

A Guide To Robust Navigation Filters Under Multi-Fault Environments

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Abstract—Global Navigation Satellite Systems (GNSS) positioning is severely compromised in urban environments by multipath effects and non-line-of-sight (NLOS) reception, which introduce non-Gaussian measurements errors and generate multiple simultaneous outliers. These challenging conditions significantly degrade the performance of conventional Precise Point Positioning with Ambiguity Resolution (PPP-AR). Multiple Hypotheses Solution Separation (MHSS)-based schemes are considered the standard approach for Fault Detection and Exclusion (FDE). However, they become computationally infeasible when applied to real-time, multi-constellation, multi-frequency GNSS systems due to their combinatorial complexity, particularly under complex urban scenarios. This paper builds upon a Robust Kalman Filter (R-KF), which integrates M-estimators into the KF update step to mitigate the impact of faulty or contaminated observations without the need of enumerating fault hypotheses. This paper provides practical guide to implementing a MHSS scheme and compares its performance against the R-KF to showcase its potential as a alternative method for resilient PPP-AR under multi-fault conditions. Simulation results under controlled fault injections scenarios show that the robust filter achieves performance comparable to MHSS while offering significant advantages in computational efficiency and scalability for future real-time deployment.

I. INTRODUCTION

It is well-known that GNSS performance in urban environments is challenged by multipath effects and non-line-of-sight (NLOS) reception, leading to signal fading, accuracy loss and even temporary outages that compromise the operational reliability of GNSS systems. The use of carrier phase measurements is crucial to overcome the limitations of the standard code-base positioning and enabling decimeter- to centimeter-level accuracy. At the core of high-precision GNSS techniques lies PPP-AR [1], which significantly improves positioning accuracy, availability and convergence speed with respect to standard PPP, at the cost of dealing with real- and integer-valued states (i.e., mixed model). Unfortunately, this estimation process is highly sensitive to poor stochastic modeling, outliers, and multiple simultaneous faults, leading to gross positioning errors or system unavailability. In such challenging conditions, FDE mechanisms are essential for ensuring the reliability and robustness of GNSS-based positioning. Traditional FDE methods primarily rely on redundant measurements and statistical tests on residuals to detect faulty satellite signals and exclude them from the navigation solution. These methods have been widely used due to their simplicity and effectiveness in detecting single faults. However, they suffer from critical

limitations when multiple simultaneous faults occur. A conventional approach in aviation and safety-critical applications is the Multiple Hypotheses Solution Separation (MHSS) scheme [2]. More recently, MHSS has been adapted to high-accuracy GNSS techniques such as PPP-RTK integrity monitoring [3]. By assuming that a specific measurement is faulty and excluding it, MHSS evaluates all possible fault scenarios and selects the most reliable solution. While this approach enhances robustness against multiple simultaneous faults, it suffers from a major drawback: its computational complexity grows combinatorially with the number of observations and potential faults [4]. This makes MHSS impractical for real-time applications, where computational resources are limited. In contrast, Robust statistics-based filtering methods, such as M-estimators and adaptive weighting schemes [5], [6], offer a computationally efficient, scalable solution by down-weighting large residuals without discarding them. Some works exist in the literature on robust filtering techniques for Real-Time Kinematic (RTK) positioning [7]. However, these techniques still remain underexplored for PPP-AR since the mixed real-integer nature of the estimation problem poses a big challenge, together with the lower measurement redundancy compared to RTK and the higher noise levels inherent in PPP. Alternatively, there exist variational Bayesian (VB)-based techniques which enhance robustness and filtering accuracy through adaptive iterative state and noise estimation [8]. However, the iterative nature of VIKF can also end up being very computationally intensive, particularly in real-time applications.

This paper evaluates the performance of robust M-estimation-based filters for robust PPP-AR estimation, introduced in [9], against traditional FDE techniques (i.e., test-statistics-based methods and MHSS scheme) under realistic fault injection scenarios. The study serves as a practical guide to implementing both the classical MHSS scheme and the proposed R-KF within the PPP-AR framework. This comparison highlights the trade-off between robustness, scalability, and real-time feasibility, demonstrating that R-KF provides a practical and efficient alternative to MHSS for resilient high-precision navigation in complex environments.

