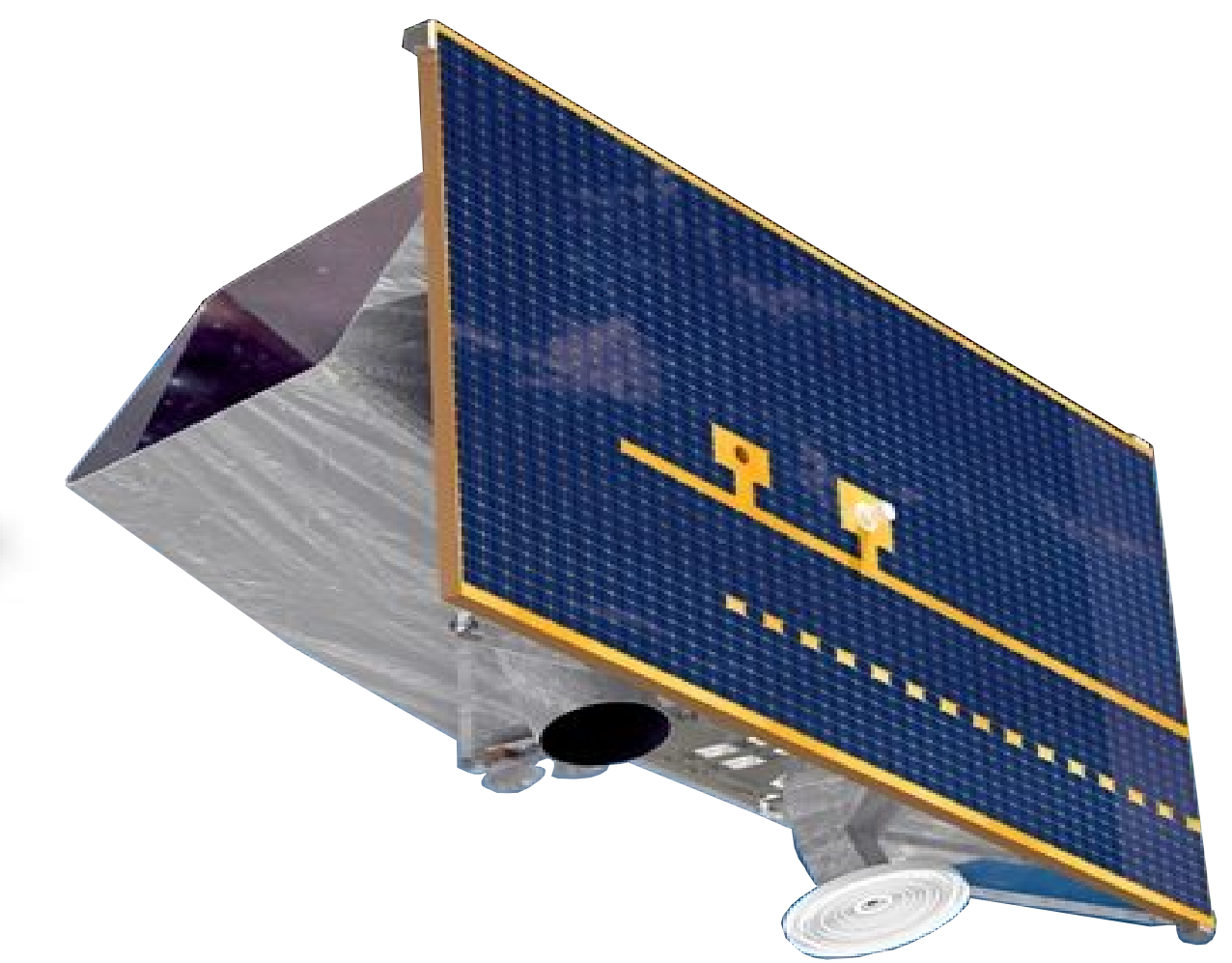
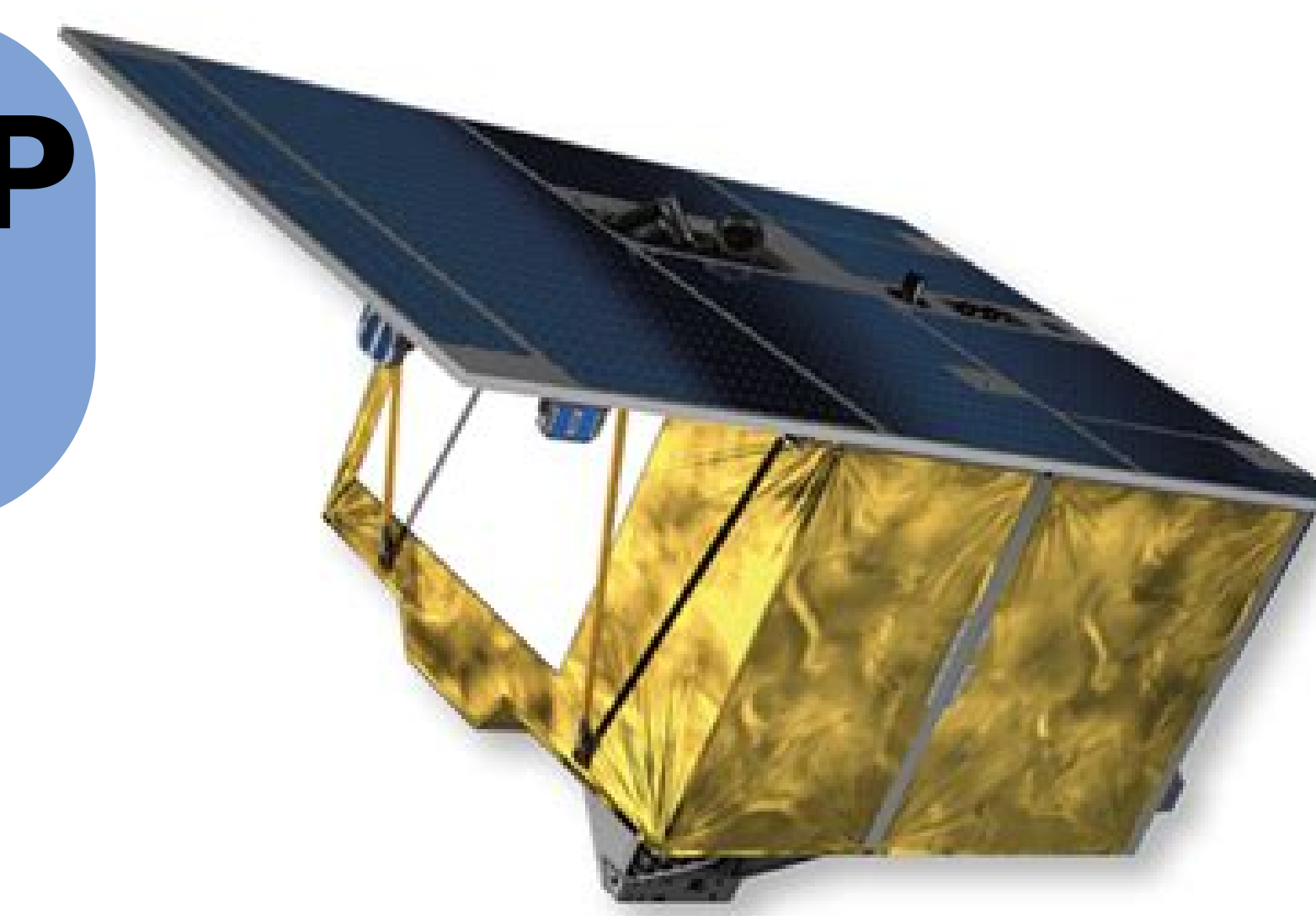


