Simultaneous Localization and Calibration (SLAC) for Cooperative Radio Navigation: When Is It Possible?

Alexis Marino, Robert Pöhlmann and Emanuel Staudinger

***Abstract*— Cooperative radio navigation is essential in GNSS-denied or infrastructure-free environments. Methods based on signal propagation time, such as round-trip time ranging, require sub-nanosecond Time-of-Flight accuracy. However, factors such as temperature variations in RF components or reconfigurable antenna surroundings can affect radio transceiver group delays, introducing ranging bias. Simultaneous Localization and Calibration (SLAC) aims to jointly estimate node positions and ranging biases, correcting them online. The success of this process critically depends on observability, i.e., whether the measurements contain enough information to uniquely estimate all desired parameters. In this work, we analyze the observability of SLAC for cooperative radio navigation through a Fisher Information Matrix (FIM) approach and derive necessary and sufficient conditions that determine when SLAC is feasible. The results characterize the minimum number of static and mobile nodes required and identify geometric configurations that yield strong, weak, or no observability. We validate the analysis via extensive simulations.**

I. INTRODUCTION

Multi-robot navigation demands robust and accurate state estimation to ensure safe autonomous operation and achieve mission objectives [1]. In environments where Global Navigation Satellite System (GNSS) is unreliable or unavailable, cooperative radio navigation can significantly improve localization performance.

An aspect that we consider as important as cooperation for a practical system, but which has received much less attention in the community, is calibration. An improperly calibrated system leads to biased estimates and, consequently, impaired performance. To highlight the role of calibration in cooperative localization, we briefly compare to the GNSS architecture: high-grade hardware in the space segment and a dedicated control segment enable the use of low-cost commercial off-the-shelf (COTS) receivers on the user side [2]. In cooperative navigation, by contrast, an agent to be localized relies largely on signals transmitted by its neighbors. Static anchors with known positions may exist, but the infrastructure is typically sparse, and in many applications the RF hardware consists of low-cost COTS components.

Simultaneous Localization and Calibration (SLAC) addresses this challenge by jointly estimating node states and calibration parameters, enabling teams of robots to improve positioning accuracy while compensating for systematic sensor biases [3]. In such networks, inter-node distances are commonly obtained by exchanging radio messages using, e.g., UWB radios and timestamp-based two-way ranging (TWR) protocols [4]. Such methods require sub-nanosecond Time-of-Flight (ToF) accuracy [5]. However, environmental factors, e.g., temperature variations, can change the group delays of radio transceivers, introducing hardware-dependent range biases.

While SLAC can correct these biases online, its success fundamentally depends on the observability of the parameters to be estimated, i.e., position and ranging bias. This paper studies the observability of SLAC in cooperative radio navigation using the Fisher Information Matrix (FIM) [6]: we exploit the equivalence between full column rank of the FIM and the measurement Jacobian, which directly characterizes parameter estimability [7]. We derive conditions that determine the minimum number of static and mobile nodes required for SLAC in practice and identify geometric configurations that yield strong, weak, or non-observable systems. This work builds upon our previous study on observability of SLAC in cooperative and non-cooperative radio navigation [8].

II. SYSTEM MODEL

A. State definition

The network is composed of two types of nodes, represented by the set $\mathcal{N} = \mathcal{A} \cup \mathcal{S}$ and $\mathcal{A} \cap \mathcal{S} = \emptyset$, where $\mathcal{A}$ denotes static anchors with known positions and $\mathcal{S}$ denotes mobile agents, e.g., robots, to be localized, for a total of $|\mathcal{N}|$ nodes. The node state vector of an agent $i \in \mathcal{S}$ consists of the kinematic states $\mathbf{x}^k_{i,\text{loc}}$ and the calibration parameter states $\mathbf{x}^k_{i,\text{cal}}$ for snapshot k,

$$\mathbf{x}^k_i = [(\mathbf{x}^k_{i,\text{loc}})^\top, (\mathbf{x}^k_{i,\text{cal}})^\top]^\top. \tag{1}$$

