Fast Parametric ISAR Autofocussing Using Quantum Search

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Abstract

Parametric autofocussing in inverse synthetic aperture radar compensates for phase shifts in return echoes caused by target motion which is unpredictable a priori. On top of enabling effective azimuth focussing, it facilitates the estimation of the target motion parameters. Traditionally, the parametric autofocussing problem is tackled with a combination of heuristic methods and exhaustive local search. The latter component constitutes an unstructured database search, which is exactly the type of problem for which quantum search is advantageous. This work describes the problem in the light of quantum search and then lays out a theoretical implementation. After this, a complexity analysis is performed which concludes that, while far beyond the reach of current quantum hardware, an asymptotic improvement in time can be achieved.

1 Introduction and Motivation

Inverse synthetic aperture radar (ISAR) is a technique employed for the imaging of a target moving relative to a radar sensor. Typically, targets are not co-operative, meaning that the specific details of their motion are not known a priori. Azimuth focussing relies on predictable Doppler shifts across slow time, but the unknown motion of the target makes these Doppler shifts unpredictable. As such, it is necessary to correct for this motion to prevent the target appearing blurred in the final image. Parametric autofocussing, which takes place after data acquisition and range compression, does this by estimating the parameters of a motion model associated with the target, from which the necessary phase shifts for each cell can be computed and corrected before azimuth focussing is performed [1]. Here, matched filtering (MF) is performed on the slow time signals for several reference scatterers on the target that have particularly strong backscattering characteristics with a number of templates constructed from different motion model parameter instantiations. The parameter instantiation associated with the template which results in the highest MF response is then the most accurate in the search space. This motion information can then be used to correct the phases of all cells in the image. Beyond facilitating imaging, the motion model parameters, which are a byproduct of parametric autofocussing, can be passed to a tracking algorithm to bootstrap a state estimate. The whole process is depicted in Figure 1. Steps which come before autofocussing, the autofocussing steps, and the returned information are enclosed in blue, yellow, and green respectively.

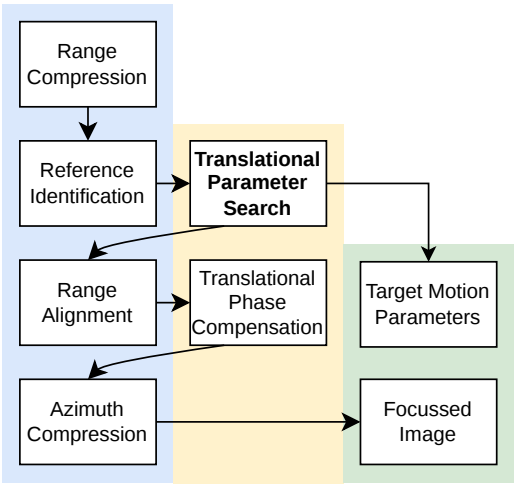


Figure 1 Steps of ISAR image generation and motion parameter estimation. Steps which come before autofocussing, the autofocussing steps, and the returned information are enclosed in blue, yellow, and green respectively. Steps which are subject to quantum search are in bold.

Given the fast paced nature of airborne targets, these steps must be performed quickly. Parametric autofocussing can be a bottleneck as it involves performing a search over a discretised set of parameters, thus severely limiting the speed at which targets can be imaged. As a result, the target state could change significantly before the processing is complete, rendering both the image and the computed parameter estimates outdated. As such, it makes sense to investigate alternative modes of computation to try to alleviate some of this processing load. It should be noted that nonparametric autofocussing methods [1] also exist, but they do not provide the target motion parameters. Also noteworthy is the existence of heuristic methods for the parameter search. Typically, these are used to identify promising regions in the parameter space over which

a locally exhaustive search follows [1]. The approach proposed in this work leads to a faster parameter search which can be used in one of two ways. The first option is to search over all parameters and retain optimality, and the second option is to still use a heuristic to identify promising regions and then speed up the local search.

