Observability Analysis of Simultaneous Localization & Calibration for Cooperative Radio Navigation

Alexis Marino, Robert Pöhlmann, and Emanuel Staudinger
Institute of Communications and Navigation
German Aerospace Center (DLR)
Oberpfaffenhofen, 82234 Wessling, Germany
{alexis.marinosalguero, robert.poehlmann, emanuel.staudinger}@dlr.de

Abstract—**Future robotic surface exploration missions, e.g. on Mars, demand robust and accurate navigation solutions. Since no Satellite Navigation or infrastructure exists on Mars, accurate navigation remains challenging. Cooperative radio navigation emerged as a key technology in such scenarios. Methods based on signal propagation time, such as round-trip time ranging, require sub-nanosecond Time-of-Flight measurement accuracy. However, environmental factors, e.g. temperature changes, can affect group delays of radio transceivers, introducing ranging bias. Simultaneous Localization and Calibration (SLAC) has been proposed to jointly estimate node positions and ranging bias as a calibration parameter, correcting such biases online. However, the success of this calibration process relies on the observability of the calibration and localization parameters, ensuring they can be accurately estimated and corrected using available measurements. To cope with that, we conduct an observability analysis of SLAC, leveraging the equivalence between the full column rank of the Fisher Information Matrix and the measurement Jacobian matrix. This analysis leads to the derivation of necessary and sufficient conditions for observability, which determine the minimum number of static and mobile nodes required for SLAC, and identifies geometric configurations that result in strong, weak, or non-observable systems. We further highlight scenarios where observability is inherently impossible and propose practical solutions for both cooperative and non-cooperative navigation. The analysis is validated through extensive simulations and real-world experiments using radio nodes integrated into static and mobile platforms. The experiments confirm the theoretical findings and demonstrate that SLAC can achieve observability under realistic deployment conditions.**

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1. Introduction

Future robotic surface exploration missions, demand robust and accurate navigation solutions to ensure safe autonomous operation, avoid hazards, and achieve mission objectives [1]. Since no Global Navigation Satellite System (GNSS) exists in such environments, accurate navigation remains challenging. Cooperative radio navigation emerged as a key technology in such scenarios. It enables precise localization in infrastructure-free environments by exchanging radio signals among networked nodes to estimate inter-node distances [2].

Methods based on signal propagation time, such as Round-Trip Time (RTT) ranging, require sub-nanosecond Time-of-Flight (ToF) measurement accuracy [3]. However, environmental factors, e.g. temperature changes, can affect group delays of radio transceivers, especially when using low-cost Commercial Off-The-Shelf (COTS) radio frequency (RF) hardware, introducing ranging bias. Transceiver group delays require precise calibration; pre-launch calibration alone is insufficient for extraterrestrial environments, making in-situ online calibration essential. To address this need, Simultaneous Localization and Calibration (SLAC) for cooperative radio navigation has been proposed, allowing teams of robots to enhance their positioning accuracy, compensate for ranging biases, and calibrate antennas [4]. Nevertheless, the observability of the SLAC framework has not yet been thoroughly analyzed, i.e., under what conditions calibration parameters and states can be estimated and with what accuracy. Accounting for observability is not only critical to guarantee reliable performance, but it also provides a foundation for optimizing measurement strategies.

SLAC is often considered as a part of Simultaneous Localization and Mapping (SLAM) [5], where observability has been extensively studied. The existing literature has shown that most SLAM formulations are not observable from a control theoretic perspective [6]. Moreover, observability of multi-robot cooperative localization is also considered in [7] where using the observability rank condition for nonlinear systems [8] shown that the cooperative localization in general has three unobservable degrees of freedom, corresponding to the global position and orientation. A useful alternative method is to examine the observability formulations in the form of a parameter estimation problem [6], which can be addressed using the concept of parameter observability [9].

To keep terminology consistent with the existing literature on SLAM, we will term the parameter estimability question as observability analysis here, more specific, we will focus on the SLAC framework for the case where positions and nodes ranging biases are parameters to estimate. A classical approach is to analyze the Fisher Information Matrix (FIM), which directly characterizes parameter observability [9]. In particular, in the literature, by treating SLAM as a classical parameter estimation problem and examining the structure of the FIM, researchers have made much progress in its observability analysis [10], [11], [12]. As it has been summarized in [6], when SLAM is formulated as a nonlinear least-squares parameter estimation problem, the full column rankness of the associated FIM governs its observability (i.e., parameter estimability). Given the fact that full column rankness of the FIM and the Jacobian matrix are equivalent,

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observability analysis of the corresponding SLAM problem is often transformed into the study of conditions guaranteeing the full column rankness of the Jacobian as shown in [12].

By using the strong foundation in the SLAM community, in this paper, we will conduct a systematic observability analysis, using a FIM approach, for the problem of SLAC for cooperative radio navigation, although our analysis follows a conceptually similar procedure with those outlined in [10], [11], [12]. The considered problem of this paper is mainly motivated by the use of cooperative radio navigation in exploration applications, To the best of our knowledge, this is the first systematic work on observability analysis in SLAC for cooperative radio navigation.

The objective of this paper is to establish necessary and sufficient conditions for SLAC observability by exploiting the equivalence between the full column rank of the FIM and the Jacobian of the measurement model, along with the structure of the SLAC formulation. We identify cases where observability is fundamentally impossible in both cooperative and non-cooperative navigation, determine the minimum number of static and mobile nodes required for practical implementation, and characterize geometric configurations that lead to strong, weak, or non-observable systems. The analysis is validated through simulations with varying node configurations, where the Cramér-Rao Bound (CRB) and eigenvalues of the FIM are computed to quantify observability, and through real-world experiments with static and mobile radio nodes, which confirm that geometric arrangements critically affect observability and that, under appropriate geometric arrangements and a sufficient number of nodes, SLAC remains observable in realistic deployments conditions.

The remainder of this paper is organized as follows. First, we present the system model for SLAC, including the range-based measurement model. Next, we perform the observability analysis by introducing the FIM and deriving the necessary and sufficient conditions for observability. Finally we report results from both simulations and real-world experiments accompanied by the conclusions.