II. ESTIMATION PROBLEM FOR PPP-AR

This section provides a brief overview to the PPP-AR observation model, the process for the recursive estimation

of the state estimate and short review on the most relevant approaches for FDE.

A. PPP-AR Observation Model

Let us denote the code and carrier phase measurements from a satellite i on frequency f at a particular time k as

$$\rho_{r,f}^i = \rho_r^i + c(dt_r - dt^i) + m^i T_r + \gamma_f I_r^i + b_{r,f} - b_f^i + \varepsilon_\rho \quad (1)$$

$$\Phi_{r,f}^i = \rho_r^i + c(dt_r - dt^i) + m^i T_r - \gamma_f I_r^i + \lambda_f (N_{r,f}^i + B_{r,f} - B_f^i) + \varepsilon_\Phi \quad (2)$$

where $\rho_r^i = \sqrt{(x^i - x_r)^2 + (y^i - y_r)^2 + (z^i - z_r)^2}$ is the geometric distance between the satellite position (x^i, y^i, z^i) and receiver position (x_r, y_r, z_r) ; cdt_r, cdt^i the receiver and satellite clock offsets, respectively, scaled by the speed of light c ; T_r is the zenith tropospheric delay and m^i maps the zenith delay to slant delay [10]; $\gamma_f = \lambda_f^2 / \lambda_1^2$ the ratio of wavelengths to scale the slant ionospheric delay I_r^i to the target f frequency; $b_{r,f}$ and b_f^i denote the receiver and satellite code biases respectively; and $B_{r,f}$ and B_f^i are the receiver and satellite phase biases. Finally, λ_f is the signal's wavelength, $N_{r,f}^i$ is the integer ambiguity for that carrier phase measurement, and $\varepsilon_\rho, \varepsilon_\Phi$ are the code and carrier phase noise terms, respectively, which gather unmodeled errors and thermal noise.

Let us define a vector of measurements comprising the complete set of dual-frequency code and carrier phase pseudorange for N visible satellites at a particular time k as

$$\mathbf{y}_k = \begin{bmatrix} \rho_k \\ \Phi_k \end{bmatrix}, \quad \mathbf{y}_k \in \mathbb{R}^{4N}. \quad (3)$$

The mixed real- and integer-valued model for PPP-AR reads as

$$\mathbf{y}_k \sim \mathcal{N}(\mathbf{h}_k(\mathbf{a}_k, \mathbf{b}_k), \Sigma_k), \quad \mathbf{a}_k \in \mathbb{Z}^{2N}, \quad \mathbf{b}_k \in \mathbb{R}^P \quad (4)$$

where $\mathbf{h}_k(\cdot)$ is a known observation function, $\Sigma_k = \text{diag}(\sigma_\rho^2 \mathbf{I}_{2N}, \sigma_\Phi^2 \mathbf{I}_{2N})$, $\Sigma_k \in \mathbb{R}^{4N \times 4N}$ as the observations' covariance matrix and σ_ρ^2 and σ_Φ^2 being the noise variance for code and phase measurements, respectively; $\mathbf{a}$ and $\mathbf{b}$ are the unknown parameters to be estimated: $\mathbf{a}$ is the vector of integer ambiguities (for both frequencies) and $\mathbf{b}$ corresponds to the real-valued unknowns (e.g., receiver's position, clock offset, atmospheric delays).

B. Estimation Process for PPP-AR

Consider the following nonlinear discrete state-space model (SSM)

$$\mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}), \quad (5)$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k. \quad (6)$$

with $\mathbf{f}(\cdot)$ being the *process model* function which describes the evolution of the states $\mathbf{x}_k$ over time, $\mathbf{w}_{k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k-1})$ as the process noise and $\mathbf{Q}_{k-1}$ as the process noise covariance matrix. The state vector $\mathbf{x}_k$ in equation 5 includes the following components:

$$\mathbf{x}_k^\top = [\mathbf{p}_k^\top, \mathbf{v}_k^\top, dt_k^\top, T_k^\top, \mathbf{I}_k^\top, \mathbf{a}_k^\top], \quad \mathbf{x}_k \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^1 \times \mathbb{R}^1 \times \mathbb{R}^N \times \mathbb{Z}^{2N} \quad (7)$$

with $\mathbf{p}_k$ and $\mathbf{v}_k$ being the three-dimensional position and velocity for the receiver antenna expressed in the ENU-frame; dt_k representing the receiver clock offset, T_k meaning the zenith tropospheric delay; $\mathbf{I}_k$ representing the slant ionospheric delays for N for satellite-receiver pairs; and $\mathbf{a}_k$ being the float ambiguities for all N satellites across different frequencies.

