

A comprehensive Rossiter–McLaughlin modelling framework in TLMC: application to HD 2685 = TOI-135 system

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ABSTRACT

We present an updated Rossiter–McLaughlin modelling framework in the exoplanet analysis code TLMC. We describe our model in detail. The model and the code were validated by nine systems, where we use *Transiting Exoplanet Survey Satellite* (TESS) photometric measurements and archival radial velocity data (WASP-15, HAT-P-1, HAT-P-3, HAT-P-6, HAT-P-7, HAT-P-11, HAT-P-14, HAT-P-20, HAT-P-32). In addition, new observations were obtained from HD 2685 = TOI-135 which is an evolved, hot ($T_{\text{eff}} = 6801$ K), single star hosting a ~ 1.15 Jupiter-mass transiting hot Jupiter in a ~ 4.1 d orbit. We obtained new HARPS radial velocity measurements in-transit and out-of-transit. Also, there are new photometric observations from six additional, not-yet-analysed TESS-sectors since the time of the discovery of its transits. This allowed us to refine the planetary, orbital and system parameters and to detect Rossiter–McLaughlin effect in it. We find an intermediate sky-projected obliquity, $\lambda = 55.6^{+10.9}_{-11.9}$, for our fiducial spectroscopic $V \sin I_*$ prior. Tests with broader $V \sin I_*$ priors show that the central value of λ remains stable, although the uncertainty increases.

Key words: methods: data analysis – techniques: photometric – techniques: radial velocities – exoplanets – planets and satellites: fundamental parameters.

1 INTRODUCTION

The Rossiter–McLaughlin (RM) effect was originally identified in eclipsing binaries (D. B. McLaughlin 1924; R. A. Rossiter 1924) and was later applied to transiting exoplanets as a means of measuring the sky-projected spin-orbit angle, λ (D. Queloz et al. 2000; Y. Ohta, A. Taruya & Y. Suto 2005; B. S. Gaudi & J. N. Winn 2007; A. H. M. J. Triaud 2018, and references therein). The effect and its line-profile generalization via Doppler tomography (A. Collier Cameron et al. 2010; V. Bourrier et al. 2021) thus provide direct access to measure the projected spin-orbit angle (λ) of transiting exoplanets and eclipsing binaries. As 298 planets have RM measurements in TEPcat (J. Southworth 2026), population-level diagnostics of planet migration, dynamical excitation, and tidal evolution is possible. A central empirical result is the apparent $T_{\text{eff}} - \lambda$ trend (J. N. Winn et al. 2010a): cool stars preferentially host aligned hot Jupiters, whereas hot stars exhibit a wide range of obliquities. This is also supported by the observed stellar mass- λ trend: higher mass (i.e. hotter) stars have more misaligned planets (K. C. Schlaufman 2010). The limit between cool/hot stars in this sense is often related to the so-called Kraft-break located at $T_{\text{eff}} = 6250$ K. The Kraft-break (R. P. Kraft 1967) means

an abrupt change in the stellar rotational velocities and often related to the disappearance of the magnetic braking at this temperature from hotter stars. Stellar magnetism is probably caused by convective motions in their envelope. The lack of magnetic braking of rotation in hot stars is suspected to be due to their radiative envelope while cooler stars have convective one, and the change of this envelope property also occurs around this effective temperature.

In the standard interpretation (K. C. Schlaufman 2010; J. N. Winn et al. 2010a; S. Albrecht et al. 2012), obliquities are excited by processes such as planet–planet scattering and Kozai–Lidov forcing, while tides damp λ efficiently only for stars with substantial convective envelopes. Early studies revealed that large spin-orbit misalignments are preferentially found around more massive, hotter stars. K. C. Schlaufman (2010) provided independent evidence for this trend using projected stellar rotation as a diagnostic of possible line-of-sight misalignment around massive stars ($M > 1.2M_{\odot}$), while J. N. Winn et al. (2010b) showed that hot-Jupiter hosts hotter than ~ 6250 K exhibit a broader obliquity distribution. This empirical pattern is commonly interpreted in terms of the different tidal realignment efficiencies of cool and hot stars (J. N. Winn et al. 2010b; S. Albrecht et al. 2012).

However, the uniqueness of this explanation has been questioned by both theory and new demographic cuts. Several

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other factors can contribute to the observed scatter of spin-orbit angle and trends, like:

- (i) stellar multiplicity
- (ii) planet-to-star mass ratio
- (iii) primordial distribution from tilted or warped protoplanetary discs
- (iv) swap planets in dense stellar fields

(i) Recent work argues that stellar multiplicity can bias the inferred transition temperature, and that restricting to single stars shifts the obliquity ‘break’ towards ~ 6500 K, closer to the single-star Kraft-break (X.-Y. Wang, S. Wang & J. M. J. Ong 2026). A. M. Rossi et al. (2026) points out that the sample of RM-measurements are still not big enough to study the distribution of spin-orbit angles, especially in certain parameter ranges (e.g. planetary mass, period).

(ii) Beyond stellar T_{eff} and stellar multiplicity, planet-to-star mass ratio ($M_{\text{planet}}/M_{\text{star}}$) has very recently emerged as a potentially independent organizing parameter. An updated compilation indicates that systems with high $M_{\text{planet}}/M_{\text{star}}$ (in the very wide mass-range from sub-Saturns to brown dwarfs) tend to be aligned even around hot stars, pointing to a primordial origin or enhanced dissipation in a restricted parameter space (J. Ruzsna et al. 2025). This motivates renewed scrutiny of tidal physics, including mechanisms not captured by simple equilibrium-tide scalings, such as dissipation driven by precessional instabilities in misaligned rotating fluids (N. B. de Vries, A. J. Barker & R. Hollerbach 2025) and the limits of tidal realignment as a universal explanation (G. Li & J. Winn 2015). On the observational side, new RM and tomographic measurements continue to populate the full obliquity phase space, ranging from aligned warm Jupiters (e.g. J.-V. Harre et al. 2023) to polar and intermediate configurations in classic hot-Jupiter systems (see recent results of Z. Balkóová et al. 2026), and extending analogous methods to short-period eclipsing binaries (e.g. T. Wells et al. 2025).

(iii) Theoretical pathways are capable of producing misalignment span both primordial and dynamical channels. Primordial misalignment can arise if the stellar spin and protoplanetary disc angular momentum are misaligned, for example through chaotic accretion, magnetic star-disc torques, or binary-induced disc tilting (M. R. Bate, G. Lodato & J. E. Pringle 2010; D. Lai, F. Foucart & D. N. C. Lin 2011; K. Batygin 2012). Dynamical excitation through planet–planet scattering and high-eccentricity migration has long been discussed as a route to misaligned hot Jupiters (F. A. Rasio & E. B. Ford 1996; Y. Wu & N. Murray 2003; D. Fabrycky & S. Tremaine 2007; M. Nagasawa, S. Ida & T. Bessho 2008). Dynamical channels that include scattering and Kozai–Lidov migration, potentially driven by wide companions whose orbits evolve under Galactic tides over Gyr timescales (E. Grishin, J. Winter & J. A. Alvarado-Montes 2025), and can operate even in hot-Jupiters located in ‘twin binaries’ (two stars of equal mass orbiting each other) via mirrored secular migration (Y. Liu, T. Lu & M. Rice 2026). Stellar evolution can further reshape the observed architecture, as illustrated by proposed engulfment-driven spin-up and internal misalignment in evolved hosts (T. Tokuno 2025).

(iv) In dense stellar fields, like in young, rich open clusters very close stellar encounters often occur. During these encounters, planets can swap their host stars which may lead to change of the inclination not only of the captured, stolen or swapped planets

between the two stars, but also can change the inclination of the preserved planets as well (E. C. Daffern-Powell, R. J. Parker & S. P. Quanz 2022) that might be followed up by RM-measurements.

Motivated by these fresh developments, we decided to increase the sample of RM-measurements by observing HD 2685b. This system was detected photometrically by M. I. Jones et al. (2019) via its transits in Sector 1 of *Transiting Exoplanet Survey Satellite* (TESS). Extending TESS photometry with ground-based photometry and velocimetry, they derived $P = 4.12688^{+0.00005}_{-0.00004}$ d orbital period, $e = 0.091^{+0.039}_{-0.047}$ eccentricity, $1.17 \pm 0.12 M_{\text{Jup}}$ planetary mass, and $1.44 \pm 0.05 R_{\text{Jup}}$ planetary radius. According to them, the star has a mass of $1.43^{+0.05}_{-0.04}$ solar masses, a radius of 1.56 ± 0.05 solar radius. The host star is a subgiant-giant transition object with spectral type of F2III/IV (SIMBAD), with large surface temperature and luminosity ($T_{\text{eff}} = 6801 \pm 76$ K and $L_* = 4.66^{+0.43}_{-0.42}$ solar luminosity) and a projected rotational velocity of $V \sin I_* = 15.3 \pm 0.2 \text{ km s}^{-1}$. L. Bourges et al. (2017) and P. Cruzalèbes et al. (2019) have reported a limb-darkened apparent diameter of $\theta = 0.076 \pm 0.002$ mas for the host star, using infrared magnitudes and stellar models. Combining this with the known *Gaia*-parallax ($\pi = 5.0667 \pm 0.0105$ mas), one can get a true radius of $1.61 \pm 0.04 R_{\odot}$ for the star, in excellent agreement with the value of M. I. Jones et al. (2019). They also noted that the planet’s low-eccentricity orbit and brightness ($V = 9.6$ mag) make it an ideal target for RM studies.

Therefore, we carried out an RM measurement during the primary transit with HARPS. Since the work of M. I. Jones et al. (2019), the system was additionally observed by TESS in Sectors 27, 28, 67, 68, 94, and 95. While Sector 1 provided 18 271 good-quality (flag equals to zero) photometric points, now we have 80 936 such photometric points. This fact and the new radial velocity data collected during primary transit justifies a new joint photometric + radial velocity study and parameter rederivation for this system.

Several packages are available for the joint modelling of transit photometry and radial velocities, including EXOFASTv2 (J. Eastman 2017), *juliet* (N. Espinoza, D. Kossakowski & R. Brahm 2019), and *allesfitter* (M. N. Günther & T. Daylan 2021) beyond TLCM (S. Csizmadia 2020) which latter on was first used in H. Rauer et al. (2009). An additional aim of the present work is to describe and validate the updated RM implementation within the TLCM framework. The new RM module differs in several practical respects. First, the RM anomaly is computed by direct numerical integration over the occulted part of the stellar disc, so the implementation is not tied to analytic formulae derived for one particular limb-darkening law. In the present version, seven limb-darkening laws are available, including the quadratic, power-2, logarithmic, square-root, Sing-3, and Claret four-parameter laws. Second, the RM calculation is coupled to the existing TLCM joint photometric and radial-velocity model, which includes eccentric orbits, multiple RV zero-point offsets, optional additional Keplerian signals, ellipsoidal RV corrections, occultations and phase-curve terms, and convective blueshift. Third, the photometric likelihood can be combined with the wavelet-based correlated-noise model of TLCM (S. Csizmadia 2020; S. Csizmadia et al. 2023; S. Kálmán, G. M. Szabó & S. Csizmadia 2023). This provides a computationally economical alternative to dense Gaussian-process modelling for long, high-cadence space-based light curves, while

Table 1. Summary of *TESS* observations of HD 2685.

Sector	Start date	End date
1	2018 Jul 25	2018 Aug 22
27	2020 Jul 05	2020 Jul 30
28	2020 Jul 31	2020 Aug 25
67	2023 Jul 01	2023 Jul 29
68	2023 Jul 29	2023 Aug 25
94	2025 Jun 29	2025 Jul 25
95	2025 Jul 25	2025 Aug 20

retaining information on short-time-scale correlated photometric noise.¹

2 OBSERVATIONS

2.1 Photometric data

TESS (G. R. Ricker et al. 2015) observed HD 2685 = TOI-135 = TIC 267263253 ($V = 9.59$ mag) in its Sectors 1, 27, 28, 67, 68, 94, and 95, in total in 7 sectors (see Table 1). We downloaded the reduced, calibrated *TESS*-SPOC light curves from the *Mikulski Archive for Space Telescopes*.² We used only the so-called short cadence data which have an exposure time of 120 s. We kept only data with quality flag = 0 (meaning good quality data) the others were excluded. We had 80 936 such data points.

Every sector was normalized individually. Our approach was to take the quantity f_{SAP} (Simple Aperture Fluxes) and we corrected them to $F_{\text{normalized}}$ via

$$F_{\text{corrected}} = \frac{f_{\text{SAP}} - (1 - \text{CROWDSAP} \times \text{MEDIAN}(f_{\text{SAP}}))}{\text{FLFRCSAP}} \quad (1)$$

and

$$F_{\text{normalized}} = \frac{F_{\text{corrected}}}{\text{MEDIAN}(F_{\text{corrected}})}, \quad (2)$$

where the parameters CROWDSAP and FLFRCSAP are calculated by the *TESS* Science Processing Operations Center (SPOC) and can be found in the data files' headers for every sector. The first quantity measures the flux coming from the target relative to all flux in the aperture; the second characterizes how much light of the target remains inside the aperture due to the fact that part of the Point Spread Function might be outside the photometric mask. The quantity $F_{\text{corrected}}$ is the flux corrected for these two effects while $F_{\text{normalized}}$ is the normalized flux. The uncertainties

¹For example, P. F. L. Maxted (2018) noted that evaluating the GP at all Kepler data points was computationally slow and therefore evaluated the GP on every 1000th points only. This illustrates a practical difficulty of dense GP treatments for long high-cadence light curves. The wavelet likelihood used in TLCDM was developed as a computationally less demanding way of treating correlated photometric noise in such data sets. Such approximations may suppress or average over correlated-noise components on time-scales shorter than the adopted GP sampling grid. The wavelet approach instead keeps the original time sampling of the photometric data and models the correlated component through scale-dependent noise amplitudes. The detailed discussion of this kind of performance check is out of the scope of the present paper and it will be discussed in a separate study.

²<https://archive.stsci.edu/>

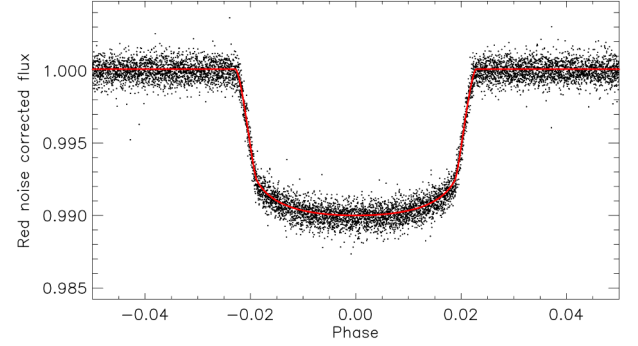


Figure 1. Zoom to the transit of HD 2685b. The dots represent the red noise corrected fluxes (observed SAP flux – red noise curve based on wavelet-based noise model) and the solid curve is the transit model fit.

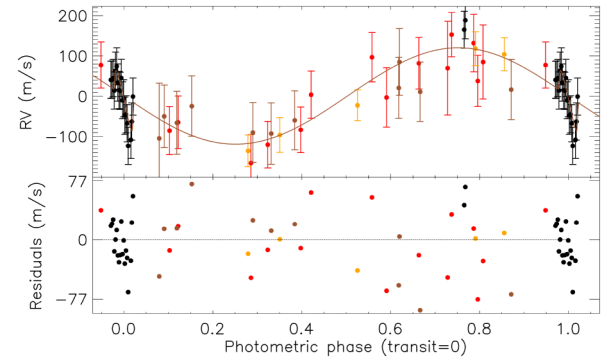


Figure 2. Upper panel: Radial velocity data and their model fit of HD 2685b. Black dots represent new HARPS-data from this study while red ones are from CORALIE, brown ones are from CHIRON and orange are from FEROS. The vertical lines denote the 1σ error bars. Brown curve is the model fit. Lower panel: residuals of the fit.

of the flux measurements were simply corrected as

$$\Delta F_{\text{corrected}} = \frac{\Delta f_{\text{SAP}} - (1 - \text{CROWDSAP} \times \text{MEDIAN}(\Delta f_{\text{SAP}}))}{\text{FLFRCSAP}}, \quad (3)$$

where the photometric error bars Δf_{SAP} were also given by SPOC in the downloadable data files.

We further selected segments of the light curves around each transit observed: 1.5 times the transit duration before and after the predicted transit times were cut and kept. This resulted 24 174 remaining points.

The phase-folded transit light curve and radial velocity measurements together with their respective model fits (see Section 4) can be seen in Figs 1–3.