Assessment of Methane Retrieval Algorithms for EnMAP and PRISMA Shortwave Infrared Observations

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EnMAP
 Kamyshlydza, Turkmenistan
 G&O Facility, Homogeneous Albedo

PRISMA
 Aix-En-Provence, France
 Cattle Farm, Inhomogeneous Albedo

Albedo-Reweighted Matched Filter (AR-MF)

The Albedo-Reweighted Matched Filter (AR-MF) is a linear detection method for identifying CH₄ enhancements by projecting each observed spectrum onto a predefined target signature. To suppress background variability, the method maximizes the signal-to-noise ratio assuming Gaussian noise.

The standard MF enhancement factor α_i for pixel i is computed as:

$$\alpha_i = \frac{\mathbf{t}^T \mathbf{C}^{-1} (\mathbf{y}_i - \boldsymbol{\mu})}{\sqrt{\mathbf{t}^T \mathbf{C}^{-1} \mathbf{t}}}$$

α_i : CH₄ enhancement estimate for pixel i
 \mathbf{y}_i : observed spectrum
 $\boldsymbol{\mu}$: mean background spectrum
 \mathbf{t} : target spectrum
 \mathbf{C} : covariance matrix of background spectra
 r_i : reflectance normalization factor

A key enhancement over the standard matched filter is albedo normalization: observed spectra are scaled by their overall reflectance to reduce false positives over bright surfaces (e.g., sand, concrete, snow). An iterative reweighting scheme improves robustness by down-weighting pixels with high variance in background components:

$$\alpha_i = \frac{\mathbf{t}^T \mathbf{C}^{-1} (\mathbf{y}_i - r_i \cdot \mathbf{t})}{\sqrt{\mathbf{t}^T \mathbf{C}^{-1} \mathbf{t}}} \quad r_i = \frac{\mathbf{y}_i^T \boldsymbol{\mu}}{\boldsymbol{\mu}^T \boldsymbol{\mu}}$$