The state vector of an anchor $i \in \mathcal{A}$ includes only the calibration parameter states $\mathbf{x}^k_i = \mathbf{x}^k_{i,\text{cal}}$.
The node kinematic states are defined as

$$\mathbf{x}^k_{i,\text{loc}} = [\boldsymbol{p}^k_i] = [x^k_i,\ y^k_i]^\top, \tag{2}$$

representing the 2D position of agent i. Node calibration state for anchors and agents is given by

$$\mathbf{x}^k_{i,\text{cal}} = [\delta^k_i], \tag{3}$$

where δ^k_i is the ranging bias present at each radio node. The complete network state vector is formed by stacking the individual node states, ordered as

$$\mathbf{x}^k = [\,\boldsymbol{p}^k_1 \dots\ \boldsymbol{p}^k_{|\mathcal{S}|}\ ,\ \delta^k_1 \dots \delta^k_{|\mathcal{S}|}\ ,\ \delta^k_1 \dots \delta^k_{|\mathcal{A}|}\,]^\top, \tag{4}$$

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Alexis Marino, Robert Pöhlmann and Emanuel Staudinge are with the German Aerospace Center (DLR), Institute of Comminications and Navigation, Oberpfaffenhofen, 82234 Wessling, Germany {alexis.marinosalguero, robert.poehlmann, emanuel.staudinger}@dlr.de

with overall dimension $\mathbf{x}^k \in \mathbb{R}^{3|\mathcal{S}|+|\mathcal{A}|}$.

B. Measurement Model

We consider measurements obtained through Two-Way Ranging (TWR) scheme. The inter-node distance between nodes i and j for snapshot k is modeled as

$$d_{i,j}^k = \left\|\boldsymbol{p}_j^k - \boldsymbol{p}_i^k\right\| = \frac{c\left(\tau_{i,j}^k - \tilde{\tau}_{i,j}^k\right)}{2} + \delta_i^k + \delta_j^k, \quad (5)$$

where $\boldsymbol{p}_i^k$ and $\boldsymbol{p}_j^k$ represents the node positions, c is the speed of light, $\tilde{\tau}_{i,j}^k$ is the transmit timestamp of the forward signal, $\tau_{i,j}^k$ is the reception timestamp of the backward signal, both measured at node i, and δ_i^k and δ_j^k are node-specific ranging biases, defined in the distance domain. We assume that the processing time at node j is compensated by a DS-TWR scheme, which mitigates the impact of relative clock offset; otherwise, clock dynamics must be explicitly modeled [9]

A snapshot k denotes one ranging round in the cooperative case, in which each considered link (agent–anchor and agent–agent) provides one range measurement. The range measured between node $i \in \mathcal{N}$ and a neighboring node $j \in \mathcal{N}$ is

$$r_{i,j}^k = \left\|\mathbf{p}_j^k - \mathbf{p}_i^k\right\| - \delta_i^k - \delta_j^k + \epsilon_{i,j}^k, \quad (6)$$

where $\epsilon_{i,j}^k$ is zero-mean Gaussian measurement noise, $\epsilon_{i,j}^k \sim \mathcal{N}(0, \sigma^2_{r_{i,j}^k})$, and we assume independence across links and snapshots.

The measurements at snapshot k are stacked into

$$\mathbf{z}^k = h(\mathbf{x}^k) + \mathbf{v}_z^k, \quad (7)$$

with $\mathbf{v}_z^k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}^k)$ and

$$\mathbf{R}^k = \mathrm{diag}\{\ldots, \sigma^2_{r_{i,j}^k}, \ldots\}. \quad (8)$$

To compute the FIM in the next section, we linearize the measurement model around the state in (4) obtaining the Jacobian $\mathbf{H}^k$. Let $l(i,j)$ be a function returning the index of the node j whose range is measured by node i. Then $\mathbf{H}^k$ is built from block rows $\mathbf{H}^k_{l(i,j),i}$, one per measurement.

The Jacobian depends on the link type. For agent–anchor measurements ($i \in \mathcal{S}, j \in \mathcal{A}$),

$$\mathbf{H}^k_{i,l(i,j)} = \left[\ldots (\boldsymbol{u}_{i,j}^k)^\top \ldots, \text{-1} \ldots, \text{-1} \ldots\right], \quad (9)$$

where $(\boldsymbol{u}_{i,j}^k)^\top$ is defined in (11) and placed in the position states of agent i, the entries -1 correspond to the bias parameters of the involved nodes, and all other elements are zero.