2.2 Radial velocity data

We observed HD 2685 with the HARPS spectrograph (M. Mayor et al. 2003) mounted on the ESO 3.6-m telescope at La Silla Observatory, Chile. A total of 19 spectra, each with an exposure time of 900 s, were taken on the night 2025 August 08/09, spanning a transit of HD 2685 b. A further two spectra were taken on 2025 September 05/06. The data were obtained under Programme 115.27UY (PI: Harre). Radial velocities were extracted using the standard DRS pipeline, employing cross-correlation with a G2

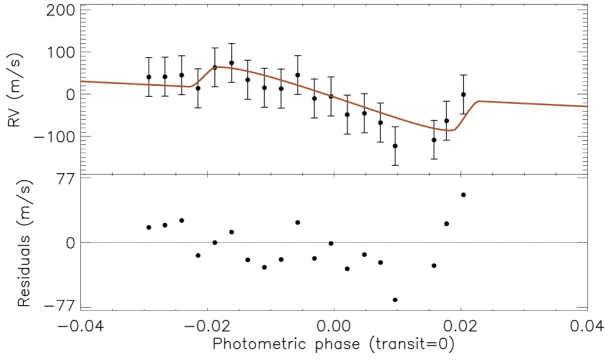


Figure 3. Upper panel: Zoom to the radial velocity measurements and their fit around the transit event. Meaning of colours is the same as in Fig. 2. The RM effect is clearly visible. Brown curve is the model fit. The error bars are inflated with the finally adopted jitter value. Lower panel: residuals of the fit.

mask. All radial velocity data of HD 2685 used in this work are collected in Table 2.

3 METHODS

Hereafter, we describe the RM-effect model in TLMC as we updated it. All other parts of the code were described in S. Csizmadia (2020), S. Csizmadia et al. (2023), and S. Kálmán et al. (2024). Transit and Light Curve Modeller (TLMC; S. Csizmadia 2020; S. Csizmadia et al. 2023) is a complex modelling code which is able to fit either only the photometric light curve, or to perform a joint fit of the radial velocity (RV) and light curve of a transiting exoplanet. It is able to model simultaneously the transit, occultation and phase curve (beaming, reflection, ellipsoidal effect are included) with or without a gravity darkened star. It has a wavelet-based noise model to remove the correlated noise from photometry. Circular and eccentric orbits are included. Only its RM effect modelling part is updated here.

Earlier we used the analytic equations of A. Giménez (2006), but that was limited for one specific limb darkening law (quadratic). Here, we generalize the RM component of the code to seven different limb darkening laws. The generalization includes the power-2 and the logarithmic laws, too, which were found more effective than the quadratic law by other investigators (P. F. L. Maxted 2018; T. Borkovits et al. 2018; N. M. Kostogryz et al. 2022).

3.1 Sky-projected planetary orbit

The calculation of the position of the planet consists of the following steps. The input data are the epoch T_0 , period P , eccentricity e , argument of periastron ω , scaled semimajor axis a/R_{star} , inclination i . We use the unit $R_{\text{star}} = 1$ in this modelling procedure.

As is explained in Y. D. Martynov (1973), A. Gimenez & J. M. Garcia-Pelayo (1983), and S. Csizmadia (2020), the mid-transit time is given as follows for eccentric orbits. In the eccentric, inclined orbit case, the exact time of the moment where the planet is crossing the central-meridian does not coincide with the moment of the minimum sky-projected distance of the centres of the star and of the planet. But the sidereal (observable) period is measured from central-meridian crossing to central meridian crossing. First, by minimization of the sky-projected star-planet

Table 2. Radial velocity data used in this work. HARPS data were obtained by us while CORALIE, CHIRON, and FEROS data are taken from M. I. Jones et al. (2019).

BJD _{TDB} – 2450000	RV (km s ⁻¹)	RV uncertainty (km s ⁻¹)	Instrument
10896.724600	2.14394	0.00264	HARPS
10896.735075	2.14467	0.00276	HARPS
10896.745966	2.14834	0.00301	HARPS
10896.756649	2.11716	0.00314	HARPS
10896.767644	2.16664	0.00271	HARPS
10896.778535	2.17741	0.00265	HARPS
10896.789009	2.13716	0.00296	HARPS
10896.799900	2.11845	0.00308	HARPS
10896.810792	2.11651	0.00307	HARPS
10896.821579	2.14863	0.00302	HARPS
10896.832470	2.09307	0.00282	HARPS
10896.843257	2.09804	0.00271	HARPS
10896.853835	2.05498	0.00276	HARPS
10896.865039	2.05817	0.00307	HARPS
10896.875513	2.03598	0.00284	HARPS
10896.885050	1.98013	0.00356	HARPS
10896.910374	1.99509	0.00380	HARPS
10896.918557	2.04036	0.00269	HARPS
10896.929552	2.10247	0.00281	HARPS
10924.765877	2.26829	0.00194	HARPS
10924.775935	2.29167	0.00206	HARPS
8382.6803119	2.18885	0.05257	CORALIE
8384.7397412	1.89132	0.05121	CORALIE
8390.6927155	2.12678	0.10582	CORALIE
8397.6786921	2.06088	0.03367	CORALIE
8398.6790221	2.13893	0.04247	CORALIE
8401.7098130	1.97378	0.03014	CORALIE
8404.6986823	1.99229	0.04442	CORALIE
8406.6329995	2.05413	0.05574	CORALIE
8407.5289225	2.14212	0.07840	CORALIE
8408.7459082	1.97165	0.03552	CORALIE
8409.6573742	1.93685	0.03205	CORALIE
8410.6247174	2.15375	0.03801	CORALIE
8411.6066317	2.09497	0.04138	CORALIE
8419.6176664	2.20996	0.02654	CORALIE
8424.6165730	2.13447	0.03143	CORALIE
8369.6043	0.0186	0.0308	CHIRON
8369.8027	0.0089	0.0260	CHIRON
8370.6479	0.0144	0.0284	CHIRON
8371.6716	-0.0680	0.0366	CHIRON
8371.8098	-0.0266	0.0295	CHIRON
8372.7658	-0.0618	0.0244	CHIRON
8373.7375	0.0829	0.0458	CHIRON
8375.6807	-0.0519	0.0359	CHIRON
8379.7618	-0.1066	0.1184	CHIRON
8380.8013	-0.0947	0.0322	CHIRON
8384.7577	-0.0921	0.0270	CHIRON
8378.8372	2.2920	0.0335	FEROS
8380.8811	2.0919	0.0360	FEROS
8382.6972	2.3061	0.0348	FEROS
8384.7130	2.0525	0.0316	FEROS
8385.7289	2.1655	0.0302	FEROS

centres distance, we define the auxiliary quantity:

$$\tan \vartheta = -\frac{e \cos \omega \cos^2 i}{\cos \vartheta \sin^2 i + e \sin \omega}. \quad (4)$$

This equation can be solved iteratively (A. Gimenez & J. M. Garcia-Pelayo 1983), starting with $\vartheta = 0^\circ$. If eccentricity is zero, then ϑ is also zero. Independently of the value of the eccentricity, for edge-on orbits ($i = 90^\circ$) this auxiliary quantity is zero, too. Next step is to set the true anomaly (v) at the mid-transit to

$$v_{\text{transit}} = 90^\circ - \omega + \vartheta. \quad (5)$$

The eccentric (E) and mean (M) anomalies at the moment of central-meridian cross is

$$\tan \frac{E_{\text{transit}}}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{v_{\text{transit}}}{2} \quad (6)$$

and

$$M_{\text{transit}} = E_{\text{transit}} - e \sin E_{\text{transit}}. \quad (7)$$

Here, M_{transit} is the mean anomaly at the epoch, and the epoch is taken to be equal to the time of some arbitrarily chosen, well observed transit at time T_0 .

The mean motion n is, by definition,

$$n = \frac{2\pi}{P}. \quad (8)$$

The mean anomaly can be calculated for every time t where we have an observation as

$$M = M_{\text{transit}} + n(t - T_0). \quad (9)$$

The eccentric anomaly is given by the Kepler-equation as

$$E = M + e \sin E, \quad (10)$$

which can be solved iteratively again, starting with $E = M$. The true anomaly at any time moment t is

$$\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}. \quad (11)$$

The true distance between the centres of the star and the planet is

$$r = \frac{a}{R_{\text{star}}} \frac{1-e^2}{1+e \cos v} \quad (12)$$

We define a coordinate system where the origin is in the stellar centre, the z coordinate is oriented towards the observer and xy axes are in the tangential plane of the sky. Then the planetary coordinates are

$$\begin{aligned} x &= r \cdot (\cos(\omega + v) \cos \lambda - \sin(v + \omega) \cos i \sin \lambda), \\ y &= r \cdot (\cos(\omega + v) \sin \lambda + \sin(v + \omega) \cos i \cos \lambda), \\ z &= r \cdot \sin(\omega + v) \sin i, \end{aligned} \quad (13)$$

assuming that the stellar rotational axis is oriented towards the y -axis. Here, λ is the spin-orbit angle, i.e. the angle between the sky-projected normal vector of the planetary orbit and the projected angular momentum vector of the star. λ is measured from the star's projected angular momentum vector to the sky-projection of the planetary orbit's normal vector.

3.2 Convective blueshift

The convective blueshift (CB) plays an important role in the analysis of the RM effect when the host star has a convective envelope. CB can easily cause 10–20 degrees systematic error in the measured value of λ which often is higher than the measured error bar; that is why it is important to take into account

(A. Shporer & T. Brown 2011; H. M. Cegla et al. 2016; D. J. A. Brown et al. 2017). This effect is due to the motion of the convective cells. The upward motion causes a blue-shifted surface area, while the downward motion causes red-shifted one. The area ratio between the upward and downward material motions is far from unity, the upward-moving cells have much bigger area. This yields that the net effect integrated to the stellar surface or behind the planet is blue-shifted. The effect can cause a local radial velocity between ca. 100 m s^{-1} to several km s^{-1} , depending on the effective temperature of the star. Since the RM effect is scaled with the $V \sin I$ amplitude of the star which is often in the range of from a few hundred m s^{-1} to a few km s^{-1} , the amplitude of the CB-effect is comparable to the RM-effect, and therefore not negligible.

F. Liebing et al. (2021) used high-resolution spectroscopy and carried out a careful analysis of the CB-effect. We fitted simple functions to their tabulated stellar effective temperature - convective blueshift data pairs, and derived an analytic formula. This is used in TLCM to characterize the convective blueshift phenomenon as follows:

$$V_{\text{CB}} = \frac{-83.7 - 92.73 \cdot ((T_{\text{eff}} - 4400.0)/1000.0\text{K})^3}{1000.0} \text{ km s}^{-1}. \quad (14)$$

Here, V_{CB} is the radial velocity of the local surface point into the direction of the surface normal vector caused by the convective blueshift; T_{eff} is the stellar effective temperature measured in kelvins. Equation (14) reproduces the results of F. Liebing et al. (2021) within 15 cm s^{-1} in the range of 4000–7000 K. We, therefore, apply equation (14) in the temperature range of 4000–7300 K. Below this range, we take a constant

$$V_{\text{CB}} = -0.08 \text{ km s}^{-1}, \quad T_{\text{eff}} \leq 4000 \text{ K}. \quad (15)$$

For high stellar effective temperatures, TLCM applies an exponential cut-off as

$$V_{\text{CB}} = -2.3 \cdot e^{-(T_{\text{eff}} - 7300.0 \text{ K})/100.0 \text{ K}} \text{ km s}^{-1} \quad T_{\text{eff}} \geq 7300 \text{ K}. \quad (16)$$

3.3 Radial velocity field on the stellar surface

We denote the stellar inclination with I . Any point on the visible hemisphere of the star has a radial velocity due to rotation as

$$V_{\text{rotational}} = -x_* \cdot (V \sin I), \quad (17)$$

where $V \sin I$ is measured from the rotational broadening, and it is fixed or has a prior during the fitting procedure. Here, x_* is the x -coordinate on the stellar surface.

We calculate the foreshortening angle μ via

$$\mu = \sqrt{1 - x_*^2 - y_*^2}. \quad (18)$$

The total radial velocity in a certain x_*, y_* is the sum of the CB and rotational effect, weighting CB appropriately with the viewing angle:

$$v_R(x, y) = -x \cdot V \sin I + \mu \cdot V_{\text{CB}}. \quad (19)$$

3.4 Integration procedure

The model value of the radial velocity (denoted V_R) is the sum of the following weighted integral and additional terms:

$$\begin{aligned}
 V_{R,j}(t_i) = & \frac{\iint_{\text{visible area}} v(x, y) L_D(\mu) dx dy}{\iint_{\text{visible area}} L_D(\mu) dx dy} \\
 & - \frac{\iint_{\text{visible hemisphere}} v(x, y) L_D(\mu) dx dy}{\iint_{\text{visible hemisphere}} L_D(\mu) dx dy} \\
 & + V_\gamma + \Delta V_{\text{offset},j} + K(e \cos \omega + \cos(v + \omega)) \\
 & + \Delta K_{\text{ellipsoidal}}.
 \end{aligned} \quad (20)$$

Here, j is the index for the j th spectrograph used for the observations and t_i is the mid-time of the i th time-moment. We assume that the RV-observation has reasonably short exposure times so binning effects does not play a role due to a too long exposure during which the radial velocity change is too much.

The first row of the r.h.s. gives the radial velocity change due to the eclipse of a planet. This integral is extended to the whole visible area. This integral is time-variable during transit and it equals to the second integral outside transit.

The second row of the r.h.s. gives a correction to the CB-effect which is constant in all phases as the integral is always extended to the whole hemisphere facing us.

The additional terms have the following meaning:

V_γ : the radial velocity of the barycenter of the exoplanetary system relative to the barycenter of the Solar system.

ΔV_{offset} : the zero-point offset of the different spectrographs used for radial velocity measurements. We set the first spectrograph's offset to be zero (its zero-point offset is included into V_{gamma}), only the second, third etc. instrument has non-zero offset. Note that instrument offset difference can be not only constant between instruments but also a function of stellar colour, spectral type or effective temperature, activity level; and the existing colour/ T_{eff} calibration shows usually larger error bars than the uncertainties of the offset-fits of individual systems (e.g. C. Soubiran et al. 2013; I. Boisse et al. 2012; C. Soubiran et al. 2018). Also, changes in the instrument-setup (e.g. fiber changes, opening the vacuum chamber for maintenance etc) can cause also sudden and spectral type dependent changes in the radial velocity offset (e.g. M. Marmier et al. 2013; K. R. Sreenivas et al. 2022). All these justify that one needs the option to fit (or at least apply a prior to) the radial velocity offset values.

$K(e \cos \omega + \cos(v + \omega))$: this term describes the RV-variation due to the Keplerian orbit of the planet. In cases where additional planets contribute to the RV-curve, TLCDM allows us to add another Keplerian to the r.h.s. of equation (20), but no gravitational interaction between the two planets is considered in the present version.

K : the radial velocity semi-amplitude, a direct fit-parameter. Note that this parameter is an abbreviation only and it is related to other parameters via $K = \frac{2\pi a \sin i}{P \sqrt{1-e^2}} \frac{M_{\text{planet}}}{M_{\text{star}} + M_{\text{planet}}}$.

$\Delta K_{\text{ellipsoidal}}$: this describes an apparent effect in the radial velocity curves which is due to the ellipsoidal effect caused by a massive, close-in planet on the star's surface: we see a variable stellar area during the orbital revolution. This is often important in short-period, massive planets as it can mimic an eccentric orbit.

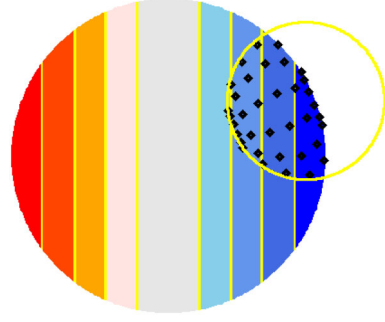


Figure 4. Distribution of quadrature points during a partial transit phase (black diamonds) for the $N = 6$ case. The yellow circle represents the contours of the planet. The stellar disc is represented by vertical stripes, where the colours in the vertical stripes denote schematically where we have red-shifted or blue-shifted velocity field due to rotation.

We note that the $\Delta K_{\text{ellipsoidal}}$ term is given in P. Arras et al. (2012) for non-rotating stars. We use a more realistic expression that takes stellar rotation into account. This is based on the work of Z. Kopal (1959) and is given by equation (10) of L. M. Bernabò et al. (2024).

Here L_D is the so-called limb-darkening function. In TLCDM now the user can choose between seven different limb darkening functions, linear, power-2, quadratic, logarithmic, claret-4, sing-3 and squareroot:

$$\begin{aligned}
 L_D &= 1 - u_{\text{linear}}(1 - \mu), \\
 L_D &= 1 - C(1 - \mu^\alpha), \\
 L_D &= 1 - u_a(1 - \mu) - u_b(1 - \mu)^2, \\
 L_D &= 1 - u_{l1}(1 - \mu) - u_{l2}\mu \log \mu, \\
 L_D &= 1 - c_1(1 - \mu^{1/2}) - c_2(1 - \mu) - c_3(1 - \mu^{3/2}) - c_4(1 - \mu^2), \\
 L_D &= 1 - s_1(1 - \mu) - s_2(1 - \mu^{3/2}) - s_3(1 - \mu^2), \\
 L_D &= 1 - q_1(1 - \mu) - q_2(1 - \sqrt{\mu}).
 \end{aligned} \quad (21)$$

Here, u_j , C , α , c_j , s_j , and q_j are the so-called limb-darkening coefficients. Their value can be fitted or can be fixed or prioritized based on theoretical limb darkening calculations.