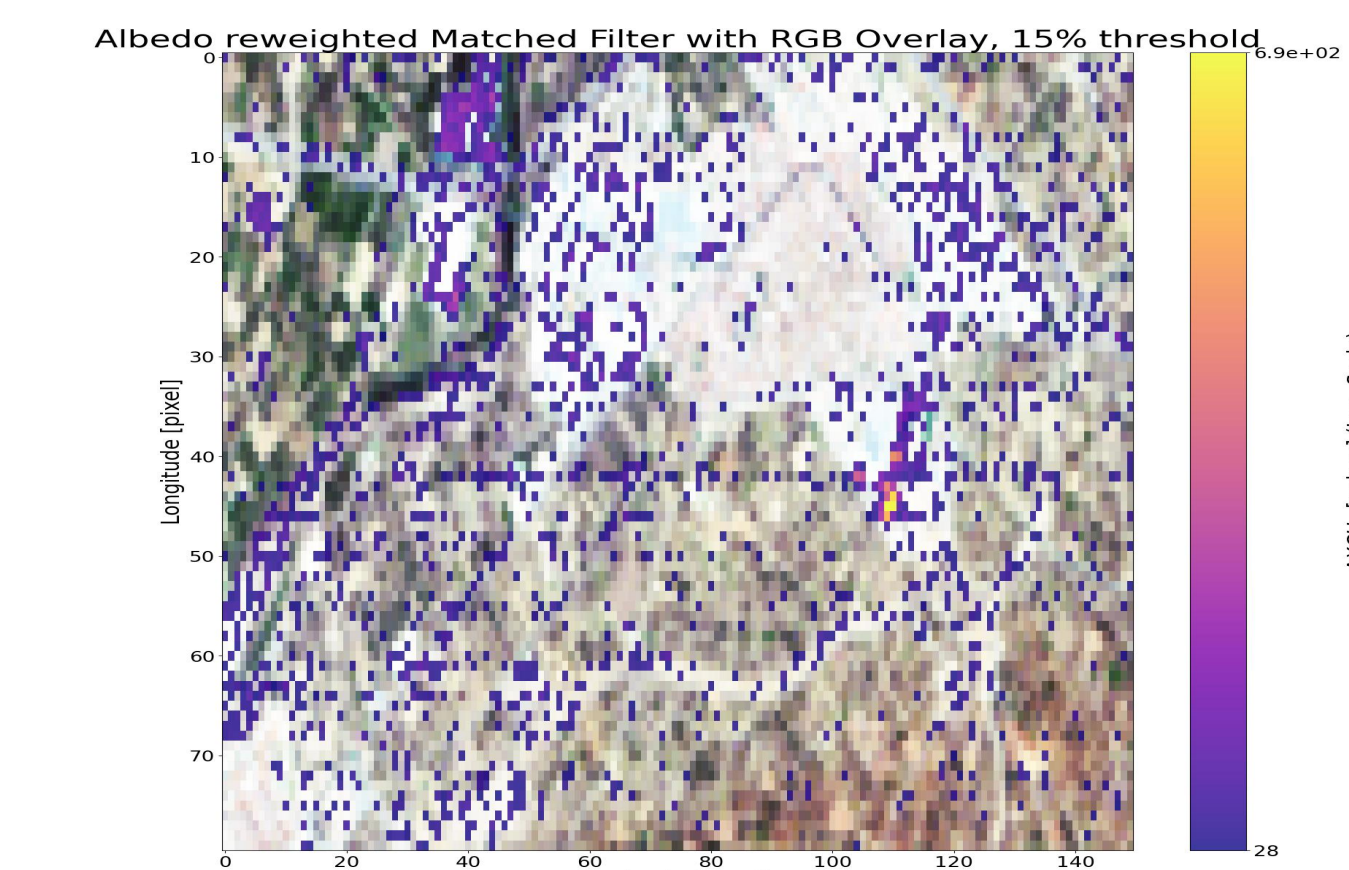
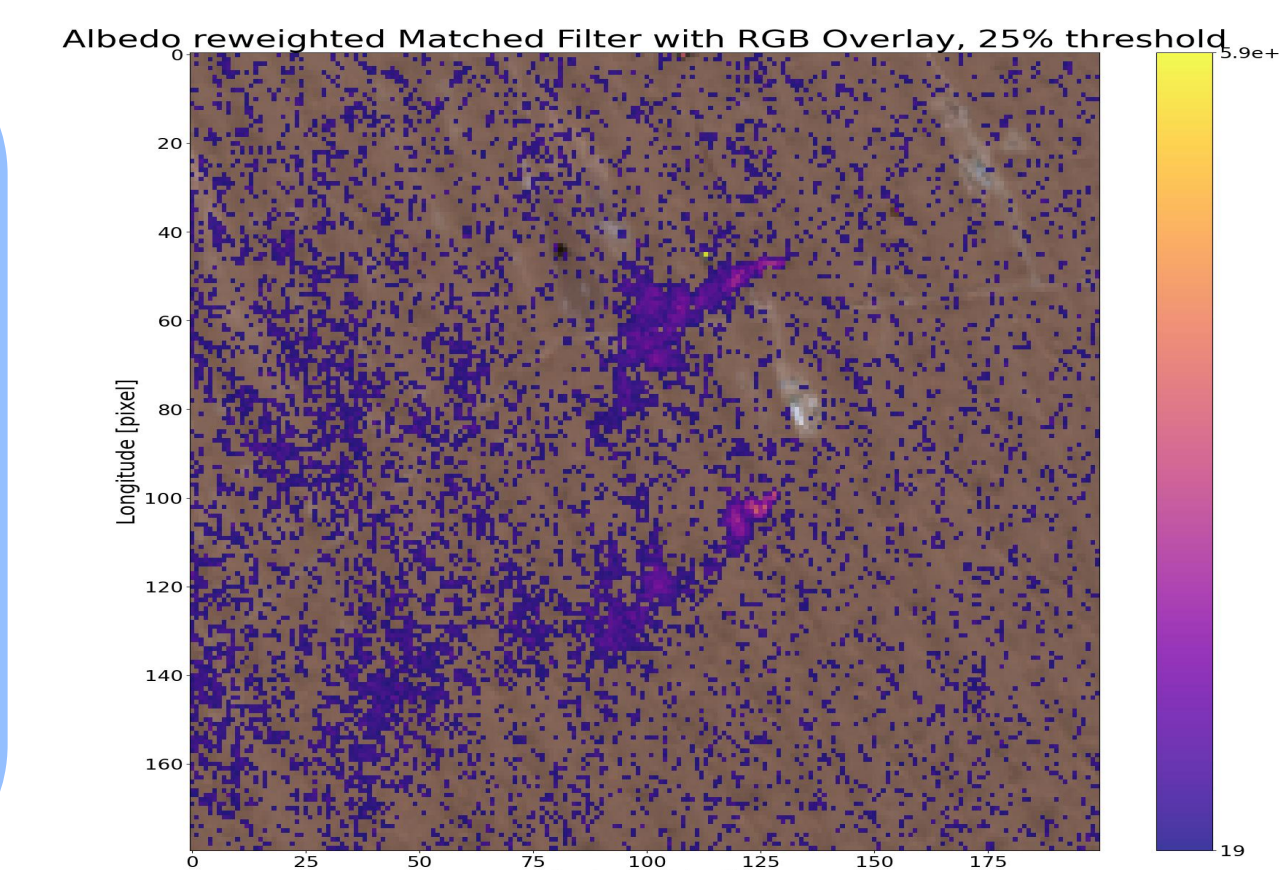
The target spectrum is derived from synthetic radiative transfer models (Py4CATs & HITRAN).

Pros

- Fast and efficient: Suitable for processing millions of pixels quickly.
- Well-suited for large datasets: Enables regional-scale plume surveys.
- Compatible with EnMAP and PRISMA: Works well with moderate spectral resolution and high spatial detail.
- Improved robustness: Reweighting and albedo correction reduce background bias.

Cons

- False positives: Can produce spurious signals over high-albedo surfaces.
- Water surfaces: May misinterpret specular reflections as methane enhancements.
- No absolute quantification: Outputs relative enhancement, not calibrated column values.