Despite there being many promising algorithms for quantum computers, relatively few have been formally proven to have an advantage over classical algorithms. Grover's algorithm is an example of a quantum computing algorithm for which there is a proven advantage, namely a theoretical quadratic speedup for an unstructured database search [2]. Taking advantage of this technique, Gao et al. were able to carry the speedup over to a gravitational wave MF template database search [3]. In the case of parametric ISAR, a template is created from each parameter instantiation being searched over, leaving a similar unstructured search over a template bank for MF. Components of Figure 1 which are subject to a quantum speedup are written in bold.

The contributions of this work can be summarised as

- identification of the potential of quantum search for parametric autofocussing,
- high level quantum circuit design,
- and discussion of asymptotic complexities in space and time,

and the sections are organised as follows. Section 2 describes the role of parametrised autofocussing as a means to prepare data for azimuth focussing through phase compensation. Generally, it describes the steps outlined in Figure 1, before Section 3 specifies the target motion model for which parameters are to be estimated. Then, Section 4 conceptually introduces Grover's algorithm, which is well established for quantum search. This section also covers the extension of Grover's algorithm to searching through MF templates in the context of ISAR parametrised autofocussing. Penultimately, an asymptotic space and time complexity is derived in Section 5 after which concluding remarks are made, limitations are stated, and interesting avenues for further research are described in Section 6.

2 Problem Description

In parameterised autofocussing, range compression takes place as normal, leaving us with an image unfocussed in azimuth. The target's motion has two important consequences. These are range cell migration and phases which evolve over slow time due to Doppler shifts. Here, the target's rotation is taken as the source of the synthetic aperture and the target's translation as the source of blur. As such, translational effects must be compensated for. Range cell migration is dealt with using the target's translation to align cells such that scatterers stay in the same range bin across slow time, and then the phase shifts induced by the target's translation from pulse to pulse are calculated and removed from the aligned slow time signals. Because translational motion has the same effect on all scatterers, range alignment can be done by shifting the full fast time signals such that scatterers line up across slow time. Another consequence of this is that the same phase correction term can be applied to across the range bins at a given slow time. This leaves aligned slow time signals whose phase evolution is caused by the target's rotation only.

Both range alignment and phase correction are based on the range deviation of the scatterers, the calculation of which requires knowledge of the target's motion parameters. At some slow time τ_k, the range deviation of a scatterer originally in range bin l is given by (1), where the required translational motion parameters $\hat{\boldsymbol{\theta}}$ are to be found by means of quantum search.

$$\hat{R}_l\left(\tau_k;\hat{\boldsymbol{\theta}}\right) - R_l\left(\tau_0\right) \tag{1}$$

This is the measured range before the motion subtracted from the range calculated based on the target's translation up until τ_k. After using these range deviations to perform range alignment, phase compensation can take place. These are shifts away from the phases which would be expected from a target undergoing rotation only. Each cell receives a phase shift to compensate for these discrepancies. For each slow time index k, there is a compensation term $\hat{\phi}_k$ which is the estimated phase correction for the cells at that slow time. The corresponding phase compensated echo then takes the form shown in (2), in which i denotes the imaginary unit and A the amplitude of the backscatter from the cell given by the accompanying slow time index k and range index l.

$$\hat{s}_{kl} = A_{kl}e^{i\left(\phi_k - \hat{\phi}_k\right)} \tag{2}$$

The phase compensation for a cell depends on how far the scatterer in question has deviated in range from where it would be should the target only be rotating. With wavelength denoted λ, the relationship between phase and range R is

$$\phi = \frac{4\pi}{\lambda}R$$

which, when (1) is plugged in for R, leads to (3). Note that phase compensation terms do not depend on range. The subscript l is kept to show that the difference between the two ranges is for the same scatterer, but it does not actually matter which scatterer this is.

$$\hat{\phi}_k = \frac{4\pi}{\lambda}\left(\hat{R}_l\left(\tau_k;\hat{\boldsymbol{\theta}}\right) - R_l\left(\tau_0\right)\right) \tag{3}$$

With the range profiles aligned and the effects of translation compensated for, slow time signals with phase evolutions induced by rotation alone are left. Under the assumption that the target's rotation does not cause additional blur, azimuth focussing can now take place.

