Multi-Objective Cost Function for Mid-Course Target Assignment in Air-Defense Cooperative Guidance

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Air-defense systems are challenged by highly-dynamic attacks such as hypersonic vehicles and heterogeneous coordinated swarms. Cooperative guidance enables dynamic reactions of effectors in-flight to increase efficiency and effectiveness of the air-defense and better protect friendly assets. A major building block for cooperation is a continuous collaborative evaluation and adjustment of Weapon Target Assignments (WTAs). This increased autonomy requires informed decision routines. Therefore, this paper proposes a domain-informed multi-objective cost function for cooperative mid-course WTA. Instead of abstract and overly simple cost models, we systematically incorporated mission-relevant domain knowledge into the WTA optimization scheme. The cost structure accounts for effector dynamics with limited propulsion reserves, protected assets risk mitigation, and time-critical intercept feasibility based on predictive forward simulations. The resulting architecture is successfully tested through numerical evaluations across scenarios with both symmetric and asymmetric force compositions. It dynamically adapts to evolving threat scenarios and heterogeneous target sets. Real-time capability is positively assessed for applicability to embedded systems. The results contribute to the development of next-generation, cooperative air-defense architectures.

Nomenclature

δ	weighting factor for cost parameter.
ε	minimum curve radius.
ζ	flight path length.
θ	heading angle.
σ	value of a target (significance).
τ	timestep.
c	scalar cost values for assignments.
d	euclidean distance.
p	position.
q	cost parameter.
r	line of sight.
s	switching cost.
t	time-to-go.
v	velocity.
w	value of an asset (significance).
x_{ij}	binary variable representing the assignment of effector i to target j.
z_{ij}	binary variable representing prohibited assignment of effector i to target j.

I. Introduction

In modern air-defense, attacker strategies tend to use coordinated swarm attacks with heterogeneous agile targets for saturation of air-defense systems [1–3]. Highly-automated support systems for the human operators may help with these increasingly complex scenarios. Cooperative guidance, as described in [4], lets effectors effectively collaborate against

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complex swarms. The cooperation of effectors throughout the flight allows to dynamically react to erratically evolving environments which include:

- hardly predictable target flight paths (pushed further by technical advancements towards faster and more maneuverable [1] missiles),
- the dynamic addition and removal of targets/effectors to the scenario, or
- threat level assessment updates from surveillance (e.g. recognition of target type, depleted ammunition, etc.).

Ultimately, the partial-autonomy achieved through cooperative guidance promises to increase effectiveness for the air-defense while additionally saving on costly resources. Cooperation is enabled by increasing technical capabilities in terms of computation and communication with developments towards network-centric warfare [1, 5, 6]. Accordingly, the topic of cooperative guidance recently gained attention in research [7].

Cooperative guidance includes both a continuous collaborative Weapon Target Assignment (WTA) during the mid-course phase as well as cooperative trajectories, particularly in case of numerical superiority against the threat. Both aspects are interconnected and pose a continuous-time mixed-integer optimal control problem. This combined problem is complex to solve [8, 9] and so far not expected to be feasible for online solution, particularly on embedded systems in a distributed approach. In most WTA literature (including the references in this work), the two aspects are separated. Effectors are allocated in a WTA optimization and trajectories are subsequently adjusted. Yet, both aspects are coupled by considering flight dynamics and trajectory guidance laws in the WTA optimization's cost function. This paper focuses on the WTA optimization as a major building block of a cooperative guidance.

While extensive research exists on WTA formulations and fast solvers [10–12], publications on the continuous decentralized application of WTA for cooperative missile guidance are scarce. Literature remains abstract and largely detached from flight-mechanical or operational constraints as outlined in the following paragraph. There remains a significant gap in integrating rich domain-knowledge into the problem formulation itself - particularly in shaping cost functions that reflect realistic engagement dynamics in air-defense. Leveraging such knowledge can significantly reduce computational effort and is critical for ensuring real-time applicability in embedded systems while still achieving the high-level objectives. As stated in [13], most WTA literature considers only one objective, either the asset survivability or the hit-kill probability of effectors on targets. The single-objective approaches often oversimplify the complexities of air-defense scenarios and may not account for relevant factors such as asset survivability, relative significance/value of assets and targets or operational constraints.

For instance, an approach for dynamic and continuous WTA in air-defense is presented in [14]. For the WTA cost function, they introduced the Earliest Intercept Geometry (EIG) concept, which is presented and utilized later in this work as well. However, in the referenced work, flight dynamics other than the current velocity vectors for EIGs, or operational knowledge, are not considered. In [15] decentralized, continuous WTA is investigated with unmanned underwater vehicles but the constraints differ from the flight-mechanical and operational constraints with missiles in air-defense. Underwater, much worse communication capabilities as well as lower speeds are assumed and the focus does not lie on informed cost functions. The same holds for a similar approach in [16] for Unmanned Aerial Vehicle (UAV) cooperation. Dynamic, continuous WTA is also applied for multi-agent attacks in [17]. In their attack scenarios, the enemy is outnumbered while for each effector-target combination the kill probability is limited. Hence, a combination of objectives for sequenced arrival and of achieving the desired probability of kill by allocating enough weapons to one target, is employed. Another example with UAV is presented in [18], where operational knowledge (battery and payload status) are added to the pure consideration of positions in the cost function. Several parameters are weighted and their weighting factors are tuned by regression. In [13], a dynamic real-time WTA approach for the assignment of missiles in air-defense with multiple objectives is provided. The multiple objectives are handled with evolutionary game theory. Nevertheless, the utilized cost function is less complex and comprises limited domain-knowledge. The approach is not used for cooperation with reassignments during the flight. Therefore, the same author published an approach for in-flight WTA in missile air-defense [19], solely using the previously mentioned EIG for the assignments.