The double-integral in equation (20) is carried out with Gauss-Legendre quadrature. TLCDM allows us to use $N = 6 \times 6$, 10×10 , 20×20 , 48×48 , and 96×96 integration points. According to our experiences, $N = 20 \times 20$ setting is enough to get good results. For the integration, first we derive the joint contour of the planet and the star which is the boundary of the transited area on the star. Then we set for every observation the integration points according to the 2D Gauss-Legendre quadrature rule. Two examples of the distribution of quadrature points in case of $N = 6$ on the stellar surface can be seen in Figs 4 and 5. Then we calculate the integrand at these points according to equation (20). The integral is calculated in practice as

$$F(t) = \frac{\Delta x}{4\pi} \sum_{j=1}^J \sum_{k=1}^K w_k w_j \Delta y_j L_{D,jk} V_R(x_j, y_k), \quad (22)$$

where j is the index of the quadrature points. Δx and Δy are the distances between the integration limits which can be easily found on the contour limiting the transited area. $J = K = N$ are the number of integration points in x and y directions (6, 10, 20, 48, or 96 in the present implementation). w_s are the Gauss-Legendre integral weights, their values can be found in mathematical textbooks.



Figure 5. Distribution of quadrature points during a total transit phase (black diamonds) for the $N = 6$ case. See Fig. 4 for more explanation.

The advantage of this approach is that one can easily include any future limb darkening law, no need to calculate complicated integrals analytically, and even gravity darkening effect and stellar spots can be added in the future. Its disadvantage is the somewhat longer computational time (see Appendix for the CPU-time records).

3.5 Validation of the model and the code

We validated our code with nine systems. These systems cover a wide variety of possible cases - sometimes very pathological cases: aligned systems (HAT-P-1, HAT-P-20), intermediate systems (HAT-P-3, HAT-P-6, WASP-15), polar orbits (HAT-P-11, HAT-P-32) and retrograde orbits (HAT-P-6, HAT-P-7, HAT-P-14). The validation sample also contain single planet systems on circular orbits with RV-offsets between the used spectrographs (WASP-15b, HAT-P-1, HAT-P-3, HAT-P-6, HAT-P-7) as well as single planets on eccentric orbits and RV-offsets between instruments (HAT-P-14, HAT-P-20, HAT-P32) and a system on eccentric orbit with additional second planet in the system in eccentric orbit (HAT-P-11). The details of the data sources and the modellings as well as the references to the results of these other investigators can be found in Appendix.

Fig. 6 shows the comparison of the results obtained by TLCM to the results obtained by A. H. M. J. Triaud et al. (2010), J. A. Johnson et al. (2008), V. Bourrier et al. (2023), L. Mancini et al. (2018), G. Hébrard et al. (2011), J. N. Winn et al. (2009, 2010a, 2011), R. Sanchis-Ojeda & J. N. Winn (2011), T. Hirano et al. (2011), M. Esposito et al. (2017), and S. Albrecht et al. (2012). See Appendix for the results in tabular form where one can find the visualization of the light curve and radial velocity solutions. The TLCM results reproduce the values obtained in other studies to within the level expected from the published uncertainties and the different data selections and modelling assumptions. This provides an empirical validation of the RM implementation over a broad range of projected obliquities.

4 ANALYSIS OF HD 2685

4.1 Fit with strict prior on $V \sin I_*$

The photometric data of HD 2685 were obtained by *TESS* and described in Section 2. We have obtained new radial velocity measurements in- and outside the transit of HD 2685 (see Section 2). These data were extended by CORALIE, CHIRON, and FEROS measurements taken from (M. I. Jones et al. 2019). We have 21

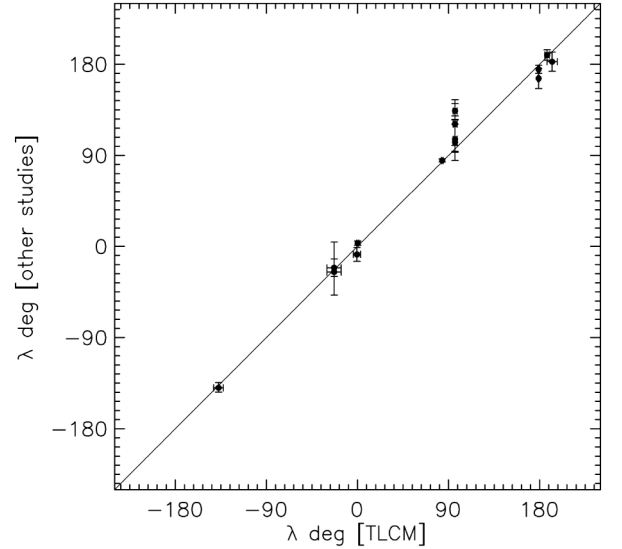


Figure 6. Comparison of the spin-orbit angle obtained by TLCM and other studies with their respective error bars. The solid line represents the 1:1 relationship. For the modelling results and for the input literature values from other studies see the Appendix. The scattered literature data at $\lambda_{\text{TLCM}} = 96^\circ$ is due to HAT-P-11 case. That system appears to have an additional planet in the system which complicates the RV-analysis. It seems that the earlier unknown second planet might have an impact of earlier λ -determination. See the Appendix for a discussion of that case.

HARPS, 15 CORALIE, 11 CHIRON, 5 FEROS data points, in total 52 RV-observations (Table 2).

We used the following stellar parameters for HD 2685: stellar effective temperature of $T_{\text{eff}} = 6801 \pm 76$ K and a metallicity of $Z = 0.019 \pm 0.005$ (based on M. I. Jones et al. 2019). These two parameters and the observed mean-stellar density – which can be derived from the fitted a/R_{star} value (e.g. S. Csizmadia et al. 2015) – are used to select isochrones for the determination of stellar mass and radius described in detail in S. Csizmadia (2020).

We used a $V \sin I_* = 15.4 \pm 0.2$ km s $^{-1}$ prior for the projected rotational velocity of the host star. This value is based on the results of M. I. Jones et al. (2019). To test how sensitive is the solution for the adopted prior, we repeated the analysis with different priors on $V \sin I_*$ in Section 4.2. We report the results of different approaches. One uses a uniform prior $U(15.2, 15.6)$ km s $^{-1}$ and the other one uses a Gaussian prior $N(15.4, 0.2)$ km s $^{-1}$. We call the solution based on the normal prior as fiducial solution.

The fitted (free) parameters and their uniform or normal priors can be found in Tables 3 and 4. Only three parameters need explanation as the others are standardly used in the field. The h parameter is a correction to all flux values to correct the flux level that at phase 0.25 we have a normalized flux 1.0. This parameter acts as $(1 + h)^{-1}$ multiplicative factor on the total model flux. For details, see S. Csizmadia (2020). For the sake of less degeneracy in the quadratic limb darkening coefficients, we used parameters A and B which are connected to the limb darkening coefficients via the equations (3)–(4) of S. Kálmán et al. (2025). For the convenience of the reader, we transform TLCM’s A and B to the usual u_a and u_b limb darkening coefficients (see the list of the ‘deduced parameters’ in Table 3). Note that the theoretical limb darkening calculations of A. Claret (2018) yields $u_a \sim 0.32$ and $u_b \sim 0.25$. The observed values (0.23 and 0.29) are not far from the theoretical numbers.

Table 3. Joint light curve and radial velocity solution results for HD 2685 system by TLCM, using a uniform prior on $V \sin I_*$. The priors are the uniform (U) or normal (N) priors on the fitted parameters during the MCMC analysis. In the last column, we list the results of M. I. Jones et al. (2019). Note that Jones et al. used a different parametrization of limb darkening and fitted the gamma-velocities separately for the different instruments rather than their zero-point differences.

Parameter	Notation (unit)	Prior	Value and uncertainty range	(M. I. Jones et al. 2019)
Fitted parameters				
Scaled semimajor axis	a/R_{star}	$U(0, 30)$	$7.62^{+0.32}_{-0.41}$	$7.6974^{+0.0689}_{-0.0541}$
Planet-to-star radius ratio	$R_{\text{planet}}/R_{\text{star}}$	$U(0, 1)$	0.09405 ± 0.00023	$0.09467^{+0.00033}_{-0.00028}$
Impact parameter	b	$U(0, 2)$	0.088 ± 0.052	
	$\sqrt{e} \sin \omega$	$U(-1, 1)$	$0.02^{+0.20}_{-0.19}$	
	$\sqrt{e} \cos \omega$	$U(-1, 1)$	$0.06^{+0.16}_{-0.18}$	
Epoch ($BJD_{\text{TDB}} - 2450000.0$)	T_0	$U(8725.99, 8726.19)$	8726.09345 ± 0.00012	8325.78297 ± 0.00020
Period	P (d)	$U(4.125, 4.128)$	$4.126904695^{+0.000000352}_{-0.000000372}$	$4.12688^{+0.00005}_{-0.00004}$
Barycentric velocity	V_γ (km s $^{-1}$)	$U(-300, +300)$	2.103 ± 0.012	
RV semi-amplitude	K (m s $^{-1}$)	$U(-10000, +10000)$	$119.6^{+12.3}_{-12.7}$	117.4 ± 12.0
h-correction	h	$U(-1, 1)$	-0.000106 ± 0.000116	
red noise factor	σ_r (ppm)	$U(0, 2 \cdot 10^6)$	42265^{+372}_{-376}	
white noise level	σ_w (ppm)	$U(0, 2 \cdot 10^6)$	634 ± 30	
RV-offset (CORALIE - HARPS)	offset1 (m s $^{-1}$)	$U(-10^5, 10^5)$	-45.9 ± 20	
RV-offset (CHIRON - HARPS)	offset2 (m s $^{-1}$)	$U(-10^5, 10^5)$	-2113 ± 25.7	
RV-offset (FEROS - HARPS)	offset3 (m s $^{-1}$)	$U(-10^5, 10^5)$	$+73.4^{+21.0}_{-22.2}$	
Projected rotational velocity	$V \sin I$	$U(15.2, 15.6)$	15.40 ± 0.13	15.40 ± 0.2
Projected spin-orbit angle	λ (deg)	$U(-180^\circ, +180^\circ)$	$55.6^{+10.4}_{-11.7}$	
Limb darkening coefficient A	A	$U(-5, 5)$	1.20 ± 0.05	
Limb darkening coefficient B	B	$U(-5, 5)$	1.13 ± 0.03	
Deduced parameters				
Inclination	i (deg)		89.3 ± 0.4	$89.252^{+0.415}_{-0.444}$
Eccentricity	e		0.0045 ± 0.064	$0.0910^{+0.0390}_{-0.0470}$
Argument of periastron	ω (deg)		18 ± 117	$184.36^{+6.26}_{-6.56}$
Transit duration	D (h)		$4.76^{+0.27}_{-0.19}$	
Stellar mass	$M_{\text{star}} (M_\odot)$		1.33 ± 0.34	$1.44^{+0.05}_{-0.04}$
Stellar radius	$R_{\text{star}} (R_\odot)$		1.57 ± 0.20	1.56 ± 0.05
Mean density of the star	$\rho_{\text{star}} (\text{g cm}^{-3})$		491 ± 71	–
Planetary mass	$M_{\text{star}} (M_{\text{Jupiter}})$		1.15 ± 0.23	1.17 ± 0.12
Planetary radius	$R_{\text{star}} (R_{\text{Jupiter}})$		1.44 ± 0.19	1.44 ± 0.05
Linear limb darkening coefficient	u_a		0.23 ± 0.03	
Quadratic limb darkening coefficient	u_b		0.29 ± 0.03	

There are significant jitter values in this radial velocity data set (M. I. Jones et al. 2019). Since we have additional HARPS-measurements, we re-derived the radial velocity jitter values. We defined four subsets of the RV-data: a subset is the RV-data from the same instrument. First, we run several hundreds of light curve fits where all possible combinations of the following fixed RV-jitter values were used: 0, 15, 30, 45, and 75 m s $^{-1}$. Then, by multilinear interpolation we searched for those jitter-combinations which yielded a reduced $\chi^2 = 1$ for every subset. This yielded the following jitter values:

- (i) $\sigma_{\text{jitter, HARPS}} = 46 \text{ m s}^{-1}$
- (ii) $\sigma_{\text{jitter, CORALIE}} = 48 \text{ m s}^{-1}$
- (iii) $\sigma_{\text{jitter, CHIRON}} = 69 \text{ m s}^{-1}$
- (iv) $\sigma_{\text{jitter, FEROS}} = 23.5 \text{ m s}^{-1}$

Note that M. I. Jones et al. (2019) found jitter values of 16^{+17}_{-12} , 22 ± 15 , 32^{+27}_{-23} , for CORALIE; CHIRON and FEROS, respectively. All values are compatible with ours within 3σ error bars. We used the aforementioned values for the subsequent work.

TLCM performs a Genetic Algorithm optimization first. The found best-fitting values are used as starting point for the subsequent Differential-Evolution Markov Chain Monte Carlo (DE-MCMC) procedure. We did 6000 steps for the burn-in and we repeated it six times iteratively as the DE-MCMC needs a good starting prior sample. Then we started the DE-MCMC analysis. For this step, we used 20 chains and no thinning. The convergence was monitored via checking the Gelman-Rubin R-parameter and checking the Effective Sample Size. When the convergence criteria (see S. Csizmadia 2020) were not fulfilled, we automatically extended the chain length. In Table 3, we report the median values and the 1σ uncertainty ranges based on the 16–84 per cent rule. The results of joint light curve and radial velocity fits are presented in Figs 1–3.

We note a much more precise period value in our fit. We derived an eccentricity and argument of periastron different from Jones et al. The system looks compatible with a circular orbit now.

All other parameters are in remarkable good agreement between our results and M. I. Jones et al. (2019). We notice the excellent agreements between the two works especially in scaled

Table 4. Joint light curve and radial velocity solution results for HD 2685 system by TLMC, using a normal prior on $V \sin I_*$. The priors are the uniform (U) or normal (N) priors on the fitted parameters during the MCMC-analysis. In the last column we list the results of M. I. Jones et al. (2019). Note that Jones et al. used a different parametrization of limb darkening and fitted the gamma-velocities separately for the different instruments rather than their zero-point differences.

Parameter	Notation (unit)	Prior	Value & Uncertainty range	(M. I. Jones et al. 2019)
Fitted parameters				
Scaled semi-major axis	a/R_{star}	$U(0, 30)$	$7.60^{+0.33}_{-0.45}$	$7.6974^{+0.0689}_{-0.0541}$
Planet-to-star radius ratio	$R_{\text{planet}}/R_{\text{star}}$	$U(0, 1)$	$0.09402^{+0.00024}_{-0.00023}$	$0.09467^{+0.00033}_{-0.00028}$
Impact parameter	b	$U(0, 2)$	0.086 ± 0.051	
	$\sqrt{e} \sin \omega$	$U(-1, 1)$	$0.04^{+0.20}_{-0.20}$	
	$\sqrt{e} \cos \omega$	$U(-1, 1)$	$0.07^{+0.17}_{-0.18}$	
Epoch ($BJD_{\text{TDB}} - 2450000.0$)	T_0	$U(8725.99, 8726.19)$	8726.09345 ± 0.00012	8325.78297 ± 0.00020
Period	P (d)	$U(4.125, 4.128)$	$4.1269046928^{+0.000000364}_{-0.000000374}$	$4.12688^{+0.00005}_{-0.00004}$
Barycentric velocity	V_γ (km s $^{-1}$)	$U(-300, +300)$	2.103 ± 0.013	
RV semi-amplitude	K (m s $^{-1}$)	$U(-10000, +10000)$	$121.1^{+12.9}_{-12.8}$	117.4 ± 12.0
h-correction	h	$U(-1, 1)$	-0.000109 ± 0.000123	
red noise factor	σ_r (ppm)	$U(0, 2 \cdot 10^6)$	42276^{+382}_{-378}	
white noise level	σ_w (ppm)	$U(0, 2 \cdot 10^6)$	634 ± 30	
RV-offset (CORALIE – HARPS)	offset1 (m s $^{-1}$)	$U(-10^5, 10^5)$	-45.9 ± 20	
RV-offset (CHIRON – HARPS)	offset2 (m s $^{-1}$)	$U(-10^5, 10^5)$	-2113 ± 25.7	
RV-offset (FEROS – HARPS)	offset3 (m s $^{-1}$)	$U(-10^5, 10^5)$	$+72.7^{+22.1}_{-21.4}$	
Projected rotational velocity	$V \sin I$	$N(15.4, 0.2)$	15.40 ± 0.19	15.40 ± 0.2
Projected spin-orbit angle	λ (deg)	$U(-180^\circ, +180^\circ)$	$55.6^{+10.9}_{-11.9}$	
Limb darkening coefficient A	A	$U(-5, 5)$	1.20 ± 0.05	
Limb darkening coefficient B	B	$U(-5, 5)$	1.13 ± 0.03	
Deduced parameters				
Inclination	i (deg)		89.3 ± 0.4	$89.252^{+0.415}_{-0.444}$
Eccentricity	e		0.0038 ± 0.075	$0.0910^{+0.0390}_{-0.0470}$
Argument of periastron	ω (deg)		36 ± 112	$184.36^{+6.26}_{-6.56}$
Transit duration	D (h)		$4.53^{+0.29}_{-0.18}$	
Stellar mass	M_{star} ($M_\odot$)		1.45 ± 0.37	$1.44^{+0.05}_{-0.04}$
Stellar radius	R_{star} ($R_\odot$)		1.59 ± 0.23	1.56 ± 0.05
Mean density of the star	ρ_{star} (g cm $^{-3}$)		488 ± 76	–
Planetary mass	M_{star} (M_{Jupiter})		1.23 ± 0.25	1.17 ± 0.12
Planetary radius	R_{star} (R_{Jupiter})		1.46 ± 0.22	1.44 ± 0.05
Linear limb darkening coefficient	u_a		0.23 ± 0.03	
Quadratic limb darkening coefficient	u_b		0.29 ± 0.03	

semi-major axis, planet-to-star radius ratio and inclination. While our stellar mass is slightly smaller than theirs, the stellar radius, planetary mass and radius are in good agreement.