The calculation of the range deviation in (1), and therefore also the calculation of the phase drift in (3), depend on

the target's motion. Specifically, they depend on parameters which are not known a priori for a motion model of the target which is known a priori. A discretised set of parameters are searched over to find the ones which best predict the phase shifts in the slow time signals. Specifically, the parameters $\hat{\boldsymbol{\theta}}$ for

$$\hat{R}_{l_0}\left(\tau_k;\hat{\boldsymbol{\theta}}\right)$$

are searched for. This function, which appears in (1), provides the range along the line of sight (LOS) to a scatterer in range bin l_0 after the target has moved. Here, however, l_0 holds a reference scatterer which is used to fit the parameters, and not some arbitrary scatterer l which may have poor backscattering characteristics. The parameters which best explain the phase histories of all of the reference scatterers are taken to be representative of the target's motion. This, in turn, facilitates the computation of the required phase correction terms which appear in (2) with (3).

The parameter optimisation is an MF template search in which the range aligned reference scatterer's phase histories across slow time are the signals to match and the templates are constructed by predicting these phase histories using the motion model with different discretised parameter values. This is analogous to an exhaustive search in which a matched filter is applied using each template in turn, with the found values serving as the parameters for (3). For a single reference scatterer in range bin l_0, the best template is the one which maximises

$$\left|y_l\left(\hat{\boldsymbol{\theta}}\right)\right| = \left|\sum_{k=0}^{K} s_{kl} h_{kl}^{*}\left(\hat{\boldsymbol{\theta}}\right)\right|,$$

which happens when the template's phase evolution is based on a close match with that of the actual slow time signal. Here, s is the signal and h is the template. In an ideal setting, the effect of the translational component of the target's motion is the same on each scatterer so only one reference is necessary. In practice, however, using multiple references is more robust. Because all templates have the same constant amplitude and only differ in phase evolution, no normalisation is necessary and the MF response can be formulated as in (4). Note that as opposed to a typical MF operation which makes use of a sliding window to align the signal with the template, here $\hat{\boldsymbol{\theta}}$ determines the alignment in slow time so this operation returns the magnitude of the complex correlation between the two functions, which is a scalar, rather than a function of offset.

$$\left|y_l\left(\hat{\boldsymbol{\theta}}\right)\right| = \left|\sum_{k=0}^{K} s_{kl} e^{i\left(\phi_{kl}-\hat{\phi}_{kl}\right)}\right| \tag{4}$$

One set of motion parameters should maximise the response for *all* of the reference scatterers. As such, the parameters from which the template that maximises the sum of the MF responses, one for each reference scatterer, is constructed are taken as optimal. These parameters are the best description in the search space of the target's motion in the maximum likelihood sense [4], assuming a correct motion model. With these parameters, each cell's phase compensation term can be calculated and used to autofocus the image using (2). Searching over all templates becomes computationally intensive when there is a wide spread of possible values or the possible values are finely discretised [4].

