SIMULATING COLLECTIVE NEUTRINO OSCILLATIONS ON A QUANTUM COMPUTER

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Collective Neutrino Oscillations

- In supernova explosions, neutrinos are produced in large quantities $\sim 10^{58}$ neutrinos
- 99% of energy released is in neutrino form with energies ~ 20 MeV per neutrino
- At such densities neutrino-neutrino interactions become relevant
- The neutrino self-interactions induce a change on the neutrino spectrum and on the flavor oscillations pattern - **a challenging problem!**
- For the case of 2 neutrino mass eigenstates, one has **Isospin Invariance**

Ideal for quantum computers

We want to calculate :

- The energy spectrum (**PITE**+**QSP**)

- The survival probability of electron neutrinos (**Trotterization**)

Collective Neutrino Oscillations

- The neutrino Hamiltonian

$$H = -\sum_{p} \omega_p \vec{B} \cdot \vec{J} + \mu(r) \vec{J} \cdot \vec{J}$$

$$\vec{B}_{\text{mass}} = (0,0,-1)$$

$$\vec{B}_{\text{flavor}} = (\sin 2\theta, 0, -\cos 2\theta)$$

$$\mu(r) = \frac{G_F}{\sqrt{2}V}\left(1 - \sqrt{1 - \frac{R_\nu^2}{r^2}}\right)^2$$

- Assuming isotropy from a single spherical emission surface
- Momenta are quantized
- Each neutrino has a distinct momenta
- Expansion of H in terms of Pauli-strings
- Case for 4 neutrinos: $\begin{pmatrix} 6\mu - 5\omega & 0 & 0 & 0 & 0 \\ 0 & H^{-}_{4\times 4} & 0 & 0 & 0 \\ 0 & 0 & H_{8\times 8} & 0 & 0 \\ 0 & 0 & 0 & H^{+}_{4\times 4} & 0 \\ 0 & 0 & 0 & 0 & 6\mu + 5\omega \end{pmatrix}$

$$\begin{pmatrix} \nu_x \\ \nu_e \end{pmatrix}$$

Imaginary-Time Evolution (ITE)

Imaginary-Time Evolution used to obtain the ground state from a given Hamiltonian

$$\underbrace{e^{-\mathcal{H}\tau}}_{\text{Non-Unitary}} \overbrace{\sum_k c_k |k\rangle}^{\text{Initial State}} = \sum_k c_k e^{-E_k \tau} |k\rangle$$

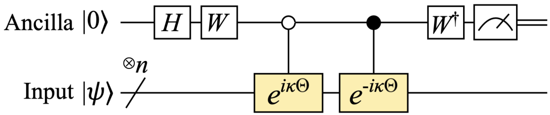
- The weights of excited states decay rapidly
- No need for an appropriate Ansatz
- We use the Probabilistic Imaginary Time Evolution to implement the non-unitary
- Excited states: QLanczos-like or a trick by taking $(\mathcal{H} - \alpha\mathbb{1})^2$

Probabilistic Imaginary Quantum Evolution (PITE)

- It implements Non-unitary transformations on a Quantum Computer
- One adds one ancilla to perform a viable Unitary operation

$$\mathcal{M} = m_0 e^{-\mathcal{H}\Delta\tau} \qquad \mathcal{U}_{\mathrm{PITE}} = \begin{pmatrix} \mathcal{M} & \sqrt{1-\mathcal{M}^2} \\ \sqrt{1-\mathcal{M}^2} & -\mathcal{M} \end{pmatrix}$$

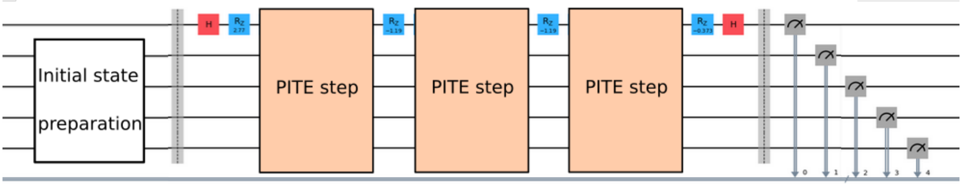
- The state $|\psi\rangle \otimes |0\rangle$ evolves to $\mathcal{M}\,|\psi\rangle \otimes |0\rangle + \sqrt{1-\mathcal{M}^2}\,|\psi\rangle \otimes |1\rangle$