Finally, the simultaneous light-curve and radial-velocity fit yields a significant, clearly visible RM signal (Fig. 3). For the fiducial spectroscopic $V \sin I_*$ prior, we obtain $\lambda = 55.6^{+10.9}_{-11.9}$. As discussed below (Section 4.2), broader $V \sin I_*$ priors increase the formal uncertainty but do not move the solution towards another configuration.

4.2 Dependence of the projected obliquity on the adopted $V \sin I_*$ prior

The projected stellar rotation velocity is an important external constraint in the interpretation of the RM signal of HD 2685b.

In our fiducial analysis, we used the spectroscopic value $V \sin I_* = 15.4 \pm 0.2$ km s $^{-1}$, reported by M. I. Jones et al. (2019), derived by them from their ZASPE spectral analysis. In Section 4.1, this external information was implemented as a Gaussian prior $N(15.4, 0.2)$ km s $^{-1}$. Fig. 7 shows that there is no correlation between $V \sin I_*$ and λ . We note that spectroscopic $V \sin I_*$ measurements are model-dependent and can be affected by

systematic uncertainties associated with macroturbulence, microturbulence, instrumental broadening, and the adopted line-broadening model. The line-broadening $V \sin I_*$ is not necessarily identical in detail to the effective velocity scale entering an anomalous-RV RM model.

We, therefore, repeated the full joint analysis with alternative priors on $V \sin I_*$. We used a uniform prior as well (Section 4.1). As a broad diagnostic test, we also used a broad uniform prior $U(5, 25)$ km s, which covers a range $\pm 50\sigma$ range of the quoted error bars of M. I. Jones et al. (2019). Another fit was also performed with uniform priors instead of prior ones to mimic error propagation differently. One was carried out with $U(15.2, 15.6)$, and another one with $N(15.4, 1.0)$, so the error bars were increased by a factor of five. The results are summarized in Table 5. The central value of λ remains stable under these tests: the four priors give λ values between 55.3° and 56.7° . As expected, the uncertainty increases when the external $V \sin I_*$ information is relaxed, reaching $(+13^\circ, -24^\circ)$ for the very broad diagnostic prior $U(5, 25)$ km s $^{-1}$. We conclude that the precise numerical uncertainty on λ depends on the adopted $V \sin I_*$ constraint, but the inference that HD 2685b has an intermediate projected obliquity is robust against reasonable broadening of this prior.

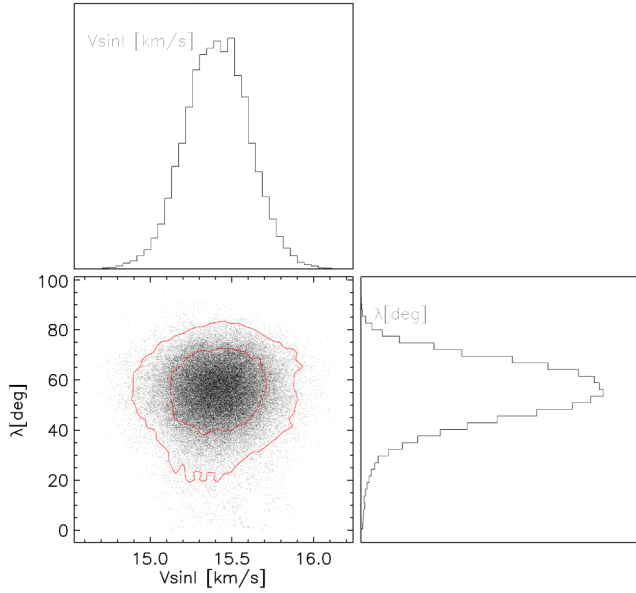


Figure 7. The figure shows the missing correlation between the adopted $N(15.4, 0.2) \text{ km s}^{-1}$ prior on $V \sin i_*$ and λ parameter. The red solid curves represent the 1 and the 2 σ limits.

Table 5. Effect of the adopted $V \sin i_*$ prior on the inferred projected spin-orbit angle. Velocities are given in km s^{-1} , and λ is given in degrees. The last row is intended as a diagnostic broad-prior test rather than as a physically motivated spectroscopic prior. For Gaussian priors, $\mathcal{N}(\mu, \sigma)$ denotes a normal distribution with mean μ and standard deviation σ while for uniform prior $U(a, b)$ denotes the limits of the prior.

Prior on $V \sin i_*$	$V \sin i_*$ posterior value	λ posterior value
$U(15.2, 15.6)$	15.40 ± 0.13	$55^\circ 6^{+10.4}_{-11.7}$
$N(15.4, 0.2)$	15.40 ± 0.19	$55^\circ 6^{+10.9}_{-11.9}$
$N(15.4, 1.0)$	15.43 ± 0.93	$56^\circ 2^{+10.0}_{-12.1}$
$U(5, 25)$	15.88 ± 6.23	$56^\circ 1^{+13.2}_{-23.7}$

The residual pattern around transit is similar in the alternative-prior runs indicating that it is not caused by the narrow $V \sin i_*$ prior. Given the adopted HARPS jitter and the limited number of in-transit points, we do not interpret this pattern as evidence for a different obliquity solution.

5 DISCUSSION

First of all, we point out that our derived stellar radius value $1.57 \pm 0.20 R_\odot$ have larger error bars than that of M. I. Jones et al. (2019) but it is still very close to the interferometrically determined $1.61 \pm 0.04 R_\odot$ value (see Section 1). This can be considered as a validation of the solution.

Fig. 8 shows the location of HD2685b on the host star’s effective temperature – $|\lambda|$ diagram. The source of the obliquity values is the TEPcat catalogue and the references thereof (J. Southworth 2026). As shown in Fig. 8, HD 2685b helps to populate a relatively sparsely sampled part of the $T_{\text{eff}}-|\lambda|$ plane, corresponding to hot host stars with intermediate projected obliquities.

HD 2685 is an evolved star with a stellar surface temperature of $6801 \pm 76 \text{ K}$ (M. I. Jones et al. 2019), above the Kraft-break

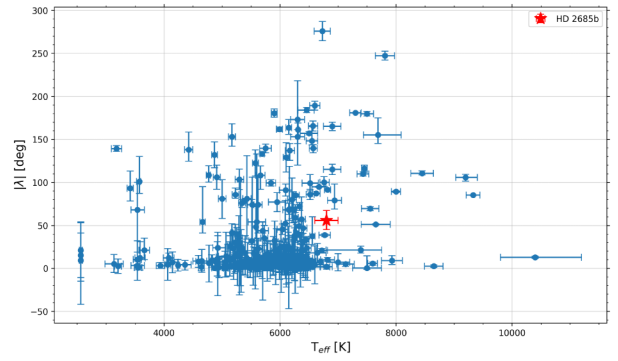


Figure 8. The absolute λ values plotted in the function of T_{eff} from TEPcat (J. Southworth 2026). During the plotting, we averaged the λ values for any system where multiple result were available. During the averaging, we excluded those measurements where a system had an outlier measurement. We also plotted our results for HD 2685b denoted by the red star.

temperature ($T_{\text{eff, Kraft}} = 6250 \text{ K}$). The measured intermediate projected obliquity, $\lambda \sim 56^\circ$, is consistent with the empirical picture established by K. C. Schlaufman (2010) and J. N. Winn et al. (2010a): hot-Jupiter systems around hotter stars show a broader obliquity distribution than those around cooler stars. The exact numerical uncertainty on λ depends on the adopted external $V \sin i_*$ constraint, but the broad-prior tests show that the system remains centred on an intermediate-obliquity solution. In the tidal interpretation discussed by J. N. Winn et al. (2010a) and S. Albrecht et al. (2012), this reflects the lower efficiency of tidal realignment in stars with radiative envelopes.

J. Ruzsna et al. (2025) found that high mass ratio ($M_{\text{planet}}/M_{\text{star}} > 0.002$) systems are tending to have spin-orbit alignment, even for hot host star systems. They pointed out that tidal re-alignment mechanism is inefficient in hot stars due to their radiative envelope. Therefore, the tendency of spin-orbit alignment in high mass ratio systems is rather primordial than consequence of tidal interaction. They also speculated on the possibility that only dynamically isolated planets can accrete enough matter from the protoplanetary discs to grow to high masses, and dynamical isolation helps them to keep their original alignment. According to them, low mass ratio systems ($M_{\text{planet}}/M_{\text{star}} < 0.002$) are dynamically excited and probably compact systems and that is why they are subject to many perturbations, causing a dispersion in the spin-orbit angle, as well as they are unable to collect more mass and gas from the protoplanetary disc. This latter mechanism does not allow the planet to grow over a mass ratio of 0.002.

The mass ratio of the HD 2685 system is 0.0008, well below the limit found by J. Ruzsna et al. (2025), and therefore we expect misalignment from their work. Our measured value ($\lambda \sim 56^\circ$) follows the trend they found.

However, this conclusion is based on the available observational material which does not include the recent work of J. Zak et al. (2025). These latter authors provide a sample of one brown dwarf and nine hot Jupiters. The brown dwarf (high mass ratio system) follows the picture proposed by J. Ruzsna et al. (2025) while eight of the nine hot Jupiters in their sample do not. These eight planets have low mass ratio but all of them are aligned. The remaining ninth hot Jupiter in the sample of J. Zak et al. (2025) is a high-mass object on an aligned orbit.

6 SUMMARY

We presented a new, numerical approach to calculate the RM effect during transit. It can be used with seven different limb-darkening laws, including the most important ones: the quadratic and the power-2 laws. It has an up-to-date analytic description of the convective blueshift phenomenon and it is implemented into the code. The code fits several effects together: RM-effect, eccentric orbits, second Keplerian orbit, corrections due to instrumental offsets and ellipsoidal apparent variation as well as transits, occultation and full phase curves. Now the new RM-approach is part of the TLMC package. The new version can be downloaded from www.transits.hu when this paper is accepted.

The new version of TLMC was validated by nine systems which have aligned, intermediate, polar and retrograde orbits. We found excellent agreements between our modelling results and the ones of other investigators.

It is easy to include the effect of gravity darkening and stellar spots in the future.

We applied the code to HD 2685b which is a transiting hot Jupiter around a hot, evolved star. We performed the first RM measurement for this system. M. I. Jones et al. (2019) had one *TESS* sector's photometric data available at that time, but we analysed seven sectors of photometric data (sectors 1, 27, 28, 67, 68, 94, and 95) and thus we were able to update and refine the system parameters. We jointly fitted the *TESS* photometric light curves with the radial velocity data collected by us with HARPS in- and out-of-transit and with the radial velocity data obtained by CORALIE, FEROS and CHIRON taken from M. I. Jones et al. (2019). Our fiducial results can be found in Table 4 and they are illustrated in Figs 1–3. Beyond the refined planetary and system parameters, we obtain an intermediate projected obliquity. For the fiducial spectroscopic $V \sin I_*$ prior, the result is $55^{+10.9}_{-11.79}$. Repeating the analysis with broader $V \sin I_*$ priors shows that the central value remains stable within a few degrees, while the uncertainty increases to $(+13^\circ, -24^\circ)$ for the broadest diagnostic prior.

Our measurement contributes to the still sparsely sampled region of the $T_{\text{eff}} - |\lambda|$ diagram occupied by hot and evolved host stars with intermediate projected obliquities.

The measured projected spin-orbit angle was compared to all available other measurements (Fig. 8). Our measurement contributes to the still sparsely sampled region of the stellar effective temperature – spin-orbit misalignment diagram occupied by hot and evolved host stars with intermediate projected obliquities. This part of parameter space remains observationally challenging, because transit depths decrease as stellar radii increase and precise radial-velocity follow-up becomes more difficult towards earlier spectral types.

The found λ -angle does not contradict to the prediction of J. N. Winn et al. (2010a) for the λ -distribution. It is also along the line of the predictions of J. Ruzsna et al. (2025) which has different theoretical starting point for the explanation of the observed scatter in the cool/hot regimes of the $T_{\text{eff}} - |\lambda|$ diagram. However, it seems further observational and theoretical work are needed to fully explain that diagram (e.g. S. H. Albrecht et al. 2021).

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DATA AVAILABILITY

All photometric data collected by *TESS* and used in this paper are publicly available at the Mikulski Archive for Space Telescopes (<https://archive.stsci.edu/>).

The radial velocity data of the RM measurement carried out by one of us (AMSS) of HD 2685 are available in the ESO archive (<https://archive.eso.org/cms.html>) and in this paper.

All other radial velocity data used in this paper are publicly available in the cited publications as well as via NASA Exoplanet Archive (<https://science.nasa.gov/exoplanets/>).

The TLMC software can be downloaded at <http://transits.hu/>.

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APPENDIX A: VALIDATION RESULTS

We show the results of the code validation here. Since TLCDM is not able to fit the radial velocity curve alone, only a light curve fit or joint light curve fit + radial velocity curve fit can be performed with it, we used the light curves observed by the *TESS*. We used the SAP light curves and only data points with quality flag = 0 were used. The noise model was the wavelet model described in (S. Csizmadia 2020) and validated in (S. Csizmadia et al. 2023). The free and derived parameters together with their priors can be found in the subsequent tables. We show the RV and light curve fits in the following figures, too. The modellings in general were done in the same way as we did it for HD 2685 (Section 4), the deviations from that approach are noticed below.

The following *TESS* Sectors' data were used for the individual systems (only 2 min cadence data were used):

- (i) HAT-P-1b: 56, 83, 84
- (ii) HAT-P-3b: 16, 22, 49, 76
- (iii) HAT-P-6b: 57, 84
- (iv) HAT-P-7b: 14, 15, 40, 41, 54, 55, 74, 75, 81, 82

- (v) HAT-P-11b: 15, 41, 54, 55, 74, 75, 76, 82, 83
- (vi) HAT-P-14b = WASP-27b: 25, 26, 52, 53, 80
- (vii) HAT-P-20b: 44, 45, 46, 71, 72
- (viii) HAT-P-32b: 58
- (ix) WASP-15b: 15

Only quality = 0 SAP data were utilized. Then, we made a cut around every transits (except WASP-15) and one full transit length before and after every transits were kept. For WASP-15b, we modelled the whole light curve.

A constant period was always assumed.

Except WASP-15b, the starting values for the epoch and the orbital period were taken from A. Kokori et al. (2023). Results in the Tables in this Appendix also tabulates the priors we used for the DE-MCMC analysis. As before, U and N denote uniform and normal priors, respectively. D(x) means a uniform $\pm x$ priors around the values of A. Kokori et al. (2023).

The *TESS* photometric data were on the BJD_{TDB} time-scale. In some source the RV-data were already in this time-scale given; in some others not. We converted them to BJD_{TDB} via J. Eastmann's online tool at University of Ohio.³ When more than one RV-set curve was available (obtained by different instruments or by the same instruments but different authors), a constant rv-offset value was fitted to take the zero-point differences of the different spectrographs into account. When an author re-reduced some older data, we did not use the older data (in general), we based the analysis to the new, more homogeneous data set.

Limb darkening priors for the quadratic limb darkening law were taken by kriging with the known stellar parameters from A. Claret (2018).

The RV data were taken from the following sources:

- (i) HAT-P-1b: HIRES & HDS (J. A. Johnson et al. 2008), MARVELS (N. Thomas et al. 2016a), HIRES and HDS (G. Á. Bakos et al. 2007). MARVELS data set had two different data reduction method. We included both with different zero-point shifts (i.e. different rv-offset values were applied the sets).
- (ii) HAT-P-3b: HIRES (G. Torres et al. 2007), MARVELS (N. Thomas et al. 2016a), HARPS-N (L. Mancini et al. 2018).
- (iii) HAT-P-6b: HIRES (R. W. Noyes et al. 2008), HIRES (S. Albrecht et al. 2012).
- (iv) HAT-P-7b: HIRES and HDS (J. N. Winn et al. 2009). HIRES-data of A. Pál et al. (2008) were re-reduced by and included in (J. N. Winn et al. 2009).
- (v) HAT-P-11b: HIRES (S. W. Yee et al. 2018). S. W. Yee et al. (2024) reports nine new RV-points obtained for monitoring the orbit of the outer long period companion. These points were not used for this appendix.
- (vi) HAT-P-14b = WASP-27b: HIRES (G. Torres et al. 2010) re-reduced and extended by J. N. Winn et al. (2011); FIES and SOPHIE (E. K. Simpson et al. 2011). 11 HARPS-N points from A. S. Bonomo et al. (2017) were not used.
- (vii) HAT-P-20b: HIRES (G. A. Bakos et al. 2011), HARPS-N (M. Esposito et al. 2017).
- (viii) HAT-P-32b: HIRES (J. D. Hartman et al. 2011), HIRES (S. Albrecht et al. 2012).