3 Motion Model Specification

In this work, a stationary sensor observes a fixed area through which a single target moves. The motion model consists of a translational component $\hat{R}_{\text{tra}}$ and a rotational component $\hat{R}_{\text{rot}}$ as shown in (5), where the parameter vector only pertains to translation. This is because it is the translational motion which must be compensated for while the rotational motion is left untouched so that it can form the synthetic aperture. Both components describe the range along the LOS to a scatterer originally in range bin l.

$$\hat{R}_l\left(\tau_k;\hat{\boldsymbol{\theta}}\right) = \hat{R}_{\text{tra},l}\left(\tau_k;\hat{\boldsymbol{\theta}}_{\text{tra}}\right) + \hat{R}_{\text{rot},l}\left(\tau_k\right) \tag{5}$$

Translation can be approximated with a Taylor series [5]. For a scatterer at slow τ_k, such an approximation to the second order,

$$\hat{R}_{\text{tra},l}\left(\tau_k\right) = R_{\text{tra},l}\left(0\right) + \dot{R}_{\text{tra}}\left(0\right)\tau_k + \frac{1}{2!}\ddot{R}_{\text{tra}}\left(0\right)\tau_k^2,$$

is an expression in terms of the zeroth, first, and second range derivatives which correspond to the range, velocity, and acceleration of scatterer l in the LOS. Translation has the same effect on the ranges of all scatterers at the same slow time, so the only scatterer dependent part of this equation is the original range $R_{\text{tra},l}$. Defining the model parameters as

$$\dot{R}_{\text{tra}} = \alpha,\ \frac{\ddot{R}_{\text{tra}}\left(0\right)}{2!} = \beta \tag{6}$$

$$\hat{R}_{\text{tra},l}\left(\tau_k\right) = R_{\text{tra},l}\left(0\right) + \alpha\tau_k + \beta\tau_k^2 \tag{7}$$

leaves (7), an approximation of the target's translation in terms of α, β. Importantly, the optimal parameters for the reference scatterer are optimal for all other scatterers as well. This means that it is possible to search for the parameters which explain the phase evolution of the reference scatterers and use them for arbitrary scatterers. With this, the parameter vector can be defined as $\hat{\boldsymbol{\theta}} = [\alpha, \beta]$.

4 Quantum Search

While a full theoretical introduction to quantum search and its application to MF is beyond the scope of this work, this section outlines the general algorithmic concepts. On top of this, the necessary conditions under which the quantum search is applicable are described and then how ISAR autofocussing meets these conditions is stated. Nielsen and Chuang give a comprehensive overview of quantum

search [7], and the work of Gao et al. on quantum search for MF [3] describes in detail certain algorithmic concepts adopted here, albeit in a different domain.

4.1 Grovers Algorithm

Grover's algorithm [2] is an unstructured database search algorithm. The entries of the database are binary strings, each with N elements. Consider a function f which accepts these binary strings and evaluates to 1 if the string meets a predefined search criterion and evaluates to 0 otherwise. The former are the sought after entries, and from here on out are referred to as "good", while the latter are referred to as "bad". Grover's algorithm searches through these binary strings and returns good strings with high probability. It runs faster than the best possible classical algorithm by a quadratic factor [2], which is owed to the ability to perform operations on all binary strings simultaneously through quantum superposition. Such a superposition has an amplitude tied to each string. Making a measurement destroys the superposition leaving one of the strings. Each string's measurement probability depends on the corresponding amplitude in the superposition. Thus, by amplifying good amplitudes, which are marked as such through f, the probability of measuring a string adhering to the search criteria can be made high. Mathematically, the application of the operator $\mathbf{O}$ implementing f has the effect shown in (8), from which it can be seen that a -1 marks a quantum state $|x\rangle$ involved in the superposition if that state represents a good string. Using this, the operator $\mathbf{G}$ first marks good strings and then amplifies their amplitudes through (9) [7], in which $|\psi\rangle$ is the maximally superimposed state.

$$|x\rangle \overset{\mathbf{O}}{\mapsto} (-1)^{f(x)} |x\rangle \tag{8}$$

$$\mathbf{G} = (2 |\psi\rangle \langle\psi| - \mathbf{I})\, \mathbf{O} \tag{9}$$

Each application of $\mathbf{G}$ to the superposition of all strings amplifies the amplitudes of good strings and suppresses the amplitudes of bad strings. The result of multiple applications, then, is a high probability of the quantum circuit returning the correct database entry. By performing multiple runs, the sought after entries will be measured more frequently.

