

# Secular Acceleration as a Deterministic Long-Term Trend in Precision Exoplanet Radial-Velocity Time Series

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## ABSTRACT

Slow linear or even curved trends can sometimes be detected in long-baseline radial velocity curves. One such trend is due to a purely geometric effect: the proper motion of the target which causes a continuously variable line of sight to a moving star. The effect can be parametrized by Gaia proper motion and parallax data. This RV-drift is characterized via

$$\dot{v}_{\text{sec}} = 2.298 \times 10^{-5} \frac{\mu_{\alpha*}^2 + \mu_{\delta}^2}{\varpi} \text{ m s}^{-1} \text{ yr}^{-1},$$

where the proper motions and parallax are in milliarcsecond (mas) yr<sup>-1</sup> and mas, respectively. The accumulated trend can reach or exceed meter-per-second level over several years for nearby high-proper-motion targets. This contribution should be reported in precise RV analyses and is relevant for future multi-year ground-based RV follow-up of PLATO targets as well as it can be important for planetary systems hosting Earth- and SuperEarth-like planet in the habitable zones of Solar-like stars.

*Keywords:* Astrometry (80); Exoplanet detection methods (489); Proper motions (1295); Radial velocity (1332); Stellar kinematics (1608)

## 1. INTRODUCTION

Precise radial-velocity (RV) surveys often model long-term trends for distant planets, brown dwarfs, stellar companions, or long-period instrumental or other astrophysical effects. For nearby stars with substantial proper motion, however, a geometric contribution to the measured RV drift exists even if the star moves uniformly through the Galaxy. Due to the proper-motion of the host star, the projection of its constant space-velocity vector onto the observer's line of sight changes. This leads a secular variation of the line-of-sight velocity (radial velocity). The effect can be important when one searches for small (10-100 cm/s) RV-amplitude in long RV-series. Hereafter I present the equation that characterizes this radial velocity drift and I estimate the amount of the effect in some specific systems.

## 2. SECULAR RV-VARIATION DUE TO PROPER-MOTION

Figure 1 illustrates the geometry of the effect. The time derivative of the radial-velocity component is

$$\dot{v}_{\text{sec}} = \frac{v_{\text{tan}}^2}{d}. \quad (1)$$

where  $d$  is the distance to the star and  $v_{\text{tan}}$  the instantaneous tangential velocity.

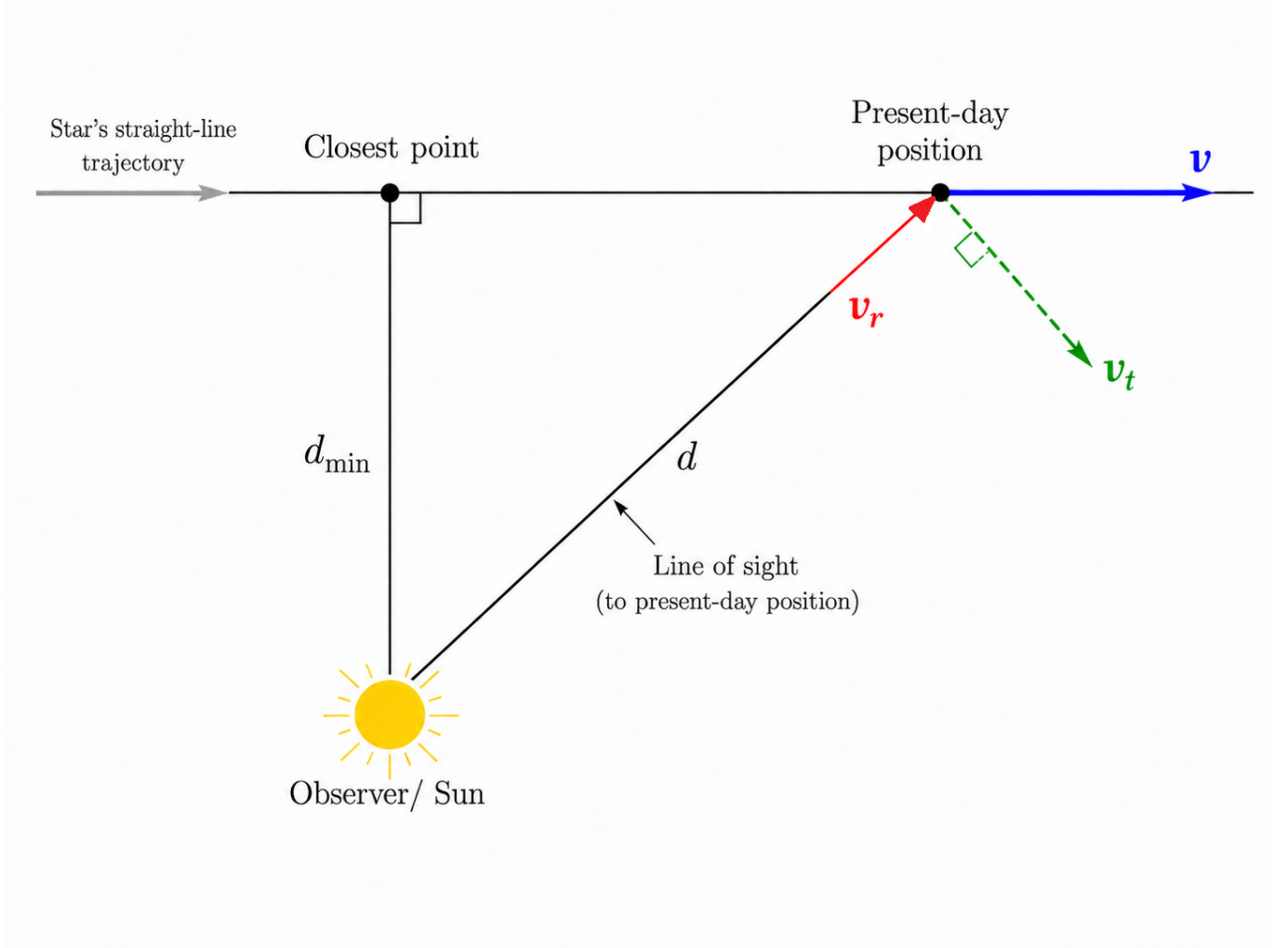
This acceleration is always positive: an approaching star exhibit less blueshifted with time, reaches zero radial velocity at closest approach, and subsequently features increasingly redshifted RV.