Classical state estimation in a nonlinear system can be formulated as a minimization problem, with the optimal state being obtained by minimizing the following quadratic cost function:

$$\hat{\mathbf{x}}_k = \arg \min_{\mathbf{x}_k} (\|\mathbf{x}_k - \mathbf{f}_{k-1}(\hat{\mathbf{x}}_{k-1})\|_{\mathbf{P}_k}^2 + \|\mathbf{y}_k - \mathbf{h}_k(\mathbf{x}_k)\|_{\Sigma_k}^2) \quad (8)$$

Note that a *explicit* estimation process for (8) is not known, since we cannot simultaneously estimate real and integer parameters. Instead, we can resort to the well-known three-step estimation process [11]: 1) *float solution*, for which we disregard the integer nature of the ambiguities; 2) *Integer Ambiguity Resolution (IAR)* to map the vector of estimated real ambiguities to integer ones; and 3) *fixed solution*, for which the real parameters are constrained based on the knowledge of the integer ambiguities.

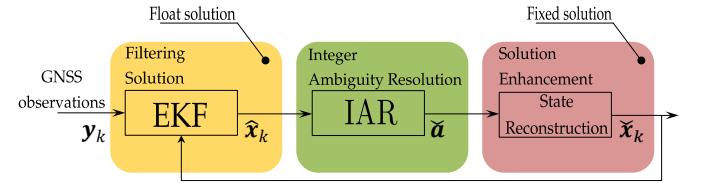


Fig. 1: Diagram flow for the three-step decomposition of the recursive estimation for the mixed model.

Details on the derivation for the above-described estimation for the mixed model filtering can be found in [12, Ch. 2.2]. The conventional approach relies on the Extended Kalman Filter (EKF) for float solution estimation, which is optimal for (quasi)linear problems when model assumptions hold. However, EKF is highly sensitive to outliers and model mismatches, which can deteriorate its performance significantly, leading to inconsistent estimates or even divergence. Such degradation in the accuracy directly impacts the IAR and float solution processes, potentially compromising the reliability of the overall estimation.

C. Fault Detection and Exclusion for PPP-AR

To mitigate the deficiencies of the EKF, FDE techniques are employed. Two widely used approaches are test statistics-based methods and MHSS schemes. In test statistics-based methods for filtering-based estimation, the residuals (or innovation vector) are typically used to compute the test statistics [13], allowing for the identification and exclusion of faulty measurements, but limited to a single fault event at each time step. On the other hand, MHSS mitigates faults by running multiple parallel filters [13], each assuming a different hypothesis on a specific measurement being faulty and excluded.

When performing MHSS is crucial to determine the fault models or subsets, which depend on the prior probability of

fault and N_f being the maximum number of simultaneous faults that need to be monitored [14]. Then the number of subsets (or hypothesis) N_h is defined based on all the possible combinations of faults among all the visible satellites and a bank of EKFs is implemented accordingly. The subset $j = 0$ corresponds to the all-in-view hypothesis (all visible satellites are assumed to be fault-free), and the rest $j > 0$ subsets correspond to the fault-mode hypothesis. After initialization, the state prediction and measurement update is performed independently for each subset using the only the measurements from the satellites considered for each subset at every epoch. Since N_h solutions are obtained (i.e., one for each subset filter), a FDE process is applied next to identify the fault-free subset, that is the subset which successfully excluded the faulty satellites and the solution is computed using only fault-free measurements (i.e., fault-tolerant solution $\hat{\mathbf{x}}_k^{FT}$ and its associated covariance matrix $\mathbf{P}_k^{FT}$). The test statistics q at a particular time k is implemented according to the Innovation-based (IB)-test, presented in [15], such that