2. System Model

State Space Definition

We define a state space for cooperative radio navigation. The network is composed of two types of nodes, represented by the set $\mathbb{N} = \mathbb{A} \cup \mathbb{M}$, where anchors $\mathbb{A}$ are static nodes with known positions, and agents $\mathbb{M}$ are mobile nodes, such as robots, that require localization. This configuration results in a total of $|\mathbb{N}|$ nodes in the network, where $|\cdot|$ represents the cardinality of the set. The node state vector of an agent $i \in \mathbb{M}$ consists of the kinematic states $\mathbf{x}_{i,\text{loc}}^k$ and the calibration parameter states $\mathbf{x}_{i,\text{cal}}^k$ for snapshot k,

$$\mathbf{x}_i^k = [(\mathbf{x}_{i,\text{loc}}^k)^\top, (\mathbf{x}_{i,\text{cal}}^k)^\top]^\top. \quad (1)$$

The node kinematic states are defined as

$$\mathbf{x}_{i,\text{loc}}^k = [\,\boldsymbol{p}_i^k], \quad (2)$$

where $[\,\boldsymbol{p}_i^k] = [x_i^k,\ y_i^k]^\top \in \mathbb{R}^2$ represents the 2D position.

The state vector of an anchor $i \in \mathbb{A}$ includes only the calibration parameter states $\mathbf{x}_i^k = \mathbf{x}_{i,\text{cal}}^k$. Node calibration states for anchors and agents are given by

$$\mathbf{x}_{i,\text{cal}}^k = [\delta_i^k], \quad (3)$$

where δ_i^k is the ranging bias present at each node. The complete network state vector is formed by stacking the individual node states, ordered as

$$\mathbf{x}^k = [\,\boldsymbol{p}_1^k \dots \boldsymbol{p}_{|\mathbb{M}|}^k \mid \delta_1^k \dots \delta_{|\mathbb{M}|}^k \mid \delta_1^k \dots \delta_{|\mathbb{A}|}^k\,]^\top, \quad (4)$$

with overall dimension $\mathbf{x}^k \in \mathbb{R}^{3|\mathbb{M}|+|\mathbb{A}|}$.

Measurement Model

We consider ranging measurements obtained through RTT protocols, which inherently compensates for transmitter and receiver clock offsets [13]. We assume that the processing time between forward and backward transmission is already compensated and sufficiently short, so that the impact of relative clock frequency offsets can be neglected. Otherwise, clock dynamics must be explicitly modeled using an appropriate clock model [14]. The distance between two nodes i and j for snapshot k, considering their ranging bias, is given by

$$d_{i,j}^k = \left\|\boldsymbol{p}_j^k - \boldsymbol{p}_i^k\right\| = \frac{c\left(\tau_{i,j}^k - \tilde{\tau}_{i,j}^k\right)}{2} + \delta_i^k + \delta_j^k, \quad (5)$$

where c denotes the speed of light, $\tilde{\tau}_{i,j}^k$ is the transmit time of the forward signal, and $\tau_{i,j}^k$ is the Time-of-Arrival (ToA) of the backward signal, both measured at node i, and δ_i^k, δ_j^k represent the ranging bias of the nodes i and j defined in the distance domain. From (5), the measured range between node $i \in \mathbb{N}$ and its neighbor $j \in \mathbb{N}$ can be written as

$$r_{i,j}^k = \left\|\mathbf{p}_j^k - \mathbf{p}_i^k\right\| - \delta_i^k - \delta_j^k + \epsilon_{i,j}^k, \quad (6)$$

where $\epsilon_{i,j}^k$ is zero-mean white Gaussian measurement noise

$$\epsilon_{i,j}^k \sim \mathcal{N}(0, \sigma_{r_{i,j}^k}^2), \quad (7)$$

with $\sigma_{r_{i,j}^k}$ representing the range error standard deviation between the two nodes i and j for snapshot k.

The measurement model for all pairwise observations, under the assumption that agents are capable of performing ranging between each other (cooperation) and with anchors, can be expressed as a vector

$$\mathbf{z}^k = h(\mathbf{x}^k) + \mathbf{v}_z^k \quad (8)$$

of dimension $\mathbf{z}^k \in \mathbb{R}^{|\mathbb{M}|(|\mathbb{M}|-1)+|\mathbb{A}||\mathbb{M}|}$. Notice that, the measurement vector (8) depends on the state vector (4) through the range measurements (6). The observation noise in (8) is zero mean white Gaussian noise

$$\mathbf{v}_z^k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}^k), \quad (9)$$

with covariance matrix

$$\mathbf{R}^k = \text{diag}\{\dots, \sigma_{r_{i,j}^k}^2, \dots\}, \quad (10)$$

where diag$\{\cdot\}$ creates a diagonal matrix of its elements.

To compute the FIM in the next section, we require the Jacobian matrix $\mathbf{H}^k$ representing the linearization of the observation model about the state vector (4). Let $l(i,j)$ be a function returning the index of node j from which node i measures the range. Then, $\mathbf{H}^k$ is composed of blocks $\mathbf{H}^k_{l(i,j),i}$ corresponding to each measurement.

The Jacobian matrix can be split into two parts based on the link types. For agent-to-anchor links ($i \in \mathbb{M}, j \in \mathbb{A}$), the elements of the Jacobian are:

$$\mathbf{H}^k_{i,l(i,j)} = \Big[\dots (\boldsymbol{u}^k_{i,j})^\top \dots | \text{-}1 \dots \text{-}1 \dots \Big], \tag{11}$$

where the unit vector $\boldsymbol{u}^k_{i,j}$ is defined in (13), placed in the corresponding position variables of the state vector, and the entries -1 mark the bias parameters of the participating nodes. All remaining entries in the row are zero.

For agent-to-agent links $(i, j \in \mathbb{M}, i \neq j)$

$$\mathbf{H}^k_{i,l(i,j)} = \Big[\dots (\boldsymbol{u}^k_{i,j})^\top \dots \text{-}(\boldsymbol{u}^k_{i,j})^\top \dots | \text{-}1 \dots \text{-}1 \dots \Big]. \tag{12}$$

The unit vector is given by:

$$\boldsymbol{u}^k_{i,j} = \frac{p^k_j - p^k_i}{\|p^k_j - p^k_i\|}. \tag{13}$$

By combining (11) and (12), we obtain the complete Jacobian for cooperative navigation (see (22) for an illustrative example). In the non-cooperative case (agents only measure range to anchors), only (11) is used.