Since cost function design bounds the decision quality for any optimization algorithm, the assignment optimization is assumed to be predominantly driven by a sound exploitation of flight mechanical and operational knowledge as previously stated in [4]. Therefore, the focus and main contribution of this work is the thorough analysis of the mission to develop a cost function, that represents the problem in a best possible way. Building on the insights from the analysed literature as well as the conceptual cooperative guidance framework introduced in [4], this work comprises a detailed, domain-informed multi-objective WTA cost function design to address the challenges and limitations identified above. Flight-mechanical and operational insights (domain-knowledge) are systematically incorporated, ultimately enabling more efficient and informed automated decision-making. The goal is to enable dynamic evaluation and optimization of assignments in real-time to maximize intercept success probabilities. Our work builds on the concept of WTA, extending it for continuous mid-course cooperation under realistic operational and flight-mechanical constraints. In the following, the WTA problem is mathematically described, the requirements for continuous WTA in cooperative guidance outlined and the detailed cost function design presented. The paper then presents exemplary simulation results to show the effect of different cost factors for successful cooperation and finishes with a conclusion.

II. The Weapon Target Assignment Problem

In the following, the WTA problem is introduced and approximated as a linear assignment problem, for which we then state how we solved it. For this paper, targets are defined as the hostile incoming threats, effectors are the missiles (interceptors) of the air-defense system and assets are objects that the defender wants to protect (e.g. surveillance radar, command centre, critical infrastructure).

A. Linear Assignment Problem Approximation

The WTA problem can be approximated as a linear assignment problem. The best combination of all possible effector/target assignments is evaluated by a combinatorial optimization. This was described in [4] and is repeated and slightly extended here for completeness. To solve the linear assignment problem mathematically, scalar cost values c_{ij} for the assignment of each effector $i \in [\![1, N]\!]$ to each target $j \in [\![1, M]\!]$ are assumed to be available. They are schematically shown in Figure 1 and will be constructed in section IV. The sum of these costs ought to be minimized by

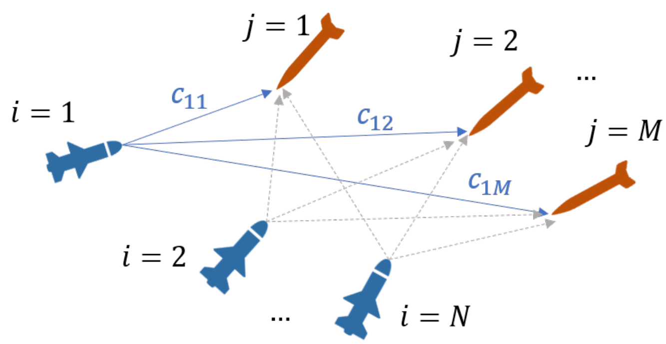

Fig. 1 Assignment Costs Illustration [4].

selecting an optimal assignment which is described by

$$\min_{x_{ij,\tau}} \left(\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij,\tau} + s \left| x_{ij,\tau} - x_{ij,\tau-1} \right| \right) \tag{1}$$

where the binary variables x_{ij} represent the assignments of effectors to targets either at time τ or the previous timestep $\tau - 1$ and a switching cost s is added to penalize changes in the assignments between consecutive time steps to avoid unnecessary reassignments. This optimization shall be subject to the following constraints.

Optimization constraints:

$$\begin{aligned}
&\text{C1 (binary assignment variables):} \quad \forall i \in [\![1, N]\!], \forall j \in [\![1, M]\!], \quad x_{ij} \in \{0, 1\}, \\
&\text{C2 (exactly one target assigned per effector):} \quad \forall i \in [\![1, N]\!], \sum_{j=1}^{M} x_{ij} = 1, \\
&\text{C3 (bounds on effectors per target):} \quad \forall j \in [\![1, M]\!], \quad \left\lfloor \frac{N}{M} \right\rfloor \le \sum_{i=1}^{N} x_{ij} \le \left\lceil \frac{N}{M} \right\rceil \\
&\text{C4 (prohibit infeasible assignments):} \quad \forall i \in [\![1, N]\!], \forall j \in [\![1, M]\!], \quad x_{ij} \le z_{ij}, \quad z_{ij} \in \{0, 1\}.
\end{aligned} \tag{2}$$

The first constraint (C1) enforces that x_{ij} is binary and the second one (C2) ensures that exactly one target is assigned to each effector. The third constraint (C3) limits the number of effectors assigned to any target between the floor and ceiling of the ratio $\frac{N}{M}$ for an even distribution of effectors across targets. Other uneven distributions are imaginable but would require additional distribution logic based on operational insights and are not considered here. In the fourth constraint (C4), the binary variable z_{ij} allows to prohibit assignments between individual effector/target combinations. For instance, assignments of effectors could be prohibited when they cannot reach a target with a required minimum speed or before the target hits its expected destination (i.e. asset).

