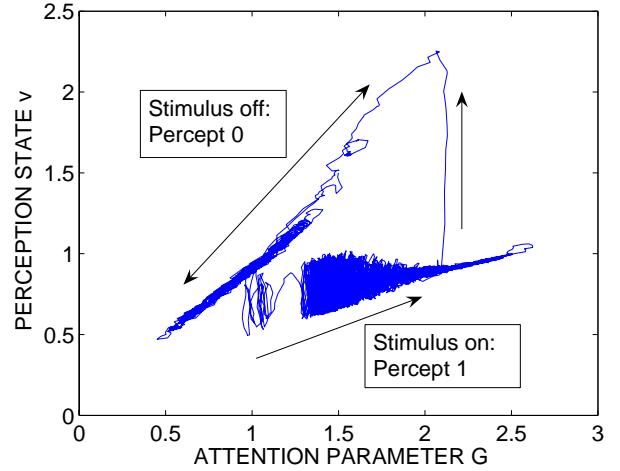


Figure 6: Phase space plot of perception – attention dynamics with attention bias $v_b = 0.9$.

Reversal Time Statistics

Figure 7 depicts the relative frequencies of the perceptual duration times of simulations as obtained by averaging 10 time series of $N = 50000$ iterations each, with $T = 2$, $\tau = 0.2$, $\gamma = 60$, $\tau_G = 500$, $v_b = 1.5$ and $s^2 = 0.03$.

Plotted are the two distributions of the perceptual durations $\Delta(P1)$ of percept 1 and $\Delta(P2)$ of percept 2. As suggested by a number of experimental results (e.g. (Borsellino et.al. 1972 & 1982)(Zhou et.al.2004)((Nakatani & van Leeuwen 2005)) the relative frequencies are fitted by a Γ – distribution as probability density with shape parameter α and scale parameter λ . Mean and variance are given by $\Delta_m = \alpha/\lambda$ and $\sigma^2 = \alpha / \lambda^2$ respectively. For percept P1 and P2 mean and standard deviation are respectively $\Delta_m = 159 T_s$, $\sigma = 95 T_s$ and $\Delta_m = 134 T_s$, $\sigma = 62 T_s$. The ratios $\sigma/\Delta_m \approx 0.4 - 0.6$ are in good agreement with the experimental findings reported in the literature. In contrast to Fürstenau (2003) with purely deterministic time series, and in agreement with Fürstenau (2004) the addition of the stochastic attention noise $L(t)$ in equation (1b) leads to a significant increase of the variance, whereas the mean values remain roughly the same, indicating the dominating influence of the deterministic dynamics on Δ_m . The chaotic contribution to the total variance in the present delay – differential model is significantly smaller as compared to the recursive approximation (Fürstenau 2004). This is in agreement with (Lehky 1995) who detected no significant chaotic contribution in the reversal time statistics of binocular rivalry. It appears that the chaotic "noise" of the perception states P1, P2 is hardly detectable by analysis of reversal time measurements because the variance is dominated by the stochastic (attention) noise.

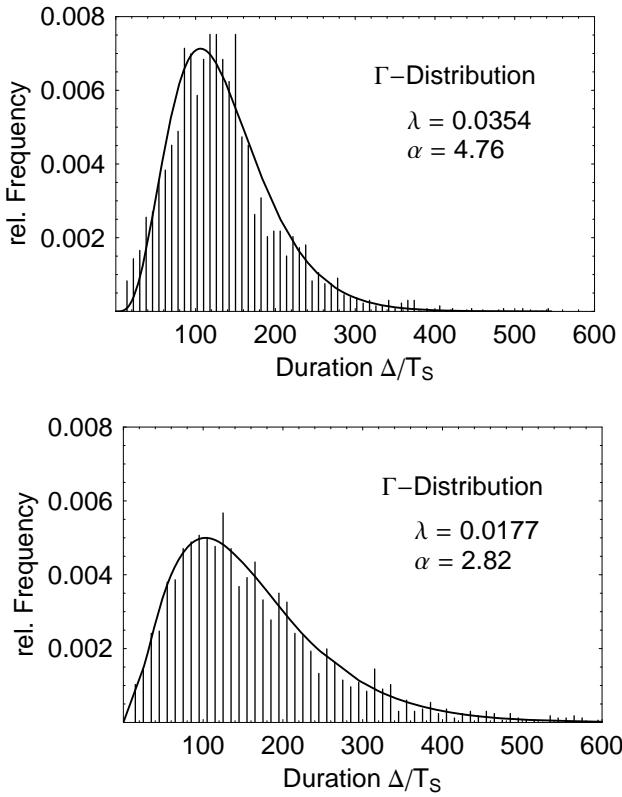


Figure 7: Relative frequencies of perceptual duration time Δ , in units $T_S = T/2$. Simulation parameters $\mu = 0.6$, $v_b = 1.5$, $\tau/T = 0.1$, $\gamma = 60$, $\tau_G = 500$, $s^2 = 0.03$. Fit with Γ -distribution (solid line). Bottom: Percept P1 (lower stationary level), top: Percept P2 (second level).