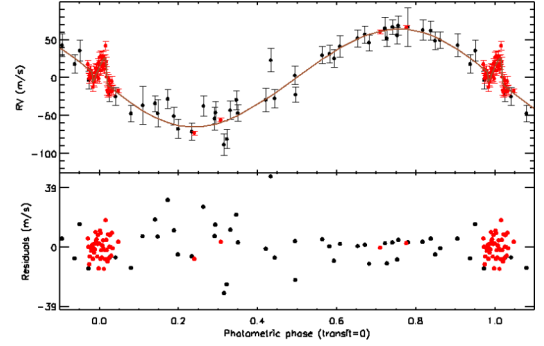


Figure A1. Upper panel: the radial velocity curve of WASP-15b and its fit. Black points represent the CORALIE data, red points represent the HARPS-N data, both data sets obtained by Triaud et al. (2010). Brown solid curve is the radial velocity fit. Lower panel: residuals of the fit.

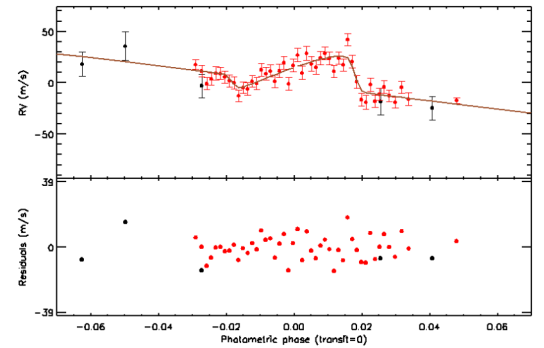


Figure A2. WASP-15b RV-fit result, zoom to the transit and its vicinity. See Fig. 3 for the explanation of the symbols.

A1 WASP-15b

The radial velocity data are taken from (A. H. M. J. Triaud et al. 2010). The radial velocity and light curve fits are illustrated in Figs A1–A4. The results are tabulated in Table A1. We fitted circular and eccentric orbit models to the data, too. Note that the fitted eccentricity ($e = 0.0315 \pm 0.0185$) differs from zero by only 1.7σ . More RV-measurements and well-measured occultation event (maybe with *HST* or *JWST*) would be needed to get the exact value of the eccentricity. Spitzer measured a very significant occultation (secondary transit) for WASP-15b but the eccentricity was not studied (B. M. Kilpatrick et al. 2017).

We find good agreement between our results and the ones of (A. H. M. J. Triaud et al. 2010).

A2 HAT-P-1

(J. A. Johnson et al. 2008) measured $\lambda = 3:1 \pm 2:1$ (J. A. Johnson et al. 2008) for HAT-P-1 while we have found $\lambda = 0:2 \pm 1:0$. The agreement is excellent. Result of TLCM joint LC + RV fit in graphical form can be seen in Fig. A5 and in tabulated form in Table A2. A circular orbit was fitted to the data. Runtime of TLCM on an older cluster took 2 h 54 min.

A3 HAT-P-3

V. Bourrier et al. (2023) measured $\lambda = -25:3^{+29:4}_{-22:8}$ with the RM Revolutions technique (also called RM tomography). With the standard RM-technique L. Mancini et al. (2018) found $\lambda = 21:2 \pm$

³<https://astroutils.astronomy.osu.edu/time/hjd2bjd.html>

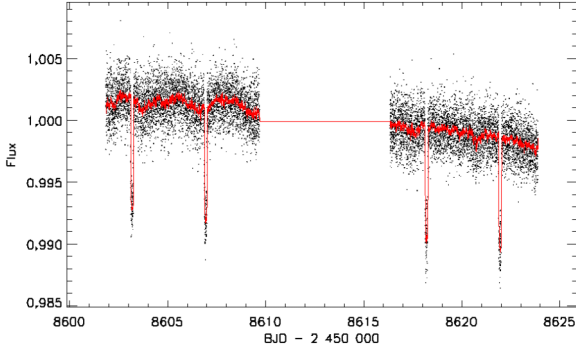


Figure A3. WASP-15b transit fit result. The black points represent the *TESS* SAP observations with quality flag of zero. The red curve is the combined transit model + wavelet based red noise model.

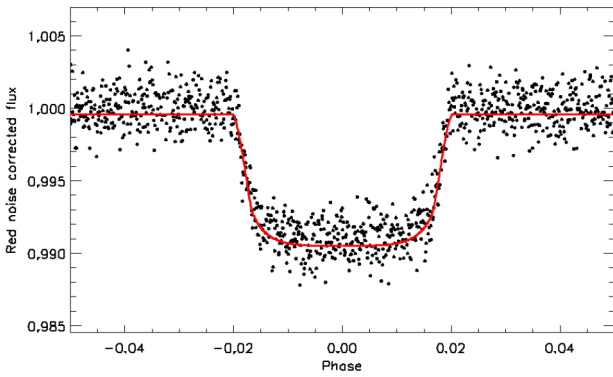


Figure A4. Zoom to the transit of WASP-15b. The black dots represent the red noise corrected fluxes (observed SAP flux – red noise curve based on wavelet-based noise model) and the red curve is the transit model fit.

8:7. Our fit resulted $\lambda = -23^\circ \pm 7^\circ$ which is in good agreement with the previous results. (Note the sign-convention difference between these works.) Fig. A6 and Table A3 present the results. We find perfect agreement with the results of V. Bourrier et al. (2023).

As the stellar and planetary radii and masses are not available in V. Bourrier et al. (2023), we compare them to L. Mancini et al. (2018) (Table A4).

Runtime of TLMC on an older cluster took 2 h and 16 min.

A4 HAT-P-6

Earlier RM-measurements indicated $\lambda = 166^\circ \pm 10^\circ$ (G. Hébrard et al. 2011, their RV-data are not used here) or $\lambda = 175^\circ \pm 4^\circ$ (S. Albrecht et al. 2012). We have got $\lambda = 179^\circ \pm 1^\circ$, in good agreement with S. Albrecht et al. (2012) value but they and we are a little bit away from the value got by (G. Hébrard et al. 2011). Results of TLMC joint LC + RV fit in graphical form can be seen in Fig. A7 and are tabulated in Table A5.

We note that TEPcat lists $\lambda = 165^\circ \pm 6^\circ$ as the value of S. Albrecht et al. (2012). However, this is the resulted value in S. Albrecht et al. (2012) without prior on the radial velocity amplitude K. When S. Albrecht et al. (2012) uses a prior on K, they have got $\lambda = 175^\circ \pm 4^\circ$ and this value must be used for comparison because TLMC fitted the full radial velocity curve (K was determined from that). NASA Exoplanet Archive repeats this 165° value as well.

We also note that an eccentric orbit fit (not detailed here) yielded $\lambda = 150^\circ$.

Runtime of TLMC on an older cluster took 51 min.

A5 HAT-P-7

J. N. Winn et al. (2009) obtained $\lambda = 182^\circ \pm 9^\circ$ while our fit yielded $\lambda = 192^\circ \pm 5^\circ$. The result of the TLMC joint LC + RV fit in graphical form can be seen in Fig. A8 and in tabulated form in Table A6. The stellar and planetary radii and masses are not reported in J. N. Winn et al. (2009) so we compare them to (A. S. Bonomo et al. 2017) (Table A7).

Because of a third body, a significant RV-trend fit was fitted. We used linear and quadratic terms. Also, stellar radius constrain from Gaia DR3 was needed to get the right fit.

Note that (J. N. Winn et al. 2009) likely has an outlying radius ratio and scaled semi-major axis ($R_{\text{planet}}/R_\star \sim 0.0834$ and $a/R_\star 3.82$) which are not compatible with TLMC107, I. Wong et al. (2016) or any other solution mentioned in the NASA Exoplanet Archive Page (see Table A8).

The difference can be understood from the figures of J. N. Winn et al. (2009) that show they missed the first contact of the transit. That lead to a spurious scaled semi-major axis as well as to potentially biased impact parameter (as they are related to each other as $b = a/R_{\text{star}} \times \cos i$, and a biased inclination means that the planet sweeps different part of the stellar disc, and this is compensated by a slightly off lambda-value.

That is why we think TLMC107 up-to-date value must be used in subsequent works for HAT-P-7 because it is based on a comprehensive, complete re-analysis of the *TESS* photometry and earlier RV-data. The case also shows that a homogeneous re-analysis of RM-data complemented with *TESS*, Kepler, CoRoT, CHEOPS and/or PLATO photometry - or very high quality ground-based photometry - would be necessary.

Runtime of TLMC on an older cluster took 12 h 54 min.

A6 HAT-P-11

HAT-P-11b has several measurements of the spin-orbit angle as the reader can see below:

- (i) $\lambda = 133^\circ \pm 8.3^\circ$ (V. Bourrier et al. (2023), RMR)
- (ii) $\lambda = 121^\circ \pm 21^\circ$ (R. Sanchis-Ojeda & J. N. Winn (2011), spots, solution 2)
- (iii) $\lambda = 106^\circ \pm 15^\circ$ (R. Sanchis-Ojeda & J. N. Winn (2011), spots, solution 1)
- (iv) $\lambda = 103^\circ \pm 10^\circ$ (J. N. Winn et al. (2010b), RM)
- (v) $\lambda = 103^\circ \pm 18^\circ$ (T. Hirano et al. (2011), RM)
- (vi) $\lambda = 96.4^\circ \pm 1.9^\circ$ (TLMC, this study, RM)

RMR here means the RM Revolution technique (V. Bourrier et al. 2021) and note that R. Sanchis-Ojeda & J. N. Winn (2011) provided two different solutions based on stellar spot crossing events by the planet. Result of TLMC joint LC + RV fit in graphical form can be seen in Fig. A9 and in tabulated form in Table A9. Since several parameter estimates exist for the system, we collected them in that table. We see a good agreement between other sources and our work in the case of most of the parameters. Since the system look like to have a second planet in the system with eccentric orbit and several years of orbital period (S. W. Yee et al. 2018, 2024), the RV-baseline looks complicated which might have an impact on earlier RV-based works when this second planet was not known. However, the presence of a

Table A1. Results of the joint radial velocity and light curve fit of WASP-15b. Note that A. H. M. J. Triaud et al. (2010) fixed the limb darkening values according to the tables of Claret (2000) – using V -band values – for the analysis of the RM data. As they did not use wavelet-based noise model for the photometric data, the red noise-parameter σ_r and σ_w appears only in our results.

Parameter	Prior	Value and uncertainty Circular orbit This letter	Value and uncertainty Eccentric orbit This letter	Triaud et al.
a/R_{star}	$U(1, 19)$	$7.83^{+0.25}_{-0.23}$	$7.53^{+0.19}_{-0.18}$	$7.45^{+0.15}_{-0.21}$ ^a
$R_{\text{planet}}/R_{\text{star}}$	$U(0, 1)$	$0.0915^{+0.0010}_{-0.0017}$	$0.0915^{+0.0007}_{-0.0012}$	$0.09842^{+0.00067}_{-0.00058}$
b	$U(-1.75, 1.75)$	$0.46^{+0.08}_{-0.05}$	$0.46^{+0.05}_{-0.07}$	$0.525^{+0.037}_{-0.028}$
Epoch (BJD _{TDB} – 245000)		$8603.1967^{+0.0003}_{-0.0002}$	$8603.1968^{+0.0003}_{-0.0003}$	$4584.69819^{+0.00021}_{-0.00020}$ ^b
Period (d)		$3.7520997(1)$	$3.7520999(1)$	$3.752100^{+0.000009}_{-0.000011}$
Normalization constant h (ppm)	$U(0, 1)$	392 ± 85	419^{+119}_{-97}	–
A	$U(-5, 5)$	1.17 ± 0.80	1.18 ± 0.74	–
B	$U(-5, 5)$	0.88 ± 0.18	0.87 ± 0.19	–
$\sqrt{e} \cdot \sin \omega$	$U(-1, 1)$	–	$0.174^{+0.045}_{-0.052}$	–
$\sqrt{e} \cdot \cos \omega$	$U(-1, 1)$	–	$-0.035^{+0.068}_{-0.054}$	–
e	–	0 (fixed)	0.0315 ± 0.0185	–
ω	–	90 (fixed)	$101^\circ \pm 10^\circ$	–
σ_r (ppm)	$U(0, 10^6)$	25111^{+726}_{-597}	25035^{+517}_{-631}	–
σ_w (ppm)	$U(0, 10^6)$	1095 ± 7	1097 ± 5	–
$V \sin I$ (km s ^{–1})	$U(2, 6)$	5.08 ± 0.30	4.92 ± 0.29	$4.27^{+0.26}_{-0.36}$
V_γ (m s ^{–1})	$U(-20, 20)$	$-2309.0^{+2.1}_{-1.9}$	$-2309.6^{+1.5}_{-1.7}$	N/A
K (m s ^{–1})	$U(-2000, 2000)$	$64.5^{+0.9}_{-1.1}$	$64.6^{+0.8}_{-0.9}$	$64.6^{+1.20}_{-1.25}$
λ (°)	$U(-180^\circ, +180^\circ)$	-135.2 ± 5.1	$-137.3^{+4.7}_{-4.9}$	$-139.6^{+5.2}_{-4.3}$ ^c
RV offset (m s ^{–1})	$U(-10\,000, 10\,000)$	14.6 ± 2.0	15.7 ± 2.0	– ^d

^a Triaud et al. reported $a/R_{\text{star}} = 0.1342^{+0.0039}_{-0.0028}$. From these values we calculated the scaled semimajor axis.

^b Triaud et al. has written only BJD – 2450000, not specifying that this is BJD_{UTC} or BJD_{TDB} time-standard.

^c Triaud et al. reported $\beta = -\lambda$ angle. We converted it to λ .

^d Not reported in Triaud et al. (see their Section 3.3).

second planet in the system does not have an impact on the spin-orbit measurements from stellar spot crossings. Our solution is closer to the first solution of R. Sanchis-Ojeda & J. N. Winn (2011) although it is compatible with both of their solution.

The case clearly shows that RM-analysis must be repeated when new, relevant knowledge on the system is revealed.

Runtime of TLCDM on an older cluster took 8 h 23 min.

Identifiers to the sources of data in Table A9: B: V. Bourrier et al. (2023), Y1: S. W. Yee et al. (2018), Y2: S. W. Yee et al. (2024), A: Q. An et al. (2024), X: J. W. Xuan & M. C. Wyatt (2020), W: J. N. Winn et al. (2010b), H11: T. Hirano et al. (2011), B24: M. Basilicata et al. (2024), K23: A. Kokori et al. (2023), B18: T. A. Berger et al. (2018), S: K. G. Stassun et al. (2017), H: T. Holczer et al. (2016), M: T. D. Morton et al. (2016), K: H. A. Knutson et al. (2014), SW: R. Sanchis-Ojeda & J. N. Winn (2011), S11: J. Southworth (2011), B10: G. Á. Bakos et al. (2010).

A7 HAT-P-14

J. N. Winn et al. (2011) obtained $\lambda = 189^\circ 1 \pm 5^\circ 1$ for this system while we have got $\lambda = 187^\circ 4 \pm 2^\circ 2$ from our fit. The results are visualized in Fig. A10 and tabulated in Table A10.

Because of a close companion star HAT-P-14B, we added an RV-drift to the fit.

Note that HAT-P-14b exhibits grazing transits which causes difficulties in LD-determination. That is why applying LD-priors is important in this case.

Runtime of TLCDM on an older cluster took 6 h 32 min.

Results in tabulated form (this target has no modern, *TESS*-based, joint solution, so we are the first to do it) are in Table A.10.

A8 HAT-P-20

M. Esposito et al. (2017) reported $\lambda = -8^\circ 0 \pm 6^\circ 9$ for this system while we have obtained $\lambda = -0^\circ 5 \pm 3^\circ 6$. Results are visualized in Fig. A11 and tabulated in Table A11. There is a significant discrepancy in argument of periastron ω , but all other parameters show good agreement with M. Esposito et al. (2017).

Runtime of TLCDM on an older cluster took 5 h 29 min.

A9 HAT-P-32

Our last validating object is HAT-P-32b which is on polar orbit. S. Albrecht et al. (2012) measured $\lambda = 85^\circ 0 \pm 1^\circ 5$ while we obtain $\lambda = 83^\circ 7 \pm 0^\circ 8$, consistent with their result within the quoted uncertainties.

One RV-point at BJD 2 454 337.9309 was deleted from the RV-data set. The results are shown in Table A12 and in graphical form in Fig. A12.

Interestingly, the limb darkening coefficients do not converge to the priors.

Runtime of TLCDM on the TU cluster: 3 h 2 min.