B. Solving the Weapon Target Assignment Problem

As previously reasoned in [4], the Hungarian-Algorithm [20] (with extension to rectangular cost matrices [21]) is selected to solve the WTA. With the Hungarian-Algorithm, switching costs to penalize undesired high-frequency reassignments can be included in the cost function (cf. Equation 1) in a slightly adapted manner, by adding costs to all previously not-assigned combinations directly in the cost factor c as

$$\hat{c}_{ij} = c_{ij} + s\left(1 - x_{ij,\tau-1}\right). \tag{3}$$

Consequently, Equation 1 is simplified to

$$\min_{x_{ij,\tau}} \left(\sum_{i=1}^{N} \sum_{j=1}^{M} \hat{c}_{ij}\, x_{ij,\tau} \right). \tag{4}$$

Furthermore, for the Hungarian-Algorithm, hard constraints like C4 in Equation 2 must be transferred to soft constraints by setting respective cost function entries to infinity. Infeasibility problems can lead to non-allocated effectors which must be avoided by additional code that relaxes respective hard constraints if necessary. Mixed Integer Linear Programming (MILP) solvers could handle such hard constraints natively but since the Hungarian-Algorithm works equally well with the described workarounds, it is favored for its simplicity (light, comprehensible and efficient code).

III. Cooperative Guidance Problem Formulation

Effectors and targets can dynamically join or leave the scenario. The group of targets can be heterogeneous (e.g. Air To Air Missiles (AAMs), Air To Surface Missiles (ASMs) or different types of aircraft). Cases of numeric equality, minority and superiority of the defending effectors should be considered. The scenarios can include assets that need to be protected. Specific intentions and future manoeuvres of the targets are not known to the defender. Assets and targets can be given different prioritization, expressed by relative numeric values, which shall be evaluated for the assignments. If an effector surpasses a specific spatial proximity to its selected target, the corresponding assignment between the two is frozen to ensure a successful homing phase.

An information exchange between the effectors, as well as an uplink from a basis that provides the effectors with information about targets and assets, is necessary to collaboratively allocate effectors as outlined in [4]. In a first step, the connections are assumed as an ideal, fully connected mesh, depicted in Figure 2. Every effector has the information of all other effectors at all times. The information that each effector sends to the group and receives about the targets

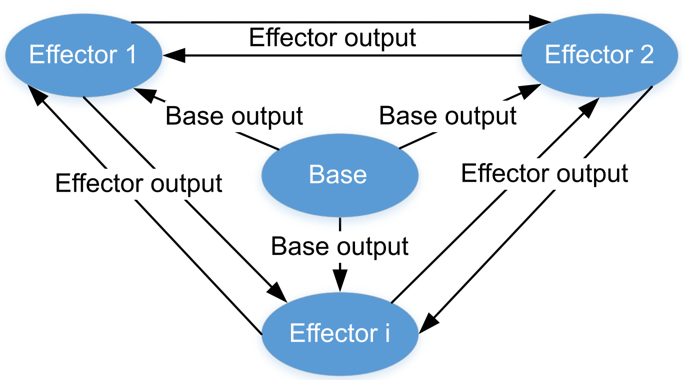

Fig. 2 Ideal Communication Scheme [4].

and assets from the base may include static (e.g. ID numbers, target/asset type) and dynamic data (e.g. position and velocity vectors as well as different statuses) in the information-consensus based cooperation approach (cf. [4]).

IV. Domain-Informed Cost Function Design

For the cost function design, we first analyze the geometry and dynamics between effectors, targets and assets. Then, we explain the desired high-level objectives and how they are represented by several cost parameters that are constructed with the flight-mechanical insights/parameters. Ultimately, the cost parameters are combined into the final cost function c for solution of Equation 4.

A. Geometry of the Air-Defense Scenario and Forward Simulation

Theoretical intercept points $\mathbf{p}_{IP_{ij}}$ between effectors i and targets j can be obtained by linear extrapolation of their current flight directions, represented by their velocity vectors $\mathbf{v}_{E_i}$ and $\mathbf{v}_{T_j}$. If the heading angles θ_{T_j} and θ_{E_i} with respect to the Line of Sight (LOS) $\mathbf{r}_{ij}$ are rotated, a locus of possible intercept points can be created which is the concept of the EIG by [22]. While this concept does not take into account the limited maneuverabilities, it is a simple and useful concept for obtaining a gross estimation of an intercept area and complimentary safe areas. Assets can be defended by keeping and moving the EIGs away from them (by adjusting the effectors heading and position). The concept combines the positions and thus the distances as well as the current flight speed of both effector and target. The approach is displayed for two dimensions in Figure 3. In three dimensions the circle becomes a sphere. Note that the position of the centre of the EIG does not coincide with the target position $\mathbf{p}_{T_j}$, but lies behind it along the LOS. For a specific effector/target pair's EIG, shortest distances d_{ijk} of any asset k to the EIG can be calculated. In Figure 3, d_{ij1} is shortest and asset $k = 1$ is thus considered to be the most critical asset.