3. OBSERVABILITY ANALYSIS

Given the above description of SLAC for cooperative radio navigation, the question to ask is whether all unknown variables in this particular SLAC problem setup are identifiable, i.e., whether the information contained in the noise-corrupted measurements is sufficient to estimate the locations and ranging bias of all nodes. This question can be addressed by analyzing the notion of parameter observability [9].

When SLAC is formulated as a nonlinear least-squares parameter estimation problem, the observability of the problem is equivalent to the non-singularity of the FIM [6]. If the FIM is singular, then the parameter estimation problem is not observable. Moreover, when the FIM is non-singular but has very small eigenvalues, the parameters are observable but the observability is "marginal" or "poor".

Fisher Information Matrix and CRB

In order to obtain the FIM for the SLAC framework, we use the model specified in (6) and (8). For non-random vector parameter estimation, the FIM of an unbiased estimator is defined as

$$\textbf{FIM} \triangleq E\left\{ \left[\nabla_{\mathbf{x}} ln\Lambda(\mathbf{x})\right] \left[\nabla_{\mathbf{x}} ln\Lambda(\mathbf{x})\right]^\top \right\}, \tag{14}$$

where $\Lambda(\mathbf{x}) \triangleq p(\mathbf{z}|\mathbf{x})$ is the likelihood function [9]. Notice that, the partial derivatives should be calculated at the true value of $\mathbf{x}$. By following the arguments as those in [10], the FIM can be formulated as

$$\textbf{FIM} = \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}, \tag{15}$$

where $\mathbf{H}$ is the Jacobian (11), (12), and $\mathbf{R}$ is the covariance matrix (10) of the measurement model.

To evaluate the variance of SLAC under different observable node configurations, we calculate the CRB, which establishes a fundamental lower bound on the variance of any unbiased estimator. For a system with a full-rank FIM, the CRB is defined as

$$\text{Var}(\hat{\mathbf{x}}) \geqslant \textbf{FIM}^{-1} = \textbf{CRB}. \tag{16}$$

The diagonal elements of the CRB matrix represent the minimum achievable variance for each state, highlighting the precision limits of the estimation process. In cases where the FIM is rank-deficient, its inverse cannot be computed, preventing the calculation of the CRB.

Observability Analysis

Examining the structure of $\mathbf{R}$ in (15), it is evident that $\mathbf{R}^{-1}$ is symmetric and positive definite. Therefore, by exploiting the factorization of $\mathbf{R}^{-1}$ through Cholesky decomposition, it can be shown that the FIM has the same rank as the Jacobian matrix, i.e.,

$$\text{rank}(\textbf{FIM}) = \text{rank}(\mathbf{H}), \tag{17}$$

see [10] for details.

The key question is to determine the conditions under which the FIM (15) is non-singular, or equivalent, the Jacobian matrix $\mathbf{H}$ is full column rank. We exploit the structure of the Jacobian $\mathbf{H}$ (22) arising from the SLAC formulation, where the unknown parameters include the agent positions $[\mathbf{x}_i, \mathbf{y}_i]^\top$ and the ranging biases $[\delta_i]$ of both agents and anchors. Based on this structure, we derive necessary and sufficient conditions for $\mathbf{H}$ to be full column rank. Furthermore, we identify particular cases in which the FIM is necessarily rank-deficient, making the system unobservable.

Necessary Condition

The Jacobian matrix has dimension $\mathbf{H} \in \mathbb{R}^{d_1 \times d_2}$, where d_1 is the total number of measurements in *cooperative* navigation and d_2 is the state dimension (4). Note that, the pair of range measures from agents $i \to j$ and $i \leftarrow j$ corresponds to two noisy observations of the same quantity, so it does not provide additional directions of observability, since their dependence structure with respect to the states $\mathbf{x}$ remains identical. Then, the dimensions are

$$d_1 = \frac{|\mathbb{M}|(|\mathbb{M}|-1)}{2} + |\mathbb{M}| \times |\mathbb{A}|, \;\; d_2 = 3|\mathbb{M}| + |\mathbb{A}|, \tag{18}$$

where $|\mathbb{M}|$ and $|\mathbb{A}|$ are the number of agents and anchors respectively. From equation (17), *a necessary and sufficient condition* for **FIM** to be non-singular is that $\mathbf{H}$ must have full column rank. A *necessary condition* for $\mathbf{H}$ to be full column rank is that

$$d_1 \geqslant d_2 \Leftrightarrow |\mathbb{A}| \geqslant \frac{3|\mathbb{M}|}{|\mathbb{M}|-1} - \frac{|\mathbb{M}|}{2}. \tag{19}$$

While the derived relationship between $|\mathbb{M}|$ and $|\mathbb{A}|$ indicates that increasing the number of agents $|\mathbb{M}|$ can reduce the anchor $|\mathbb{A}|$ requirements, this is still bounded by the fundamental need for at least three anchors with known positions to uniquely determine the agents locations in a 2D space [15]. Hence, any feasible solution must satisfy $|\mathbb{A}| \geqslant 3$ to guarantee global observability, regardless of the number of mobile agents. In particular, under a minimum configuration with three anchors, condition (19) shows that at least three cooperating mobile agents are required to achieve SLAC observability.

In the case of *non-cooperative* navigation, i.e. the agents do not measure range between them, the Jacobian matrix $\mathbf{H} \in \mathbb{R}^{d_1 \times d_2}$ is of dimension

$$d_1 = |\mathbb{M}| \times |\mathbb{A}|, \;\; d_2 = 3|\mathbb{M}| + |\mathbb{A}|. \tag{20}$$

Then, the *necessary condition* for FIM to be non-singular is that

$$|\mathbb{A}| \geqslant \frac{3|\mathbb{M}|}{|\mathbb{M}|-1} \quad \text{or} \quad |\mathbb{M}| \geqslant \frac{|\mathbb{A}|}{|\mathbb{A}|-3}. \tag{21}$$

In cases where the necessary condition is satisfied but the FIM remains rank-deficient, it is valuable to investigate why the Jacobian $\mathbf{H}$ is not full column rank. This analysis helps to identify the conditions needed to render the system observable and leads to a necessary and sufficient condition. In the following, we explore the observability of SLAC scenarios based on this conclusion.