$$q_k = \mathbf{r}_k^T \mathbf{S}_k^{-1} \mathbf{r}_k, \quad (9)$$

where $\mathbf{r}_k$ corresponds to the residuals (or innovation vector) and is expressed as $\mathbf{r}_k = \mathbf{y}_k - \mathbf{h}_k(\hat{\mathbf{x}}_{k|k-1})$ with its associated covariance matrix as $\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{\Sigma}_k$. A suitable threshold T_k for this test can be computed by using the inverse chi-square cumulative distribution given a desired probability of false alarm and a given number of degrees of freedom.

One of the main advantages of MHSS scheme is that it is able to deal with multiple simultaneous faults and FDE is performed inherently. However, its main drawback is its high implementation complexity and heavy computation load.

The overall architecture of the implemented MHSS filter is as shown in Algorithm 1, with K as the total number of time steps, $\mathbf{F}_k$ and $\mathbf{H}_k$ as the state transition and measurement Jacobians, respectively; the predicted state and predicted state covariance matrix as $\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$; the Kalman gain as $\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{\Sigma}_k)^{-1}$; the updated state vector and covariance matrix can be obtained as $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{r}_k$, and $\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}$. The superscript $(\cdot)^{-i}$ means $\{(\cdot)_k^j \mid j \in \{1, \dots, N\} \setminus \{i\}\}$ and $\mathcal{J}_{\text{pass}}$ represents the *test passed* set, which is defined as $\mathcal{J}_{\text{pass}} = \{j \in \{0, \dots, N\} \mid q_k^{(j)} \leq T_k^{(j)}\}$. The method used for IAR is Partial Ambiguity Resolution (PAR). The MHSS implementation presented in Algorithm 1 does not include any subset reduction techniques.

III. ROBUST FILTERING FOR PPP-AR

Despite the mixture of integer- and real-valued estimation in PPP-AR, robust integer estimators are not known. Thus, the key to *robustifying* PPP-AR lies in improving the performance of the float solution estimation. To do so, faulty satellite links are detected and their impact on the estimation is mitigated, enabling the guidance of the integer ambiguity search and enhancing its success ratio. Such an improvement is obtained by resorting to robust KF techniques, as presented in [9].