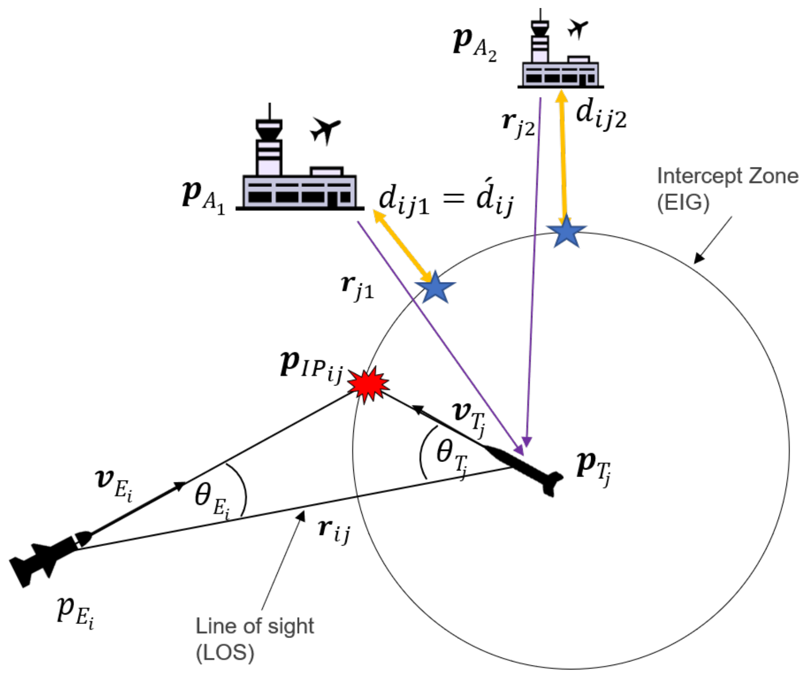

Fig. 3 Earliest Intercept Geometry (EIG) with Assets

While the EIG serves as a useful concept to consider assets with conservative assumptions on target maneuverability, more comprehensive insights are obtainable for the ability of effectors to intercept targets. Particularly the effector dynamics and guidance laws should be reasonably well known to analyze the attainability of different targets.

To predict the time-to-go for each effector to intercept any target, forward simulations are conducted with the entire effector flight dynamics model plus guidance. The target is assumed to continue to fly with its current heading. If available, other assumptions on the target flight paths could be considered as well. The linear extrapolation of the target flight path is depicted by the grey curve in Figure 4. The time-to-go t_{ij} for interception of the target is obtained with the flight path length ζ_{ij} (yellow line) and the effector velocity v_{E_i}. Forward simulations are performed for every effector/target combination. To reduce computation time, the time-step size of the forward simulation is varied throughout the simulations depending on the current accelerations and turn rates while enforcing fine resolutions when close to interception.

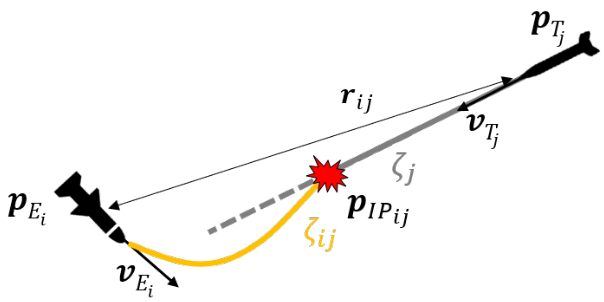

Fig. 4 Intercept Path Prediction

B. Optimization Objectives

As [13] outlines, both static and dynamic WTA optimization objective is usually either based on minimization of target survivability or maximization of asset survivability. However, for the increased autonomy with continuous mid-flight WTA pursued in this work, a multi-objective approach is desired. The following set of high-level optimization objectives were established in [4]:

1) maximization of asset-survivability (if applicable),
2) minimization of general threat level imposed by targets and
3) maximization of effector speed.

These objectives ought to be cast into cost parameters for any assignment of effectors to targets, such that the WTA problem can be optimized by selecting a combination of assignments which minimize the combined costs. The construction of cost factors to achieve these high-level objectives by exploitation of profound domain-knowledge, is outlined in the following.

The idea of *objective 1* is to keep the targets away from the assets. More specifically, it shall lead to allocations of effectors to targets, with the aim of maximizing distances of target-intercept points to the assets. The objective should take the hazard potential of the targets into account. Metrics for the evaluation of the objective include the euclidean distances of intercept geometries to the assets (Figure 3), the heading angle between targets and assets, values of the assets plus the hazard potential of the targets on the assets (e.g. evaluate preferred destinations of targets like AAM or ASM, if applicable). This yields the first cost parameter

$$q_{1_{ij}} = -\hat{d}_{ij,\hat{k}} \tag{5}$$

with the distance $\hat{d}_{ij}$ between the most critical asset to the EIG for the combination of effector i and target j. The most critical asset $\hat{k}$ is obtained by finding the asset that is closest to the EIG, scaled with value of the asset $w_k \in (0,1]$, by

$$\hat{k} = \arg\min_k \left(\frac{\hat{d}_{ijk}}{w_k}\right). \tag{6}$$

The distance $\hat{d}_{ijk}$ is the distance d_{ijk} in Figure 3 corrected for the targets limited manoeuvrability as

$$\hat{d}_{ijk} = d_{ijk} + \sphericalangle(\mathbf{v}_{T_j}, -\mathbf{r}_{jk})\,\varepsilon_{T_j} \tag{7}$$

with the targets heading angle to the asset ($\mathbf{r}_{jk}$ is the purple LOS in Figure 3), multiplied by the target's minimum curve radius ε_{T_j}, which is obtained with its maximum lateral acceleration that is assumed to be roughly provided by surveillance.