Sufficient Condition

The Jacobian matrix of SLAC with cooperative navigation, consistent with the state distribution given in (4), can be written as follows:

$$\left[\begin{array}{ccccc:ccccc:c}
\mathbf{L}_1 & \mathbf{0}_{a\times2} & \dots & \mathbf{0}_{a\times2} & \mathbf{0}_{a\times2} & -\mathbf{1}_a & \mathbf{0}_a & \dots & \mathbf{0}_a & \mathbf{0}_a & -\mathbf{I}_a \\
\mathbf{0}_{a\times2} & \mathbf{L}_2 & \dots & \mathbf{0}_{a\times2} & \mathbf{0}_{a\times2} & \mathbf{0}_a & -\mathbf{1}_a & \dots & \mathbf{0}_a & \mathbf{0}_a & -\mathbf{I}_a \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\mathbf{0}_{a\times2} & \mathbf{0}_{a\times2} & \dots & \mathbf{L}_{i\text{-}1} & \mathbf{0}_a & \mathbf{0}_a & \mathbf{0}_a & \dots & -\mathbf{1}_a & \mathbf{0}_a & -\mathbf{I}_a \\
\mathbf{0}_{a\times2} & \mathbf{0}_{a\times2} & \dots & \mathbf{0}_{a\times2} & \mathbf{L}_i & \mathbf{0}_a & \mathbf{0}_a & \dots & \mathbf{0}_a & -\mathbf{1}_a & -\mathbf{I}_a \\
\hdashline
\mathbf{T}_1 & [& \mathbf{C}_1 & &] & -\mathbf{1}_{m\text{-}1} & [& -\mathbf{I}_{m\text{-}1} & &] & \mathbf{0}_{m\text{-}1} \\
\mathbf{0}_{m\text{-}2} & \mathbf{T}_2 & [& \mathbf{C}_2 &] & \mathbf{0}_{m\text{-}2} & -\mathbf{1}_{m\text{-}2} & [& -\mathbf{I}_{m\text{-}2} &] & \mathbf{0}_{m\text{-}2} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\mathbf{0} & \mathbf{0} & \dots & \mathbf{T}_i & \mathbf{C}_i & \mathbf{0} & \mathbf{0} & \dots & -\mathbf{1} & -\mathbf{1} & \mathbf{0}
\end{array}\right] \tag{22}$$

where $\mathbf{0}_{n\times m}$ denotes the $n \times m$ zero matrix, $\mathbf{0}_n$, $\mathbf{1}_n$ are the zero and unit vector of length n, and $\mathbf{I}_n$ is the $n \times n$ identity matrix. For simplicity, let us consider the number of agents equal to m and anchors equal to a. The explicit expressions of $\mathbf{L}_i$, $\mathbf{T}_i$, and $\mathbf{C}_i$ are given in eqs. (23) to (25).

The Jacobian matrix (22) can be divided in two main blocks, separated by the horizontal dashed line. The upper block corresponds to range measurements of agent-to-anchor links (11), where each $\mathbf{L}_i$ $(i = 1, \dots, m)$ collects the contributions of agent i with respect to all anchors.

$$\mathbf{L}_i = \begin{bmatrix} (\boldsymbol{u}_{i,1}^k)^\top \\ (\boldsymbol{u}_{i,2}^k)^\top \\ \vdots \\ (\boldsymbol{u}_{i,j}^k)^\top \end{bmatrix} = \begin{bmatrix} \frac{x_1^k - x_i^k}{d_{i,1}^k}, \frac{y_1^k - y_i^k}{d_{i,1}^k} \\ \frac{x_2^k - x_i^k}{d_{i,2}^k}, \frac{y_2^k - y_i^k}{d_{i,2}^k} \\ \vdots \\ \frac{x_j^k - x_i^k}{d_{i,j}^k}, \frac{y_j^k - y_i^k}{d_{i,j}^k} \end{bmatrix}_{j\in\mathbb{A}}, \tag{23}$$

where $\boldsymbol{u}_{i,j}^k$ is defined in (13) and $d_{i,j}^k$ is the Euclidean distance between nodes i and j. The remaining entries of the upper block correspond to the bias parameters of the participating nodes.

The lower block represent the cooperative case, where the range measurements are provided by agent-to-agent links (12). The terms $\mathbf{T}_i$ $(i = 1, \dots, m-1)$ and $\mathbf{C}_i$ $(i = 2, \dots, m)$ are defined as

$$\mathbf{T}_i = \begin{bmatrix} (\boldsymbol{u}_{i,1}^k)^\top \\ (\boldsymbol{u}_{i,2}^k)^\top \\ \vdots \\ (\boldsymbol{u}_{i,j}^k)^\top \end{bmatrix}_{j\in\mathbb{M},\, i<j}, \tag{24}$$

$$\mathbf{C}_i = \operatorname{diag}\{(\boldsymbol{u}_{i,1}^k)^\top, \dots, (\boldsymbol{u}_{i,j}^k)^\top\}_{j\in\mathbb{M},\, i<j}. \tag{25}$$

The Jacobian matrix (22) is of full column rank if and only if the linear system $\mathbf{Hq} = \mathbf{0}$ admits only the trivial solution $\mathbf{q} = \mathbf{0}$. In other words, full column rank is equivalent to injectivity, and the null space $\mathbf{q}$ of $\mathbf{H}$ directly characterizes the unobservable directions of the system.

In the SLAC setting, we assume fixed anchors with known positions. This configuration eliminates the unobservable global translation and rotation modes typical of SLAM [16]. Nevertheless, by analyzing the upper block of the Jacobian matrix (22), *non-cooperative case*, we find the following non-trivial subspace of the null space

$$\mathbf{q}_1 = \begin{bmatrix} \mathbf{0}_m \\ \mathbf{1}_m \\ \textbf{-1}_a \end{bmatrix} \quad \mathit{or} \quad \begin{bmatrix} \mathbf{0}_m \\ \textbf{-1}_m \\ \mathbf{1}_a \end{bmatrix}. \tag{26}$$

This corresponds to a bias-shifting ambiguity: shift all agent biases by a constant (e.g., +1) while decreasing all anchor biases by the same constant (e.g., –1), or vice versa, yields identical measurements. Hence, in the *non-cooperative case*, SLAC becomes unobservable. By contrast, in the *cooperative case*, the additional constraints introduced by inter-agent measurements remove this ambiguity, and the full Jacobian matrix (22) no longer exhibits this unobservable mode.