Linear Least Squares (LLS)

The Linear Least Squares (LLS) method fits a linear model to the observed spectrum by combining background and target basis functions. The CH₄ enhancement factor α is estimated by minimizing the squared residual between model and measurement:

$$\mathbf{y} = \sum_{j=1}^N \beta_j \mathbf{x}_j + \alpha \cdot \mathbf{s}$$

\mathbf{x}_j : Basis vectors representing the background signal
 β_j : Coefficients for the background
 \mathbf{s} : CH₄ absorption signature
 α : Enhancement coefficient to be retrieved

The system is solved in matrix form using ordinary least squares. The matrix $\mathbf{A} = [\mathbf{X}, \mathbf{s}]$ combines the background and target spectra, and the optimal coefficients are computed by minimizing the squared residual:

$$\min_{\beta, \alpha} \left\| \mathbf{y} - \mathbf{A} \begin{bmatrix} \beta \\ \alpha \end{bmatrix} \right\|^2 \quad \text{with } \mathbf{A} = [\mathbf{X} \quad \mathbf{s}]$$

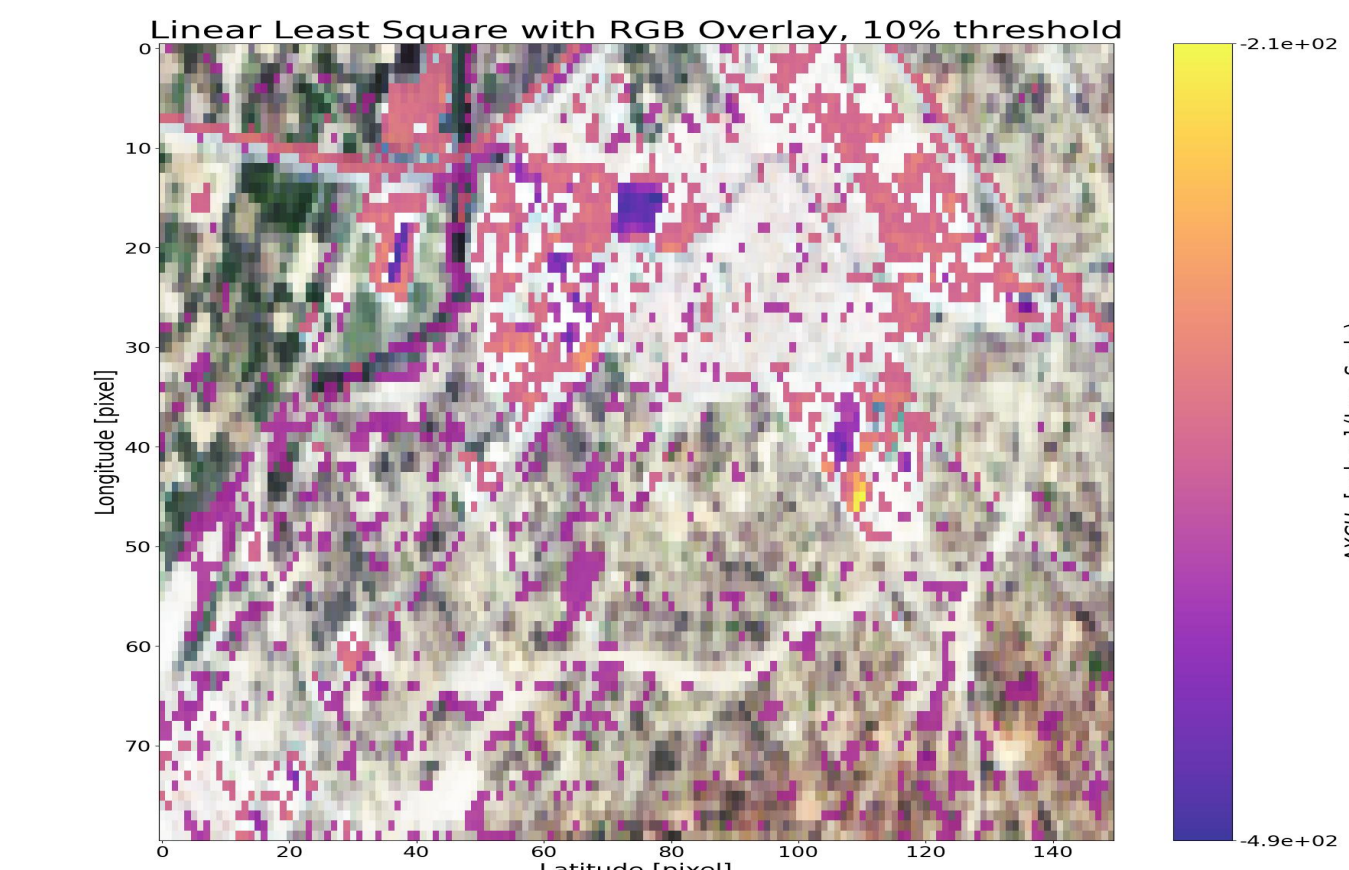
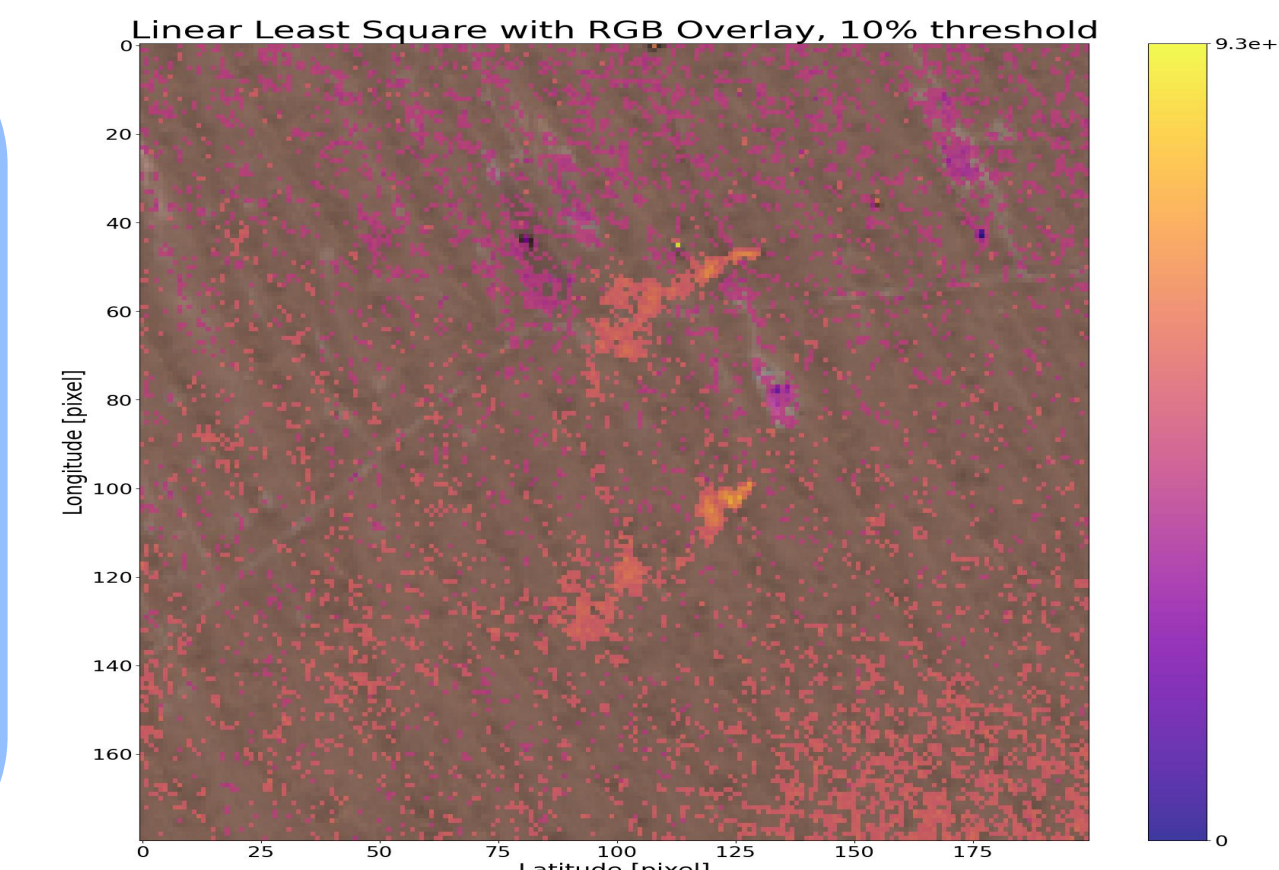
LLS is straightforward, fast, and does not require prior atmospheric knowledge. However, it assumes the CH₄ signal is orthogonal to the background, which can lead to underestimation or leakage in spectrally complex scenes.