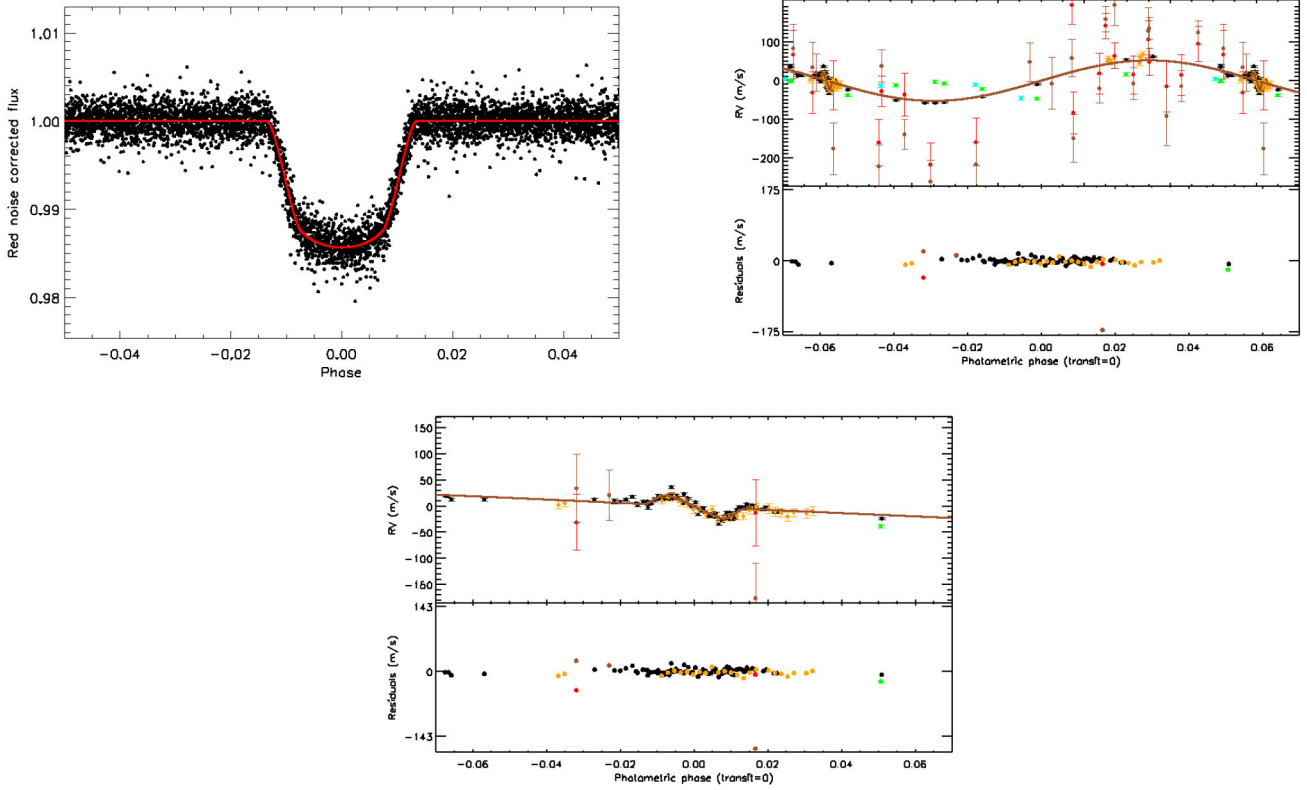


Figure A5. Results of the joint RV + LC solution of HAT-P-1. In the RV-figures, blue points are HIRES data from J. A. Johnson et al. (2008), red and violet are MARVELS (first and second data reduction) from N. Thomas et al. (2016b), orange is HDS from J. A. Johnson et al. (2008), green and cyan are HIRES and HDS from G. Á. Bakos et al. (2007).

Table A2. Priors and results of the joint RV + light curve solution of HAT-P-1 by TLCM and the comparison of the results to N. Nikolov et al. (2014) and to J. A. Johnson et al. (2008).

Parameter	N. Nikolov et al. (2014)	J. A. Johnson et al. (2008)	TLCM	Prior
a/R_*	9.853 ± 0.071	10.67 ± 0.25	9.91 ± 0.19	$U(1, 20)$
R_p/R_*	0.11802 ± 0.00018	0.11295 ± 0.00073	0.1191 ± 0.0011	$U(0, 1)$
b	$0.7501^{+0.0064}_{-0.0069}$	0.693 ± 0.023	0.745 ± 0.012	$U(0, 2)$
$\sqrt{e} \sin \omega$	n/a	0	0.0 (fixed)	–
$\sqrt{e} \cos \omega$	n/a	0	0.0 (fixed)	–
e	n/a	0	0.0 (fixed)	–
ω (deg)	n/a	0	90° (fixed)	–
Epoch (BJD _{TDB} –2450000)	3979.93202(24)	4363.94656(72)	3979.92996 ± 0.00060	$D(0.02)$
Period (d)	4.46529976(55)	4.4652934(93)	4.4653015(4)	$D(2 \times 10^{-5})$
K (m s ^{–1})	58.9 ± 1.2	59.3 ± 1.4	52.27 ± 0.60	$U(0, 1000)$
V_γ (km s ^{–1})	n/a	n/a	0.0005 ± 0.0003	$U(-100, 100)$
RV offset 2 (m s ^{–1})	n/a	n/a	0.02 ± 9.4	$U(-1000, 1000)$
RV offset 3 (m s ^{–1})	n/a	n/a	12.18 ± 7.81	$U(-1000, 1000)$
RV offset 4 (m s ^{–1})	n/a	n/a	-43.72 ± 1.11	$U(-1000, 1000)$
RV offset 5 (m s ^{–1})	n/a	n/a	49.71 ± 1.26	$U(-1000, 1000)$
RV offset 6 (m s ^{–1})	n/a	n/a	47.72 ± 2.22	$U(-1000, 1000)$
Height correction	n/a	n/a	0.0109 ± 0.0005	$U(-1, 1)$
Red noise factor	n/a	n/a	0.11487 ± 0.00093	–
White noise σ	n/a	n/a	0.001897 ± 0.000013	–
$V \sin I$ (km s ^{–1})	n/a	3.75 ± 0.58	3.39 ± 0.13	$U(2, 6)$
λ (deg)	n/a	3.7 ± 2.1	0.2 ± 1.0	$U(-180, 180)$
A	n/a	n/a	1.25 ± 0.01	$N(1.25, 0.05)$
B	n/a	n/a	1.37 ± 0.05	$N(1.37, 0.05)$
$R_*/R_\odot$	$1.174^{+0.026}_{-0.027}$	1.115 ± 0.050	1.20 ± 0.08	–
$M_*/M_\odot$	$1.151^{+0.052}_{-0.051}$	1.133 ± 0.077	1.14 ± 0.14	–
R_p/R_{Jup}	1.319 ± 0.019	1.225 ± 0.059	1.40 ± 0.10	–
M_p/M_{Jup}	0.525 ± 0.019	0.524 ± 0.031	0.46 ± 0.04	–

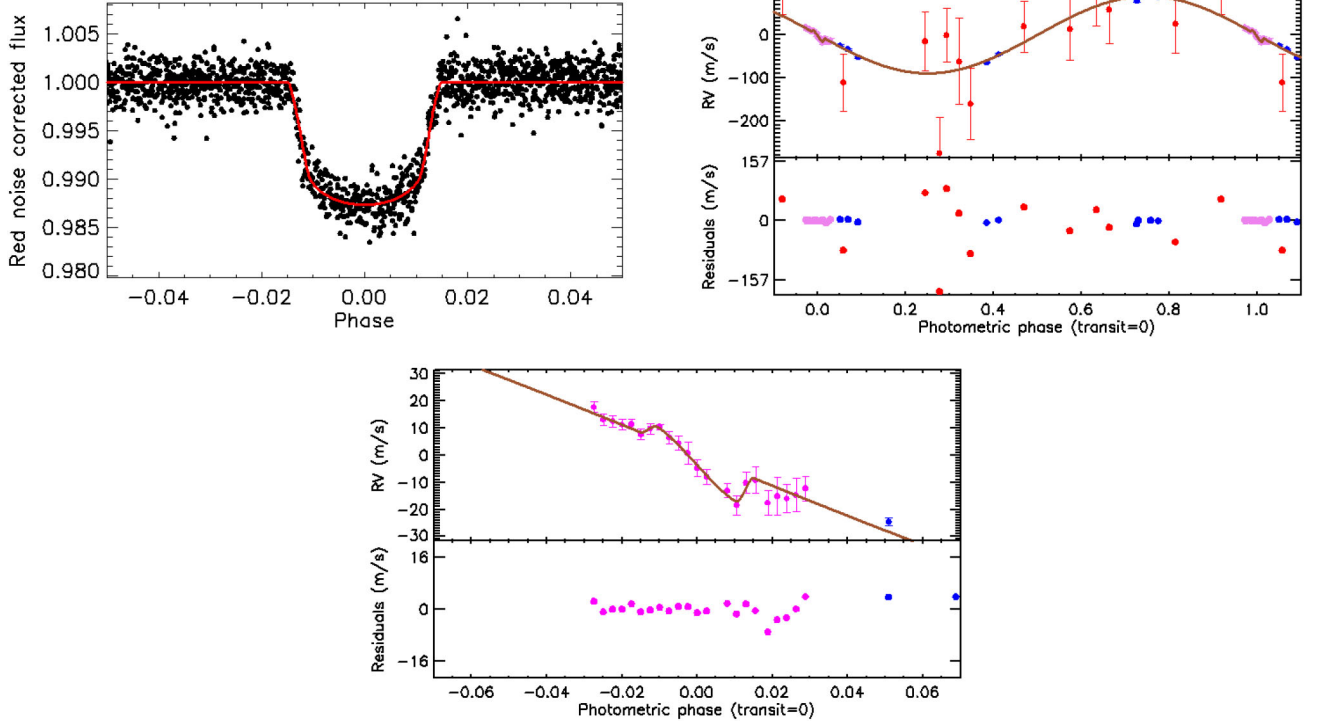


Figure A6. Results of the joint RV + LC solution of HAT-P-3. In the RV-figures above: blue points are Hires data from G. Torres et al. (2007), red are MARVELS from N. Thomas et al. (2016b), violet ones are from L. Mancini et al. (2018).

Table A3. Comparison of fitted parameters from V. Bourrier et al. (2023) and TLMC analysis for HAT-P-3b.

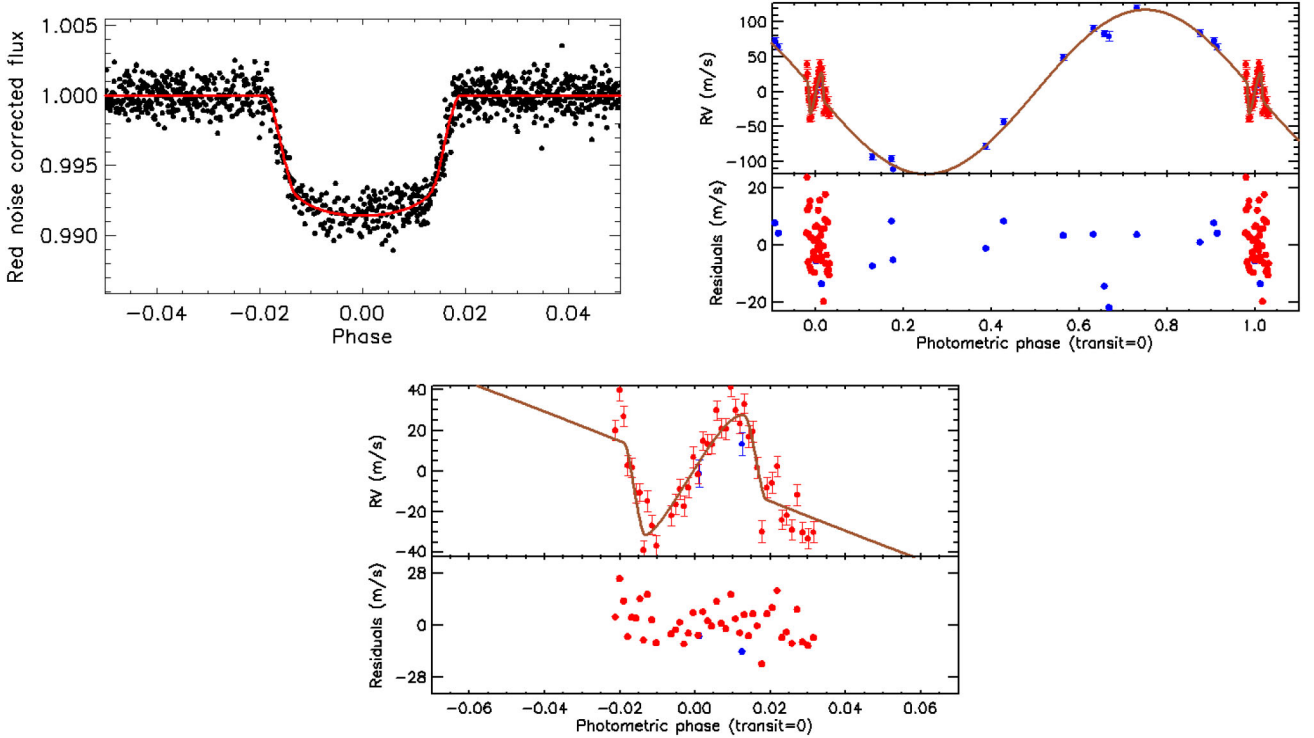
Parameter	V. Bourrier et al. (2023)	TLMC	Prior
a/R_*	9.81050 ± 0.26670	9.98 ± 0.40	$U(1, 20)$
R_p/R_*	0.11091 ± 0.00048	0.1080 ± 0.0017	$U(0, 1)$
b	0.615 ± 0.012	0.59 ± 0.05	$U(0, 2)$
$\sqrt{e} \sin \omega$	0.0 (fixed)	0.0 (fixed)	–
$\sqrt{e} \cos \omega$	0.0 (fixed)	0.0 (fixed)	–
e	0.0 (fixed)	0.0 (fixed)	–
ω (deg)	90° (fixed)	90° (fixed)	–
Epoch (BJD _{TDB} – 2450000)	4218.75960(16)	6843.02259(78)	$D(0.02)$
Period (d)	2.89973797(38)	2.899737(99)	$D(2 \times 10^{-5})$
K (m s ^{–1})	90.63 ± 0.58	89.71 ± 0.67	$U(0, 1000)$
V_y (km s ^{–1})	n/a	-0.0142 ± 0.0005	$U(-100, 100)$
RV offset 2 (m s ^{–1})	n/a	47.93 ± 20.21	$U(-1000, 1000)$
RV offset 3 (m s ^{–1})	n/a	-23370.60 ± 0.88	$U(-1000, 1000)$
Height correction	n/a	-0.00021 ± 0.00017	$U(-1, 1)$
Red noise factor	n/a	0.033186 ± 0.000737	–
White noise σ	n/a	0.001530 ± 0.000008	–
$V \sin I$ (km s ^{–1})	n/a	1.26 ± 0.17	$U(0, 2)$
λ (deg)	$-25.3^{+29.4}_{-22.8}$	-23.1 ± 7.1	$U(-180, 180)$
A	n/a	1.14 ± 0.01	$N(1.31, 0.05)$
B	n/a	1.34 ± 0.75	$N(1.46, 0.05)$

Table A4. Comparison of stellar and planetary radii, masses between our results and (L. Mancini et al. 2018) for HAT-P-3A and b.

Parameter	L. Mancini et al. (2018)	TLMC
$R_*/R_\odot$	0.850 ± 0.023	0.84 ± 0.06
$M_*/M_\odot$	0.925 ± 0.046	0.90 ± 0.08
R_p/R_{Jup}	0.911 ± 0.034	0.88 ± 0.08
M_p/M_{Jup}	0.595 ± 0.024	0.59 ± 0.03

Table A5. Results for HAT-P-6b.

Parameter	R. W. Noyes et al. (2008)	TLCM	Prior
a/R_*	7.69 ± 0.22	7.38 ± 0.16	$U(1, 20)$
R_p/R_*	0.09338 ± 0.00053	0.0903 ± 0.0007	$U(0, 1)$
b	0.602 ± 0.030	0.668 ± 0.019	$U(0, 2)$
$\sqrt{e} \sin \omega$	0.0 (fixed)	0.0 (fixed)	–
$\sqrt{e} \cos \omega$	0.0 (fixed)	0.0 (fixed)	–
e	0.0 (fixed)	0.0 (fixed)	–
ω (deg)	90° (fixed)	90° (fixed)	–
Epoch (BJD _{TDB} –2450000)	4035.67652 ± 0.00196	6100.8847 ± 0.0006	$D(0.02)$
Period (d)	3.852985	$3.8529976(60)$	$D(2 \times 10^{-5})$
K (m s ^{–1})	115.5 ± 4.2	117.8 ± 1.7	$U(0, 1000)$
V_γ (km s ^{–1})	n/a	-0.0170 ± 0.0012	$U(-100, 100)$
RV offset 2 (m s ^{–1})	n/a	12.99 ± 1.54	$U(-1000, 1000)$
Height correction	n/a	0.00034 ± 0.00008	$U(-1, 1)$
Red noise factor	n/a	0.019675 ± 0.000466	–
White noise σ	n/a	0.000795 ± 0.000004	–
$V \sin I$ (km s ^{–1})	7.8 ± 0.6^a	9.12 ± 0.34	$U(8, 12)$
λ (deg)	175 ± 4^a	179 ± 1	$U(-180, 180)$
A	n/a	1.18 ± 0.01	$N(1.19, 0.05)$
B	n/a	1.23 ± 0.05	$N(1.24, 0.05)$
$R_*/R_\odot$	1.46 ± 0.069	1.53 ± 0.13	–
$M_*/M_\odot$	1.290 ± 0.066	1.32 ± 0.31	–
R_p/R_{Jup}	$1.330^{+0.064}_{-0.058}$	1.35 ± 0.13	–
M_p/M_{Jup}	$1.059^{+0.053}_{-0.052}$	1.09 ± 0.17	–

^aThe $V \sin I$ and λ value is from S. Albrecht et al. (2012) and not from R. W. Noyes et al. (2008).**Figure A7.** Legend for RV figures and transit light curves of HAT-P-6b above: blue points are HIRES data from R. W. Noyes et al. (2008), red points also are HIRES data from S. Albrecht et al. (2012).