However, another well-known condition leading to unobservability in localization, for both cooperative and non-cooperative navigation, is the collinearity between agents and anchors [9]. Let's consider the following sub space of the null space

$$\mathbf{q}_2 = [1, -k, 1, -k, \dots, 1, -k \mid \mathbf{0}_m, \mathbf{0}_a]^\top, \tag{27}$$

where k is a real number. Multiplying this vector by the Jacobian matrix (22) yields the set of equations

$$\begin{bmatrix} x_1^k - x_2^k = k(y_1^k - y_2^k) \\ x_1^k - x_3^k = k(y_1^k - y_3^k) \\ \vdots \\ x_i^k - x_j^k = k(y_i^k - y_j^k) \end{bmatrix}, \tag{28}$$

with $i \in \mathbb{M}, j \in \mathbb{M} \cup \mathbb{A}, i < j$ if $j \in \mathbb{M}$. This condition is satisfied if and only if the constant k corresponds to the slope of the line on which all agents and anchors lie, i.e., a collinear configuration.

In the *non-cooperative* case, it is sufficient for a single agent to be collinear with all anchors for the Jacobian to lose rank. In contrast, in *cooperative navigation,* all agents and anchors must be collinear simultaneously, a condition that is much harder to satisfy in practice. Consequently, cooperative navigation is inherently more robust against rank deficiency in the Jacobian, thus reducing the occurrence of unobservable situations.

We can therefore conclude that a *necessary and sufficient condition* for SLAC with *cooperative navigation* to be observable is that the number of anchors and agents satisfies (19), and that their positions are not collinear.

In contrast, the *non-cooperative* case is more challenging due to the additional unobservable direction (26) in the Jacobian matrix. To overcome this issue, we propose a practical requirement: at least one node must be equipped with reliable self-calibration capabilities, enabling the estimation of calibration parameters for the remaining nodes. Under this condition, the *necessary and sufficient condition* for SLAC with *non-cooperative* navigation becomes: the number of anchors and agents must satisfy (21), at least one node must be calibrated, and anchors and agents must not be collinear.

To assess the degree of observability of the SLAC framework, we evaluate the eigenvalues of the FIM (15). In particular, the smallest eigenvalue corresponds to the least observable direction of the system and thus provides a quantitative measure of observability, indicating when the system approaches unobservability. However, the raw eigenvalues of the FIM may span inappropriate range of values and lack dimensional homogeneity. Following the approach in [17], we therefore apply a normalization that renders the parameters dimensionless and bounds the eigenvalue spectrum, preventing it from being dominated by parameter units.

Let the state vector be $\mathbf{x} \in \mathbb{R}^{\mathbb{N}}$ (4), and define the scaled, dimensionless parameters

$$\phi = \mathbf{T}^{-1}\mathbf{x}, \qquad \mathbf{T} = \mathrm{diag}\{t_1, t_2, \dots, t_{|\mathbb{N}|}\},\ t_i > 0, \tag{29}$$

where t_i are user-chosen scaling factors for the parameters. The corresponding rescaled Jacobian is

$$\mathbf{H}_\phi = \mathbf{H}\mathbf{T}, \tag{30}$$

which yields the dimensionless FIM

$$\mathbf{FIM}_\phi = \mathbf{H}_\phi^\top \mathbf{R}^{-1}\mathbf{H}_\phi = \mathbf{T}^\top \mathbf{FIM}\,\mathbf{T}. \tag{31}$$

4. Simulation Results

First, we evaluate the observability of the SLAC problem through simulation. The objective is to test how well the parameters, particularly the ranging biases, can be estimated in configurations where the FIM satisfies the necessary conditions eqs. (19) and (21) for cooperative and non-cooperative navigation.

Scenarios with three, four, five, and six anchors are analyzed, arranged in fixed geometric patterns. For each anchor configuration, a growing number of agents, between 2 and 50, are placed at random distributed positions within a circular area of radius $50\,\mathrm{m}$ performing ranging, see Fig. 1. For each resulting setup, the CRB is computed to determine the minimum achievable variance under the given configuration. A ranging error of $\sigma_{r^k_{i,j}} = 0.25\,\mathrm{m}$ is assumed. Moreover, in the non-cooperative case, we assume that one anchor is pre-calibrated (its bias removed from the parameter vector) to ensure observability. To obtain statistically consistent results, we perform Monte Carlo simulations with 100 independent trials for each configuration.

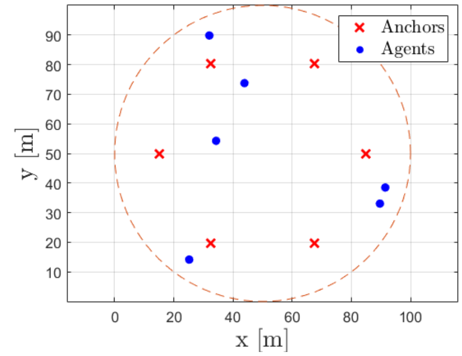


Figure 1: Example simulation scenario with six anchors and six randomly positioned agents.

Fig. 2 shows the average ranging-bias CRB across all nodes in the network for both cooperative and non-cooperative scenarios. The results confirm the validity of the necessary conditions, since the CRB can only be computed when (19) and (21) are satisfied. For instance, in cooperative navigation with three or four anchors, observability is achieved only when at least three agents are present, whereas with five anchors it is possible from two agents. In contrast, the non-cooperative case requires at least four anchors and three agents to achieve observability.