Pros

- Fast and easy to implement: Computationally efficient, ideal for quick analyses.
- Simple linear model: Requires minimal parameter tuning or prior knowledge.
- Good baseline method: Useful for comparative performance benchmarking.

Cons

- Underestimates CH₄ enhancements: Especially when CH₄ signals overlap with background spectral features.
- Sensitive to low signal-to-noise: Performance degrades in noisy measurements.
- False positives: Susceptible to retrieval errors in regions with inhomogeneous albedo (e.g., complex terrain, urban surfaces).
- No physical correction: Lacks atmospheric or surface reflectance corrections.



Generalized Least Square (GLS)

The Generalized Least Squares method estimates methane (CH₄) enhancements by fitting the observed spectrum while explicitly accounting for spectral correlations in the background. It models the observation \mathbf{y} as:

$$\mathbf{y} = \sum_{j=1}^N \beta_j \mathbf{x}_j + \alpha \cdot \mathbf{s}$$

\mathbf{x}_j : Basis vectors representing the background signal
 β_j : Coefficients for the background
 \mathbf{s} : CH₄ absorption signature
 α : Enhancement coefficient to be retrieved

What distinguishes GLS from standard LLS is that the fit is weighted by the inverse of the background covariance matrix \mathbf{C}^{-1} :

$$\min_{\beta, \alpha} \left\| \mathbf{C}^{-1/2} \left(\mathbf{y} - \sum_{j=1}^N \beta_j \mathbf{x}_j - \alpha \cdot \mathbf{s} \right) \right\|^2$$