Table 1 lists results of Γ -distribution fits to the relative frequencies of perceptual duration times as obtained from simulations with delay times $T = 0, 1, 2 T_S$, (simulation sample time T_S , noise variance $s^2 = 0.03$).

Table 1: Mean (in seconds), relative standard deviation and correlation coefficient for Γ – distribution fit to relative frequencies of perceptual duration times (separate for P1, P2) as obtained with different delay times.

	T/T_S	Δ_m / s	σ / Δ_m	R^2
P2	0	1.89	0.39	0.97
P2	1	2.62	0.37	0.98
P2	2	2.64	0.45	0.96
P1	0	2.69	0.53	0.95
P1	1	3.27	0.56	0.94
P1	2	3.22	0.63	0.93

Excellent least squares fits to the data are obtained by means of a Γ – distribution for all T values: the correlation coefficient shows that only 3 – 7 % of the total scattering is not explained by the Γ – density function. The relative standard errors of the shape and scale parameters α and λ respectively are around 3% for all simulations. All shape

parameter values are within the range $2 < \alpha < 6$ in agreement with experimental results of (Murata et.al. 2003). An important result is the fact that even with zero delay ($T = 0$) the mean reversal time $\Delta_m(T=0)$ and the variance σ^2 is of the same order of magnitude as with finite delay. This indicates that the contribution of the deterministic limit cycle oscillations and chaos to the reversal time variance is small as compared to the stochastic noise, in agreement with Lehky (1995) and Merk & Schnakenberg (2002).

Conclusion

A behavioral recursive nonlinear phase oscillator model of spontaneous perceptual switching is presented which is related to previously published models of Ditzinger & Haken (1995) and Kelso et.al. (1995). It is based on the delayed nonlinear feedback of the perception state via attention as adaptive feedback gain control parameter, and as a kind of minimum architecture, reflects the reentrant thalamocortical circuits of the dynamical core hypothesis of consciousness (Edelman 2004). Experimental results of Nakatani & van Leeuwen (2005) support the assumption that attentional effort controls switching rates. The perception state is assumed to originate from interference between two stimulus induced phase synchronized perception fields as proposed by the neuronal mean field theory (Schuster & Wagner 1990). The interference contrast value μ ($0 \leq \mu \leq 1$) is interpreted as difference of meaning of the two alternative perception states with $\mu < 0.18$ (node bifurcation threshold) for stimulus off. Relative frequencies of perceptual duration times are fitted by Γ -distributions with shape parameters in the range $2 < \alpha < 6$ in agreement with experimental results of Murata et.al. (2003). By associating feedback delay time T with the stimulus onset - primary visual cortex (V1) response delay of ~ 40 ms (Lamme 2003) absolute values of mean perceptual duration times of $\Delta_m \approx 3$ s are obtained, in reasonable agreement with published experimental results (1 – 10 s, e.g. (Borsellino et al 1972)(Zhou et al 2004)). The large inter – subject variations of Δ_m can be modeled by suitable choice of satiation (fatigue) and recovery time constants γ and τ_G respectively. The relative duration of dominance vs. suppression times, i.e. a bias towards one of the two percepts is determined by the perception bias parameter v_b . The magnitude of limit cycle and chaotic oscillations with eigenfrequencies < 100 Hz as determined by a stability analysis is controlled by the ratio τ/T of perceptual damping time constant and delay time. Because even with the physically unrealistic case of $T = 0$ the statistical evaluation yields similar results as with finite T , it is concluded that the limit cycle and chaotic oscillations which are superimposed on the stationary perception states contribute only weakly to the reversal time statistics, in agreement with experimental results of Lehky (1995) and Merk & Schnakenberg (2002). The present model thus supports the proposal of Poston & Stewart (1978) of a deterministic catastrophe topology as the basis of the perception reversal dynamics, with the higher moments of the statistics determined by a stochastic process. Ongoing work aims at quantifying the amount of long range correlations of the time series by evaluating the Hurst parameter as proposed by Zhou & Gao (2005) and at

reproducing the experimental shape parameter statistics with relative maxima at natural numbers (Murata et.al. 2003).

Acknowledgement

I am indebted to Monika Mittendorf for help in writing the Mathematica and Matlab code of the numerical simulation and analysis software and in performing the statistical analysis. I would like to thank H. Nakatani of Riken Brain Science Institute for information on some recent experimental results.