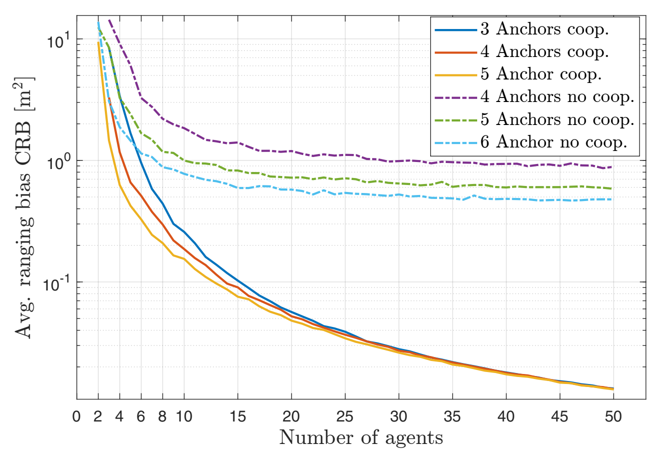


Figure 2: Average ranging bias CRB of all nodes in the network for cooperation and non-cooperative navigation, across various anchor configuration.

Fig. 2 clearly illustrates the advantage of cooperative navigation, even in large networks, the ranging-bias CRB remains significantly higher in the non-cooperative case. For small networks, the large CRB indicates that the states are poorly observable, an effect that can be mitigated once the agents move and additional measurements are collected over time.

Next, we evaluated the sufficient condition for observability in cooperative navigation, which is equivalent to the non-cooperative case. Fig. 3 shows a setup with three anchors placed in a collinear configuration and three moving agents. Although this arrangement satisfies the necessary condition (19), the agents follow trajectories that progressively bring them into collinearity with the anchors at different snapshots. For this analysis, we applied a normalization of the FIM using the scaling factors $[1\,\mathrm{m}, 1\,\mathrm{m}, 0.1\,\mathrm{m}]$.

Fig. 4 shows the rank deficiency of the FIM, the evolution of its minimum eigenvalue along the agents' trajectories, and

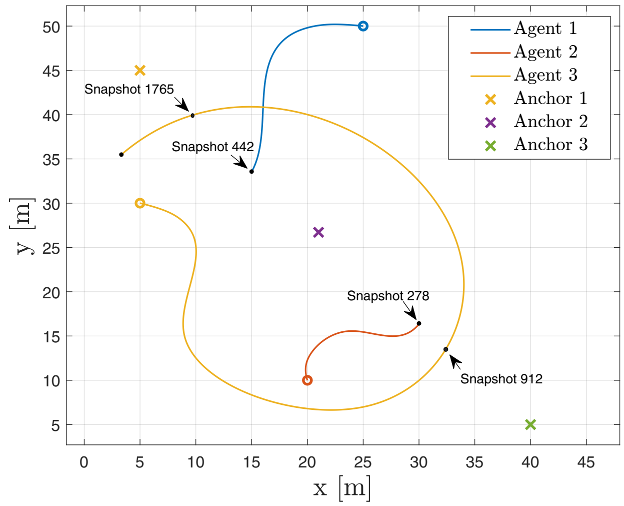


Figure 3: Simulation scenario: collinear anchors and agents approaching collinearity.

the position standard deviation of the agents, computed as $\sigma_i = \sqrt{\text{CRB}_{x_i} + \text{CRB}_{y_i}}$. At the beginning of the simulation, the minimum eigenvalue increases as the agents move, indicating an improvement in system observability. However, as agents 1 and 2 reach a collinear configuration with the anchors, observability deteriorates. Although the third agent initially helps maintain observability, the minimum eigenvalue ultimately approaches zero once all nodes become collinear and the FIM loses rank. This clearly demonstrates how geometric configurations critically reduce the system's observability.

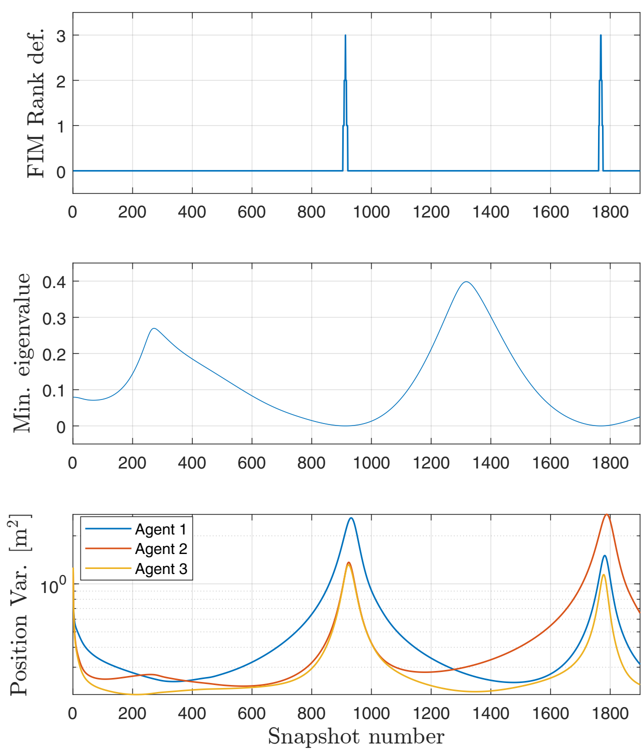
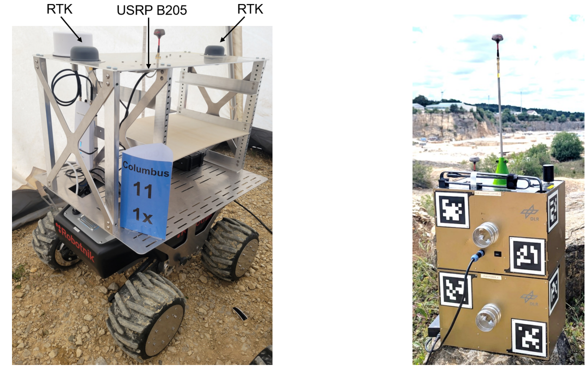


Figure 4: FIM rank deficiency, evolution of the minimum eigenvalue and position minimum variance.



(a) Agent Columbus. (b) Anchor node.

Figure 5: Pictures of an anchor node and agent Columbus, a robotic rover.

5. Measurements Results

The observability analysis for the SLAC framework is motivated by practical considerations. An important contribution of this paper is the evaluation of observability with real measurement data. To this end, two experiments were designed: the first evaluates observability in a setup with a sufficient number of anchors and agents, following trajectories that avoid collinear configurations; the second investigates a scenario where one agent crosses a collinear configuration.