The advantage offered by Grover's algorithm disappears when the database has exploitable structure. The reliance of classical parametric ISAR autofocussing algorithms on exhaustive search [4][6] is evidence of the difficulty involved in utilising structure in the search space, indicating a quantum advantage. The goal is to search over target motion parameters to find the ones which best explain the phase evolution of the reference scatterers across slow time. From each parameter instantiation, a series of predicted phase shifts emerges for each reference scatterer. This constitutes a signal in slow time which is the template used for MF with the measured signal. There is a template for each parameter instantiation which leads to an MF response value from the corresponding MF operation as described by (4). In this way, ISAR autofocussing can be described as a database search for high MF response values. Assuming that the motion model is correct, this database search is optimal in that if a set of parameters exists which results in a high enough MF response, they will be found with certainty by classical exhaustive search and with a high probability in the quantum case.

4.2 Quantum Search Matched Filtering

Quantum search requires a decision function which identifies database elements matching a search criterion. In MF, this decision function should identify high MF responses. This is achieved by comparing the MF responses resulting from templates constructed from different parameter instantiations to a threshold. Any parameter instantiation which causes the sum MF responses across the reference scatterers to exceed the threshold is marked as good. A consequence of this is that, depending on the threshold, multiple parameter instantiations can be good, or none at all. It is assumed here that a reasonable threshold can be predetermined. In practice, the number of solutions needs to be known beforehand to apply (9) the correct number of times. This is possible with quantum counting [7], which does not worsen the asymptotic time or space complexity.

Typically, each superimposed string in the database would fetch a template from quantum random access memory (QRAM) so that the MF operation could be performed. At the time of writing, QRAM is far from operational. Fortunately, it is possible to construct, rather than load, the templates from the strings in certain cases [3], of which parametric ISAR is one. Here, there are two parameters, α and β, and the goal is to find their best instantiation. First, the search space must be defined. A vector is created for each, containing evenly spaced values that the respective parameters can take on. The strings that are fed into the quantum search algorithm are composed of two substrings, each one belonging to one of these parameter vectors. Each substring encodes an index in the corresponding vector, and because these strings become superimposed, all indices of all parameters are considered simultaneously. Due to the even spacing, multiplying an index by the increase between two consecutive values and then shifting by the first element in the vector leaves the actual parameter value. This is described by $\hat{\boldsymbol{\theta}} = [a\Delta\alpha + \boldsymbol{\alpha}_0, b\Delta\beta + \boldsymbol{\beta}_0]$ where the index, increase between two consecutive values, and first element are denoted a, $\Delta\alpha$, and $\boldsymbol{\alpha}_0$ for α and b, $\Delta\beta$, and $\boldsymbol{\beta}_0$ for β. In this way, the parameter values being searched over can be computed directly from a set of superimposed indices.

After using (7) and then (3) to compute a phase compensation term for each of the N slow time samples, (2) returns, for each reference scatterer, a signal of N samples. At this point, the algorithm has access to a superposition of templates, all without relying on QRAM in any capacity. The measured slow time signals of the reference scatterers, also each of N samples, are encoded into the circuit, after which a MF response is calculated and compared to the threshold. The mechanisms of Grover's algorithm, over a number of iterations, then amplify the amplitudes of the database register indices which resulted in MF values which meet or exceed the threshold. This results in a high

probability of measuring these good indices.

Finally, what is left are the indices which correspond to the locations of the optimal parameter instantiations in their vectors. Figure 2 depicts the circuit used for the search. All of the gates in between the dashed lines implement (9). The circuit consists of a number of registers. The purpose of each one is described in Table 1. Generally, the circuit functions as follows. The indices are superimposed by Hadamard gates H before being mapped to actual candidate parameter values, stored in the $\hat{\boldsymbol{\theta}}$ value register, and then to templates. Next, matched filtering is performed on the slow time signal and the superimposed templates, and the MF response is stored in the MF response register. In the ancillary register, a -1 marks good solutions after which all entangling operations between the $\hat{\boldsymbol{\theta}}$ index register and the ancillary register are undone which is why the circuit has a U shaped structure.