References

- Attneave F (1971). Multistability in perception. *Scientific American* 225: 63 – 71
- Blake R., Logothetis N.K. (2002). Visual competition. *Nature Reviews / Neuroscience* 3: 1 – 11
- Borsellino A., de Marco A., Allazetta A., Rinesi S., Bartolini B. (1972). Reversal time distribution in the perception of visual ambiguous stimuli. *Kybernetik* 10, 139–144
- Borsellino A, Carlini F, Riani M, Tuccio M T, De Marco A, Penengo P, Trabucco A (1982). Effects of visual angle on perspective reversal for ambiguous patterns. *Perception* 11, 263 - 273
- Ditzinger T., & Haken H. (1995). A Synergetic Model of Multistability in Perception. In: *Ambiguity in Mind and Nature* (pp. 255-273). Berlin: Springer
- deGuzman G C, & Kelso J A S. (1991). Multifrequency behavioral patterns and the phase attractive circle map. *Biological Cybernetics* 64, 485–495
- Edelman, G. (2004). *Wider than the Sky*. Penguin Books
- Engel A.K., Fries P., König P., Brecht M., Singer W. (1999). Temporal binding, binocular rivalry, and consciousness. *Consciousness and Cognition* 8, 128 – 151
- Engel A.K., Fries P., Singer W. (2001). Dynamic Predictions: Oscillations and Synchrony in Top-Down Processing. *Nature Reviews Neuroscience*, 2, 704 – 718
- Fürstenuau, N. (1991). Bistable fiber-optic Michelson interferometer that uses wavelength control. *Optics Letters* 16, 1896–1898
- Fürstenuau N. (2003). Nonlinear dynamics model of cognitive multistability and binocular rivalry, *Proceedings IEEE 2003 Int. Conf. on Systems, Man and Cybernetics*, IEEE cat. no. 03CH37483C, 1081-1088
- Fürstenuau N. (2004). A chaotic attractor model of cognitive multistability. *Proceedings IEEE Int. Conf. on Systems, Man and Cybernetics*, IEEE cat. no. 04CH37583C, 853-859
- Gao, J.B., Merk, I., Tung W W, Billok V, White, K.D., Harris J G, Roychowdhury V P (2004). Inertia and memory in ambiguous visual perception. *preprint Phys. Rev. Lett.*, submitted
- Hillyard, S.A., Vogel, E.K. Luck, S.J.(1999). Sensory gain control (amplification) as a mechanism of selective attention: electrophysiological and neuroimaging evidence. In: G.W. Humphreys, J. Duncan &A. Treisman (Eds.), *Attention, Space, and Action* (pp. 31-53), Oxford University Press.
- Hupe, J.-M. & Rubin N. (2003). The dynamics of bistable alternation in ambiguous motion displays: a fresh look at plaids, *Vision Research* 43, 531 – 548
- Kelso J.A.S., Case P., Holroyd T., Horvath E., Raczszek J., Tuller B., Ding M. (1995). Multistability and metastability in perceptual and brain dynamics. In: *Ambiguity in Mind and Nature* (pp. 255-273). Berlin: Springer
- Lamme, V.A.F. (2003). Why visual attention and awareness are different. *Trends in cognitive Sciences*, 7, 12–18
- Lehky S. R. (1995). Binocular rivalry is not chaotic, *Proc. R. Soc. Lond., B* 259, 71–76
- Merk, I L K, & Schnakenberg, J (2002). A stochastic model of multistable perception. *Biol.Cybern.* 86, 111 – 116
- Murata. T., Matsui, N., Miyauchi, S., Kakita, Y., Yanagida, T. (2003). Discrete stochastic processes underlying perceptual rivalry. *NeuroReport* 14, 1347-1352
- Nakatani H., van Leeuwen C. (2005). Individual differences in perceptual switching rates; the role of occipital alpha and frontal theta band activity. *Biol. Cybern.* 93, 343 - 354
- Orbach, J, Ehrlich, D. Heath, H.A. (1963): Reversibility of the Necker Cube: An examination of the concept of satiation of orientation, *Perceptual and Motor Skills* 17, 439 - 458
- Poston T., & Stewart I. (1978). Nonlinear Modeling of Multistable Perception, *Behavioral Science* 23, 318 - 334
- Richards W., Wilson H.R., Sommer M.A. (1994): Chaos in percepts. *Biol. Cybern.* 70, 345-349
- Robinson, D. (Ed.) (1998). *Neurobiology*. Berlin, New York: Springer
- Srinivasan R., Russel D S, Edelman G M, Tononi G (1999) Increased synchronization of magnetic responses during conscious perception, *J. Neuroscience* 19: 5435 – 5448
- Schuster, H.G., & Wagner, P. A (1990): A Model for Neural Oscillations in the Visual Cortex: 1. Mean field theory and the derivation of the phase equations, *Biol. Cybernetics*, 64, 77-82
- Tononi, G. & Edelman, G.M., (1998). Consciousness and Complexity. *Science* 282, 1846 - 1851
- Watts, C., Fürstenuau, N. (1989): Multistable fiber-optic Michelson Interferometer exhibiting 95 stable states. *IEEE J. Quantum Electron* 25, 1–5
- Zhou, Y.H., Gao, J.B., White, K.D., Merk, I., Yao, K. (2004). Perceptual dominance time distributions in multistable visual perception. *Biol. Cybern.* 90, 256-263