Algorithm 1: MHSS Filter

Input : $\hat{\mathbf{x}}_{0|0}, \mathbf{P}_{0|0}, \mathbf{y}, N_h$

- 1 Define subfilters according to N_h
- 2 Initialize subfilters $\hat{\mathbf{x}}_{0|0}^{(j)} = \hat{\mathbf{x}}_{0|0}, \mathbf{P}_{0|0}^{(j)} = \mathbf{P}_{0|0}, \forall j \in \{0, \dots, N_h\}$
- for** $k = 0, 1, 2, \dots, K$ **do**
 - Estimation Block**
 - for** $j = 0, 1, 2, \dots, N_h$ **do**
 - 3 Predict $\hat{\mathbf{x}}_{k|k-1}^{(j)}$ and $\mathbf{P}_{k|k-1}^{(j)}$
 - 4 Obtain $\mathbf{H}_k^{(j)}, \mathbf{\Sigma}_k^{(j)}, \mathbf{r}_k^{(j)}, \mathbf{S}_k^{(j)}$ and $\mathbf{K}_k^{(j)}$
 - FDE Block**
 - for** $j = 0, 1, 2, \dots, N_h$ **do**
 - 5 Compute $q_k^{(j)}$ and $T_k^{(j)}$ and $\mathcal{J}_{\text{pass}}$
 - if** $q_k^{(0)} > T_k^{(0)}$ **then**
 - Test failed for subset (0) (all-in-view);
 - if** $q_k^{(1:N_h)} > T_k^{(1:N_h)}$ **then**
 - Test failed for all subsets;
 - No update; $\hat{\mathbf{x}}_{k|k}^{(j)} = \hat{\mathbf{x}}_{k|k-1}^{(j)}$ and
 - $\mathbf{P}_{k|k}^{(j)} = \mathbf{P}_{k|k-1}^{(j)}, \forall j \in \{0, \dots, N_h\}$
 - $\hat{\mathbf{x}}_k^{FT}, \mathbf{P}_k^{FT}$ unavailable;
 - else**
 - Find subset (j) such that
 - $j^* = \arg \min_{j \in \mathcal{J}_{\text{pass}}} q_k^{(j)}$
 - Update $\hat{\mathbf{x}}_{k|k}^{(j^*)}$ and $\mathbf{P}_{k|k}^{(j^*)}$
 - No update; $\hat{\mathbf{x}}_{k|k}^{(j)} = \hat{\mathbf{x}}_{k|k-1}^{(j)}$ and
 - $\mathbf{P}_{k|k}^{(j)} = \mathbf{P}_{k|k-1}^{(j)}, \forall j \in \{1, \dots, N_h\} \setminus \{j^*\}$
 - Return:** $\hat{\mathbf{x}}_k^{FT} = \hat{\mathbf{x}}_{k|k}^{(j^*)}, \mathbf{P}_k^{FT} = \mathbf{P}_{k|k}^{(j^*)}$
 - else**
 - Test passed for subset (0);
 - Update $\hat{\mathbf{x}}_{k|k}^{(j)}$ and $\mathbf{P}_{k|k}^{(j)} \forall j \in \{0, \dots, N_h\}$
 - Return:** $\hat{\mathbf{x}}_k^{FT} = \hat{\mathbf{x}}_{k|k}^{(0)}, \mathbf{P}_k^{FT} = \mathbf{P}_{k|k}^{(0)}$
 - IAR Block**
 - if** IAR enabled; **then**
 - 12 Perform $\tilde{\mathbf{a}}_k = \mathcal{S}(\hat{\mathbf{a}}_k)$ with FAR
 - 13 Compute fixed solution $\tilde{\mathbf{x}}_k$ and $\mathbf{P}_k^{\text{fixed}}$ as in [12, Ch. 2.2]
 - Return:** $\tilde{\mathbf{x}}_k$ and $\mathbf{P}_k^{\text{fixed}}$
- 14 **Return:** $\hat{\mathbf{x}}_k^{FT}, \mathbf{P}_k^{FT}, \tilde{\mathbf{x}}_k$ and $\mathbf{P}_k^{\text{fixed}}$

A. Robust Estimation Process for PPP-AR

As alternative to the conventional KF, compromised under the presence of contaminated measurements, one may resort to the framework of *Robust Filtering*. Robust Filtering provides a score function to mitigate the effect of outliers on the state estimates. Thus, the minimization problem in (8) can

be instead expressed as

$$\hat{\mathbf{x}}_k = \arg \min_{\mathbf{x}_k} \left(\|\mathbf{x}_k - \mathbf{f}(\mathbf{x}_{k-1})\|_{\mathbf{P}_k}^2 + \|\mathbf{y}_k - \mathbf{h}(\mathbf{x}_k)\|_{\bar{\Sigma}_k}^2 \right) \quad (10)$$

where $\bar{\Sigma}_k$ is the observations' covariance matrix conditional on a set of estimated weights, such that

$$\bar{\Sigma}_k = \Sigma_k^{1/2} \mathbf{W}^{-1} \Sigma_k^{\top/2} \quad (11)$$

where $\Sigma_k^{1/2}$ is the Cholesky factorization of Σ_k and $\mathbf{W}$ is a weighting matrix given by

$$\mathbf{W} = \text{diag} \left[\mathbf{w} \left(\Sigma_k^{-1/2} (\mathbf{y}_k - \mathbf{h}(\mathbf{x}_k)) \right) \right] \quad (12)$$

with $\mathbf{w}(\cdot)$ a function derived from its robust score function $\psi(\cdot)$ [12, Ch. 5.1.1]. The underlying idea behind robust functions is to down-weight or nullify the effect that observations not fitting the underlying model have over the actual estimates.

The implementation of the R-KF is realized via Robust Information Filters (RIF) using a Huber function as in [16] with tuning parameter $c_{\text{Hub}} = 1.345$. The architecture of the R-KF is as shown in Algorithm 2.