5 Complexity Analysis

The quantum template bank search offers a theoretical speedup over the classical template bank search. Despite this fact, the hardware requirements still far exceed today's capabilities, and solving problems of any realistic size will have to wait until more advanced quantum computers, both in size and error correction, are available. It should be noted that this section describes worst case asymptotic behaviour, when there is exactly one solution. With more solutions, the scaling is actually better, and counting the number of solutions, which is required, does not add a dominating factor to the time or space complexity [7]. For quantum algorithms, the time complexity is the circuit depth, which is the length of the longest path through the circuit, and the space complexity is the number of required quantum bits, known as qubits, which are units of quantum information.

The dominating factors in the classical time complexity for the parametric autofocussing approach described here depend linearly on the number of slow time samples N and the number of templates M. This leads to a time complexity of $\mathcal{O}(MN)$. The quantum time complexity is $\mathcal{O}\left(\sqrt{M}N\right)$. Both the required number of applications of (9) in a single circuit pass and the required number of full circuit passes are dominated by this time complexity. While an improvement over an already linear factor in the time complexity may seem insignificant, it is important to remember that there is a template for each parameter combination in the search space, which can be prohibitive classically. It should also be noted that this is the time complexity of the autofocussing component only, that is, the yellow box in Figure 1. The complexity becomes more severe when the preprocessing steps are also included, but the advantage in M is retained.

From Figure 2 it can be seen that two registers are needed for each parameter being estimated. The size of the index registers depends on the size of the search space as $\mathcal{O}(\log M)$ and the size of the value registers depends on the desired accuracy in their binary representation, also logarithmically. The same can be said of the MF response and threshold registers. The ancillary register is of size 2. Due to the need to represent all samples in the signals, the space complexity depends linearly on the length of these signals as $\mathcal{O}(N)$. Taking the desired accuracy as constant, the space complexity is therefore $\mathcal{O}(N \log M)$.

6 Conclusion, Limitations, & Outlook

This work makes theoretical arguments in favour of quantum search for parameterised autofocussing in ISAR. Not only does this allow faster image generation, but also faster motion model parameter estimation. Whether or not a heuristic is first applied to limit the search space size or not, the advantage is present. Using the quantum approach, a more fine grained search becomes possible, or a search over a wider domain of parameter instantiations.

Although the proposed approach reduces blur in the resulting image, micro-Doppler effects are not compensated for. On top of this, certain rotational motion, which is not dealt with here, often needs to be compensated for in practice. Additional limitations include the shortcomings of current quantum computers for practical implementation and the difficulty in choosing an appropriate threshold.

Future work will address these points. On top of this, experiments in a simulated quantum environment on toy problems will be run. Finally, an in depth discussion on parameterised autofocussing as an unstructured search problem, on which the argument in this work depends, will be included.

Table 1 Quantum circuit specifications.

Gate	Purpose
$U_{s_0} \dots U_{s_n}$	encoding of signal values $s_l(\tau_k)$
$U_{\hat{\boldsymbol{\theta}}}$	computation of $\hat{\boldsymbol{\theta}}$ from index
$U_{h_0} \dots U_{h_n}$	computation of template values $h(\tau_k)$ from parameters
U_{MF}	computation of MF responses
U_c	comparison of MF responses to threshold
$U^\dagger$	Hermitian adjoints to undo entangling operations