For agent-to-agent links ($i, j \in \mathcal{S}, i \neq j$)

$$\mathbf{H}^k_{i,l(i,j)} = \left[\ldots (\boldsymbol{u}_{i,j}^k)^\top \ldots \text{-}(\boldsymbol{u}_{i,j}^k)^\top \ldots, \text{-1} \ldots, \text{-1} \ldots\right], \quad (10)$$

whit the unit vector

$$\boldsymbol{u}_{i,j}^k = \frac{p_j^k - p_i^k}{\left\|p_j^k - p_i^k\right\|}. \quad (11)$$

Combining (9)–(10) yields the cooperative Jacobian (17).

III. Observability analysis

Given the above SLAC description, the key question is whether all unknowns in this SLAC setup are identifiable i.e., whether the noisy measurements provide enough information to estimate every node's position and ranging bias. This can be assessed via parameter observability [6].

When SLAC is formulated as a nonlinear least-squares estimation problem, observability is equivalent to a non-singular FIM [10]. A singular FIM implies an unobservable system. Moreover, if the FIM is non-singular but has very small eigenvalues, the problem is observable but only weakly.

For the non-linear observation model (7) the FIM is defined as

$$\mathbf{FIM}(\mathbf{x}) = \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}, \quad (12)$$

where $\mathbf{H}$ is the Jacobian (9), (10), and $\mathbf{R}$ is the covariance matrix (8) of the observation model.

To evaluate the variance of SLAC under different observable node configurations, we calculate the Cramér-Rao Bound (CRB), which provides a lower bound on the variance of any unbiased estimator. For a system with a full-rank FIM, the CRB is defined as

$$\mathrm{Var}(\hat{\mathbf{x}}) \geq \mathbf{FIM}(\mathbf{x})^{-1} = \mathbf{CRB}(\mathbf{x}). \quad (13)$$

The diagonal elements of the CRB matrix represent the minimum achievable variance for each state.

A. *Observability Analysis*

From the structure of $\mathbf{R}$ in (12), using its Cholesky factorization, the FIM can be shown to have the same rank as the measurement Jacobian [7]:

$$\mathrm{rank}(\mathbf{FIM}) = \mathrm{rank}(\mathbf{H}). \quad (14)$$

Hence, the full column rank of the measurement Jacobian matrix is equivalent to the non-singularity of the FIM.

We therefore assess observability of the unknown agent positions $[\mathbf{x}_i, \mathbf{y}_i]^\top$ and the ranging biases δ_i of agents and anchors by checking whether $\mathbf{H}$ is full rank. To this end, we derive necessary and sufficient conditions that guarantee $\mathbf{H}$ is full rank.

1) Necessary Condition: The Jacobian matrix has dimension $\mathbf{H} \in \mathbb{R}^{d_1 \times d_2}$, where d_1 is the total number of measurements in cooperative navigation (7) and d_2 is the state dimension (4). Note that, the two ranges $i \to j$ and $i \leftarrow j$ are two noisy observations of the same quantity and do not add independent observability directions. Thus,

$$d_1 = \frac{|\mathcal{S}|(|\mathcal{S}|-1)}{2} + |\mathcal{S}| \times |\mathcal{A}|, \;\; d_2 = 3|\mathcal{S}| + |\mathcal{A}|, \quad (15)$$

where $|\mathcal{S}|$ and $|\mathcal{A}|$ are the number of agents and anchors respectively. From (14), *a necessary and sufficient condition* for **FIM** to be non-singular is that $\mathbf{H}$ must have full column rank. A *necessary condition* for $\mathbf{H}$ to be full column rank is that

$$d_1 \geq d_2 \Leftrightarrow |\mathcal{A}| \geq \frac{3|\mathcal{S}|}{|\mathcal{S}|-1} - \frac{|\mathcal{S}|}{2}. \quad (16)$$

While the derived relationship between $|\mathcal{S}|$ and $|\mathcal{A}|$ indicates that increasing the number of agents $|\mathcal{S}|$ can reduce the required anchor $|\mathcal{A}|$, in 2D localization at least three anchors with known positions are still necessary for global observability [11]. Hence, any feasible solution must satisfy $|\mathcal{A}| \geq 3$ to guarantee global observability, regardless of the number of mobile agents.