Algorithm 2: R-KF

Input : $\hat{\mathbf{x}}_{0|0}$, $\mathbf{P}_{0|0}$, $\mathbf{y}$, Robust-estimator parameters:
 $w(\cdot), c$

for $k = 0, 1, 2, \dots, K$ **do**

Estimation Block

1 Predict $\hat{\mathbf{x}}_{k|k-1}$ and $\mathbf{P}_{k|k-1}$

2 Obtain $\mathbf{H}_k$, Σ_k

3 Initialize $\hat{\mathbf{x}}^{(0)} = \hat{\mathbf{x}}_{k|k-1}$, $\mathbf{W}^{(0)} = \mathbf{I}$

for $p = 1, 2, \dots$ **until convergence do**

4 Solve Eq. (8) to obtain $\hat{\mathbf{x}}^{(p)}$, $\mathbf{Z}^{(p)}$ as
described in Eq. (21)–(22) of [16]

5 Compute $\mathbf{W}^{(p)}$ from (12) with $\hat{\mathbf{x}}^{(p)}$

6 **Return:** $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}^{(p)}$, $\mathbf{P}_{k|k} = (\mathbf{Z}^{(p)})^{-1}$

IAR Block

if IAR enabled; **then**

Same as Lines 12, 13 from Alg. 1

Return: $\check{\mathbf{x}}_k$ and $\mathbf{P}_k^{\text{fixed}}$

7 **Return:** $\hat{\mathbf{x}}_{k|k}$, $\mathbf{P}_{k|k}$, $\check{\mathbf{x}}_k$ and $\mathbf{P}_k^{\text{fixed}}$

IV. SIMULATION RESULTS

The impact of outliers on standard EKF, MHSS and the proposed robust filter for the PPP-AR estimation problem is evaluated through simulation experiments. In this section, we analyze the behavior of three prototypes non-nominal conditions: i) EKF, a classical filter heavily affected by the presence of outliers; ii) MHSS, a bank of EKFs followed by a FDE scheme based on IB-test; and iii) the R-KF, as defined in III, based on applying robust weighting functions to the KF correction step. To properly assess the performance of the filters, a Monte Carlo (MC)-based simulation framework is

developed (i.e., GNSS simulation tool). The evaluation part consist in emulating different types and combinations of error profiles with the GNSS simulator, affecting different types of measurements, signals, and satellites, and checking the filters capabilities to detect and mitigate these error types. The selected scenario for the simulation is based on open-sky conditions, with an elevation mask of 5 degrees, since we are interested in introducing faults to the observed satellites while preserving PVT full observability and a minimum of 4 satellites per constellation. In order to reduce the implementation complexity of the MHSS, a static skyplot is being considered for the selected scenario. The profile of the faults injected is presented in Table I, where *step* error means a bias of given magnitude is added to the target measurement. The number of MC experiments is set to 20, the total duration of the experiment is 3600 seconds, with the prediction and correction steps being processed at 1 Hz. The probability of false alarm for the IB-test is set to 10^{-3} . The grey areas in the figures represent the time period where faults are present. As a metric of performance, the root mean squared error (RMSE) at each time instant for the horizontal positioning is considered.

TABLE I: Fault profile of the simulated scenario.

Type	Initial Time (s)	End Time (s)	SV ID	No. of Faulty SVs
step	500	1000	random	3
step	1500	2000	random	3
step	2500	3000	random	3

Constellation	Frequency	Observation Type	Characterization
GPS+GAL	all (L1,L2,E1,E5b)	code	Bias: 100 m
GPS+GAL	all	code	Bias: 100 m
GPS+GAL	all	code	Bias: 100 m

From the results presented in Fig. 2: i) the EKF, as expected, rapidly degrades under the presence of outliers since it has no capabilities to deal with them, leading to several meters of errors in the position domain, which consequently prevents the filter from estimating correctly the ambiguities and this results in a biased fix solution as well; ii) the MHSS is able to successfully identify and exclude the faulty satellites, yielding the most accurate solution for both float and fixed position estimates; iii) the R-KF is able to deliver a solution comparable to MHSS but it cannot mitigate completely the impact of the injected outliers. This may be because Huber-based M-estimators effectively downweight measurements with large residuals while maintaining zero-mean error distribution (e.g., caused by variance-inflated faults), whereas persistent biases can be partially absorbed by the estimated state parameters, causing the corresponding residuals to decrease and diminishing the ability of the robust weighting scheme to distinguish the biased measurements from nominal ones. Also note that the peak that appears for the MHSS in Fig. 2c around epoch 1700 is due to a time instance where the fault-tolerant solution was unavailable. Finally, the running time and memory usage