In the first experiment, the measurement setup consists of four anchor nodes named *RadioBox1*, *RadioBox4*, *Lander* and *Collumbus*, and three robotic rovers, named *daGama*, *Drake*, and *Magellan*. Both anchors and agents are equipped with dipole antennas, as shown in Fig. 5. Each rover carries a commercial dual-antenna real-time kinematic (RTK) receiver, which internally fuses inertial sensors and provides ground-truth positioning.

The physical layer signaling is based on the in-house developed DLR Swarm Communication and Navigation system [18]. It operates at a carrier frequency of 5.7 GHz, with a sampling rate of 25 MHz, transmit power of 0 dBm, and employs orthogonal frequency-division multiplexing (OFDM) with an FFT length of 1024, resulting in an occupied bandwidth of approx. 90% of 25 MHz. The system is implemented as an Software Defined Radio (SDR) using Ettus Research USRP B205mini devices with single-port antennas installed on each node.

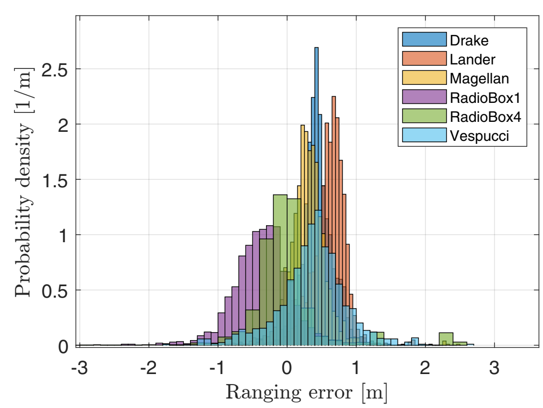


Figure 6: Ranging error calculated from rover daGama to all other nodes.

To enable accurate ranging, pre-calibration of the group delays in the radio transceivers is carried out in the laboratory before the experiment. After the actual measurement campaign, the ranging errors are compared against the ground truth. The results reveal that pre-calibration is not perfect and a residual ranging bias remains. Fig. 6 shows the histogram of ranging errors from the agent *daGama* to all other nodes. It clearly illustrates the presence of systematic biases in the ranging measurements, which directly motivate the need for SLAC, and therefore, the investigation of when SLAC is feasible through an observability analysis.

The experiment was carried out on an open uneven gravel terrain. Fig. 7 shows the site layout, including the positions of the four anchor nodes and the initial positions and trajectories of the robotic rovers. The total duration of the experiment was 3:46 min. Initially, all nodes remained stationary; after approximately 15 s, the three rovers began moving along predefined trajectories. The ranging variance $\sigma_{r^k_{i,j}}$ was directly computed from the real data, and the eigenvalues were normalized using the same parameters as in the simulation. During the entire experiment, the received signals were recorded and evaluated in post-processing.

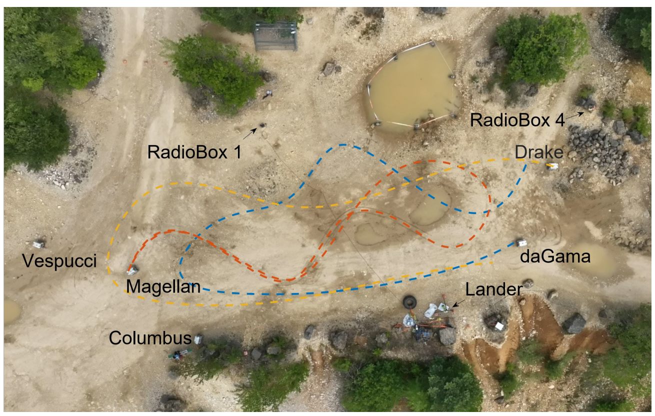


Figure 7: Top view of the measurement scenario showing anchors and agents and their trajectories.

Examining the experimental results in Fig. 8, the favorable configuration of anchors and agents led, as expected, to consistently high values of the minimum eigenvalue, indicating strong observability. The observed reductions of the minimum eigenvalue around 15:06:4 and 15:08:16 can be attributed to the temporary proximity of two moving agents at those times, however the FIM does not lose rank. When agents move too close to each other, the network geometry loses spatial diversity, and the corresponding range measurements become nearly redundant. Consequently, the Fisher Information Matrix provides less independent information about the system states, which directly reduces the minimum eigenvalue. This result highlights how geometric factors, such as agent clustering, can significantly degrade observability, underscoring that not only an adequate anchor configuration but also a careful design of agent trajectories is required. Moreover, since the minimum eigenvalue is directly related to the quality of parameter estimation, these findings provide a foundation for the development of future navigation filters that explicitly account for state observability.

Finally, the second experiment demonstrates how the FIM can lose rank in real-world scenarios. In this setup, three mobile nodes, *daGama*, *Magellan*, and *Vespucci*, and three fixed anchors with known positions, *Drake*, *Columbus*, and *RadioBox4* are considered. All nodes except *daGama* started in a collinear configuration, while *daGama* moved across the area, repeatedly crossing the line of collinearity. Fig. 9 illustrates the site layout, including the anchors and rovers. The total duration of the experiment was 3:21 min. The corresponding results are shown in Fig. 10, where it can be observed that the moving agent helps maintain system observability until it crosses the collinearity line, at which point the minimum eigenvalue drops to zero and the FIM loses rank.