*2) **Sufficient Condition**:* In cases where the necessary condition is satisfied but the FIM is not full rank, it is valuable to investigate why the Jacobian $\mathbf{H}$ is not full column rank. This analysis helps to identify the conditions needed to render the system observable and leads to a necessary and sufficient condition. In the following, we explore the observability of SLAC scenarios based on this conclusion.

The Jacobian matrix of SLAC with cooperative navigation, consistent with the state definition given in (4), can be written as follows:

$$\left[\begin{array}{ccccc:ccccc:c} \mathbf{L}_1 & \mathbf{0}_{a\times 2} & \dots & \mathbf{0}_{a\times 2} & \mathbf{0}_{a\times 2} & -\mathbf{1}_a & \mathbf{0}_a & \dots & \mathbf{0}_a & \mathbf{0}_a & -\mathbf{I}_a \\ \mathbf{0}_{a\times 2} & \mathbf{L}_2 & \dots & \mathbf{0}_{a\times 2} & \mathbf{0}_{a\times 2} & \mathbf{0}_a & -\mathbf{1}_a & \dots & \mathbf{0}_a & \mathbf{0}_a & -\mathbf{I}_a \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0}_{a\times 2} & \mathbf{0}_{a\times 2} & \dots & \mathbf{L}_{i\text{-}1} & \mathbf{0}_a & \mathbf{0}_a & \mathbf{0}_a & \dots & -\mathbf{1}_a & \mathbf{0}_a & -\mathbf{I}_a \\ \mathbf{0}_{a\times 2} & \mathbf{0}_{a\times 2} & \dots & \mathbf{0}_{a\times 2} & \mathbf{L}_i & \mathbf{0}_a & \mathbf{0}_a & \dots & \mathbf{0}_a & -\mathbf{1}_a & -\mathbf{I}_a \\ \hdashline \mathbf{T}_1 & [& & \mathbf{C}_1 &] & -\mathbf{1}_{m\text{-}1} & [& & -\mathbf{I}_{m\text{-}1} &] & \mathbf{0}_{m\text{-}1} \\ \mathbf{0}_{m\text{-}2} & \mathbf{T}_2 & [& \mathbf{C}_2 &] & \mathbf{0}_{m\text{-}2} & -\mathbf{1}_{m\text{-}2} & [& -\mathbf{I}_{m\text{-}2} &] & \mathbf{0}_{m\text{-}2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{T}_i & \mathbf{C}_l & \mathbf{0} & \mathbf{0} & \dots & -\mathbf{1} & -\mathbf{1} & \mathbf{0} \end{array}\right] \tag{17}$$

where $\mathbf{0}_{n\times m}$ is the $n \times m$ zero matrix, $\mathbf{0}_n$ and $\mathbf{1}_n$ are the length-n zero and ones vectors, and $\mathbf{I}_n$ is the $n \times n$ identity. Let $m = |\mathcal{S}|$ and $a = |\mathcal{A}|$. The matrices $\mathbf{L}_i$, $\mathbf{T}_i$, and $\mathbf{C}_i$ are defined in eqs. (18) to (20).

The Jacobian in (17) is split into two blocks, separated by the horizontal dashed line. The upper block corresponds to agent–anchor links (9), where each $\mathbf{L}_i$ $(i = 1, \dots, m)$ stacks the unit vectors of agent $i \in \mathcal{S}$ to all anchors

$$\mathbf{L}_i = \left[(\boldsymbol{u}_{i,1}^k)^\top, (\boldsymbol{u}_{i,2}^k)^\top, \dots, (\boldsymbol{u}_{i,j}^k)^\top\right]_{j\in\mathcal{A}}^\top, \tag{18}$$

where $\boldsymbol{u}_{i,j}^k$ is defined in (11). The remaining entries of this block correspond to the bias parameters of the participating nodes.