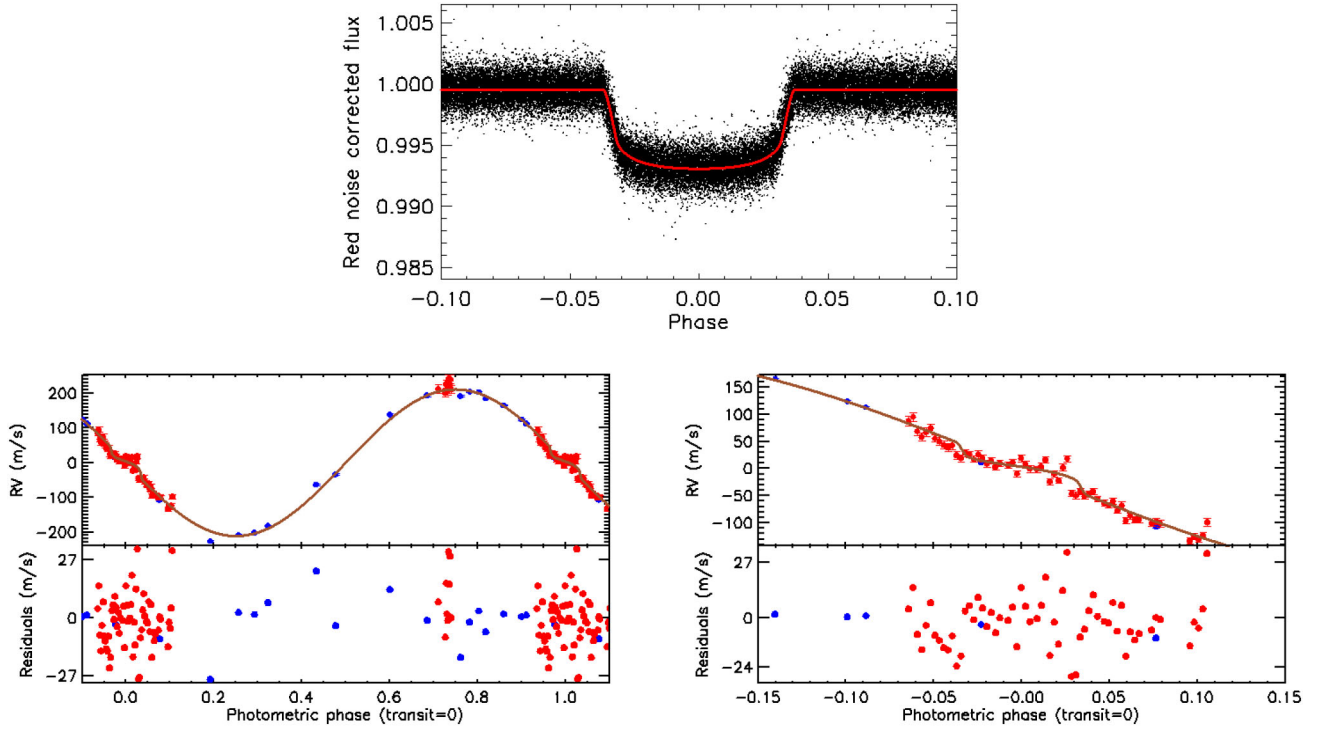


Figure A8. Results on HAT-P-7. Legend for RV figures above: blue points are HIRES data from J. N. Winn et al. (2009) that also includes the points of A. Pál et al. (2008) (HIRES-data of A. Pál et al. (2008) were re-reduced by and included in J. N. Winn et al. (2009), red points are HDS data from J. N. Winn et al. (2009).

Table A6. Comparison of HAT-P-7 system parameters from J. N. Winn et al. (2009) and TLCM analysis.

Parameter	J. N. Winn et al. (2009)	TLCM	Prior
a/R_*	4.03 ± 0.16^a	4.18 ± 0.04	$U(1, 20)$
R_p/R_*	0.0781 ± 0.0007^a	0.0769 ± 0.0002	$U(0, 1)$
I (deg)	82.2 ± 1.2^a	–	–
b	0.55 ± 0.09^b	0.473 ± 0.037	$U(0, 2)$
$\sqrt{e} \sin \omega$	0.0 (fixed) ^a	0.0 (fixed)	–
$\sqrt{e} \cos \omega$	0.0 (fixed) ^a	0.0 (fixed)	–
e	0.0 (fixed) ^a	0.0 (fixed)	–
ω (deg)	90° (fixed) ^a	90° (fixed)	–
Epoch (BJD _{TDB} –2450000)	$5739.244382(13)^a$	5739.2421 ± 0.0004	$D(0.02)$
Period (d)	$2.204736003(49)^a$	$2.204738(2)$	$D(2 \times 10^{-5})$
K (m s ^{–1})	211.8 ± 2.6^a	214.2 ± 0.6	$U(0, 1000)$
V_γ (km s ^{–1})	n/a	-0.3881 ± 0.0433	$U(-100, 100)$
RV linear drift (m s ^{–1} d ^{–1})	n/a	-0.815 ± 0.090	–
RV quadratic drift (m s ^{–1} d ^{–2})	n/a	-0.00041 ± 0.00004	–
RV offset 2 (m s ^{–1})	n/a	-5.2952 ± 1.2131	$U(-1000, 1000)$
Height correction	n/a	-0.000156 ± 0.000084	$U(-1, 1)$
Red noise factor	n/a	0.041673 ± 0.000595	–
White noise σ	n/a	0.000983 ± 0.000003	–
$V \sin I$ (km s ^{–1})	4.9 ± 1.2	4.77 ± 0.39	$U(2.9, 6.9)$
λ (deg)	182.5 ± 9.4	192.4 ± 5.1	$U(0, 360)$
A	n/a	1.20 ± 0.01	$N(1.22, 0.05)$
B	n/a	1.12 ± 0.05	$N(1.30, 0.05)$

^aValues adopted from A. Kokori et al. (2023).

^bImpact parameter calculated from published table values.

Table A7. Comparison of stellar and planetary radii and masses of the HAT-P-7 system to each other obtained by different studies.

Parameter	A. S. Bonomo et al. (2017)	TLCM (this study)
$R_*/R_\odot$	$2.00^{+0.01}_{-0.02}$	1.92 ± 0.20
$M_*/M_\odot$	$1.51^{+0.04}_{-0.05}$	1.42 ± 0.17
R_p/R_{Jup}	1.806 ± 0.036	1.44 ± 0.16
M_p/M_{Jup}	1.510 ± 0.020	1.75 ± 0.14

Table A8. Published values of R_p/R_* and a/R_* of HAT-P-7b compared to TLCM results.

Publication	R_p/R_*	a/R_*
K. G. Stassun, K. A. Collins & B. S. Gaudi (2017)	n/a	4.13 ± 0.15
I. Wong et al. (2016)	0.0781 ± 0.0007	4.03 ± 0.16
T. D. Morton et al. (2016)	$0.075408(8)$	n/a
L. J. Esteves, E. J. W. De Mooij & R. Jayawardhana (2015)	$0.077524^{+0.000017}_{-0.000022}$	$4.1545^{+0.0029}_{-0.0013}$
B. M. Morris, A. M. Mandell & D. Deming (2013)	0.07759 ± 0.00003	4.1502 ± 0.0039
J. Southworth (2011)	n/a	$4.18760^{+0.01673}_{-0.01659}$
TLCM, this study	0.0769 ± 0.0002	4.18 ± 0.04
J. N. Winn et al. (2009)	$0.0834^{+0.0012}_{-0.0021}$	$3.82^{+0.39}_{-0.16}$

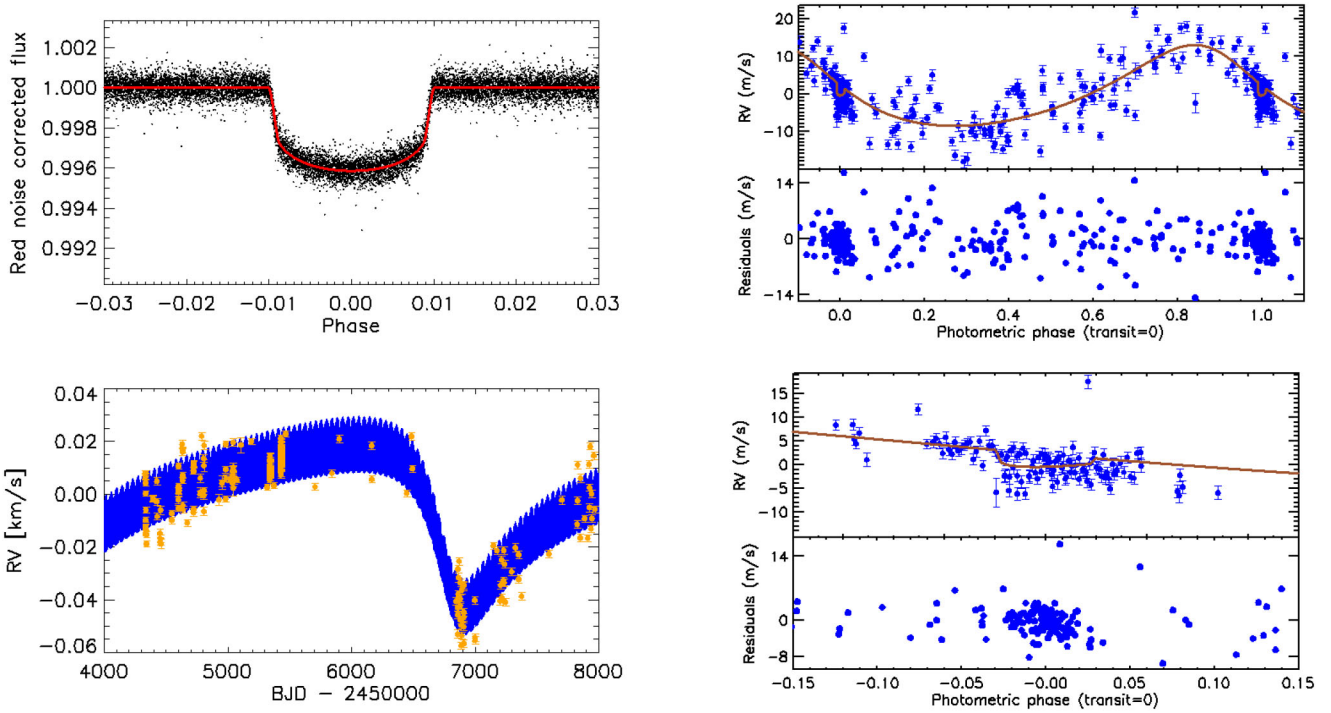
**Figure A9.** RV and LC fits of HAT-P-11. Legend for RV figures above: all data are HIRES-observations from S. W. Yee et al. (2018).⁴

Table A9. Comparison of several RV and light curve fit results. See Section A6 for resolving the abbreviations of the references.

Parameter	Value (various sources)	TLCM	Prior
a/R_*	B24: $15.05^{+0.21}_{-0.22}$ K23: $15.58^{+0.17}_{-0.82}$ B: 16.50 ± 0.18 S: 15.10 ± 0.47 SW: 15.6 ± 1.5 S11: $16.2655^{+0.2199}_{-0.2141}$ B10: $15.58^{+0.17}_{-0.82}$	14.34 ± 0.13	$U(1, 20)$
R_p/R_*	B24: $0.058993^{+0.000065}_{-0.000070}$ K23: 0.0576 ± 0.0009 B: $0.05850^{+0.00009}_{-0.00013}$ M16: $0.057989^{+0.000049}_{-0.000033}$ SW: 0.05862 ± 0.00026 B10: 0.0576 ± 0.0009	0.0591 ± 0.0001	$U(0, 1)$
b	B24: $0.227^{+0.013}_{-0.015}$ B: $0.209^{+0.019}_{-0.032}$ SW: 0.132 ± 0.045 B10: $0.347^{+0.130}_{-0.139}$	0.249 ± 0.021	$U(0, 2)$
$\sqrt{e} \sin \omega$	–	0.223 ± 0.018	$U(-1, 1)$
$\sqrt{e} \cos \omega$	–	0.430 ± 0.009	$U(-1, 1)$
e	A: $0.251^{+0.045}_{-0.047}$ B24: $0.2577^{+0.0033}_{-0.0025}$ K23: 0.20 ± 0.05 B: 0.264353 ± 0.000602 Y1: $0.218^{+0.034}_{-0.031}$ S: 0.20 ± 0.05 K14: $0.232^{+0.054}_{-0.053}$ B10: 0.198 ± 0.046	0.235 ± 0.011	
ω (deg)	A: 28 ± 11 B24: $192.0^{+2.9}_{-3.0}$ K23: 355.2 ± 17.3 B: 342.186 ± 0.179 Y1: 19^{+14}_{-16} K14: 7^{+24}_{-25} B10: 355.2 ± 17.3	27.4 ± 2.0	
Epoch (BJD _{TDB} – 2450000)	B24: 4957.8132067(53) K23: 5798.515261(23) B10: 4605.89132(32)	5798.515143(39)	$U(5798.25, 5798.50)$
Period (d)	A: 4.8880 ± 0.0001 B24: $4.887802443 \pm 3.4 \times 10^{-8}$ K23: $4.88780201 \pm 1.7 \times 10^{-7}$ S11: $4.88781501 \pm 6.8 \times 10^{-7}$	$4.88780246(48)$	$N(4.887802, 50)$
K (m s ^{–1})	B: 12.01 ± 1.38 Y1: $10.42^{+0.64}_{-0.66}$ S: 11.6 ± 1.2	10.80 ± 0.12	$U(-10000, 10000)$
V_γ (km s ^{–1})	n/a	^a -0.0021 ± 0.0002	$U(-300, 300)$
Height correction	n/a	0.000214 ± 0.000092	$U(-1, 1)$
Red noise factor	n/a	0.059595 ± 0.000261	$U(0, 1)$
White noise σ	n/a	0.000523 ± 0.000001	$U(0, 1)$
$V \sin I$	Y1: ^b 1.5 ± 1.5	2.68 ± 0.23	$U(0, 3)$
λ (deg)	B: $133.9^{+7.1}_{-8.3}$ SW: 121^{+24}_{-21} W: 103^{+26}_{-10}	$96^{+2.1}_{-1.9}$	$U(0, 360)$
K_2 (m s ^{–1})	Y2: $30.1^{+1.1}_{-1.0}$	30.7 ± 0.2	$U(-1000, 1000)$
P_2 (d)	A: 3361 ± 31	3574.09 ± 14.63	$N(3299, 50)$

Table A9 – *continued*

Parameter	Value (various sources)	TLCM	Prior
e_2	A: 0.652 ± 0.017	0.542 ± 0.011	
ω_2 (deg)	A: 142.1 ± 2.5	132.8 ± 0.6	
$R_*/R_\odot$	B24: 0.76 ± 0.01	0.80 ± 0.03	
$M_*/M_\odot$	A: 0.811 ± 0.030	0.84 ± 0.08	
R_p/R_{Jup}	B24: 0.4466 ± 0.0059	0.46 ± 0.02	
M_p/M_{Jup}	B24: 0.0787 ± 0.0048	$0.078 \pm 0.005z$	

^a *: not real gamma-velocity as the cited work subtracted the mean from the RV-values. According to B24, gamma-velocity is -63.24 ± 0.26 km/s.

^b B10 and Y1 gives $V_{\text{sin} I} = 1.5 \pm 1.5$ km/s (adopted for TLCM-analysis), B gives $V_{\text{sin} I} = 0.67 \pm 0.1$ km/s (error bars rounded).