Using the usual astrometric relation

$$v_{\text{tan}} = 4.74047 \frac{\mu}{\varpi} \text{ km s}^{-1}, \quad (2)$$

where

$$\mu = \sqrt{\mu_{\alpha*}^2 + \mu_{\delta}^2} \quad (3)$$



**Figure 1.** Schematic illustration of the geometry of secular acceleration. The star moves with constant velocity  $v$  along a straight trajectory relative to the observer. At the present-day position, the line-of-sight distance is  $d$ , and the velocity decomposes into radial and tangential components,  $v_r$  and  $v_{\text{tan}}$ . The point of closest approach lies at the perpendicular distance  $d_{\text{min}}$  from the observer. Although  $v$  is constant, the projection onto the instantaneous line of sight changes with time, producing a secular drift of the measured radial velocity.

36 is the total proper motion and  $\varpi$  is the parallax. Equation (1) can be re-written in Gaia catalogue parameters as

$$37 \quad \dot{v}_{\text{sec}} = 2.298 \times 10^{-5} \frac{\mu_{\alpha*}^2 + \mu_{\delta}^2}{\varpi} \text{ m s}^{-1} \text{ yr}^{-1}, \quad (4)$$

38 where  $\mu_{\alpha*}$ ,  $\mu_{\delta}$ , and  $\varpi$  expressed in  $\text{mas yr}^{-1}$ ,  $\text{mas yr}^{-1}$ , and  $\text{mas}$ , respectively. Here  $\mu_{\alpha*}$  stands for  $\dot{\alpha} \cos \delta$ .

39 The total velocity change over a period of  $T$  is given by

$$40 \quad \Delta v_{\text{sec}} = \dot{v}_{\text{sec}} \cdot T = 2.298 \times 10^{-5} \frac{\mu_{\alpha*}^2 + \mu_{\delta}^2}{\varpi} \left( \frac{T}{1 \text{ yr}} \right) \text{ m s}^{-1}. \quad (5)$$

41 If the total proper motion exceeds the following limit calculated from the equation above, then the effect must be  
42 taken into account in RV-analysis:

$$43 \quad \frac{\mu_{\alpha*}^2 + \mu_{\delta}^2}{\varpi} \gtrsim 4.35 \times 10^3 \left( \frac{\Delta v}{1 \text{ m s}^{-1}} \right) \left( \frac{10 \text{ yr}}{T} \right). \quad (6)$$

## 3. RELEVANCE FOR EXOPLANET RADIAL-VELOCITY ANALYSES

On a long observing baseline this secular acceleration appears as a linear RV trend and can be degenerate with the local slope of a long-period companion or of a tidal-decay signal. It may bias the linear trend term of an RV-analysis or reduce sensitivity to weak outer companions.

Several familiar examples illustrate the scale of the effect. For Barnard’s star, the predicted secular acceleration is approximately

$$\dot{v}_{\text{sec}} \simeq 4.5 \text{ m s}^{-1} \text{ yr}^{-1}, \quad (7)$$

large enough to dominate the long-term Doppler baseline if ignored. Kürster et al. (2003) measured a secular RV acceleration consistent with this expectation. For  $\pi$  Mensae, Hatzes et al. (2022) obtained

$$\dot{v}_{\text{sec}} \simeq 0.48 \text{ m s}^{-1} \text{ yr}^{-1}, \quad (8)$$

which corresponds to roughly  $8 \text{ m s}^{-1}$  across their 17 yr RV baseline.