The lower block represents the cooperative case with agent–agent links (10), defined via $\mathbf{T}_i$ $(i = 1, \dots, m-1)$ and $\mathbf{C}_i$ $(i = 2, \dots, m)$ as

$$\mathbf{L}_i = \left[(\boldsymbol{u}_{i,1}^k)^\top, (\boldsymbol{u}_{i,2}^k)^\top, \dots, (\boldsymbol{u}_{i,j}^k)^\top\right]_{j\in\mathcal{S},\ i<j}^\top, \tag{19}$$

$$\mathbf{C}_i = \mathrm{diag}\{(\boldsymbol{u}_{i,1}^k)^\top, (\boldsymbol{u}_{i,2}^k)^\top, \dots, (\boldsymbol{u}_{i,j}^k)^\top\}_{j\in\mathcal{S},\ i<j}. \tag{20}$$

The Jacobian matrix (17) is of full column rank if and only if the linear system $\mathbf{Hq} = \mathbf{0}$ admits only the trivial solution $\mathbf{q} = \mathbf{0}$. In other words, full column rank is equivalent to injectivity, and the null space $\mathbf{q}$ of $\mathbf{H}$ directly characterizes the unobservable directions of the system.

In the SLAC setting, we assume fixed anchors with known positions. This configuration eliminates the unobservable global translation and rotation modes typical of SLAM [12].

However, another well-known condition leading to unobservability in localization, for both cooperative and non-cooperative navigation is the collinearity between agents and anchors [6]. Let's consider the following sub space of the null space

$$\mathbf{q}_1 = \left[1, -k, 1, -k, \dots, 1, -k\ ,\ \mathbf{0}_{1\times m}\ ,\ \mathbf{0}_{1\times a}\right]^\top, \tag{21}$$

where k is a real number. Multiplying this vector by the Jacobian matrix (17) yields the set of equations

$$\begin{bmatrix} x_1^k - x_2^k = k(y_1^k - y_2^k) \\ x_1^k - x_3^k = k(y_1^k - y_3^k) \\ \vdots \\ x_i^k - x_j^k = k(y_i^k - y_j^k) \end{bmatrix}, \tag{22}$$

with $i \in \mathcal{S}, j \in \mathcal{S} \cup \mathcal{A}, i < j$ if $j \in \mathcal{S}$. This condition is satisfied if and only if the constant k corresponds to the slope of the line on which all agents and anchors lie, i.e., a collinear configuration.

We can therefore conclude that a *necessary and sufficient condition* for SLAC with *cooperative navigation* to be observable is that the number of anchors and agents satisfies (16), and that their positions are not collinear.

To quantify the degree of observability of the SLAC framework, we analyze the eigenvalues of the FIM in (12). The smallest eigenvalue corresponds to the least observable direction and indicates when the system approaches unobservability. However, computing FIM eigenvalues without normalization can yield values that are difficult to interpret: the spectrum is unbounded and the eigenvalues may lack dimensional homogeneity due to linear combinations of heterogeneous state components (e.g., positions and biases). Following [13], we therefore normalize the FIM to (i) bound the eigenvalue spectrum and (ii) obtain a dimensionless set of eigenvalues, enabling a fair comparison of the degree of observability across different network configurations.

Let $\mathbf{x} \in \mathbb{R}^{|\mathcal{N}|}$ be the state in (4) and define

$$\phi = \mathbf{T}^{-1}\mathbf{x}, \qquad \mathbf{T} = \mathrm{diag}\{t_1, t_2, \dots, t_{|\mathcal{N}|}\},\ t_i > 0, \tag{23}$$

with user-chosen scaling factors t_i, that represent typical magnitudes of each state component to render ϕ dimensionless.