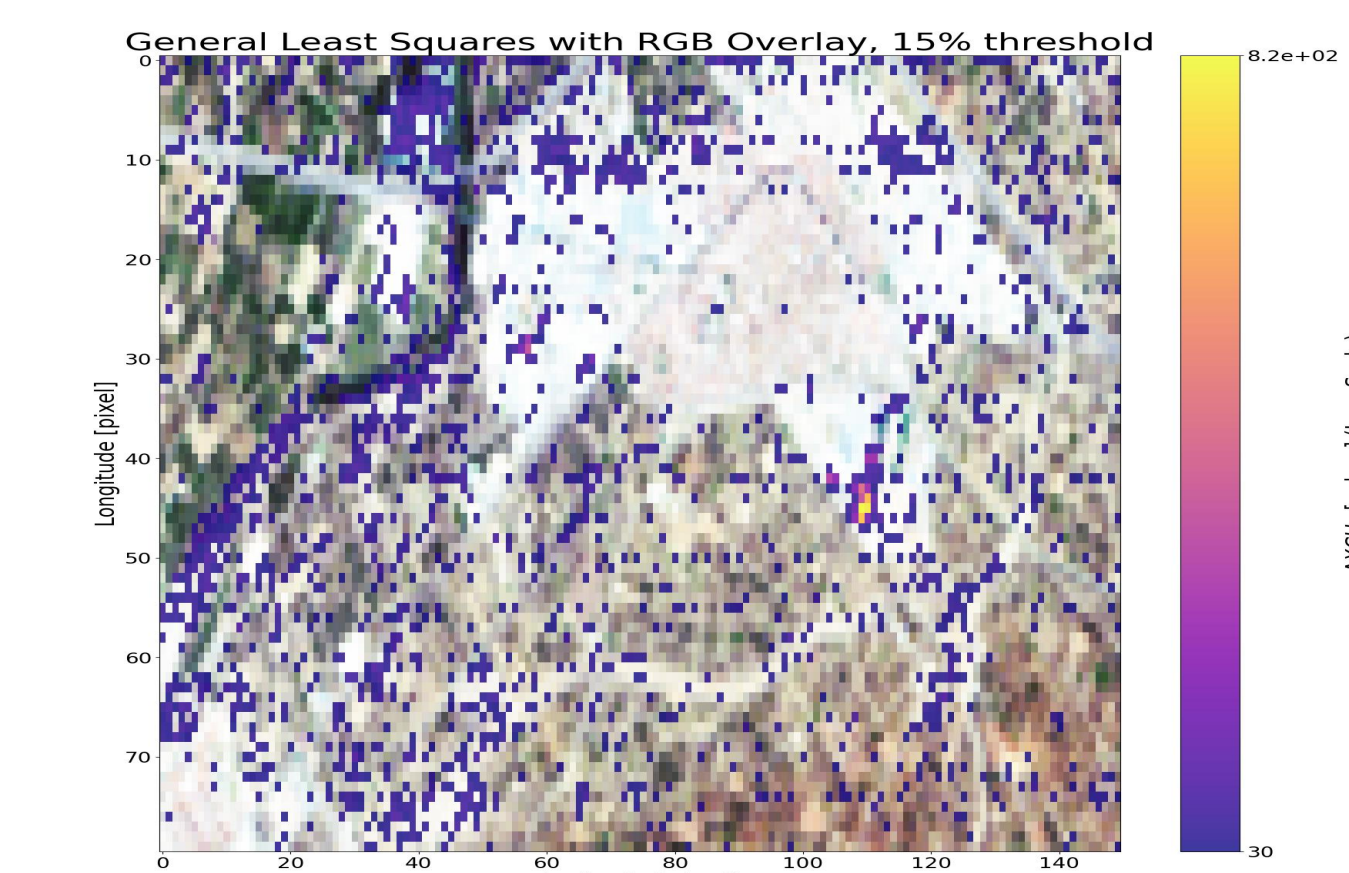
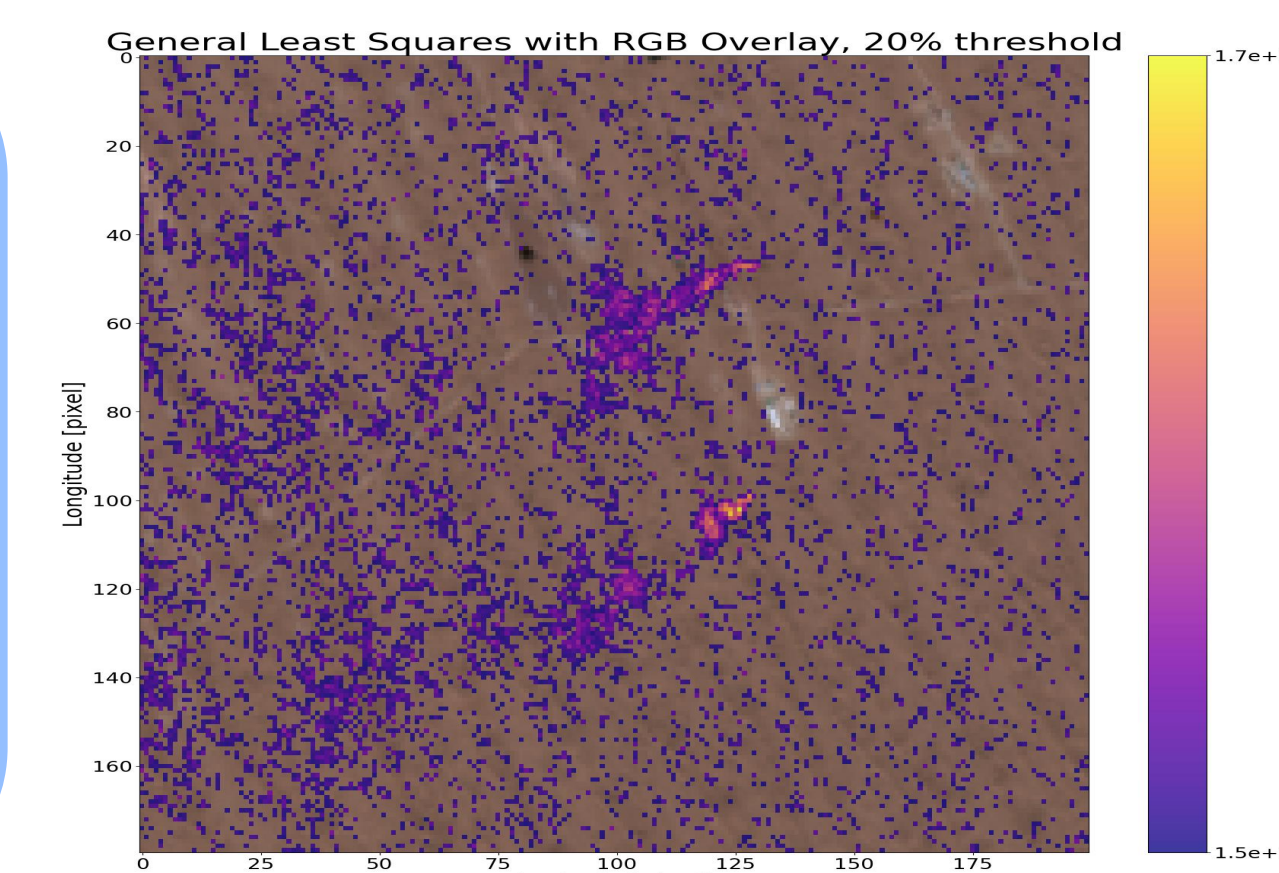
This allows GLS to downweight spectral components that exhibit strong natural variability (e.g., surface reflectance features), leading to a more reliable separation of methane absorption from background structures.