Objective 2 ought to favour assignments of the most suitable effectors to targets, taking effector manoeuvrability into account. The second cost factor is achieved using the predicted time-to-go of the effectors to intercept the targets t_{ij} scaled with values of targets as

$$q_{2_{ij}} = \frac{t_{ij}}{\sigma_j} \tag{8}$$

with target value $\sigma_j \in (0,1]$.

The last *objective 3* is supposed to lead to allocations that keep the effectors flight speed as high as possible in order to be more robust against future changes in the scenario. The intercept capabilities and maneuverability of effectors strongly depends on its speed, which must remain higher than the target's speed. Furthermore, as every re-assignment leads to manoeuvres which increase aerodynamic drag and thus reduce flight speed, this objective also prevents unnecessary swapping of assignments. This serves as a more physically meaningful threshold than a high switching cost s (cf. Equation 1) as tested in simulations. The switching cost can then be set fairly low (e.g. 0.1), merely to avoid swapping in case costs are close to equality. The metric used is the predicted final speed of the effector at intercept $v_{E_{ij}}(t_{end})$, obtained from the forward simulations for intercept of target j through effector i.

$$q_{3_{ij}} = -v_{E_{ij}}(t_{end})\,\sigma_j. \tag{9}$$

The metric is multiplied with the previously introduced target value σ_j, such that the final speed of effectors is more relevant for more important targets to ensure higher intercept probabilities.

C. Final Cost Function

The different objectives eventually result in several individual cost values. For solution of Equation 4 they need to be combined and balanced into a single parameter c for every effector-target combination. The intuitive and computationally simple weighted sum is the most common solution for multi-objective optimization. The costs for each assignment c_{ij} are obtained by the objective-function

$$c_{ij} = \sum_{h=1}^{Q} \delta_h q_{h_{ij}} \tag{10}$$

that consists of $h \in [\![1, Q]\!]$ cost parameters $q_{h_{ij}}$ with $Q = 3$ and then weighted by the tunable weighting factors δ_h. The weighting factors are manually tuned as an initial engineering approach. A systematic optimization is planned for the future. A starting point is to adjust the weights δ_h such that all cost parameters $q_{h_{ij}}$ come to a similar order of magnitude. Therefore, δ_2 and δ_3 can be set to similar values, but δ_1 should be 1-2 orders of magnitude lower.

Assignments of effectors to targets, for which the predicted final intercept speed of the effector is lower than 200 m/s, as required by its flight mechanics model, are prohibited. Corresponding entries in the cost matrix are set to infinity.

D. Extension of Cost Function for Inequality of Forces

To allocate multiple effectors to the same targets with the Hungarian Algorithm, the cost function needs to be extended. In the case of numerical superiority of the effectors, the cost matrix is extended to the right, such that the total number of (virtual) targets is an integer multiple of the number of effectors:

$$c_{ij} \to c_{i\tilde{j}}, \quad \text{with } \tilde{j} = M \cdot \left\lceil \frac{N}{M} \right\rceil, \tag{11}$$

where the ceiling function $\lceil \frac{N}{M} \rceil$ depicts the required multiple of M in N.

If assets are considered, the cost matrix extension is altered. If effectors cannot be evenly distributed on the targets, i.e. they are not an exact integer multiple of the number of targets, the $M \cdot \lfloor \frac{N}{M} \rfloor$ (note the floor function instead of the ceiling) targets are assigned as above and the remaining $mod(N, M)$ effectors are allocated to those targets with EIGs closest to our assets:

$$c_{ij} \to c_{i\tilde{\tilde{j}}}, \quad \text{with } \tilde{\tilde{j}} = M \cdot \left\lfloor \frac{N}{M} \right\rfloor + mod(N, M). \tag{12}$$

The additional $l = mod(N, M)$ targets (i.e. columns of c) are selected by searching for the l-highest values in

$$\arg\max_j \left(\min_i (q_{1_{ij}})\right). \tag{13}$$

E. Summary of the Approach

We can now let the effectors autonomously solve the domain-informed WTA problem online in a decentralized manner. The effectors collect flight-mechanical and operational insights from the network and process them to create the cost function. The efficient Hungarian Algorithm can then solve the global WTA problem (Equation 4) onboard every effector. As long as an information consensus is maintained, all effectors obtain the same solution and each effector gets a target assigned in the best interest of the entire group. To react to evolving scenarios, the assignment optimization is repeated with a constant frequency (currently 1 Hz) or when the number of effectors/targets has changed.

V. Simulation Results and Discussion

A. Simulation Setup

Exemplary many-on-many simulations outline the behaviour of the continuous mid-course WTA and prove its functionality. Multiple effectors are tasked with engaging multiple incoming targets. Simulations are conducted from launch until target interception. For the scenarios shown here, different numbers of targets are placed in the air at different positions but with identical altitudes. They fly with speeds of around 250 m/s and varying headings. Effectors are then successively activated around 100 km away with initial speeds of 400 m/s (simulating a launch from a moving platform e.g. fighter jet) and an engine burn which lasts 30 s. If a target is successfully intercepted, its pre-computed intended flight path is plotted as a dashed line from there on. Effectors use a standard Proportional Navigation Guidance (PNG) [23] for trajectory guidance. Larger numbers of participants, e.g. eight effectors vs. three targets or three effectors vs. nine targets, have been successfully tested as well but are not presented here because the intersecting trajectories quickly become incomprehensible. Stochastic and statistical analysis of such scenarios could be presented in future. In all tests, the entire scenarios including predictions and optimizations of all effectors, could be simulated within around ten seconds on a standard consumer laptop, indicating applicability on embedded systems with limited computational resources.