The scaled Jacobian is $\mathbf{H}\phi = \mathbf{HT}$, yielding the dimensionless FIM

$$\mathbf{FIM}\phi = \mathbf{H}\phi^\top \mathbf{R}^{-1}\mathbf{H}\phi = \mathbf{T}^\top \mathbf{FIM}\,\mathbf{T}. \tag{24}$$

To bound the spectrum, we further normalize by the trace,

$$\widetilde{\mathbf{FIM}}\phi = \frac{|\mathcal{N}|}{\mathrm{tr}(\mathbf{FIM}\phi)}\,\mathbf{FIM}_\phi, \tag{25}$$

so the eigenvalues are dimensionless and lie in $[0, |\mathcal{N}|]$.

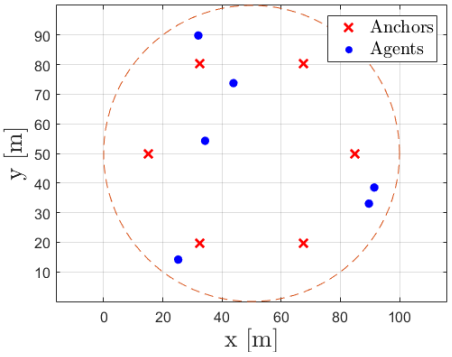

Fig. 1. Example simulation scenario with six anchors and six randomly positioned agents.

IV. Simulation Results

First, we evaluate the observability of the SLAC problem through simulation. The objective is to test how well the parameters, particularly the ranging biases, can be estimated in configurations where the FIM satisfies the necessary condition (16).

We consider scenarios with three, four, and six anchors arranged in fixed geometric patterns. For each anchor configuration, a varying number of agents from 2 to 50 is randomly deployed within a circular area of radius 50 m, see Figure 1. For each resulting setup, the CRB is computed to determine the minimum achievable variance under the given configuration. In addition, we quantify the degree of observability by evaluating the minimum eigenvalue of the normalized FIM in (25) for each configuration. To obtain statistically consistent results, we perform Monte Carlo simulations with 100 independent trials per configuration.

We assume a ranging noise standard deviation of $\sigma_{r^k_{i,j}} = 0.25\,\mathrm{m}$. For the eigenvalue-based analysis, we normalize the FIM using the scaling matrix $\mathbf{T}$ in (23), with scaling factors $t_i = [1\,\mathrm{m}, 1\,\mathrm{m}, 0.1\,\mathrm{m}]$ for the position and ranging-bias parameters, respectively.

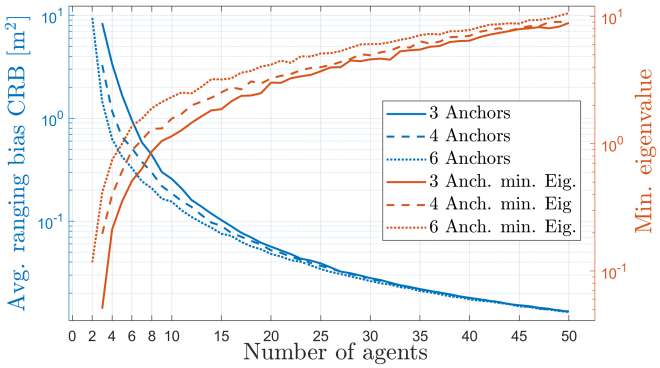

Fig. 2. Average ranging-bias CRB of all nodes in the cooperative network for different anchor configurations and the minimum eigenvalue of the normalized FIM, which quantifies the least observable direction for each configuration.

The left blue y-axis of Figure 2 shows the minimum achievable one-shot estimation variance of the ranging biases for different agent–anchor configurations using CRB. The results corroborate the necessary condition in (16), since the CRB can only be computed when the condition is satisfied. For example, with three or four anchors, observability requires at least three agents, whereas with six anchors it is achieved with only two agents. For small networks, the large CRB indicates that the states are poorly observable. This limitation can be mitigated once agents move and additional measurements are accumulated over time.

Similarly, the right orange y-axis of Figure 2 reports the initial minimum eigenvalue of the FIM, which provides a quantitative measure of the degree of observability for each configuration. In the next simulation, we show how this eigenvalue evolves depending on the trajectories executed by the mobile agents.