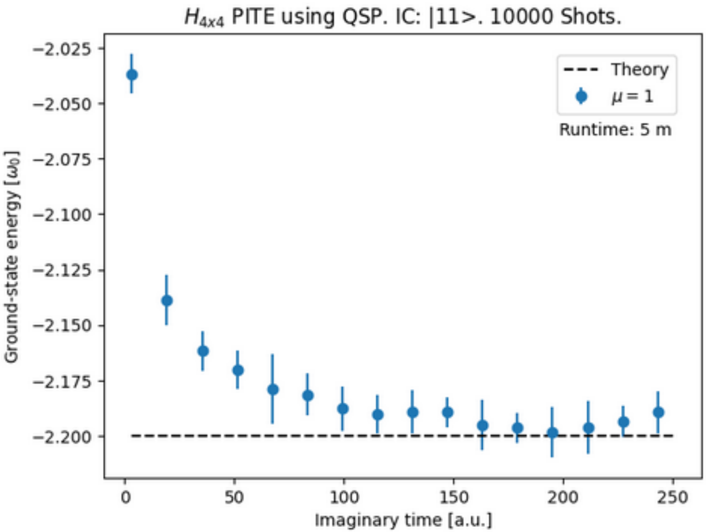
- Θ is a function of $\mathcal{M}$ and $W = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$
- If one measures the ancilla with the state $|0\rangle$ one gets the state $\mathcal{M}\,|\psi\rangle$

Quantum Signal Processing (QSP)

- It avoids mid-circuit measurements, useful devices do not have it!
- A basis transformation - we choose to execute it in X direction on the ancilla
- One R_Z rotation per step for the final result of a x-fold iteration
- Angle determination: randomly chosen at start and then optimizing the phases
- The phase landscape is very non-convex - different minimum each time
- An overall final measurement needs still to be performed



Numerical results



Real-Time evolution of the Hamiltonian in the flavor basis

- Assuming that the electron neutrinos are predominant
- One wants to calculate time evolution of the Hamiltonian for 4 neutrinos
- Method Trotterization of order four

$$H_{\text{flavor}} = 3\mu\,\mathbf{1} + H_X + H_Y + H_Z$$

$$\mathcal{U}(t) \approx \left(e^{-iH_X\Delta t} e^{-iH_Y\Delta t} e^{-iH_Z\Delta t}\right)^n$$

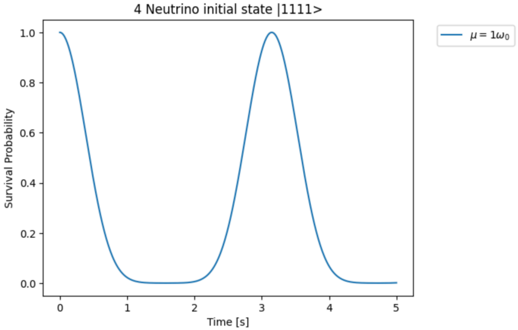
$$n = \frac{t}{\Delta t}$$

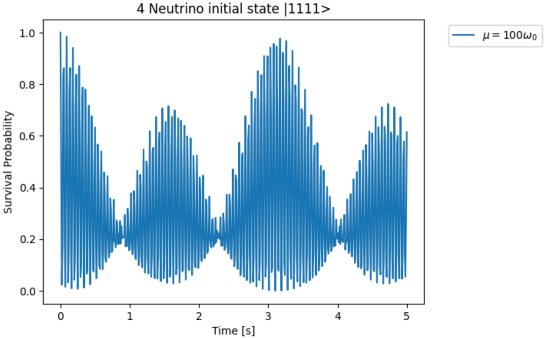
$$H_X = \frac{\sin 2\theta}{2}\sum_{i=1}^{4}\omega_i X_i + \frac{\mu}{2}\sum_{i<j}^{4} X_i X_j,$$

$$H_Y = \frac{\mu}{2}\sum_{i<j}^{4} Y_i Y_j,$$

$$H_Z = -\frac{\cos 2\theta}{2}\sum_{i=1}^{4}\omega_i Z_i - \frac{\mu}{2}\sum_{i<j}^{4} Z_i Z_j,$$

Real-Time evolution of the Hamiltonian in the flavor basis





Conclusions

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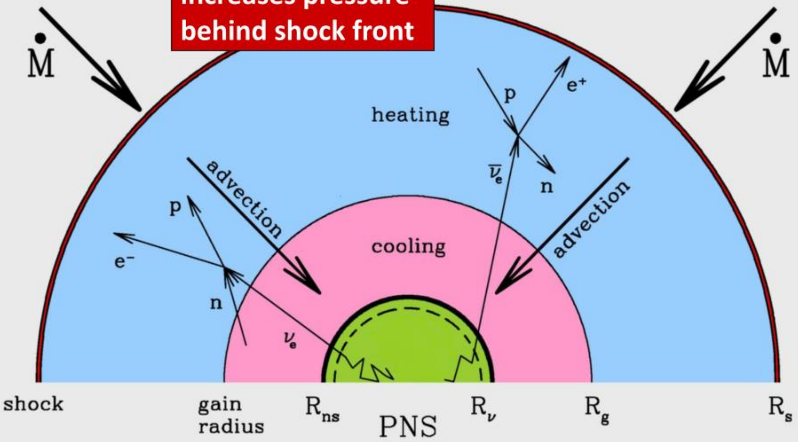
- Neutrino oscillations in dense medium is a challenge: Neutrino self-interactions
- Isospin symmetry makes the problem addressable to Quantum Computers
- Imaginary-Time Evolution is used to calculate the ground state
- PITE is an attractive quantum method to implement non-unitaries with one ancilla
- QSP reimplement PITE so that all measurements are made at the end
- Error mitigation methods need to be taken into account for real devices