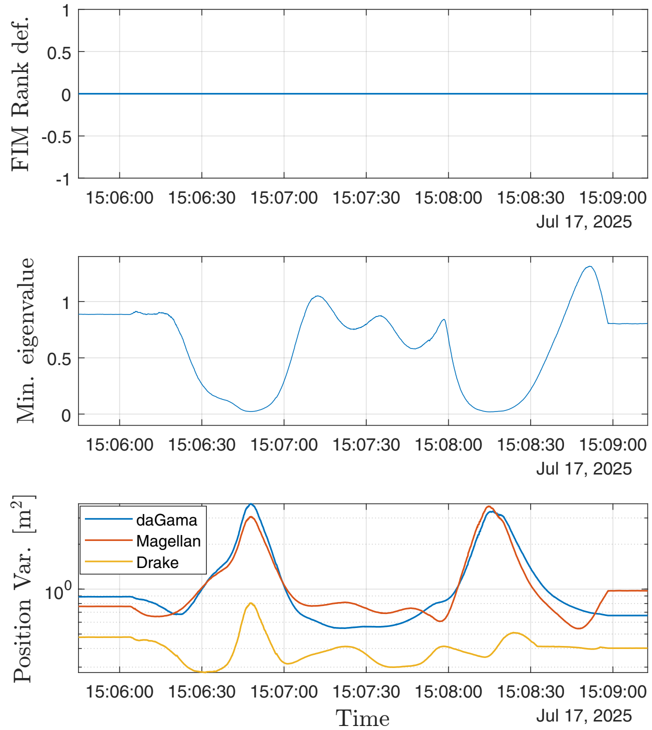


Figure 8: FIM rank deficiency, evolution of the minimum eigenvalue and position minimum variance.

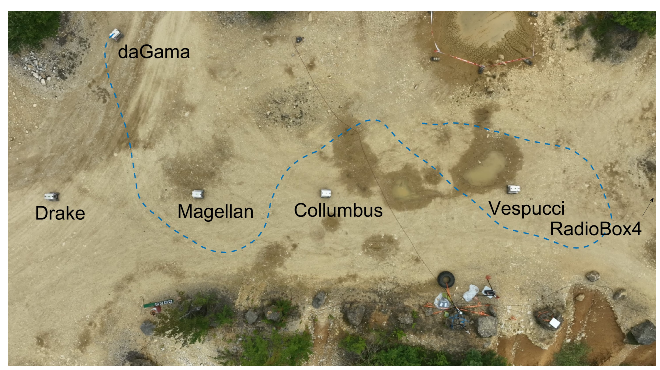


Figure 9: Top view of the measurement scenario showing anchors and agents in collinear configuration.

6. Conclusions

In this work, we carried out an observability analysis (i.e., parameter estimability) of Simultaneous Localization and Calibration (SLAC) for cooperative radio navigation, considering only radio ranging measurements and ranging biases as calibration parameters. Using the Fisher Information Matrix and its equivalence with the Jacobian's full column rank, we derived necessary and sufficient conditions for SLAC observ-

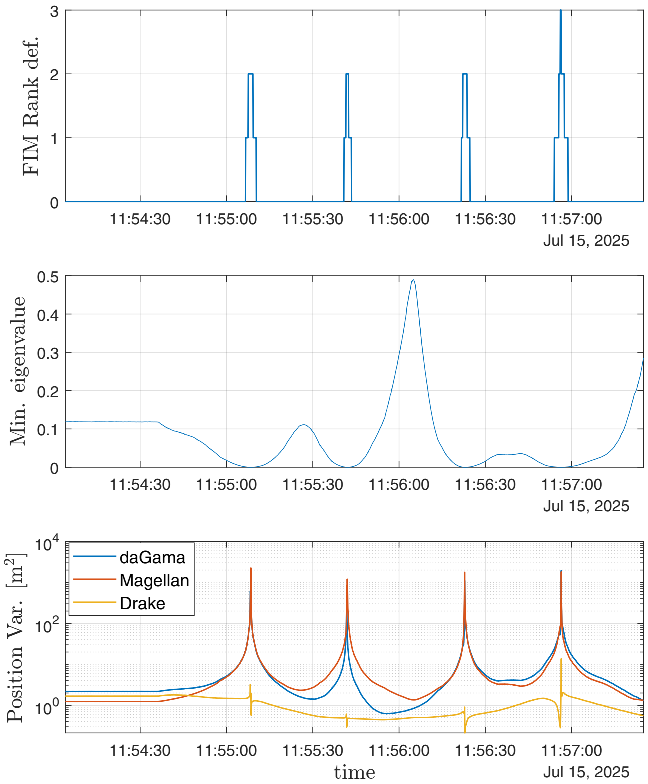


Figure 10: FIM rank deficiency, evolution of the minimum eigenvalue and position minimum variance for a collinear configuration.

ability. Our results establish the minimum number of anchors and agents required and highlight geometric configurations that yield strong, weak, or non-observable systems. These insights provide practical guidelines to increase navigation performance/accuracy given that real world ranging devices suffer from remaining biases to be estimated in-situ. We also showed that, without cooperation, SLAC is unobservable and proposed a practical solution. Theoretical insights were validated through simulations and real experiments using the DLR Swarm Communication and Navigation system. The experiments confirmed that geometric configurations strongly affect observability and demonstrated how the minimum eigenvalue of the FIM can serve as a reliable indicator of parameter estimability. Finally, we note that the analysis was limited to instantaneous measurement snapshots; extending the study to dynamic point of view remains an important future direction. Additionally, future work will focus on designing cooperative radio navigation filters that explicitly account for observability, thereby improving localization accuracy even under challenging geometric conditions.

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BIOGRAPHY

Alexis Marino *received the B.Sc. degree in Electric and Control engineering from the Polytechnic of Chimborazo, Ecuador, in 2021, his M.Sc. degree in Robotic and Automation from the Calabria University, Italy, in 2023. He then joined the Institute of Communications and Navigation, German Aerospace Center (DLR) as a Ph.D. student. His current research interests include control of mobile robots and cooperative navigation.*

Robert Pohlmann *received the B.Sc. and M.Sc. degrees in electrical engineering and information technology from the Technical University of Munich (TUM) in 2014 and 2016, respectively. He then joined the Institute of Communications and Navigation, German Aerospace Center (DLR) as a Research Staff Member. In 2022, he received the Ph.D. with distinction from the Kiel University. He is a recipient of the VDE ITG dissertation award 2023. His current research interests include cooperative and hybrid localization as well as statistical signal processing for multi-antenna systems.*

Emanuel Staudinger *received the M.Sc. degree in Embedded Systems Design from the University of Applied Sciences of Hagenberg, Austria, in 2010. Since 2010, he is with the Institute of Communications and Navigation of the German Aerospace Center (DLR), Wessling, Germany. He received a Ph.D. with distinction from the Institute of Electrodynamics and Microelectronics at the University of Bremen, Germany, in 2015. His current research interests include system design for cooperative positioning, experimental platform design based on SDRs, and experimental validation for swarm navigation.*