B. Exemplary Results

A representative example - a 3 vs. 3 engagement - is illustrated in Figure 5. Flight paths are displayed in three dimensional space (North East Down (NED) coordinate system) whereas assignments and effector speeds are plotted over simulation time. The values of the targets (i.e. relative significance) are mutually set to one and no assets are

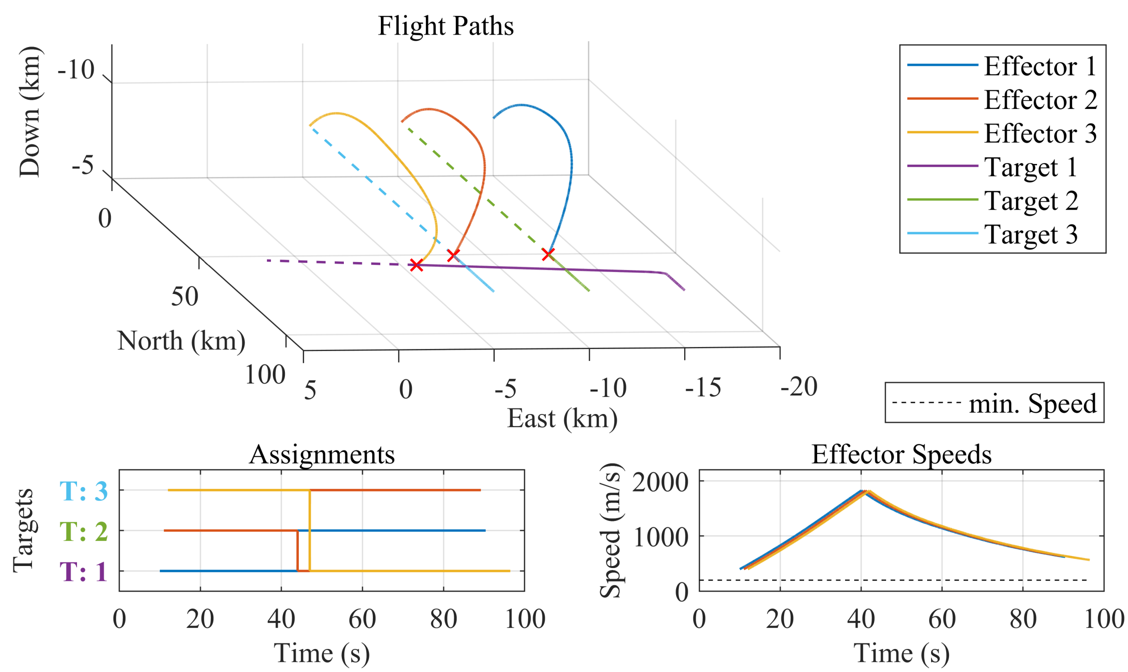

Fig. 5 Many-on-many scenario: 3 effectors vs. 3 targets

present. After ten seconds of simulation, the first effector, number one, is launched and assigned by the WTA algorithm to target number one which is predicted to come closest to the effector, thus being the one that can be intercepted the fastest. One second later, effector number two is launched and is assigned to target number two. After 12 seconds of simulation effector number three starts. The now westernmost effector three is assigned to the westernmost target number three which is expected to continue on its straight path. After 40 seconds of simulation, target number one begins a turning manoeuvre to the left with around 8.1 degrees per second which lasts eight seconds. At 44 seconds, the flight path prediction (linearly interpolated) of the target triggers a reallocation of the effectors, which swaps assignments between effectors one and two. At 47 seconds another swapping between effectors two and three occurs. Target number one is now heading to the far east. Accordingly, the easternmost effector three is assigned to it, while the westernmost effector one is assigned to the newly westernmost target two. The effector in the middle is assigned to the target in the middle. Eventually, all targets are intercepted.

The cost factor evolution of factor two q_2 and three q_3 are exemplarily shown for effector one in Figure 6. It shows how the predicted time-to-go and final intercept speed for an interception of target number one significantly increase/decrease as target one starts its turning manoeuvre (see Figure 5). For target number two, which is ultimately intercepted, the predicted time-to-go linearly decreases over time and the final intercept speed remains constant thanks to the PNG. Cost curves are cut as soon as assignment is prohibited, i.e., the minimal speed is predicted to be surpassed.

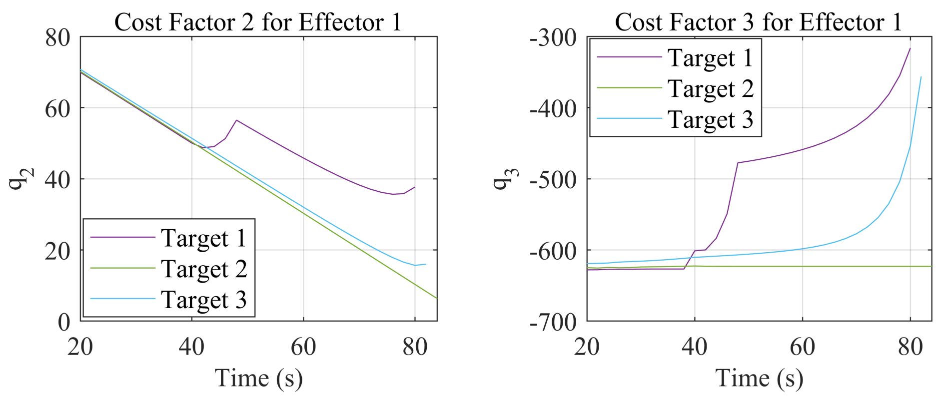

Fig. 6 Evolution of Cost Factors of Effector 1 (corresponding to Figure 5)