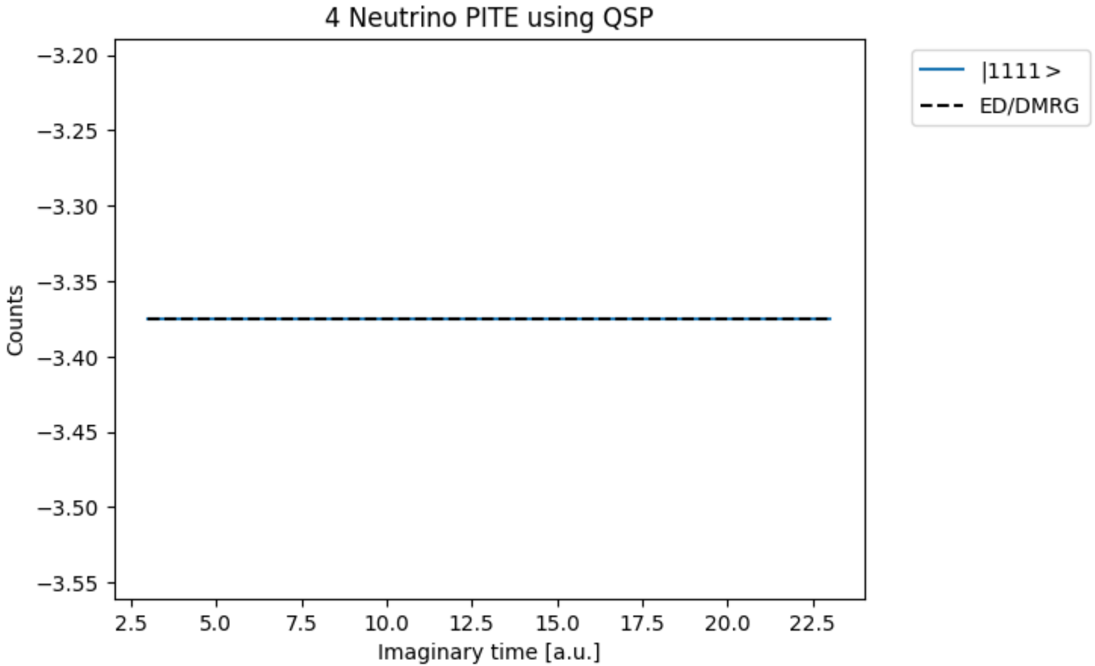
Thank you!

Neutrinos Rejuvenating Stalled Shock

Neutrino heating increases pressure behind shock front



Picture adapted from Janka, astro-ph/0008432


4 Neutrino PITE using QSP
−3.20
−3.25
−3.30
−3.35
−3.40
−3.45
−3.50
−3.55
Counts
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
Imaginary time [a.u.]
|1111 >
ED/DMRG

$$H = \begin{pmatrix} 6\mu - 5\omega & 0 & 0 & 0 & 0 \\ 0 & H^{-}_{4\times 4} & 0 & 0 & 0 \\ 0 & 0 & H_{8\times 8} & 0 & 0 \\ 0 & 0 & 0 & H^{+}_{4\times 4} & 0 \\ 0 & 0 & 0 & 0 & 6\mu 5\omega \end{pmatrix}$$

$$H_{8\times 8} = \frac{\omega_0 + 6\mu}{4}\mathbf{1} + \frac{\mu}{2} H_1 - \frac{\omega_0}{4} H_2$$

where H_1 is defined as

$$\begin{aligned} H_1 = & X_1 X_2 X_3 + Y_1 Y_2 X_3 + Z_1 X_2 X_3 \\ & + X_1 X_2 + X_2 X_3 + Y_1 Y_2 + X_1 Z_2 \\ & + Z_1 X_2 + Z_2 X_3 - Z_1 Z_2 \\ & + X_1 + X_2 + X_3 + Z_1 + Z_2 \end{aligned}$$

and H_2 as

$$H_2 = 2\, Z_1 Z_2 + Z_2 Z_3 + 4\, Z_1 + Z_2 + Z_3$$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\, e^{\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\, e^{\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\, e^{\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\, e^{\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{pmatrix}$$

Parameter	NO	IO
$\Delta m^2_{21}[10^{-5}\mathrm{eV}^2]$	$7.50^{+0.22}_{-0.20}$	
$\lvert\Delta m^2_{31}\rvert[10^{-3}\mathrm{eV}^2]$	$2.55^{+0.02}_{-0.03}$	$2.45^{+0.02}_{-0.03}$
$\sin^2\theta_{12}/10^{-1}$	3.18 ± 0.16	
$\sin^2\theta_{23}/10^{-1}$	5.74 ± 0.14	$5.78^{+0.10}_{-0.17}$
$\sin^2\theta_{13}/10^{-2}$	$2.200^{+0.069}_{-0.062}$	$2.225^{+0.064}_{-0.070}$
δ/π	$1.08^{+0.13}_{-0.12}$	$1.58^{+0.15}_{-0.16}$