Gaia astrometry of GJ 367 provides  $\mu_{\alpha*} = -462.621 \text{ mas yr}^{-1}$ ,  $\mu_{\delta} = -582.668 \text{ mas yr}^{-1}$ , and  $\varpi = 106.1727 \text{ mas}$ , so, I have got

$$\dot{v}_{\text{sec}}(\text{GJ 367}) \simeq 0.120 \text{ m s}^{-1} \text{ yr}^{-1} = 0.0328 \text{ cm s}^{-1} \text{ day}^{-1}. \quad (9)$$

This star has only  $79 \pm 11 \text{ cm/s}$  RV-amplitude (Lam et al. 2021), so the proper motion effect is comparable to the error bar of the measured RV amplitude over a year. The RV analyses of GJ 367 by Lam et al. (2021) and Goffo et al. (2023) do not explicitly state that such acorrection was applied. For the intensive HARPS data set of Goffo et al. (2023), which spans nearly three years, we expect acumulative drift of only  $\simeq 0.36 \text{ m s}^{-1}$ . This is negligible for the recovery of the 7.7 hr signal of GJ 367 b itself. However, when the RV baseline is extended by comparison with older HARPS measurements taken more than a decade earlier, the same contribution reaches the order of  $2 \text{ m s}^{-1}$ . This example shows that the proper motion effect in the RV-analysis may be therefore relevant, when we search for long period signals in long baseline data sets. One can add that such changes can appear as apparent radial velocity offsets when the RV-curve is sparsely sampled with large gaps or can be mixed with instrumental zero-point changes (e.g. due to fiber-change of HARPS in 2015 that led to  $0.5 \text{ m/s}$  offset).

I note that an Earth–Sun analog moving with a tangential speed of  $30 \text{ km s}^{-1}$  would show secular RV drifts of  $\simeq 0.092 \text{ m s}^{-1} \text{ yr}^{-1}$  at 10 pc and  $\simeq 0.0092 \text{ m s}^{-1} \text{ yr}^{-1}$  at 100 pc (100% and 10% of the expected RV-amplitude).

A related point concerns RV analyses aimed at detecting tidal orbital decay, apsidal motion, or planetary Love numbers. Such long-term linear and curved trends were already searched in long-baseline RV-data as mentioned below. Such a secular acceleration is formally additive to any fitted long-term acceleration term,  $\dot{V}_{\gamma}$  (also denoted in many works as  $\dot{\gamma}$ ). Using Gaia astrometry, the predicted secular accelerations are

$$\dot{v}_{\text{sec}}(\text{WASP-43}) \simeq 0.0064 \text{ m s}^{-1} \text{ yr}^{-1}, \quad (10)$$

$$\dot{v}_{\text{sec}}(\text{WASP-18}) \simeq 0.0030 \text{ m s}^{-1} \text{ yr}^{-1}, \quad (11)$$

and

$$\dot{v}_{\text{sec}}(\text{WASP-19}) \simeq 0.0096 \text{ m s}^{-1} \text{ yr}^{-1}. \quad (12)$$

Over representative intervals of 10, 5, and 12 yr, respectively, these correspond to accumulated shifts of only approximately 0.06, 0.02, and  $0.11 \text{ m s}^{-1}$ . These levels are negligible for the analyses of WASP-43b by Bernabò et al. (2025), WASP-18Ab by Csizmadia et al. (2019), and WASP-19Ab by Bernabò et al. (2024). Nevertheless, since the WASP-43 b and WASP-19Ab studies explicitly considered or fitted long-term RV accelerations, the deterministic secular term should in general be subtracted or included before interpreting any residual  $\dot{V}_{\gamma}$  as astrophysical.

For data analysis, there are two equivalent practical options. One may subtract

$$v_{\text{sec}}(t) = \dot{v}_{\text{sec}}(t - t_0) \quad (13)$$

from the RV measurements before fitting, or include the same term as a fixed component of the RV model. Here  $t_0$  is an arbitrary reference epoch, preferably chosen near the weighted mean observation time to minimize covariance with the RV zero point. Of course, the trend can be included as a free parameter into the RV-fit, preferably with priors taken from the latest Gaia-catalogues if the target is included.

## 4. CONCLUSION

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91 I recommend that long-baseline precision RV studies explicitly calculate Equation (4) from Gaia astrometry and state  
92 whether the corresponding secular acceleration has been removed, incorporated into the RV model, or verified to be  
93 negligible. This is especially relevant for nearby high-proper-motion stars, M-dwarf RV surveys, and multi-instrument  
94 time series spanning many years. The correction is deterministic, and can prevent a purely geometric RV drift from  
95 being interpreted as an astrophysical acceleration.

96 This recommendation is timely in the context of PLATO. The Ground-based Observations Programme (GOP) will  
97 organise follow-up observations for the PLATO prime sample including high-precision RV measurements needed to  
98 confirm candidates and determine planetary masses (Rauer et al. 2025). For terrestrial planets around bright stars,  
99 the required RV precision may reach the level of a few tens of  $\text{cm s}^{-1}$ . In multi-season observational data sets, this  
100 kind of trend should be taken into account.

101 I also add that Rafikov (2009) analyzed the effect of proper motion on transit timing and transit duration variations.  
102 That work is now extended to the radial velocity analyses.

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