7 Literature

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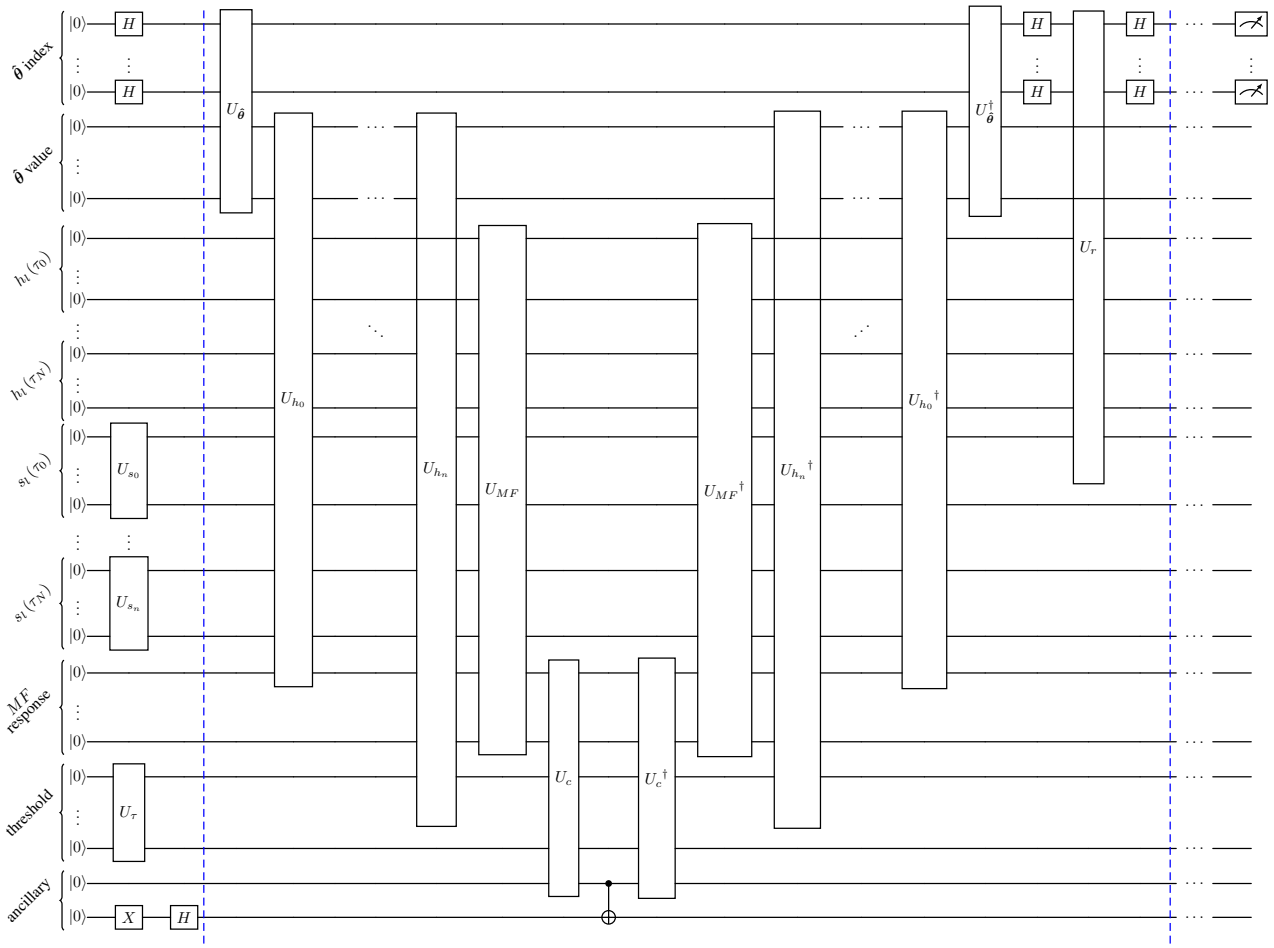


Figure 2 Circuit structure for target motion model parameter search using Grover's algorithm with matched filtering.

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