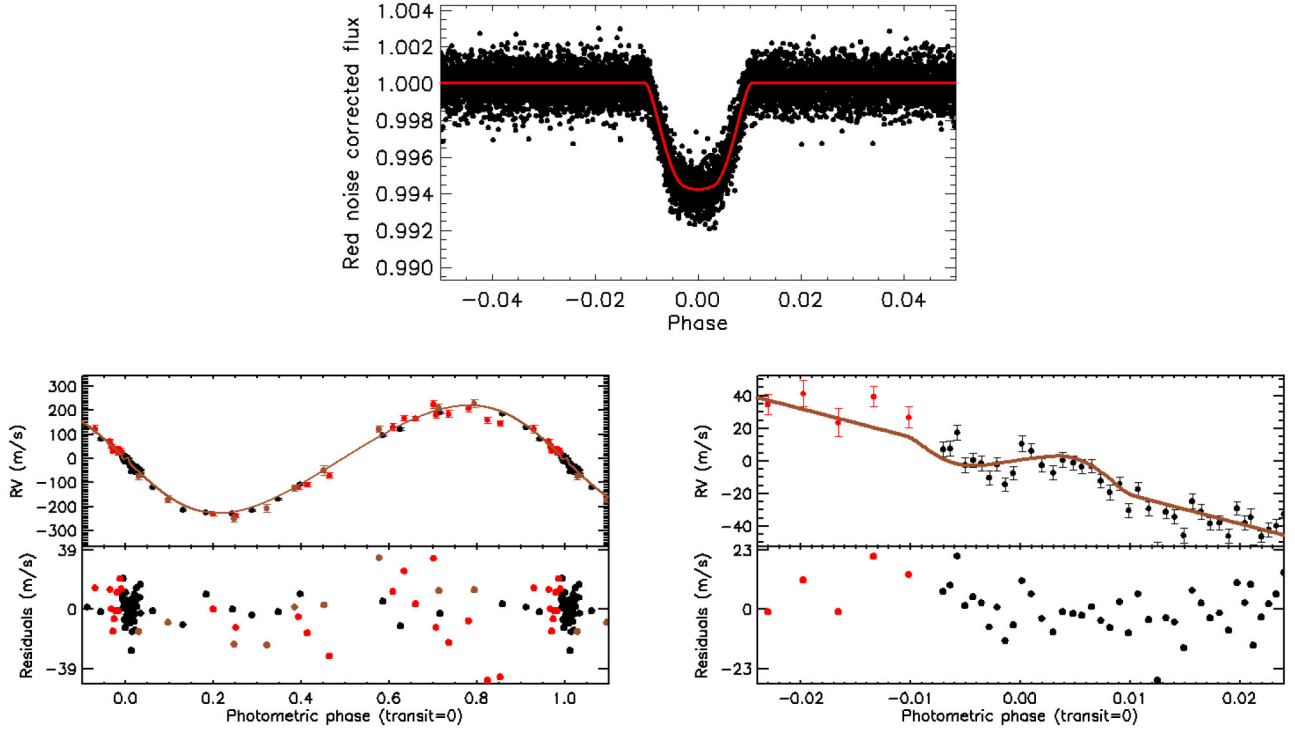
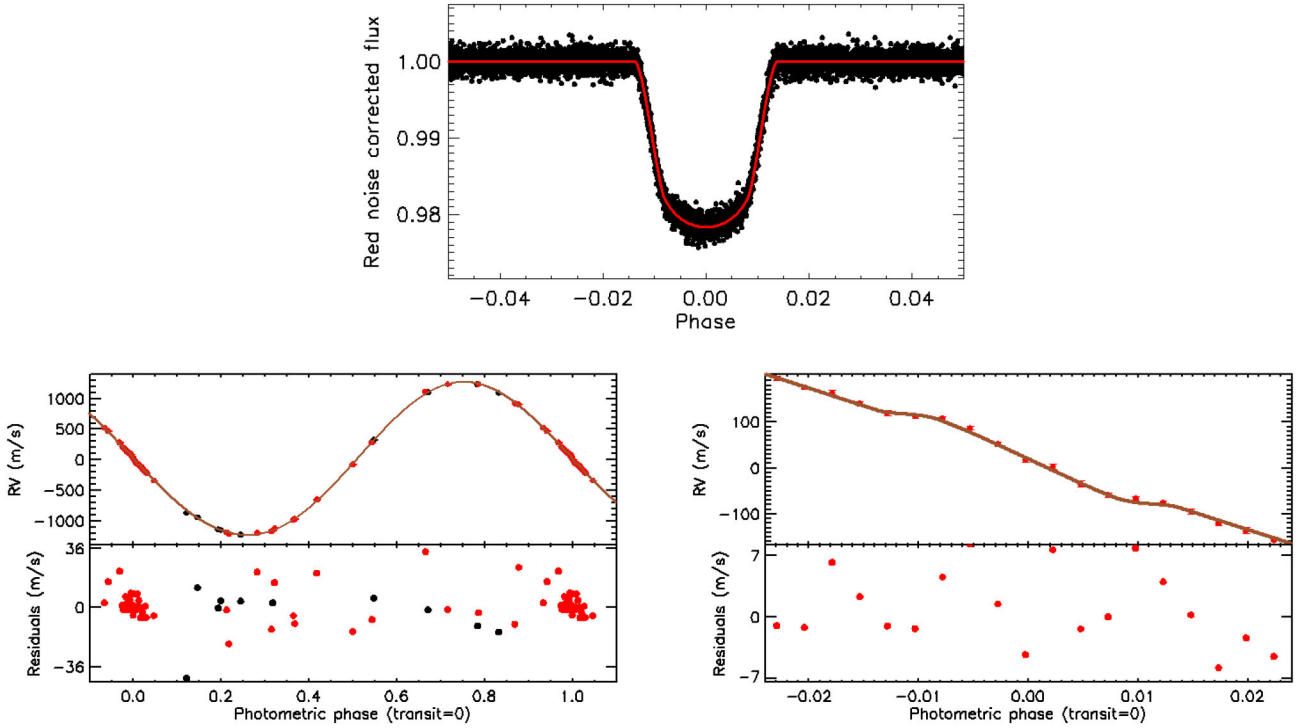


Figure A10. Data and fits of HAT-P-14. Notice the grazing transit event on the transit light curve. Legend for RV figures above: black dots are HIRES data from J. N. Winn et al. (2011), red ones are FIES and brown ones are SOPHIE data from E. K. Simpson et al. (2011).⁵

Table A10. Joint RV + LC result fits on HAT-P-14.

Parameter	J. N. Winn et al. (2011)	TLCM	Prior
a/R_*	$8.873^{+0.268}_{-0.252}$	8.40 ± 0.10	$U(1, 20)$
R_p/R_*	0.0800 ± 0.0015	0.0822 ± 0.0006	$U(0, 1)$
b	0.894 ± 0.013	0.899 ± 0.003	$U(0, 2)$
$\sqrt{e} \sin \omega$	n/a	0.332 ± 0.008	$U(-1, 1)$
$\sqrt{e} \cos \omega$	n/a	-0.024 ± 0.042	$U(-1, 1)$
e	0.107 ± 0.013	0.112 ± 0.005	–
ω (deg)	$94^\circ \pm 4^\circ$	$97.2^\circ \pm 1.8^\circ$	–
Epoch (BJD _{TDB} – 2450000)	5314.91794 ± 0.00066 (HJD)	5421.3556 ± 0.0011	$D(0.02)$
Period (d)	$4.6276690(50)$	$4.627659(12)$	$D(2 \times 10^{-5})$
K (m s ^{–1})	218.9 ± 5.7	224.9 ± 1.3	$U(0, 1000)$
V_γ (km s ^{–1})	n/a	0.0252 ± 0.0014	$U(-100, 100)$
RV offset 2 (m s ^{–1})	n/a	20.4517 ± 0.0021	$U(-1000, 1000)$
RV offset 3 (m s ^{–1})	n/a	20.4039 ± 0.0048	$U(-1000, 1000)$
Height correction	n/a	0.000033 ± 0.000086	$U(-1, 1)$
Red noise factor	n/a	0.029118 ± 0.000458	$U(0, 1)$
White noise σ	n/a	0.000755 ± 0.000003	$U(0, 1)$
RV drift (km s ^{–1} d ^{–1})	n/a	0.0000125 ± 0.0000024	$U(-0.01, 0.01)$
$V \sin I$ (km s ^{–1})	8.18 ± 0.49	$8.08^{+0.27}_{-0.13}$	$U(7.9, 8.9)$
λ (deg)	189.1 ± 5.1	187.4 ± 2.2	$U(-180, 180)$
A	n/a	1.24 ± 0.04	$N(1.24, 0.05)$
B	n/a	1.33 ± 0.05	$N(1.33, 0.05)$
$R_*/R_\odot$	1.386 ± 0.045	1.53 ± 0.15	–
$M_*/M_\odot$	1.468 ± 0.042	1.36 ± 0.34	–
R_p/R_{Jup}	1.142 ± 0.033	1.23 ± 0.13	–
M_p/M_{Jup}	2.232 ± 0.058	2.25 ± 0.38	–


Figure A11. Results of the joint RV + LC solution of HAT-P-20. Legend for RV figures above: red is HIRES data from G. A. Bakos et al. (2011), black is HARPS-N data from M. Esposito et al. (2017).

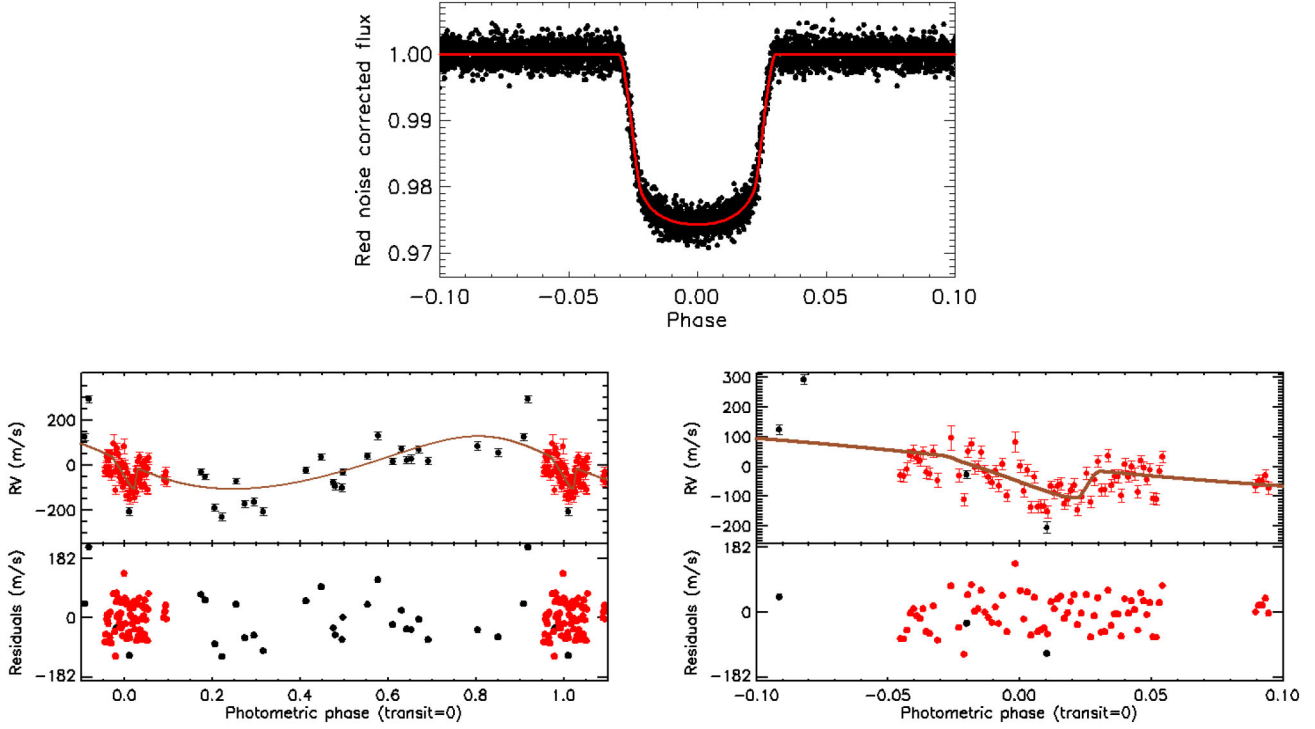


Figure A12. Results of the joint RV + LC fit of HAT-P-32b. Legend for RV figures above: red dots are HIRES data from Albrecht et al. (2012), black ones are HIRES data from J. D. Hartman et al. (2011).

Table A11. Joint RV + LC result fits on HAT-P-20.

Parameter	M. Esposito et al. (2017)	TLCM	Prior
a/R_*	11.36 ± 0.25	11.03 ± 0.11	$U(1, 20)$
R_p/R_*	0.155 ± 0.010	0.1420 ± 0.0006	$U(0, 1)$
b	0.622 ± 0.059	0.647 ± 0.01	$U(0, 2)$
$\sqrt{e} \sin \omega$	n/a	-0.071 ± 0.004	$U(-1, 1)$
$\sqrt{e} \cos \omega$	n/a	0.115 ± 0.003	$U(-1, 1)$
e	0.0172 ± 0.0016	0.0184 ± 0.0008	–
ω (deg)	$342.7^\circ \pm 7.3^\circ$	$328.0^\circ \pm 1.6^\circ$	–
Epoch (BJD _{TDB} –2450000)	5942.681 ± 0.016	7959.1202 ± 0.0003	$D(0.02)$
Period (d)	$2.875316938(190)$	$2.875318(38)$	$D(2 \times 10^{-5})$
K (m s ^{–1})	1249.5 ± 1.2	1252.9 ± 7.0	$U(0, 10000)$
V_γ (m s ^{–1})	-18087.44 ± 0.7 (in-transit) -18093.36 ± 0.8 (out-transit)	8 ± 8	$U(-100000, 100000)$
RV offset (m s ^{–1})	n/a	-18086.9 ± 11.5	$U(-100000, 100000)$
Height correction	n/a	0.001065 ± 0.000312	$U(-1, 1)$
Red noise factor	n/a	0.110060 ± 0.000658	$U(0, 1)$
White noise σ	n/a	0.001110 ± 0.000006	$U(0, 1)$
$V \sin I$ (km s ^{–1})	1.85 ± 0.27	2.31 ± 0.20	$U(7.9, 8.9)$
λ (deg)	-8.0 ± 6.9	-0.5 ± 3.6	$U(-180, 180)$
A	n/a	1.36 ± 0.04	$N(1.24, 0.05)$
B	n/a	1.56 ± 0.04	$N(1.33, 0.05)$
$R_*/R_\odot$	0.6796 ± 0.0054	0.69 ± 0.02	–
$M_*/M_\odot$	0.742 ± 0.042	0.73 ± 0.04	–
R_p/R_{Jup}	1.025 ± 0.053	0.96 ± 0.03	–
M_p/M_{Jup}	7.22 ± 0.036	7.14 ± 0.29	–

^aTime of periastron, not time of transit.

Table A12. Joint RV + LC result fits on HAT-P-32.

Parameter	S. Albrecht et al. (2012) and X.-Y. Wang et al. (2026)	TLCM	Prior
a/R_*	$5.344^{+0.040}_{-0.039}$	5.50 ± 0.15	$U(1, 20)$
R_p/R_*	$0.14886^{+0.00056}_{-0.00054}$	0.1492 ± 0.0002	$U(0, 1)$
b	$0.0830^{+0.0700}_{-0.0550}$	0.124 ± 0.009	$U(0, 2)$
$\sqrt{e} \sin \omega$	n/a	0.255 ± 0.054	$U(-1, 1)$
$\sqrt{e} \cos \omega$	n/a	0.255 ± 0.040	$U(-1, 1)$
e	$0.159^{+0.051}_{-0.028}$	0.013 ± 0.032	–
ω (deg)	50^{+27}_{-18}	$44.9^\circ \pm 7.7^\circ$	–
Epoch (BJD _{TDB} – 2450000)	5867.402743(49)	6265.14578(128)	$D(0.02)$
Period (d)	2.15000820(13)	2.15001298(70)	$D(2 \times 10^{-5})$
K (m s ^{–1})	$99.0^{+16.0}_{-15.0}$	116.9 ± 4.0	$U(0, 10000)$
V_γ (m s ^{–1})	n/a	12.6 ± 3.0	$U(-100000, 100000)$
RV offset (m s ^{–1})	n/a	-17.8 ± 5.0	$U(-100000, 100000)$
Height correction	n/a	$-0.000362^{+0.000157}_{-0.000144}$	$U(-1, 1)$
Red noise factor	n/a	$0.023093^{+0.000783}_{-0.000715}$	$U(0, 1)$
White noise σ	n/a	$0.001282^{+0.000011}_{-0.000012}$	$U(0, 1)$
$V \sin I$ (km s ^{–1})	20.6 ± 1.5	20.71 ± 0.34	$U(20.2, 21.3)$
λ (deg)	85.0 ± 1.5	83.7 ± 0.8	$U(-180, 180)$
A	n/a	1.28 ± 0.05	$N(1.26, 0.05)$
B	n/a	1.22 ± 0.03	$N(1.36, 0.05)$
$R_*/R_\odot$	$1.367^{+0.031}_{-0.030}$	1.35 ± 0.12	–
$M_*/M_\odot$	$1.132^{+0.051}_{-0.050}$	1.20 ± 0.22	–
R_p/R_{Jup}	1.980 ± 0.045	1.96 ± 0.17	–
M_p/M_{Jup}	$0.68^{+0.11}_{-0.10}$	0.83 ± 0.11	–

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