Figure 7 shows the summed up cost c of all possible assignment plans. In the legend, "E2T3" stands for the cost factor c of the combination of effector two on target three. At any point in time, the assignment plan with the lowest cost sum is the one chosen according to Equation 4. The three assignment plans that are employed over time are displayed with thicker lines. In agreement with the assignment plot in Figure 5 the lowest curve and thus the assignment plan changes two times, marked by the vertical red dashed lines. Again, cost curves are cut for prohibited assignments.

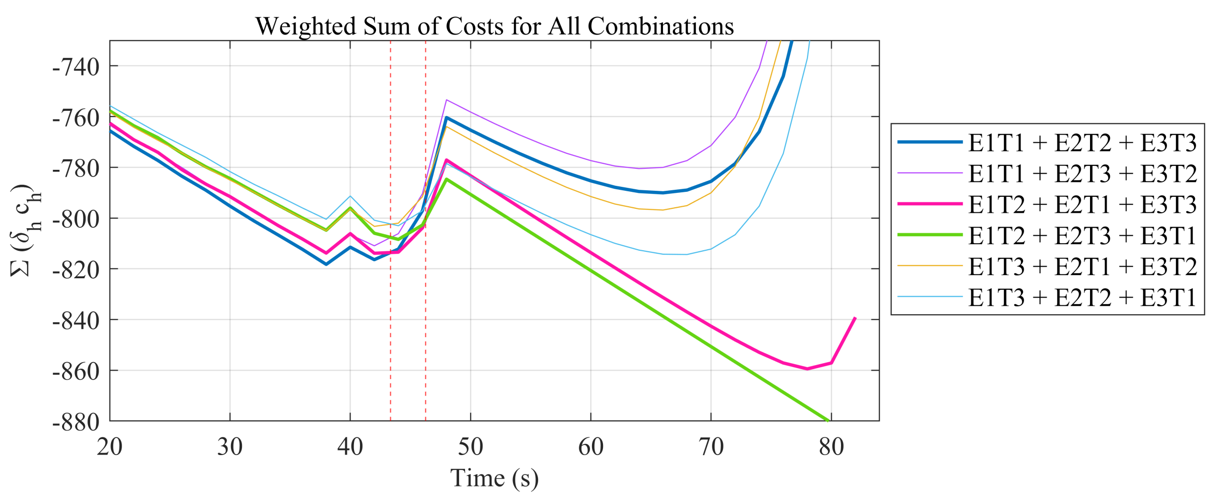

Fig. 7 Evolution of All Sum of Cost Combinations (corresponding to Figure 5)

A numerical inferiority example - a 3 vs. 4 engagement - is displayed in Figure 8. Target number two has a lower value (0.1) than the others. All targets execute turning manoeuvres at different times, causing multiple re-assignments of effectors. Due to the re-assignments the effectors avoid intersecting each others flight paths. Thereby, they preserve valuable flight time and kinetic energy. Due to the minority of effectors, not all targets can be intercepted. As desired, target number two is never assigned.

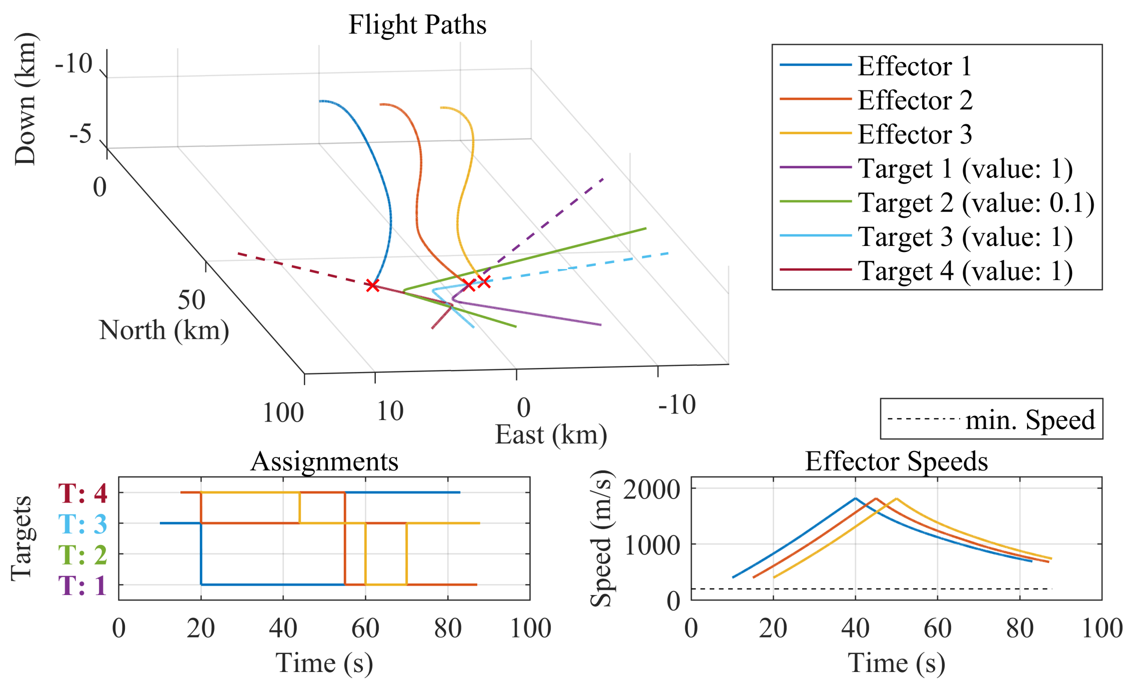

Fig. 8 Many-on-many scenario: 3 effectors vs. 4 targets

Figure 9 displays a 3 vs. 3 engagement with a pop-up target and a friendly asset. Target number one represents a