Pros

- Better signal-to-noise handling: Weights the fit using background variability, improving detection in noisy data.
- Deals well with inhomogeneous albedo: Suppresses false positives in spectrally complex regions.
- Reliable source detection: Accurately separates CH₄ signals from background structures.
- Computationally efficient: Still relatively fast despite using a covariance-weighted fit.
- Physically grounded: Makes use of the statistical structure of the background spectra.

Cons

- Requires background statistics: Needs a representative background dataset to estimate the covariance matrix.
- Sensitivity to covariance quality: Poor or unrepresentative background models can degrade performance.
- More complex than LLS: Implementation and tuning are less straightforward.



Cluster Tuned Singular Value Decomposition (SVD)

The SVD method decomposes the observed spectral data into orthogonal components, separating background features from potential CH₄ signals. It assumes the measured spectrum \mathbf{y} can be expressed as:

$$\mathbf{y} = \sum_{j=1}^K \beta_j \mathbf{u}_j + \alpha \cdot \mathbf{s}$$

\mathbf{u}_j : first K singular vectors (principal components) derived from the background spectra
 β_j : Coefficients describing the background contribution
 \mathbf{s} : CH₄ target spectrum
 α : CH₄ enhancement factor

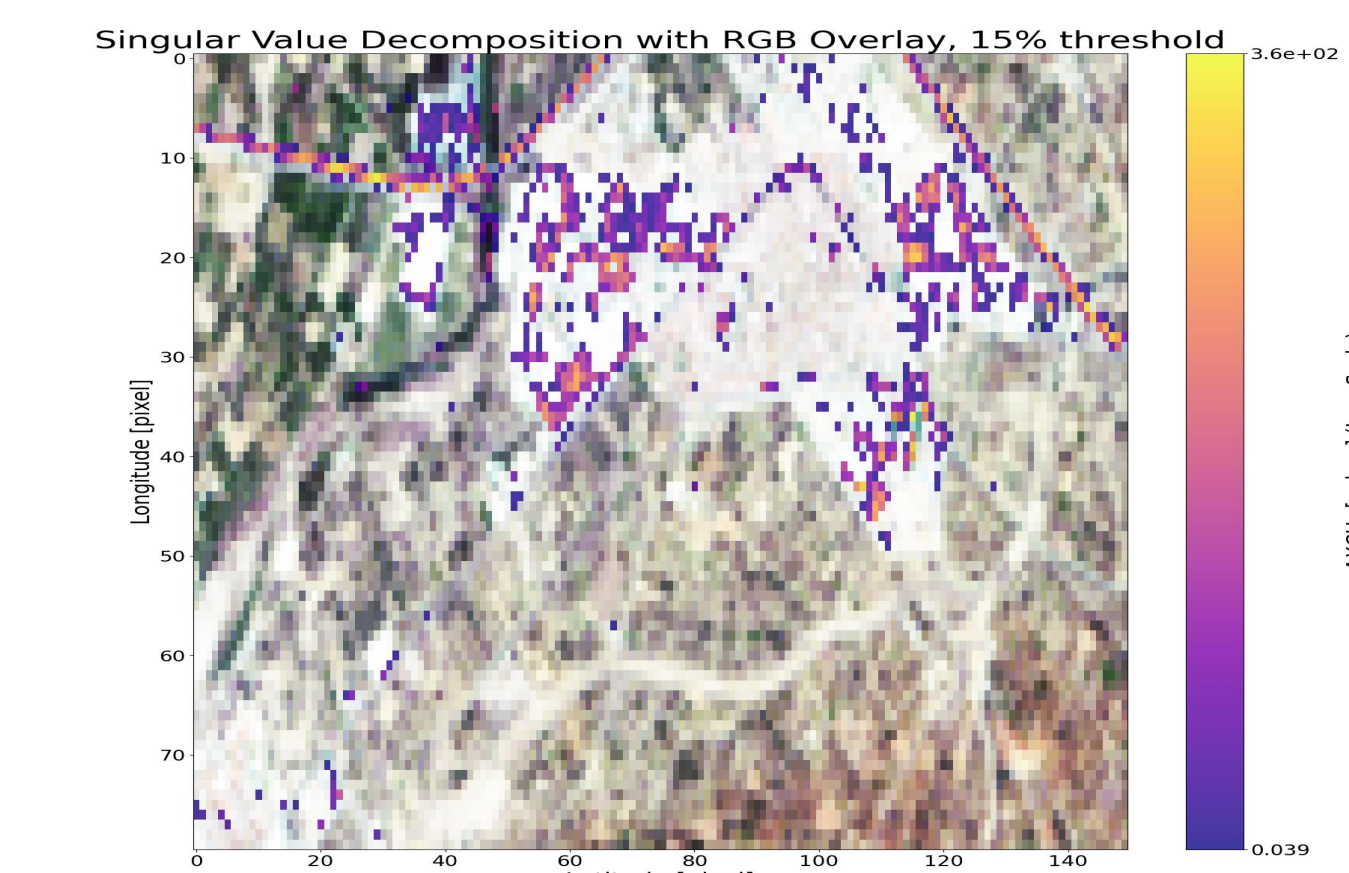
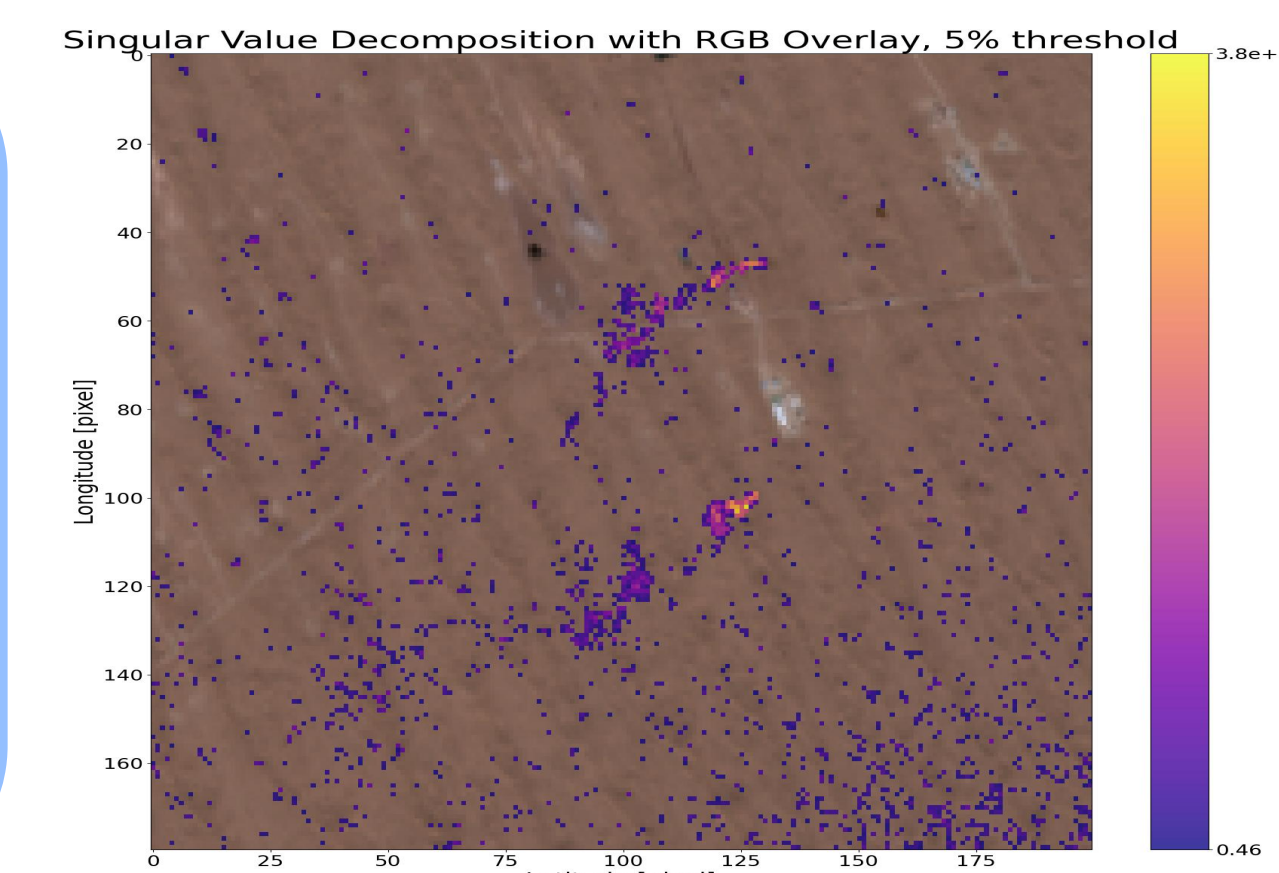
The "cluster-tuned" variant improves performance by applying SVD locally within background clusters (e.g., similar surface types), which reduces spectral variability within each group and leads to more accurate separation of CH₄ signals from the background. SVD fits the model using ordinary least squares, solving for α after subtracting the reconstructed background signal from the measurement.