Second, we simulate the sufficient condition for observability in cooperative navigation. Figure 3 depicts a scenario with five anchors and three mobile agents. This arrangement satisfies the necessary condition in (16). Since observability is evaluated snapshot-wise, the agents' trajectories matter only through the instantaneous geometry: along the chosen paths, the agents become progressively collinear, reaching collinearity at snapshot 401 of 1000 , and then move away as they continue their trajectories.

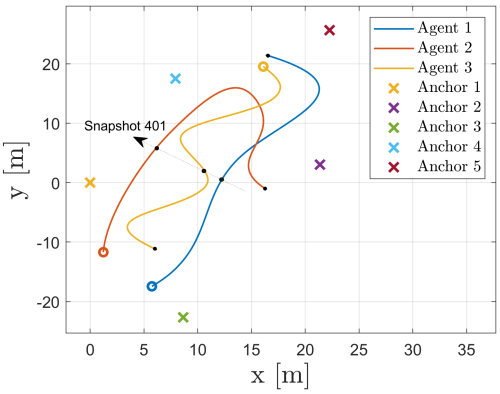

Fig. 3. Simulation scenario: five anchors and agents approaching collinearity.

Figure 4 presents the results for the scenario in Figure 3. The plot shows: (i) the FIM rank deficiency, indicating potential loss of rank; (ii) the evolution of the minimum eigenvalue along the agents' trajectories; and (iii) the agents' position standard deviation, computed as $\sigma_i = \sqrt{\mathrm{CRB}_{x_i} + \mathrm{CRB}_{y_i}}$.

At the start of the simulation, the minimum eigenvalue reflects the baseline degree of observability determined by the agent–anchor configuration, Figure 2, confirming that the system is observable with certain degree of observability. As the agents move toward a collinear formation, the minimum eigenvalue decreases, indicating degraded observability. Importantly, the FIM does not become rank-deficient because the anchors remain well distributed, preventing a complete loss of observability. Once the agents move away from collinearity, the minimum eigenvalue increases and the system recovers. This highlights how geometric configurations can severely weaken observability in cooperative radio navigation even without causing a strict loss of rank.

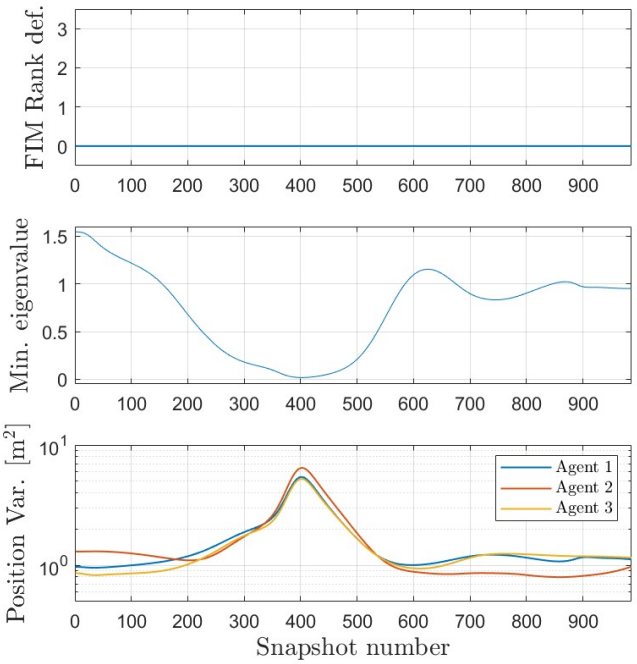

Fig. 4. FIM rank deficiency, evolution of the minimum eigenvalue and position minimum variance.

V. CONCLUSIONS

In this work, we performed an observability analysis of Simultaneous Localization and Calibration (SLAC) for cooperative radio navigation, considering only radio ranging measurements and ranging biases as calibration parameters. This study answers the central question: when is SLAC possible? We derived feasibility conditions that specify the required number of mobile agents and static anchors to achieve SLAC, and identified geometric configurations that lead to strongly, weakly, or unobservable systems using the minimum eigenvalue of the FIM. These insights provide practical guidelines for system and experiment design, demonstrating that too few nodes or poorly chosen agent trajectories can significantly degrade localization and bias calibration accuracy, even when the system remains formally observable.

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