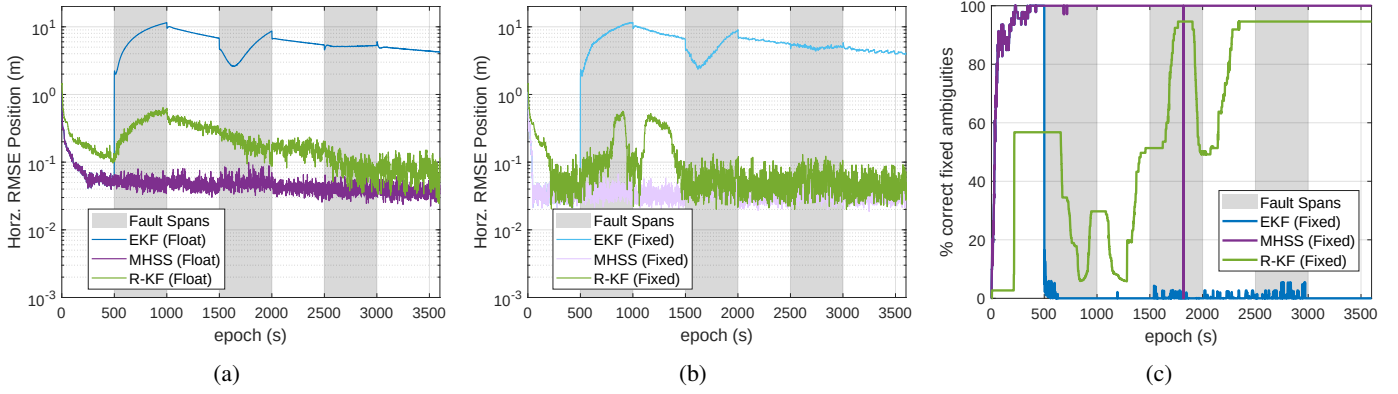


Fig. 2: Horizontal RMSE of the float and fixed position estimates (left and middle) and percentage of correctly fixed ambiguities (right) over time for three prototype navigation filters: i) Conventional EKF, ii) MHSS, and iii) Robust KF.

required by the different filters over the 20 MC runs was recorded using the MATLAB profiling application: i) EKF: 820.2s (9.7MB), ii) MHSS: 321135.6s (9.95GB), iii) R-KF: 2973.7s (9.2MB). The PC specifications used to run the MC simulations is shown in Table II as a reference. Although the R-KF does not consistently achieve the same level of accuracy as MHSS throughout the entire simulation, it provides a substantial reduction in processing time.

Specification	Details
Model	Dell Precision 7680
Operating System	Windows 11 Enterprise
Installed RAM	64 GB
Processor	13th Gen Intel(R) Core(TM) i9-13950HX, 2.20 GHz
System Type	x64-based processor

TABLE II: PC specifications.

V. CONCLUSIONS

This work presents a practical tutorial on the implementation of a conventional MHSS-based FDE technique and a Robust Kalman Filtering (R-KF) based on M-estimation within the PPP-AR estimation problem framework [9]. The MHSS scheme is implemented as a parallel filtering approach that evaluates fault hypotheses, while the R-KF is applied through robust weighting in the Kalman update step, enabling efficient outlier mitigation without hypothesis enumeration. Through a Monte Carlo simulation, the authors demonstrate that the R-KF achieves performance closely matching MHSS in terms of accuracy and solution reliability under multiple simultaneous faults, while offering a significant reduction in computational cost. The results obtained highlight the critical trade-off between robustness, scalability, and real-time feasibility, demonstrating the potential of robust estimators for GNSS high-precision applications, particularly in resource-limited environments.

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