Pros

- Separates methane enhancements from background: Effectively distinguishes CH₄ signals from spectral clutter.
- Adaptable to surface variability: Clustering allows tailored background modeling for different surface types.
- No need for external atmospheric data: Entirely data-driven using the observed spectra.
- Moderately fast: Computationally efficient once clusters and basis vectors are defined.

Cons

- Parameter tuning required: Performance depends on selecting a suitable number of K-means clusters and SVD basis vectors per cluster.
- Background leakage possible: If clusters are poorly defined, background features may be misinterpreted as CH₄.
- Assumes local linearity: May struggle in highly nonlinear or mixed-pixel environments.



Beer InfraRed Retrieval Algorithm (BIRRA)

BIRRA is a nonlinear inversion algorithm designed to retrieve trace gas concentrations from infrared spectra using line-by-line radiative transfer modeling. It is based on the Beer-Lambert law for optical absorption:

$$I(\lambda) = I_0(\lambda) \cdot \exp \left(- \sum_k \alpha_k(\lambda) \cdot c_k \cdot l_k \right)$$

$I(\lambda)$: measured radiance
 $I_0(\lambda)$: background radiance (without gas absorption)
 $\alpha_k(\lambda)$: absorption cross section of species k
 c_k : concentration of gas k
 l_k : effective path length

In practice, BIRRA retrieves the gas concentration c_{CH_4} by fitting synthetic spectra (generated from HITRAN-based optical depths) to the observed spectrum, using Levenberg-Marquardt optimization to minimize the residual between model and measurement. BIRRA explicitly models:

- Instrumental spectral response
- Surface reflectance
- Atmospheric scattering and absorption

This physically grounded approach provides quantitative trace gas estimates rather than relative enhancements.

Pros

- Physically accurate: Uses full radiative transfer modeling based on the Beer-Lambert law.
- Quantitative results: Retrieves absolute CH₄ concentrations, not just relative enhancements.
- High precision: Effective at separating CH₄ absorption from background variability.
- Accounts for multiple effects: Models surface reflectance, atmospheric scattering, and instrument response.

Cons

- Computationally expensive: Significantly slower than linear or semi-empirical methods.
- Complex implementation: Requires detailed spectroscopic data and careful setup of radiative transfer parameters.
- Sensitive to model inputs: Accuracy depends on reliable prior information (e.g., pressure, temperature, surface properties).
- Parameter tuning required: Regularization, convergence thresholds, and spectral fitting windows must be carefully selected.