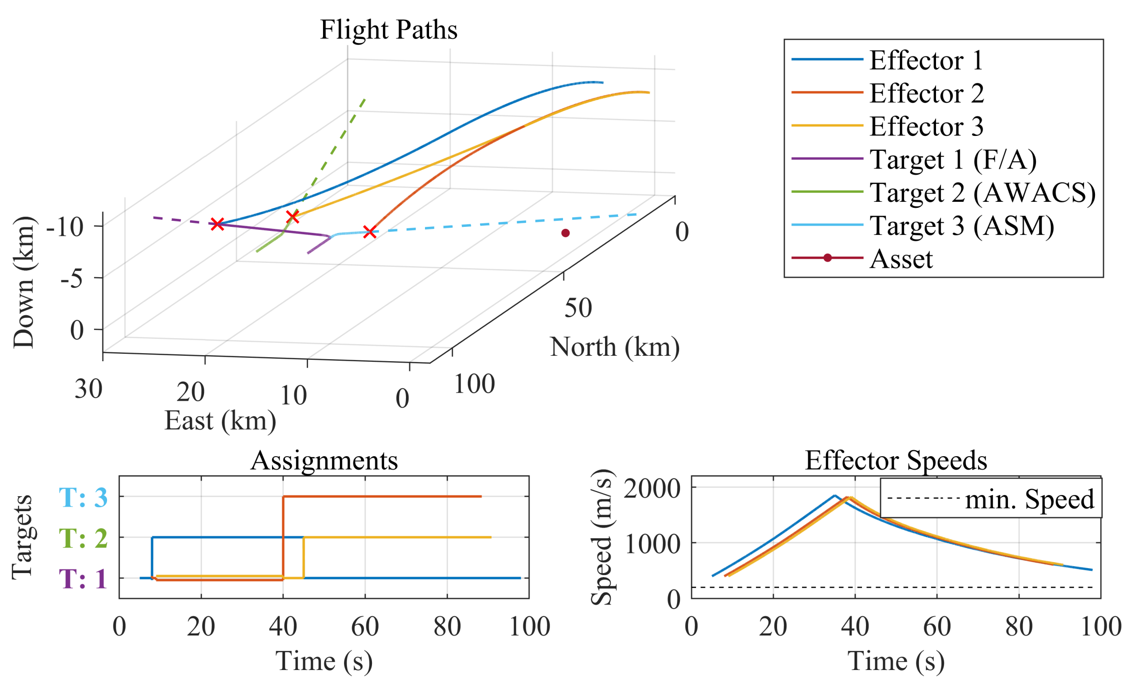

Fig. 9 Many-on-many scenario with asset: 3 effectors vs. 2 + 1 pop-up targets

fighter aircraft (F/A). After 40 s it launches a missile (ASM), i.e. target number three, and bails out to the left. The ASM turns towards the asset (downward, right). Target number two represents a larger aircraft (e.g. tanker or AWACS) that flies with more distance to the asset and the effector launch points. It slightly turns to the east during the simulation. The effectors are launched successively from the air as can be seen in the effector speeds plot. Whereas the two western effectors (number two and three) are assigned to target number one in the beginning due to its proximity to the asset, the second effector is re-assigned to the new pop-up target three once it is launched. Effector number one and three switch targets later on, as their allocated targets cross flight paths.

VI. Conclusion

The presented cost-driven WTA optimization approach focuses on the effective utilization of domain-knowledge. We consider this to add valuable benefits for obtaining optimal solutions. The multi-objective cost function adapts to evolving scenarios and utilizes both flight-mechanical and operational insights for a meaningful allocation throughout the flight. Based on the current status of the work, the approach is considered as an effective and efficient solution for envisioned onboard real-time applications with limited computational power. This is particularly the case for missiles with infrared seekers, that carry significant computational power for image processing which is only needed in the seeker-based homing phase and may be free to utilize during mid-course.

We showed a successful implementation of the WTA algorithms for cooperative guidance by the test simulations. Scenarios of different numerical force compositions, varying numbers of targets throughout the scenario and with assets were tested. Stochastic and statistical analysis of larger simulation campaigns are planned to be presented in future.

Technology maturation will require more comprehensive and realistic simulations of the entire scenarios and environment. This includes detailed simulations of missile launches, reactive targets, sensors and measurement noises and errors, including a homing-phase with seeker. Communication interruptions have the potential to represent a major challenge. Appropriate robustness measures against transmission errors, delays and bandwidth restrictions should thus be investigated in a next step. A systematic optimization of the weighting parameters in Equation 10 to balance the three high-level objectives is planned for the future.

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